HKKM Calculation of Atmospheric Neutrino Flux.

M.Honda

1. Full 3D calculation with inclusive interaction code created from established code.
   => ~a few 100 times faster than original code.
   => Easy to modify the secondary spectra.

2. Muon calibration of the inclusive interaction code.

3. Use relatively large virtual detector with virtual detector correction.

4. Realistic geomagnetic field and air profile.
   IGRF and NRLMSISE-00.

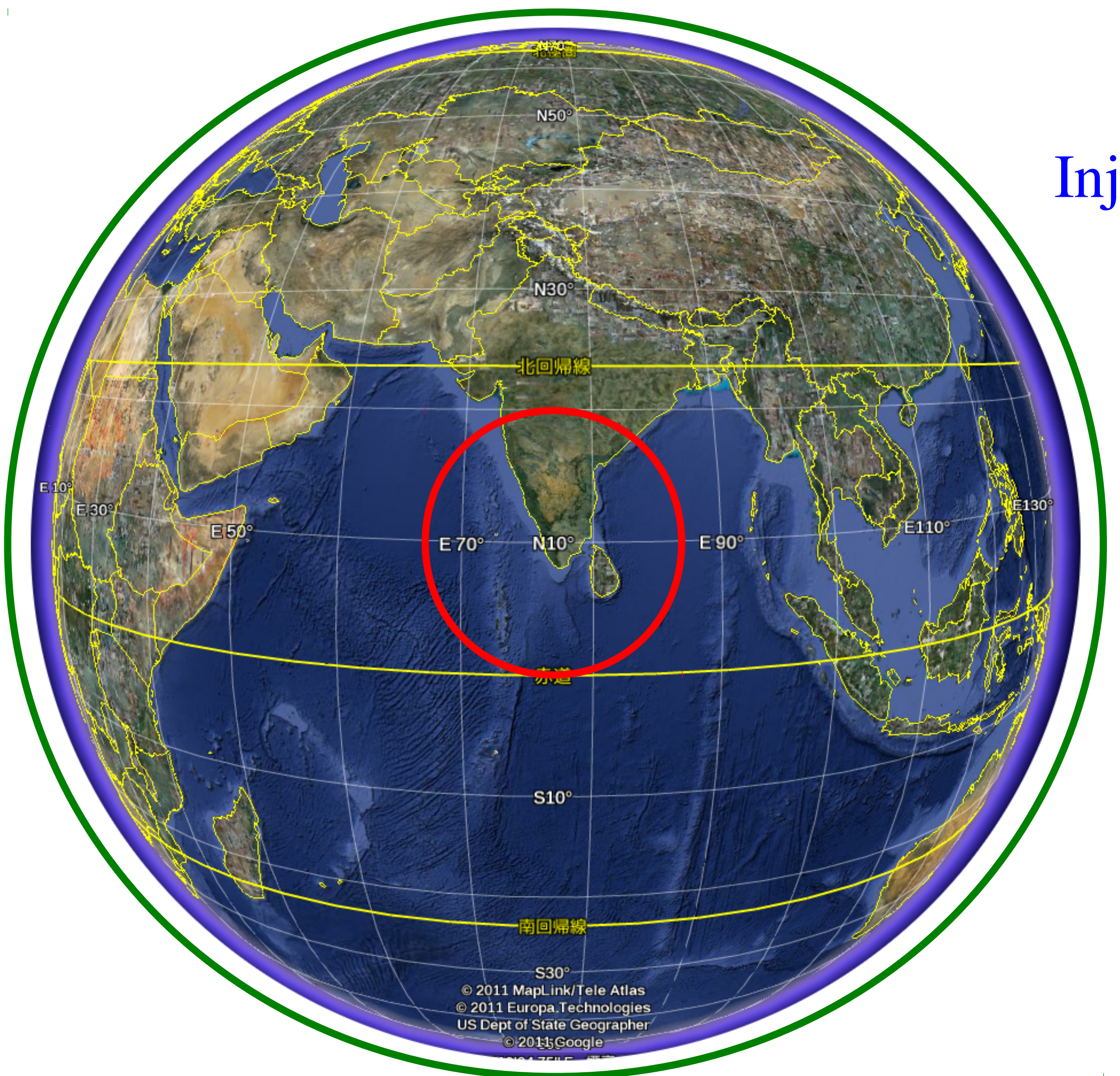
5. New Cosmic Ray Spectra Model (preliminary).

3D-Calculation Geometry

Re = 6378km

Simulation Sphere (Rs = 10 x Re)

Cosmic ray go out this sphere are discarded.
Cosmic rays go beyond are pass the rigidity cutoff test



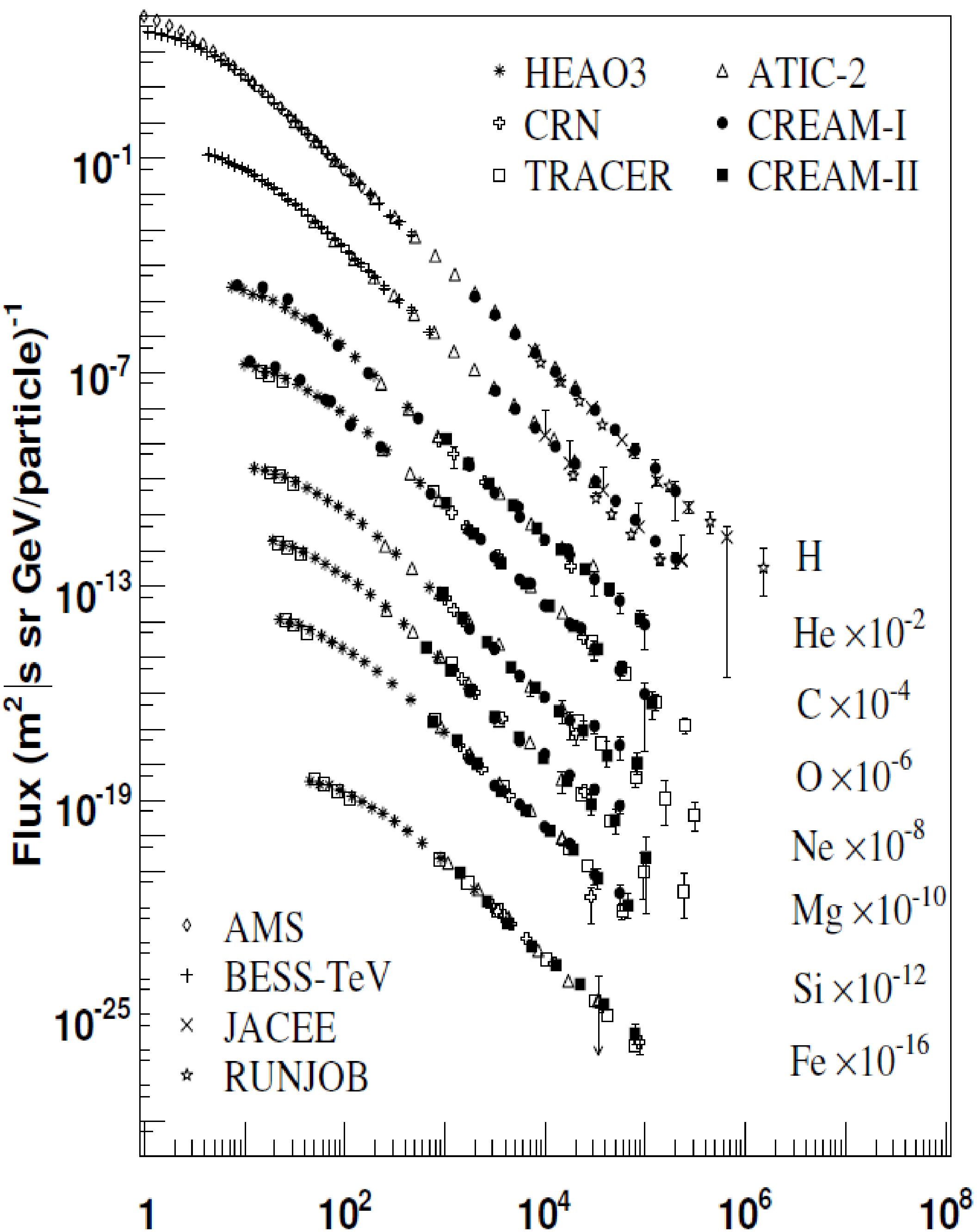
Injection Sphere (Re +100lm)

Cosmic Rays are sampled and injected here

Virtual Detector

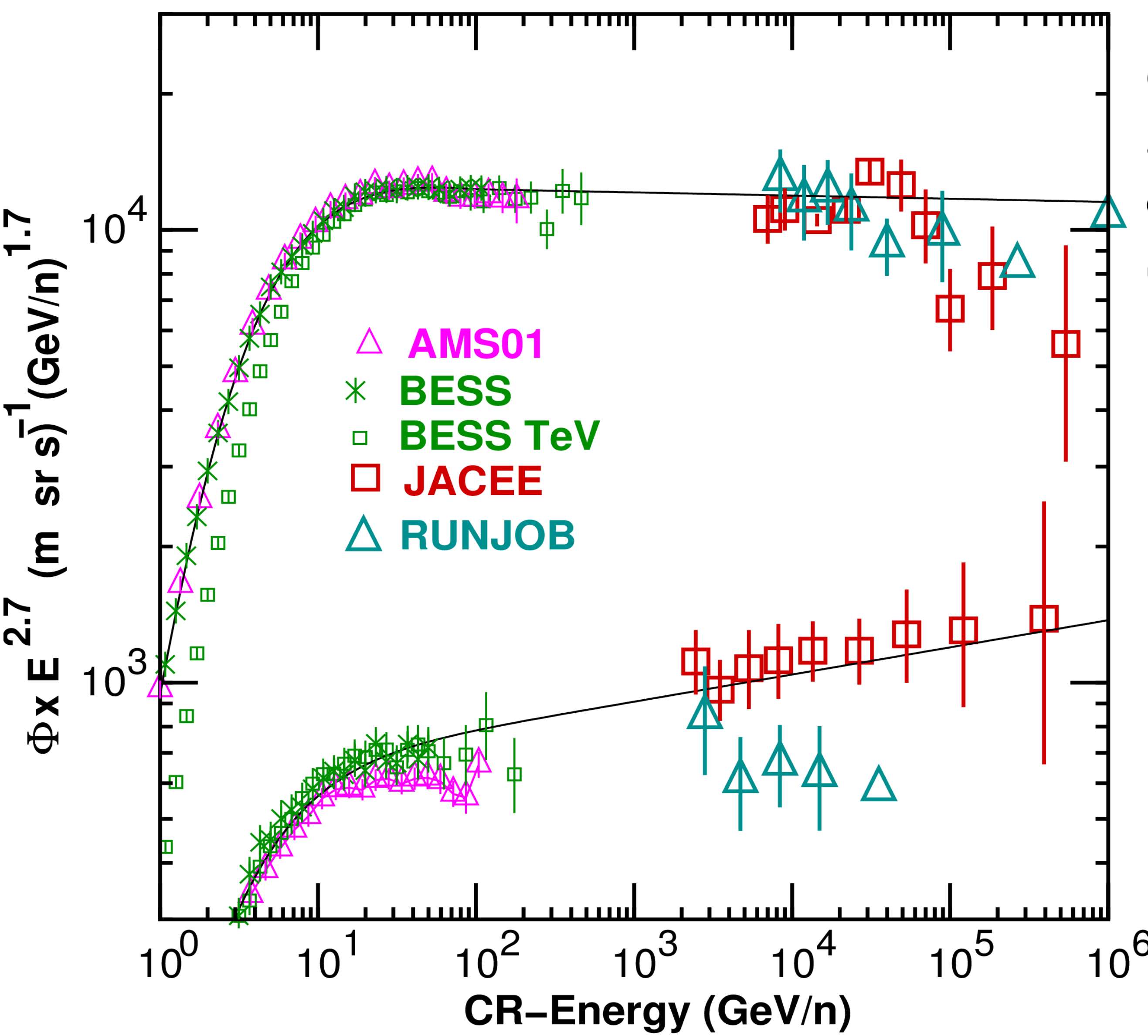
All neutrinos path through are recorded

Primary Cosmic Ray Spectra



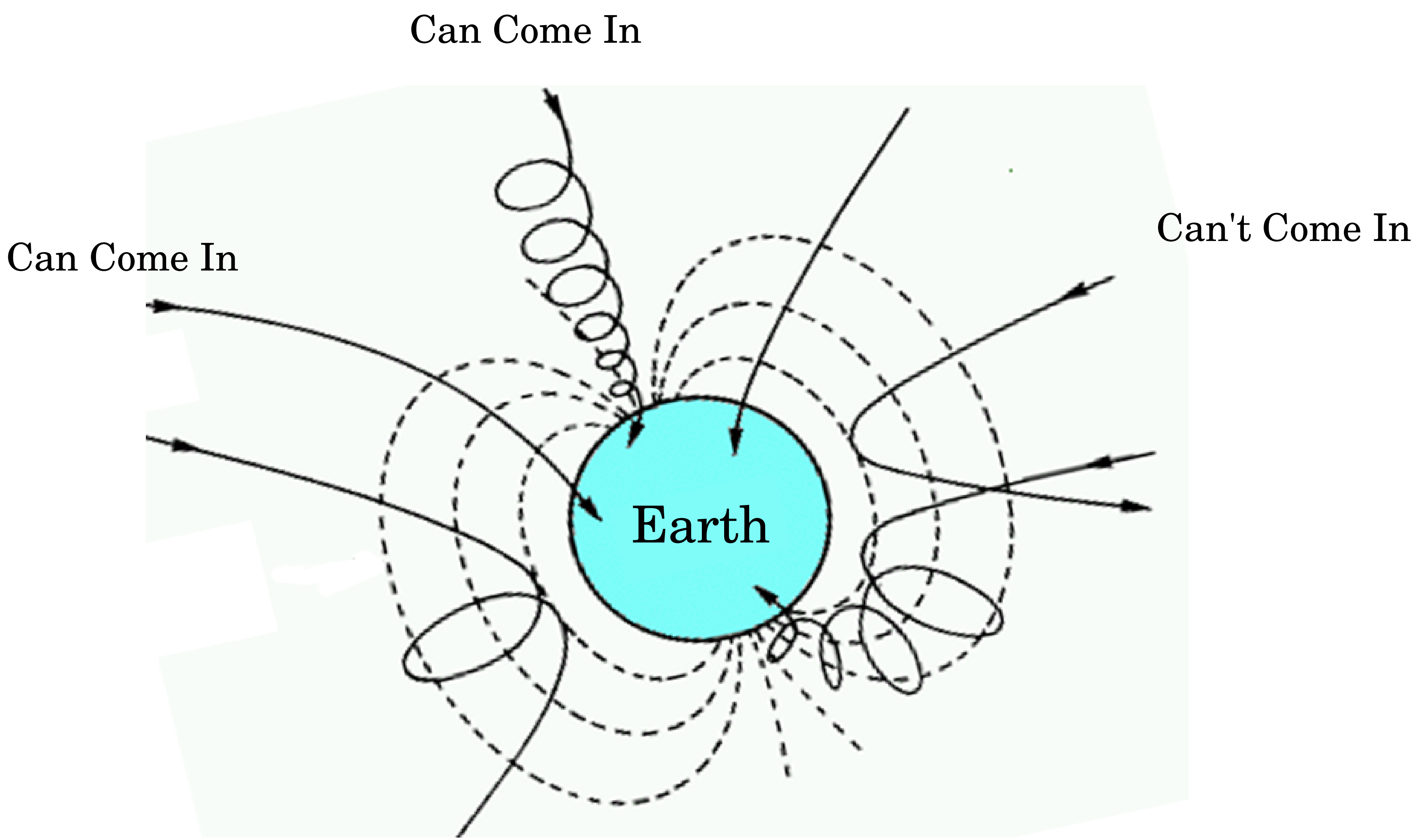
From
E.S. Seo @ ICRC2009

Primary Cosmic Ray Model and referred data



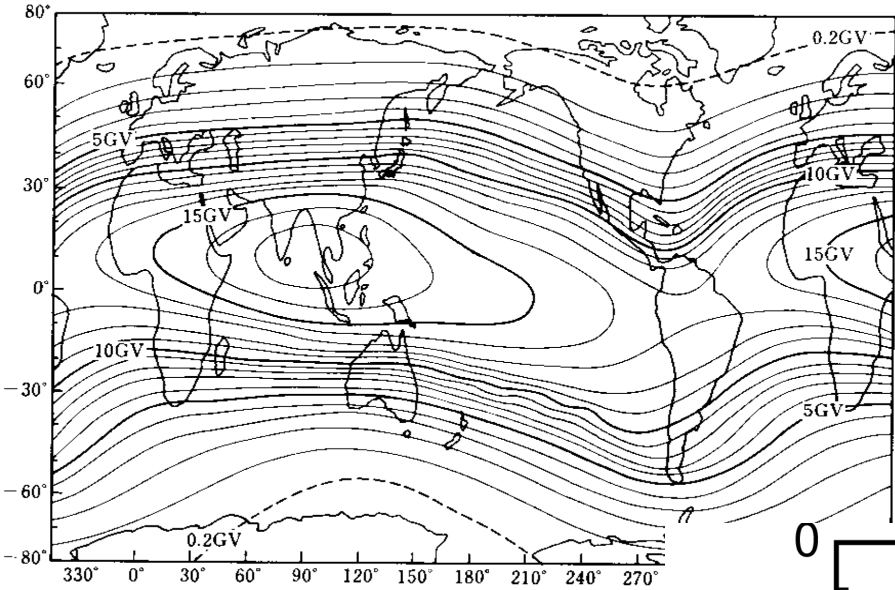
Other chemical compositions are also considered in the calculation, but they give small contributions.

Rigidity Cutoff and Geomagnetic Field (cartoon)

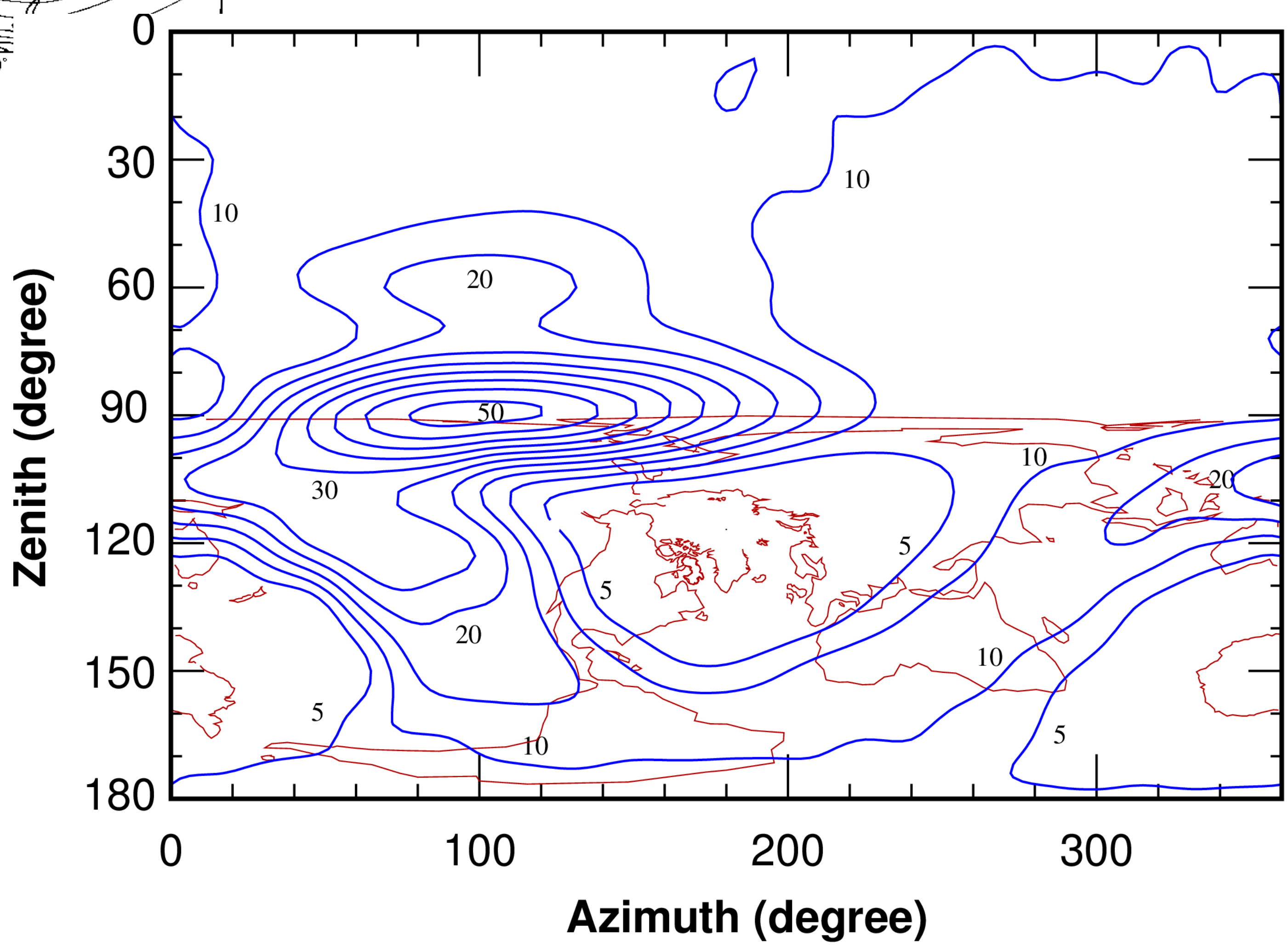


Rigidity Cut Off

Rigidity Cutoff for
Virtical direction



Rigidity Cutoff table
at SK for 1D calculation



Gaisser Formula for illustration (by T.K.Gaisser at Takayama, 1998)

$$\Phi_{\nu} = \Phi_{primary} \otimes R_{cut} \otimes Y_{\nu}$$

$$\Phi_{\mu} = \Phi_{primary} \otimes R_{cut} \otimes Y_{\mu}$$

Where

$\Phi_{primary}$: Cosmic Ray Flux

$R_{cut} = R_{cut}(R_{cr}, latt., long., \theta, \varphi)$: Geomagnetic field

$Y_{\nu} = Yield_{\nu}(h, \theta)$: Hadronic Interaction Model,

Air Profile, and meson-muon decay

$Y_{\mu} = Yield_{\mu}(h, \theta)$: Hadronic Interaction Model,

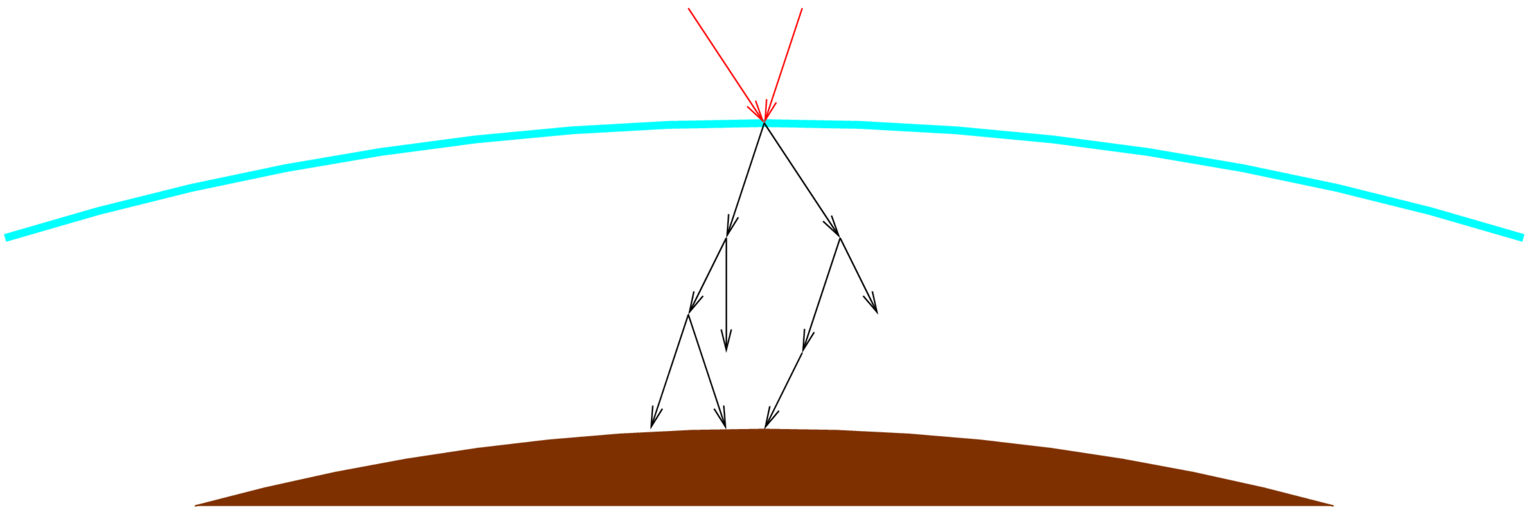
Air Profile, and meson decay

This formula illustrates 1D-calculation well

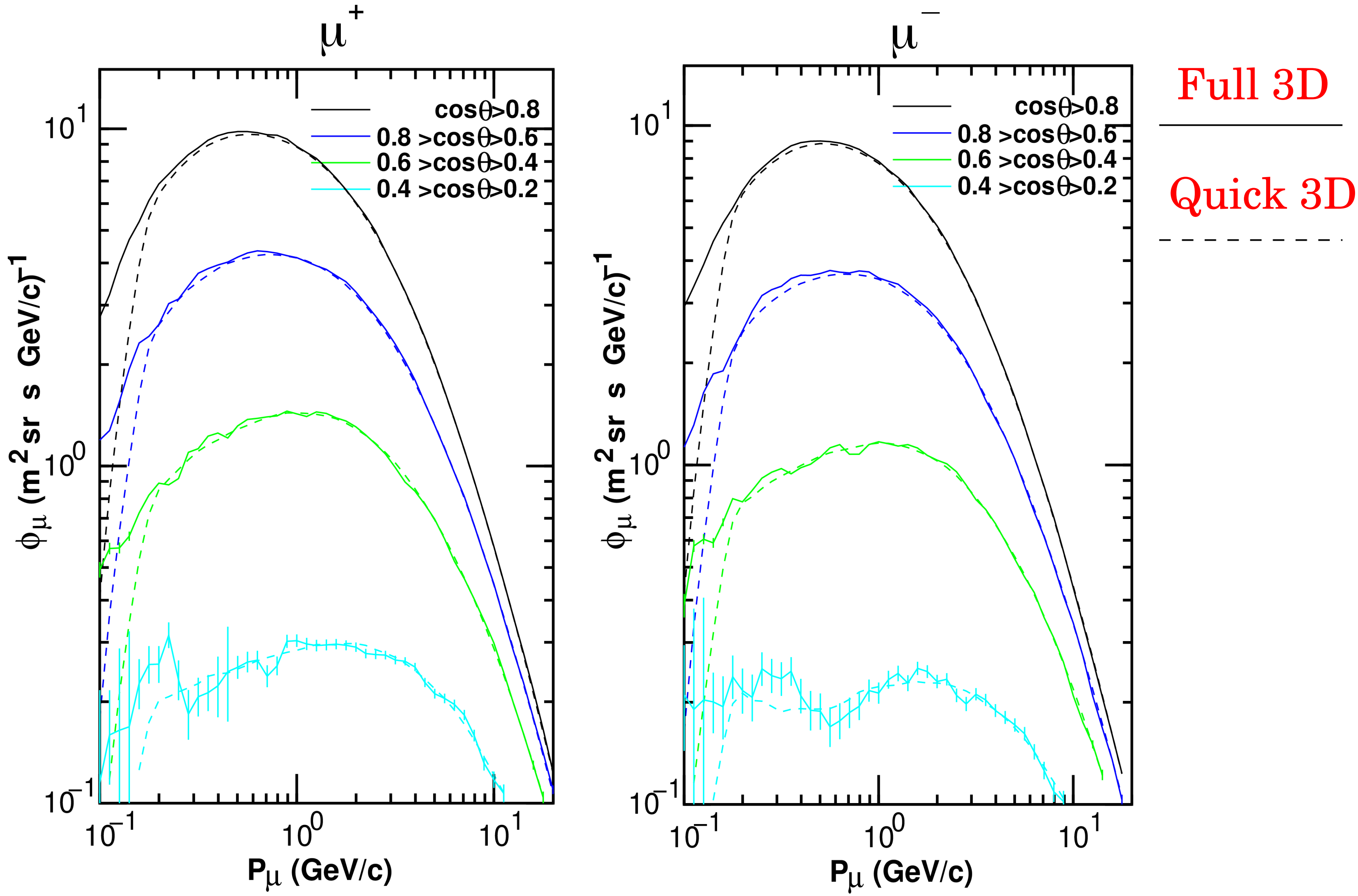
Muon Calibration of Interaction Model

Quick 3D calculation of muon flux.

As the muon flux is a "local quantity" ($\gamma ct \sim$ 60km at10 GeV),
We can calculate it in a quick calculation method:
1. Inject cosmic rays just above the observation point,
2. Analize all the muons reach the surface of Earth.



Comparison of Quick 3D calculation with Full 3D calculation

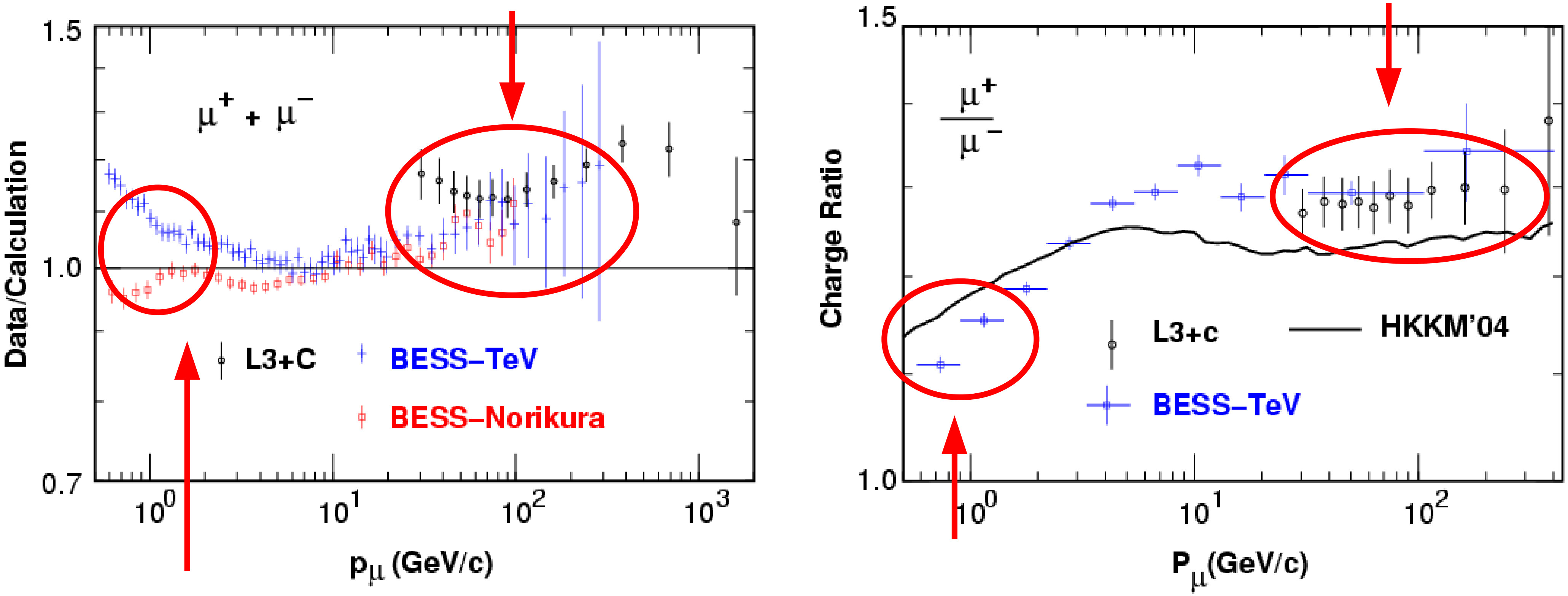


This method works above 0.2 GeV/c.

Muon Calibration of inclusive DPMJET-III

Data are larger by ~15%

Data are larger by ~0.05

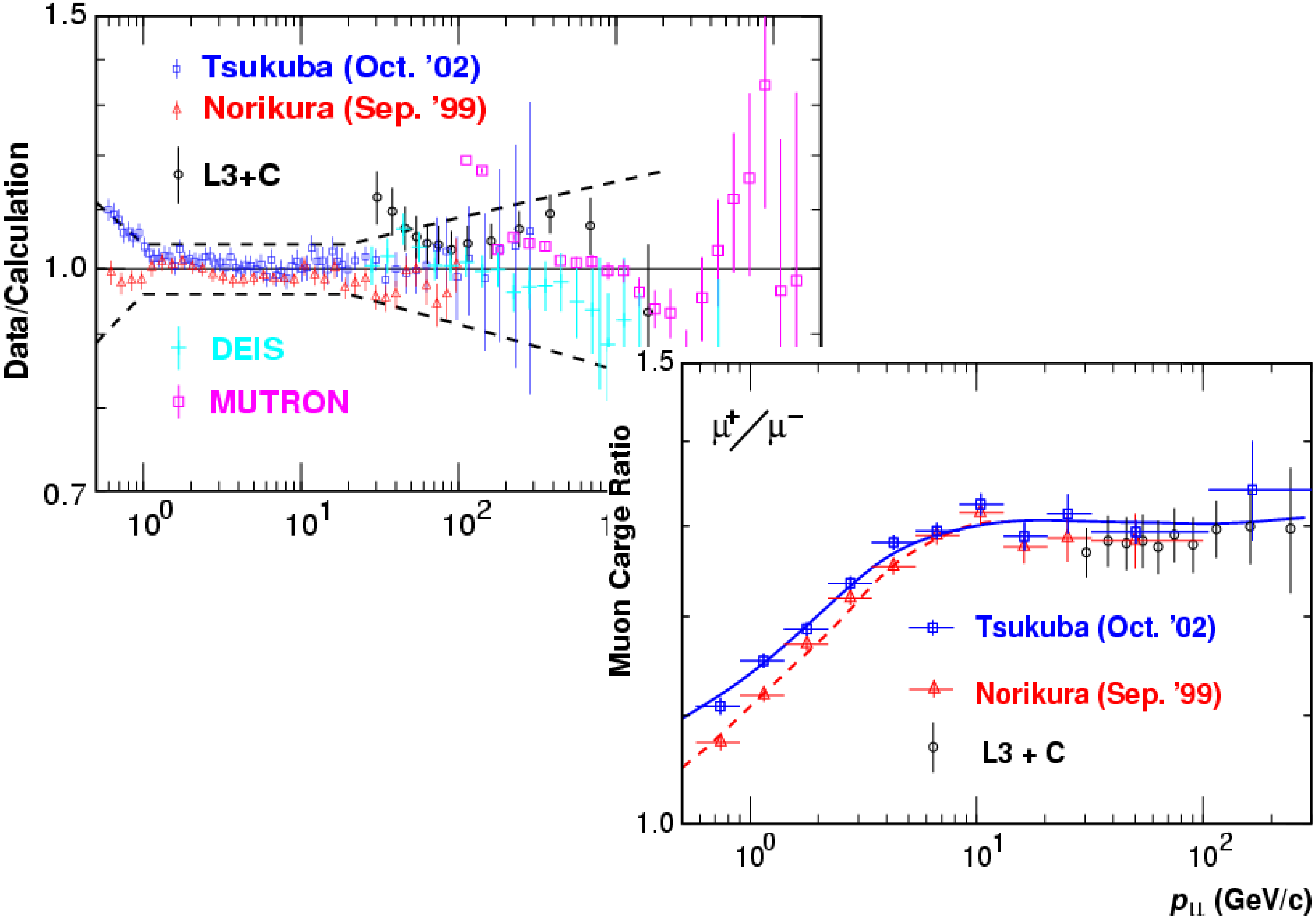


~15% scatter ?

Data are smaller by ~0.05

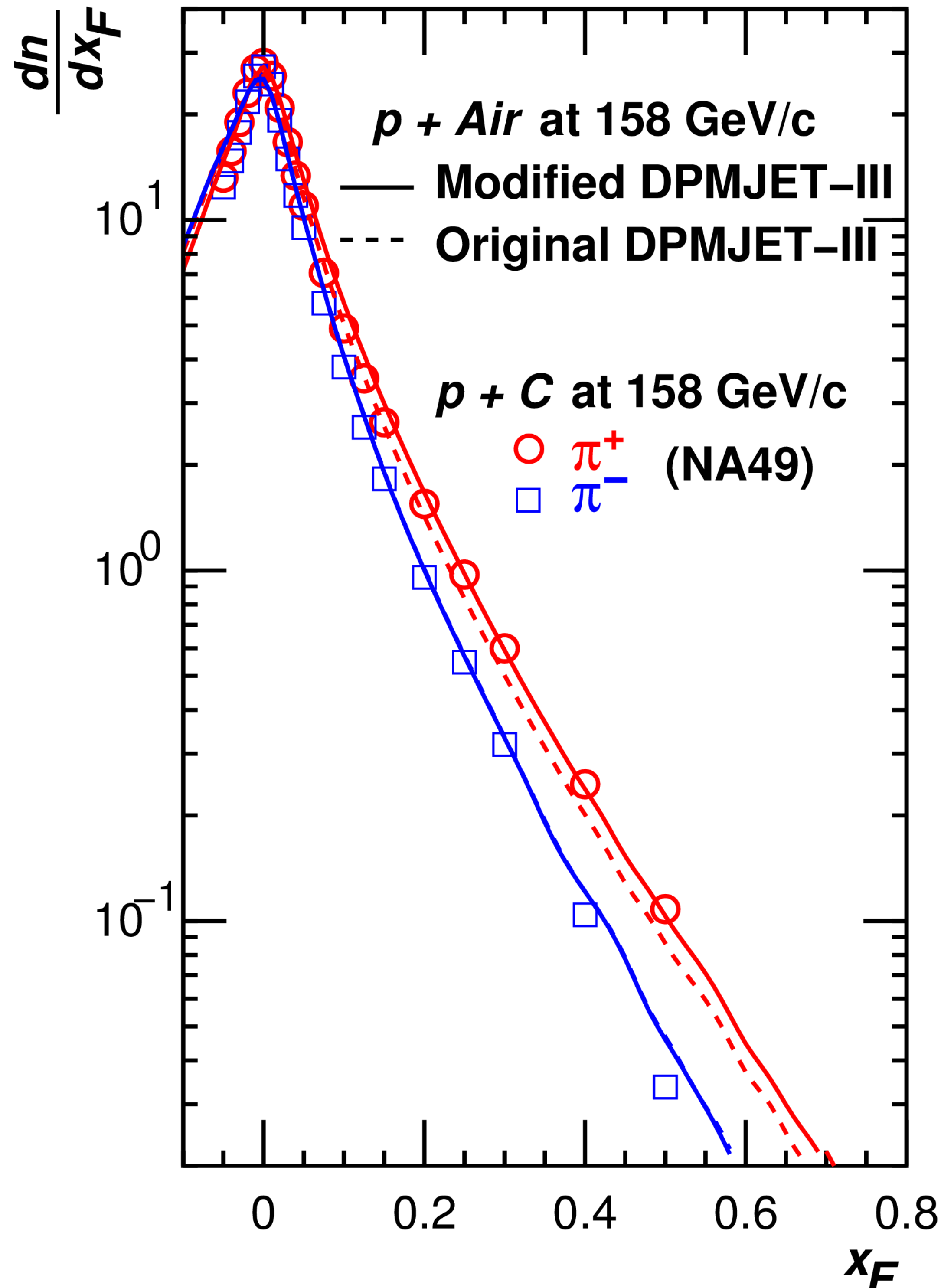
==> DPMJET-III Should be Modified

Comparison AFTER the modification

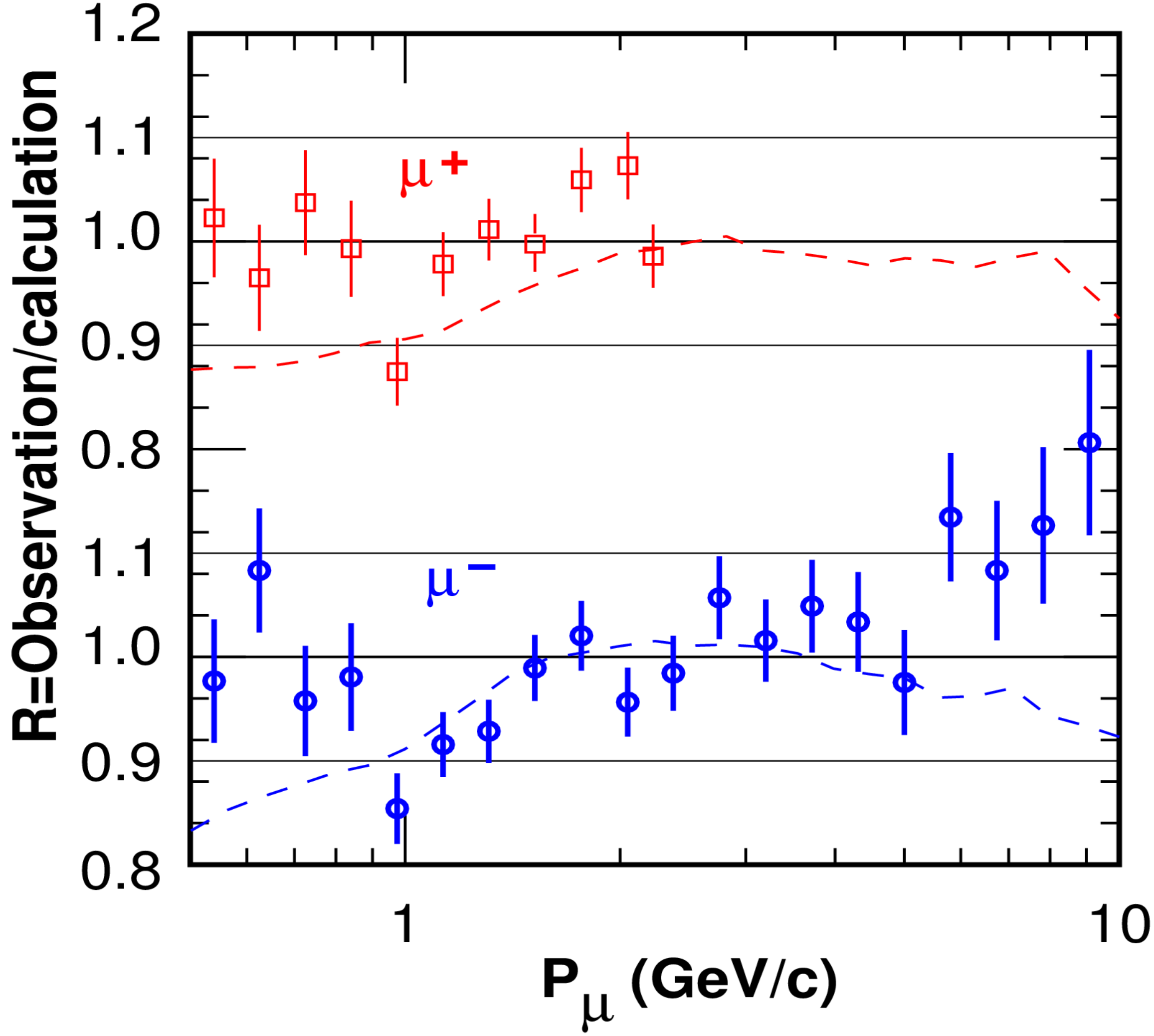


Comparison with Accelerator data

DPMJET-III vs NA49

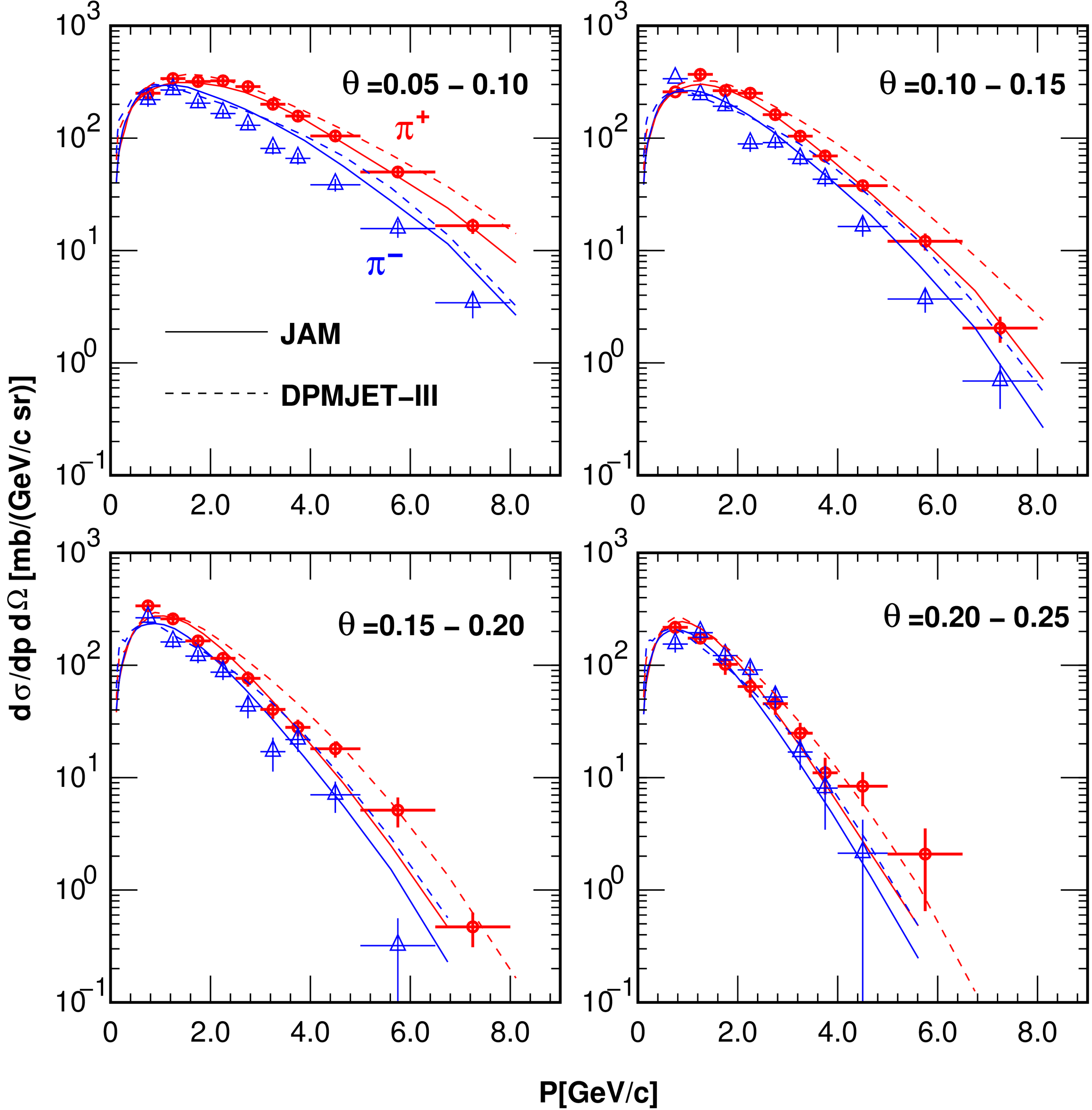


JAM + Modified DPMJET-II vs Muons at the Balloon altitude (HKKM2011)



Good agreement ! → Use DPMJET-III above 32 GeV and JAM below 32 GeV

DPMJET II
JAM
vs HARP



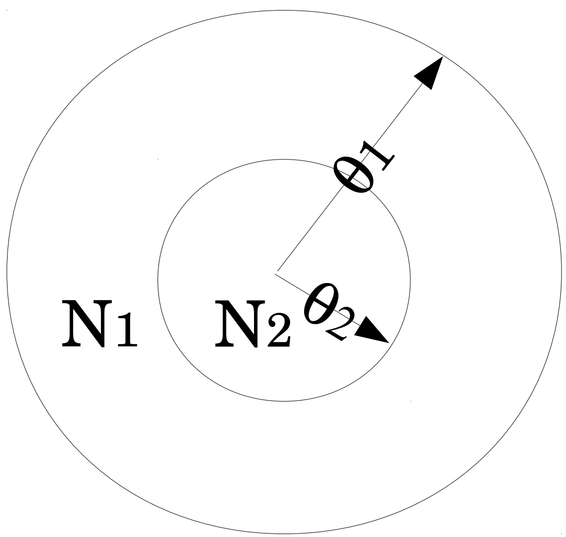
Virtual detector correction

Averages in $\theta<\theta_1$ and $\theta<\theta_2$ can be written with the central value φ_0 as

$$\phi_1 \simeq \varphi_0 + \varphi' \theta_1^2$$

$$\phi_2 \simeq \varphi_0 + \varphi' \theta_2^2$$

where φ' is a constant.



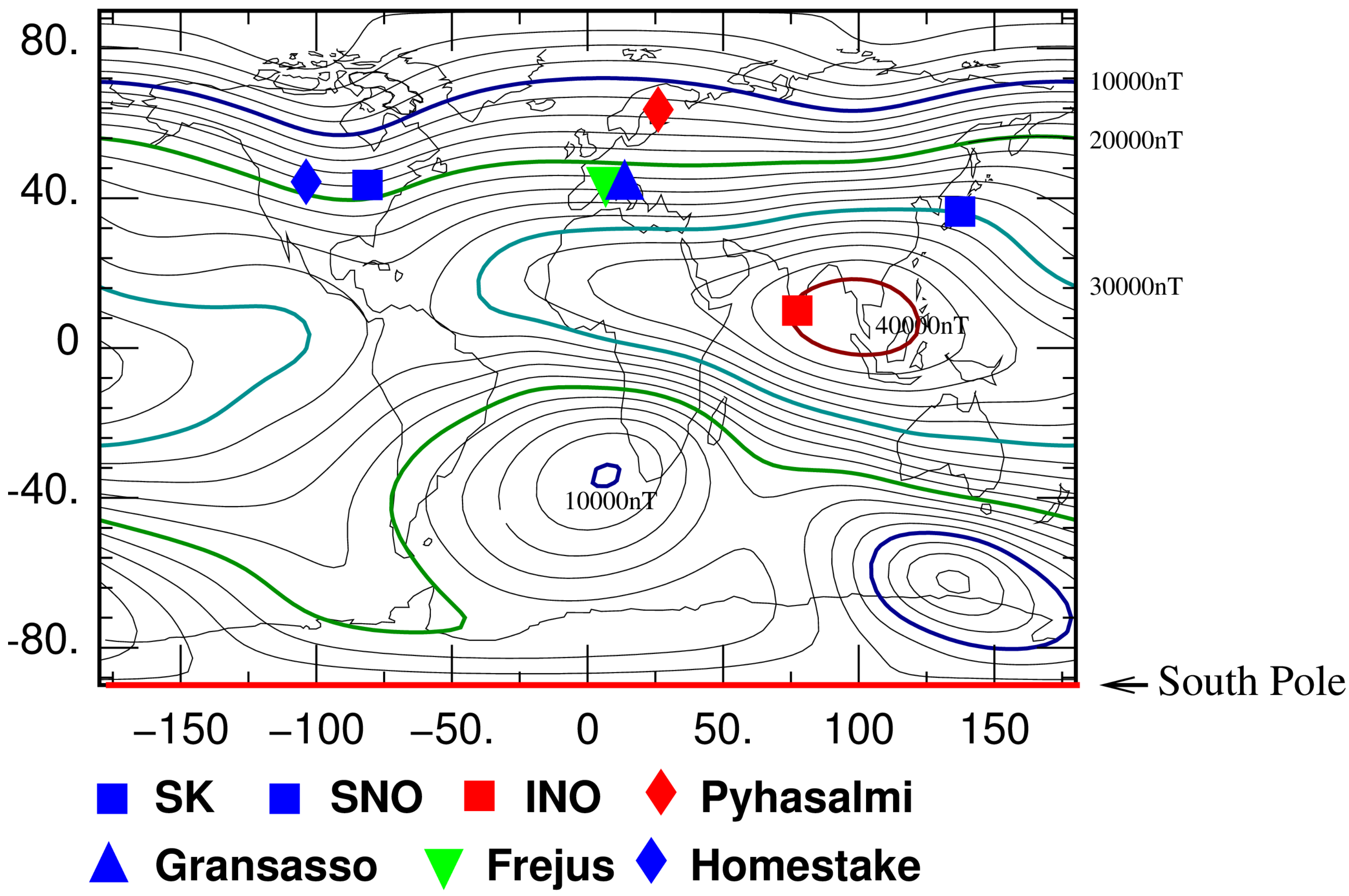
Then we can calculate the central flux value as

$$\varphi_0 \simeq \frac{\theta_1^2 \phi_2 - \theta_2^2 \phi_1}{\theta_1^2 - \theta_2^2} = \frac{\phi_2 - r^2 \phi_1}{1 - r^2} \quad \text{for} \quad r = \left(\frac{\theta_2}{\theta_1}\right), \ r < 1$$

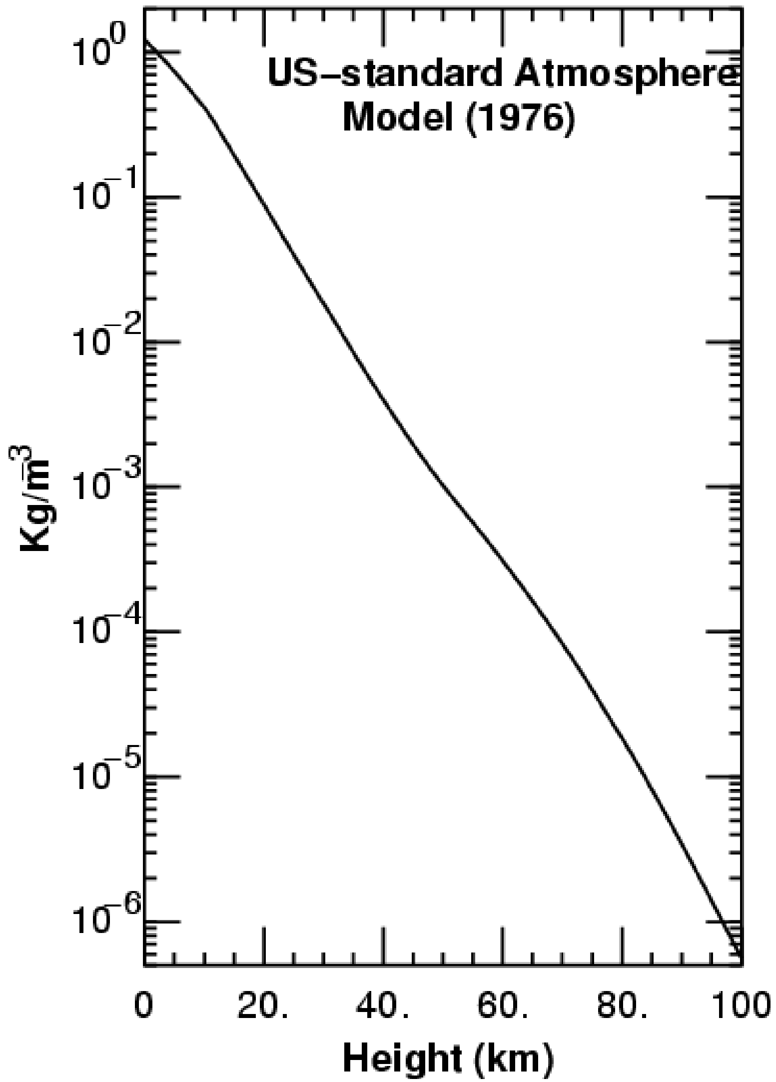
Apply this relation to the MC results

$$\phi_1 = \frac{N_1}{T \pi \theta_1^2}, \quad \phi_2 = \frac{N_2}{T \pi \theta_2^2}$$

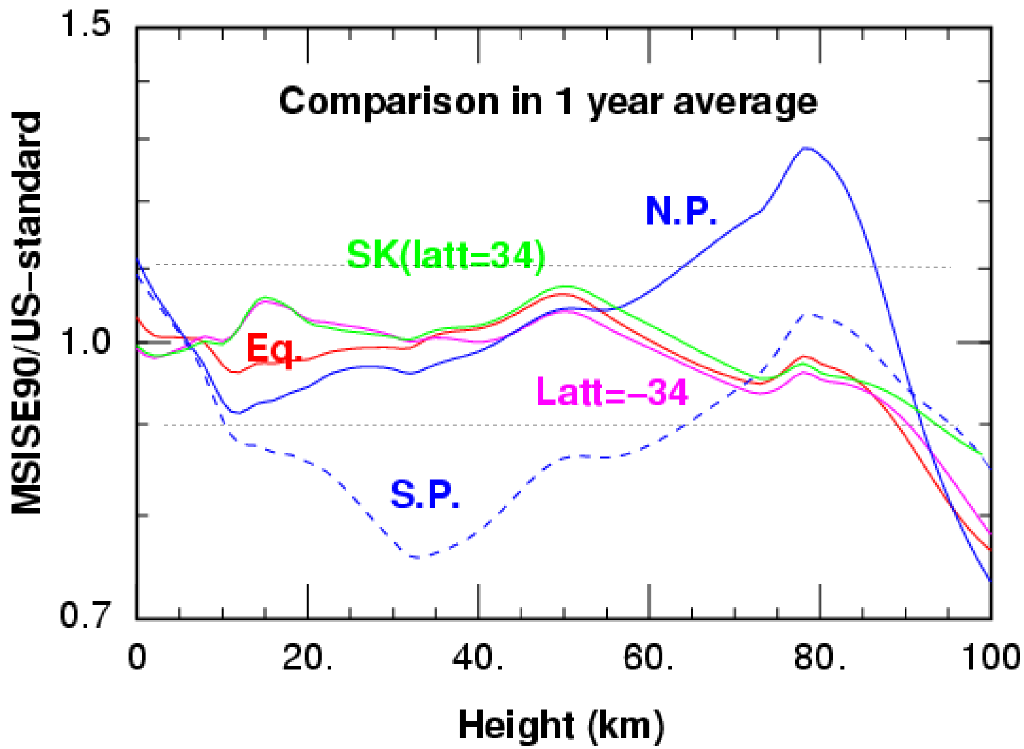
IGRF10 Geomagnetic Horizontal Field Strength



Atmosphere Model

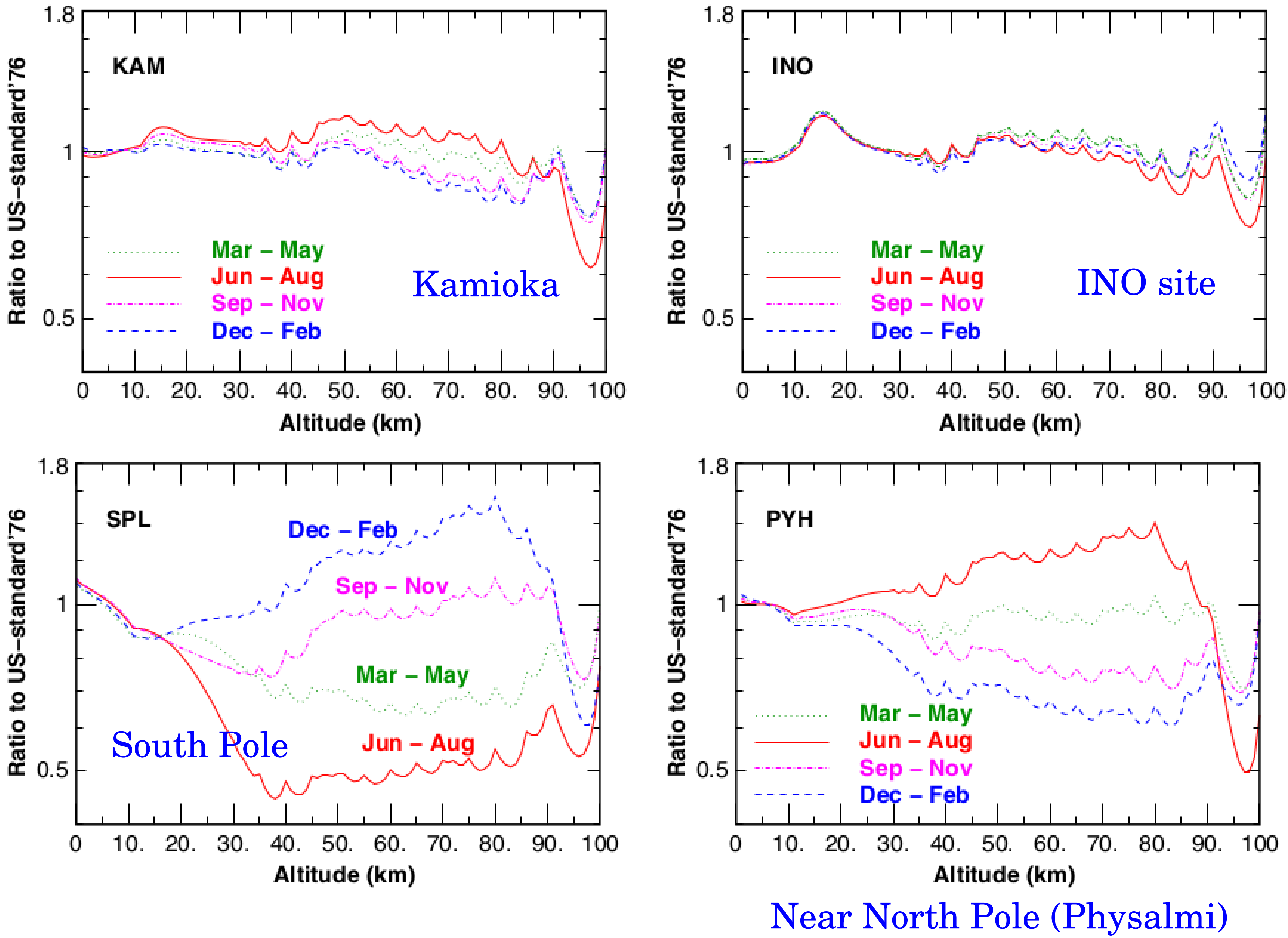


Air density comparison with MSISE90

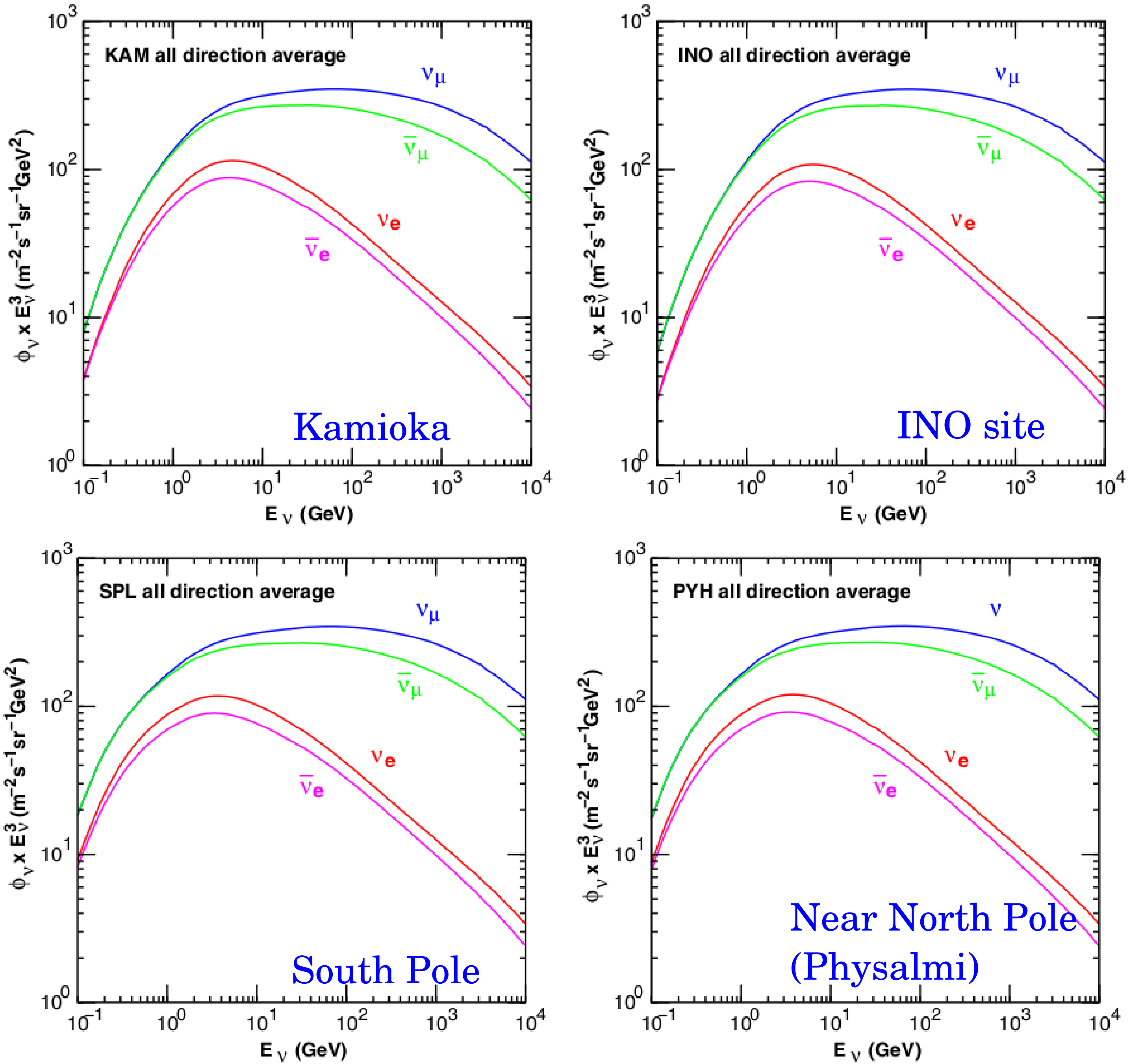


US-starndard'76 may be used as the global approximation of the Atmosphere.

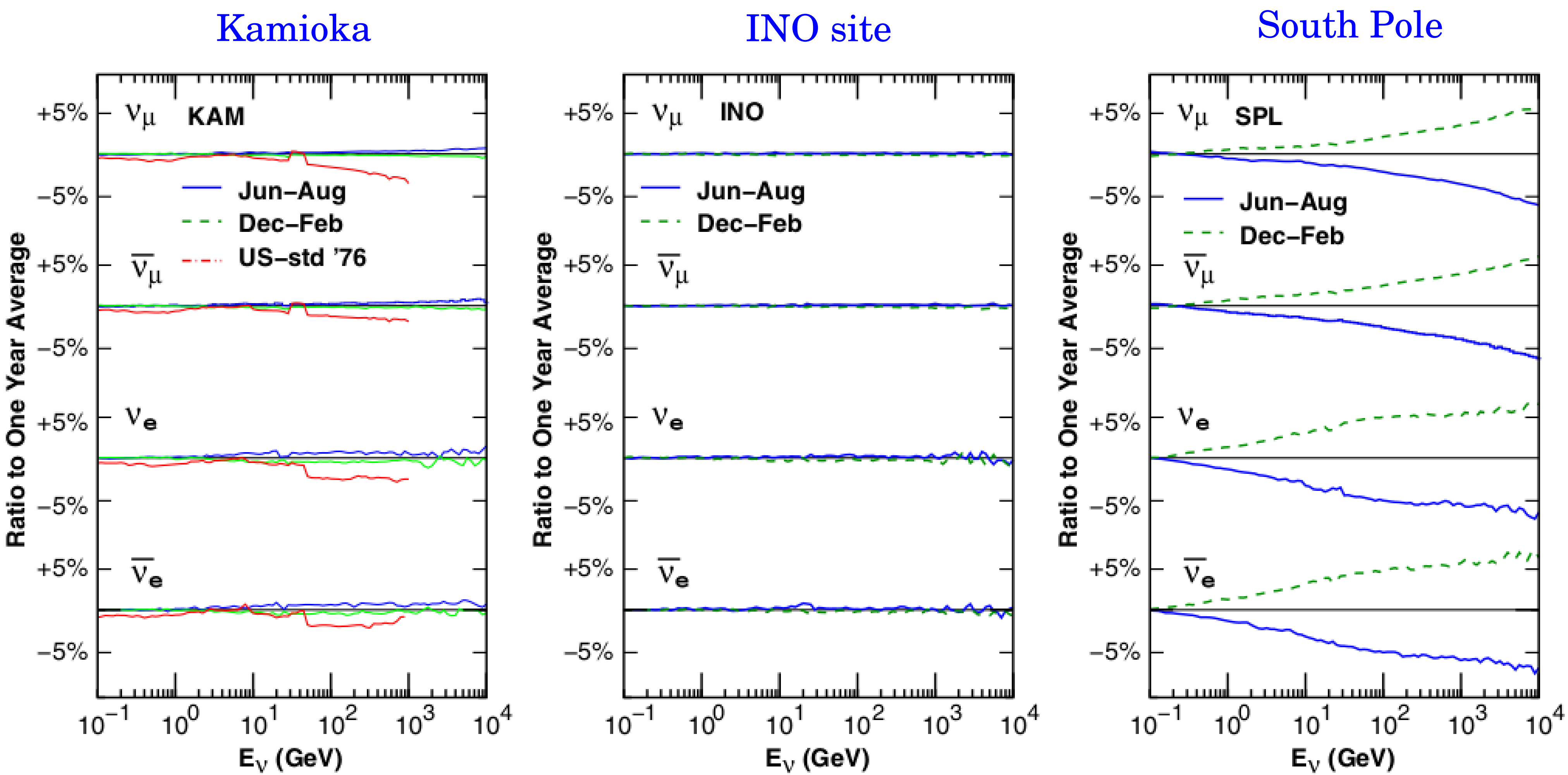
Atmosphere model (NRLMSISE-00) and seasonal variations



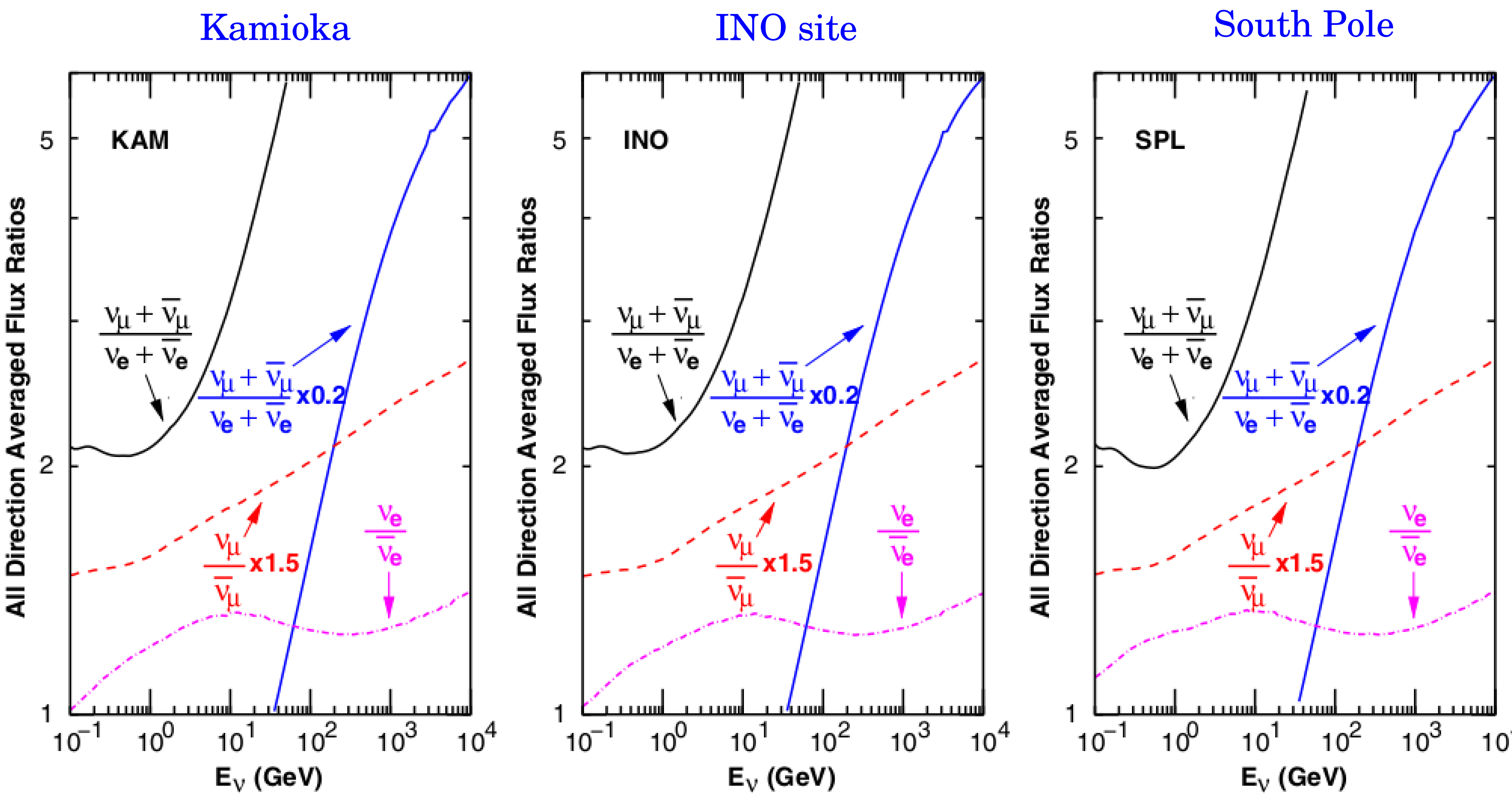
Calculated Atmospheric Neutrino Flux averaged over all directions



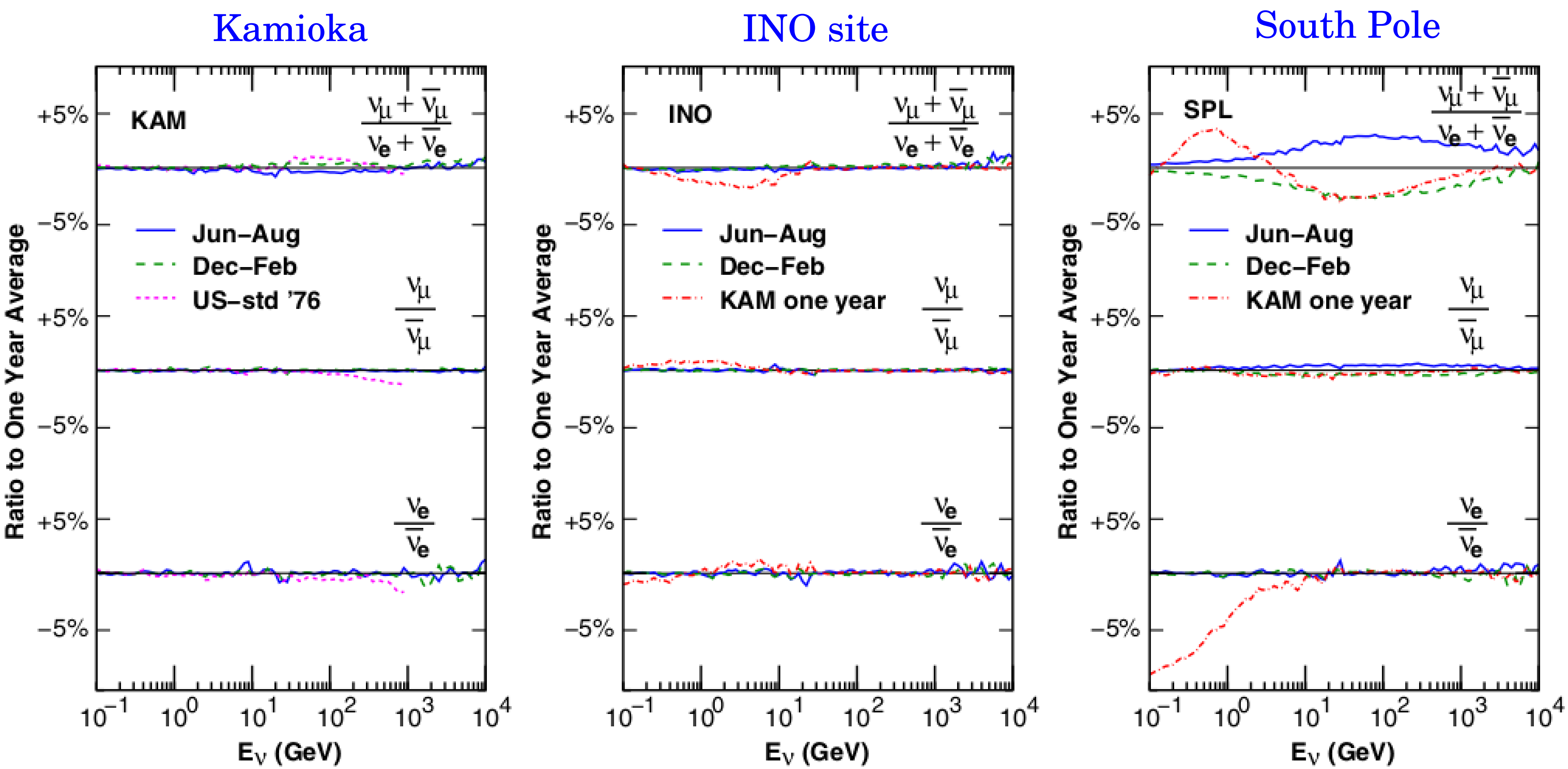
Seasonal Variation of Atmospheric Neutrino flux



Flavor Ratios of Atmospheric Neutrino Flux

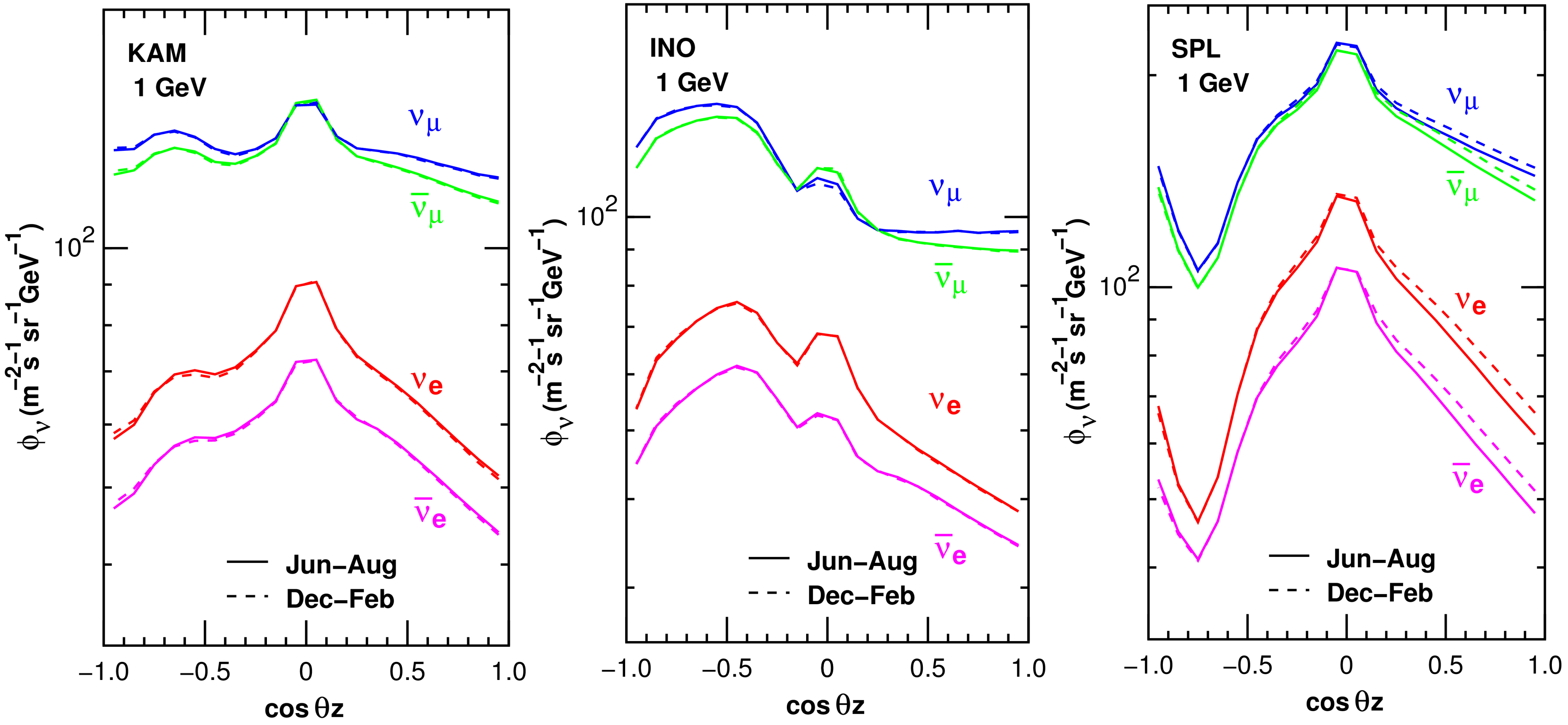


Seasonal and Site Variation of Atmospheric Neutrino Flavor Ratios

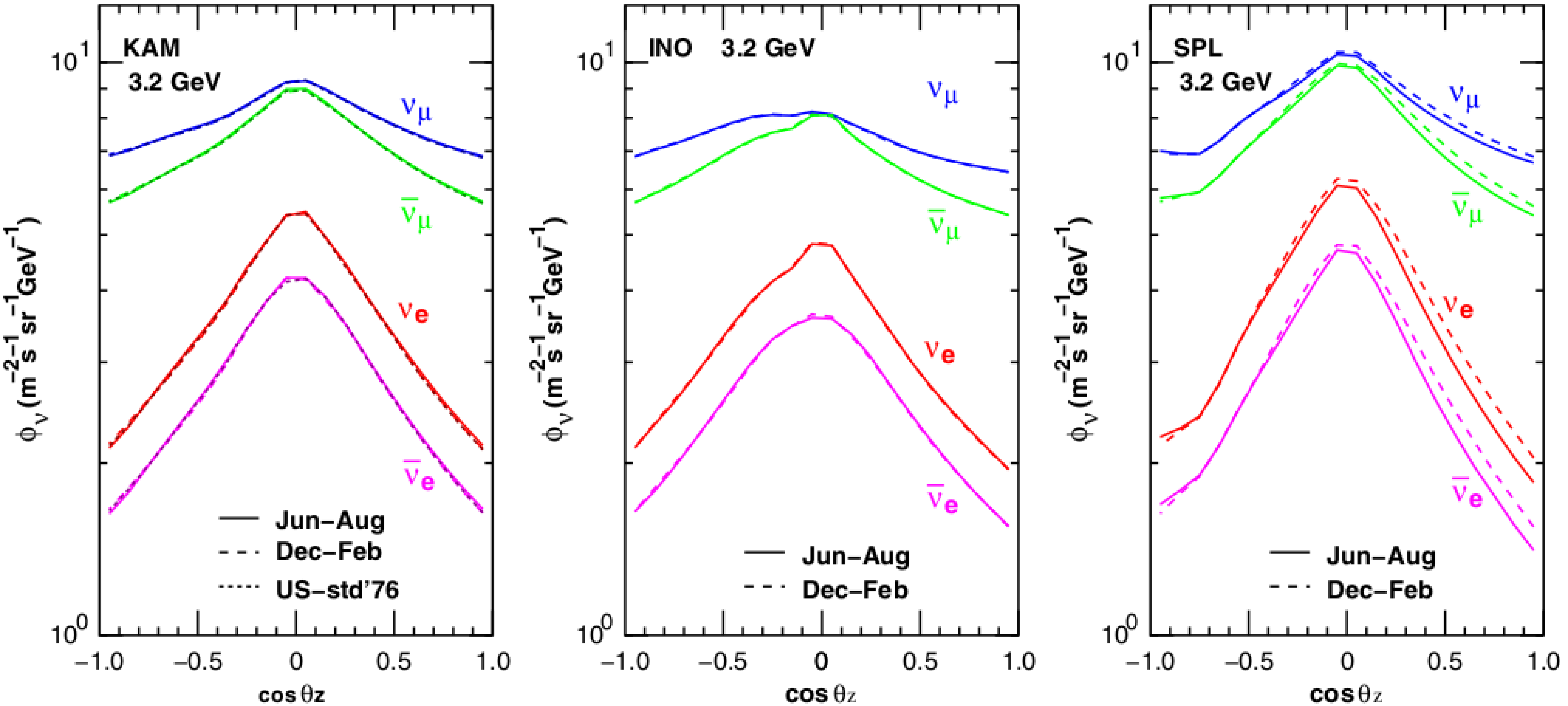


The variation of $\frac{\nu_\mu + \bar{\nu}_\mu}{\nu_e + \bar{\nu}_e}$ at South Pole and the difference from Kamioka are almost equal to the largest estimation of its uncertainty.

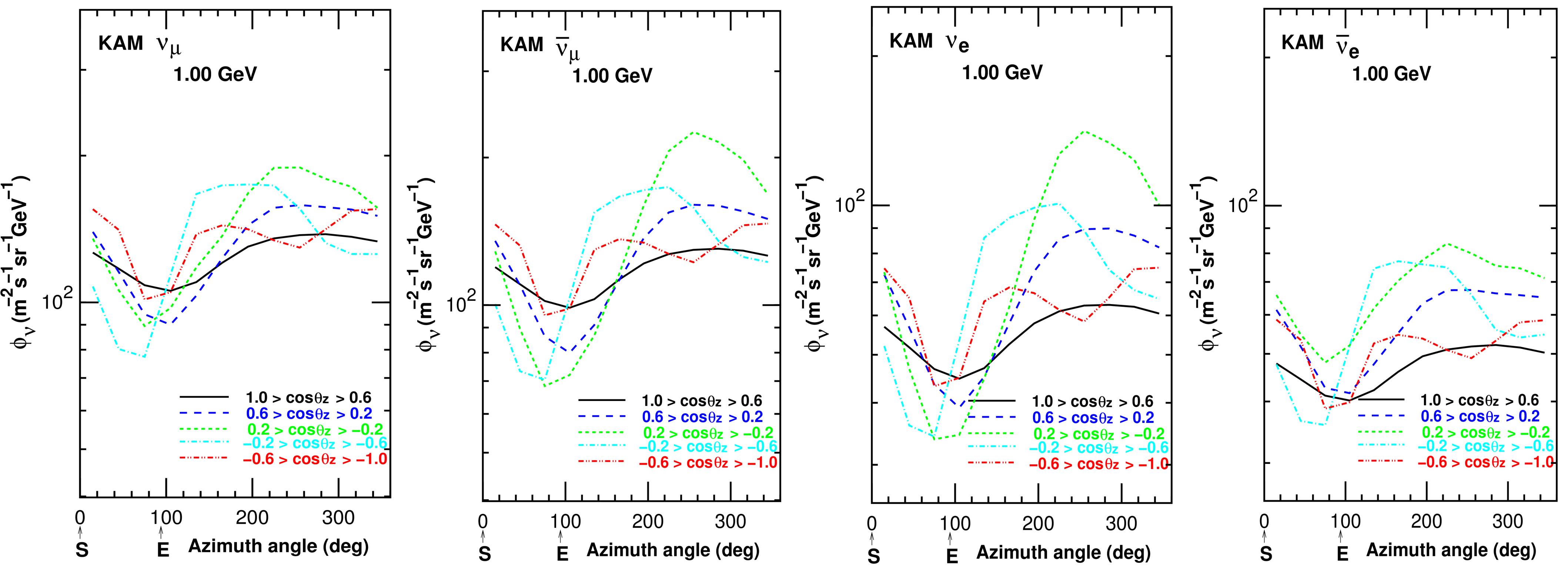
Zenith Angle Variation of Neutrino Fluxes at 1 GeV



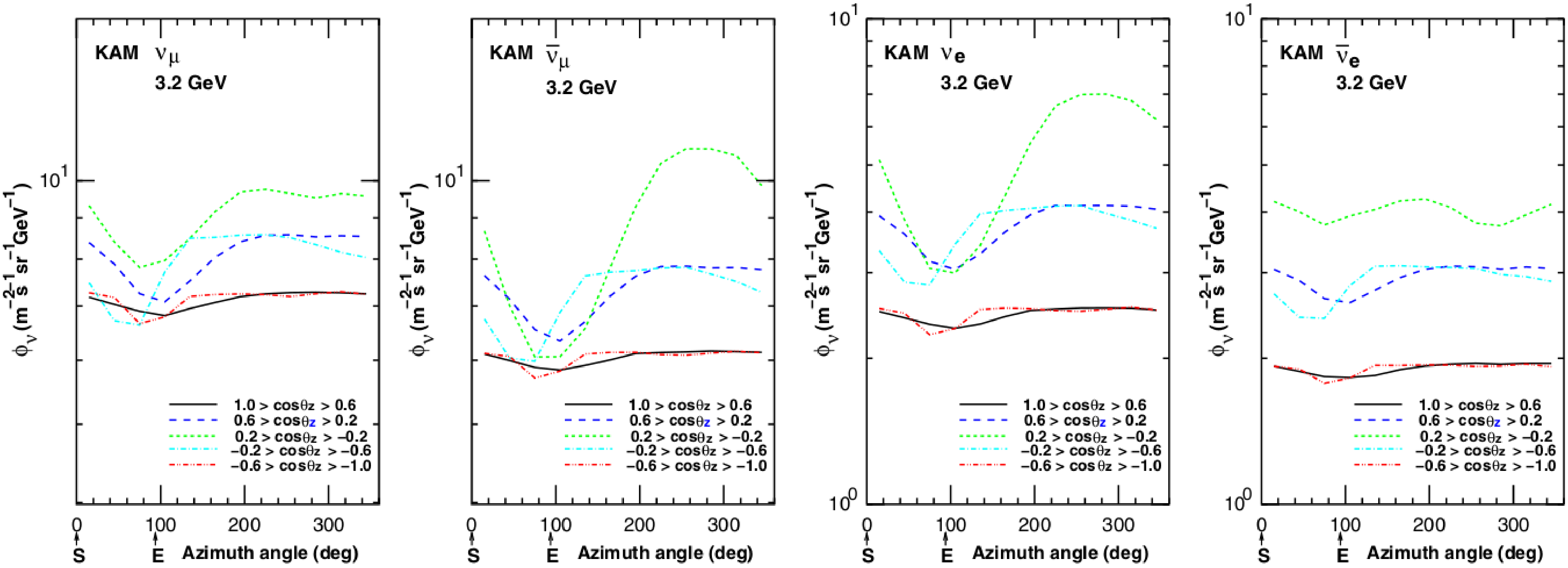
Zenith Angle Variation of Neutrino Fluxes at 3.2 GeV



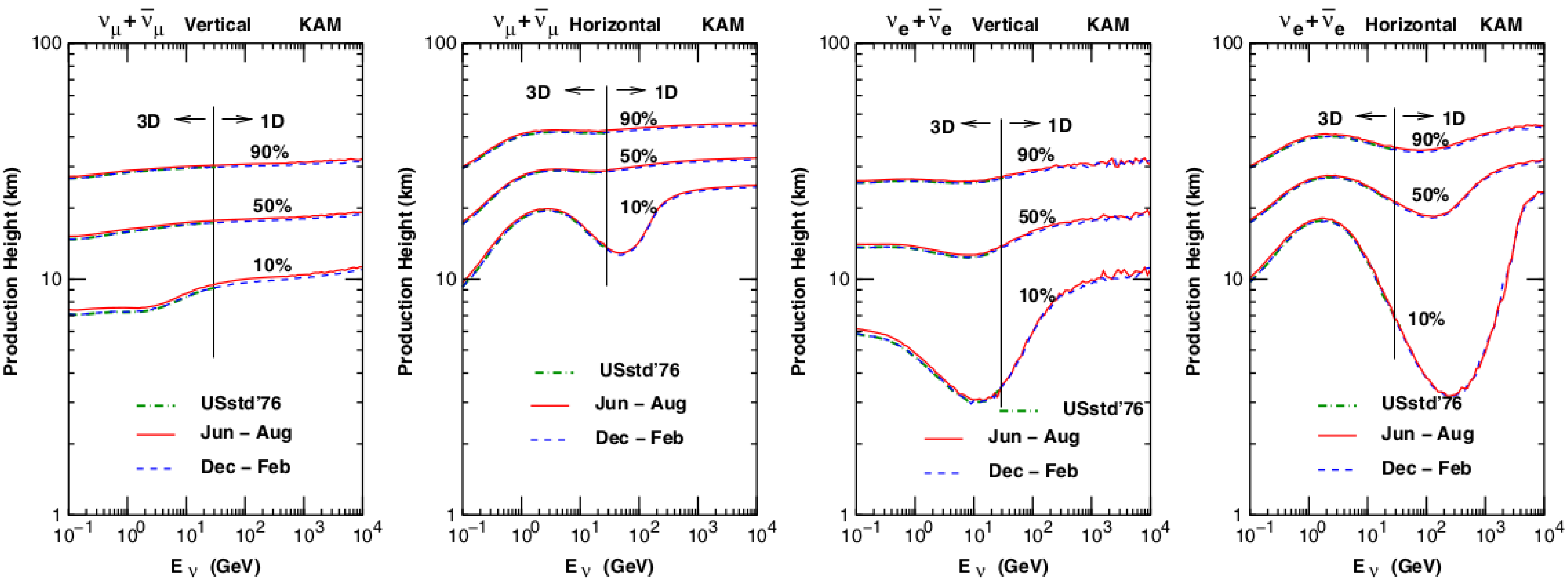
Azimuth Angle Variation of Neutrino Fluxes at 1 GeV at SK site



Azimuth Angle Variation of Neutrino Fluxes at 3.2 GeV at SK site



Cumulative Neutrino Production Height at SK site (Summed over all azimuth angles)

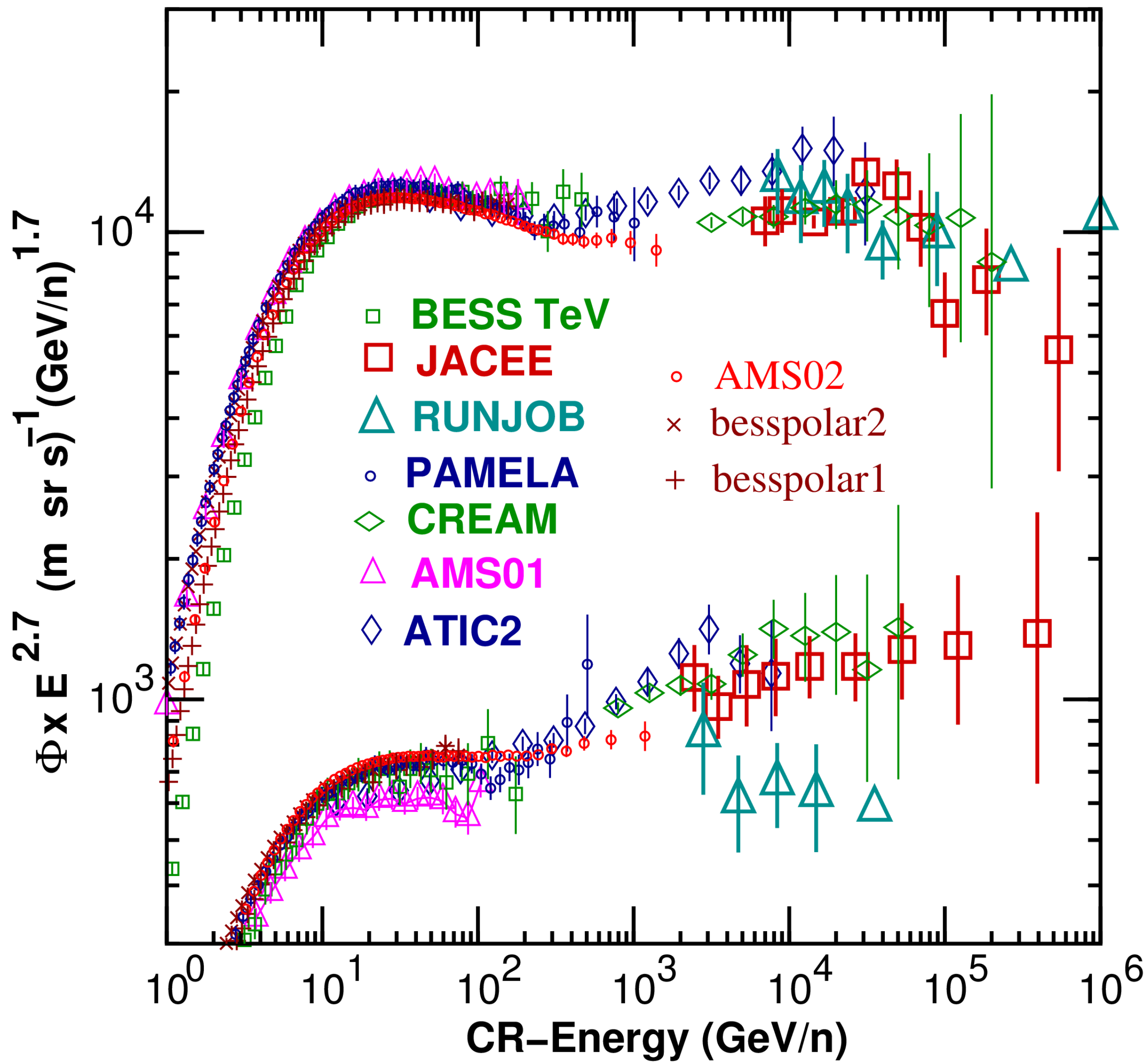


Impact of AMS02

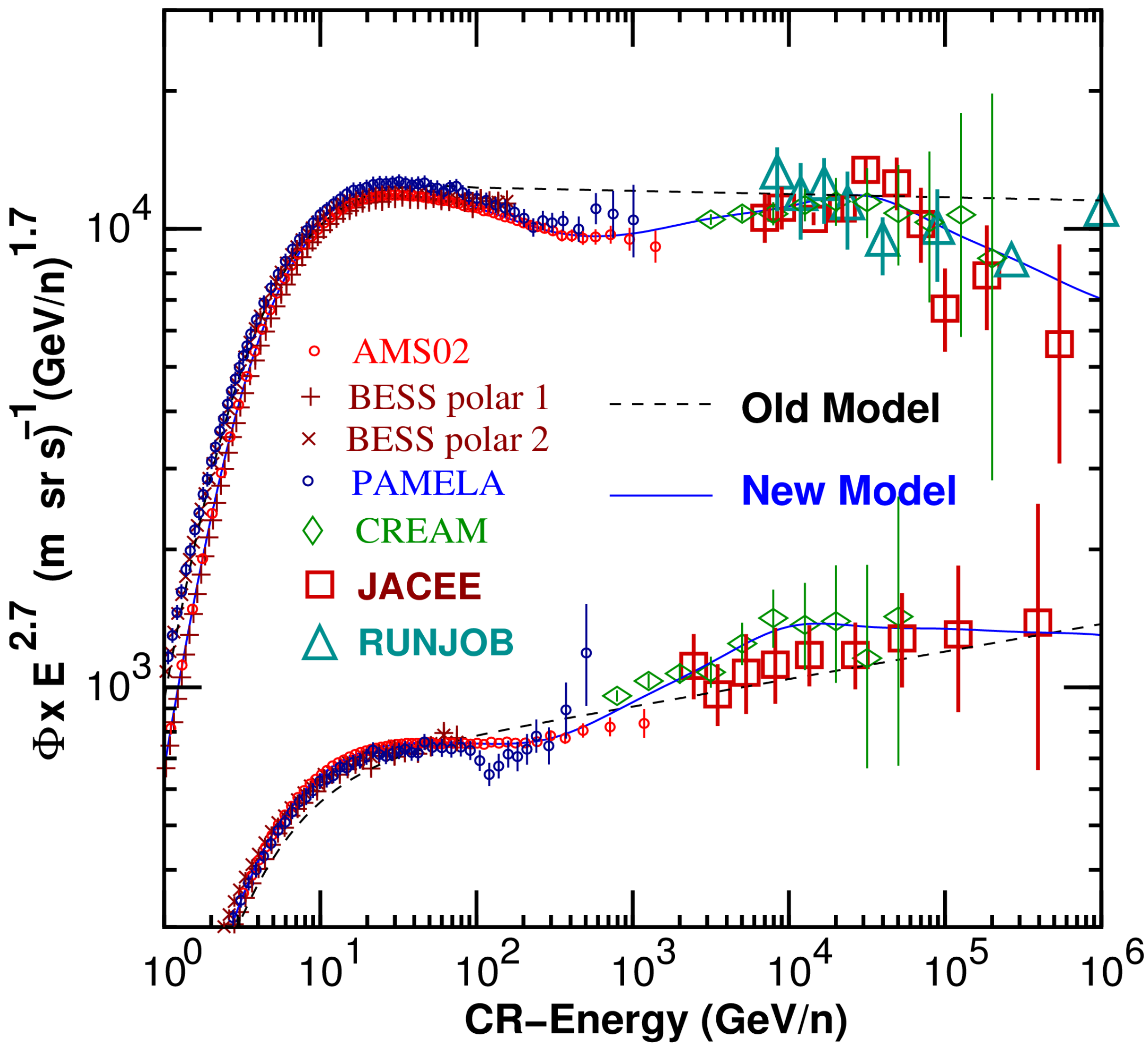
and

BESS-polar

Recent Cosmic Ray observation
and
available High Energy data

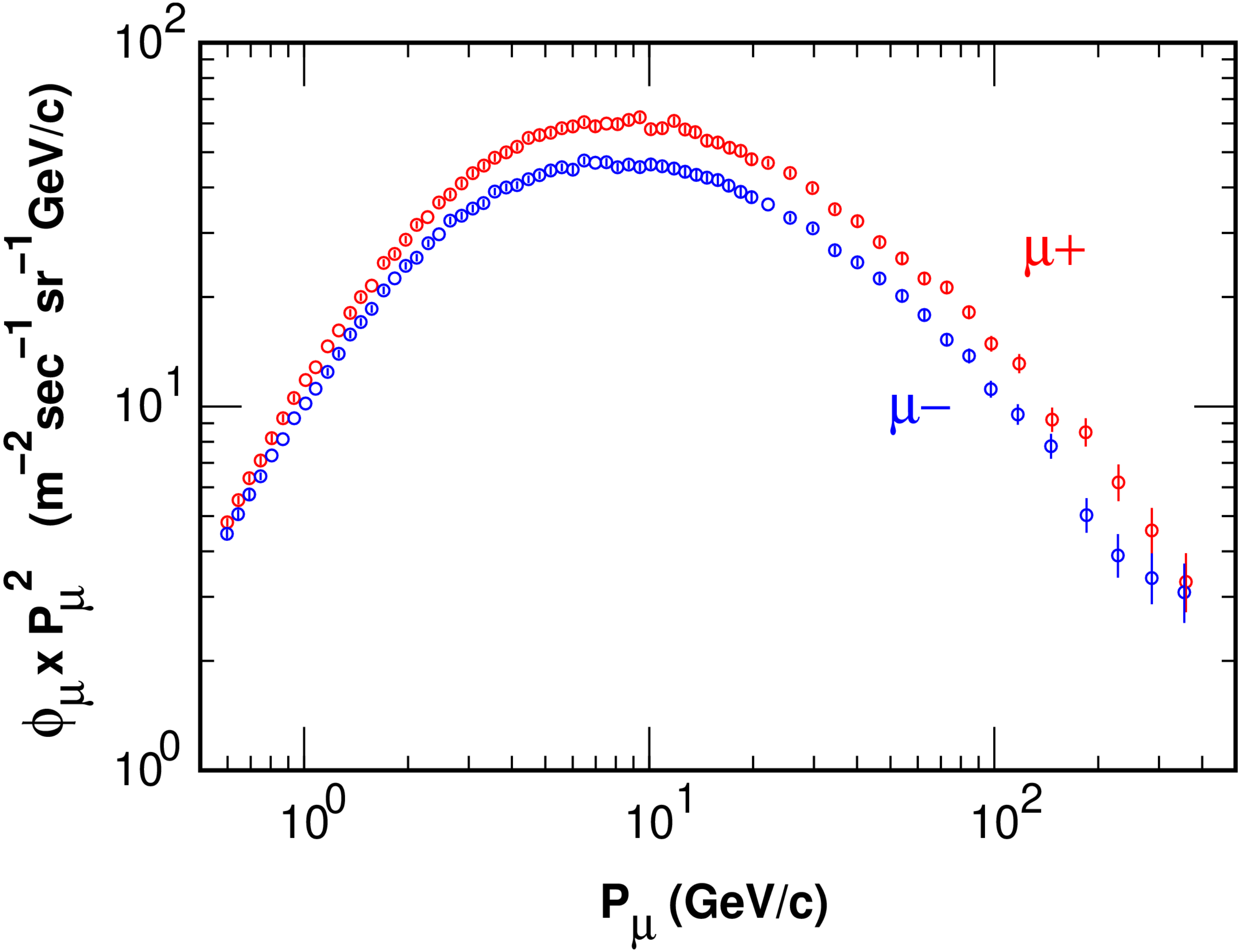


New Cosmic Ray Model with AMS02 and BESS-polar

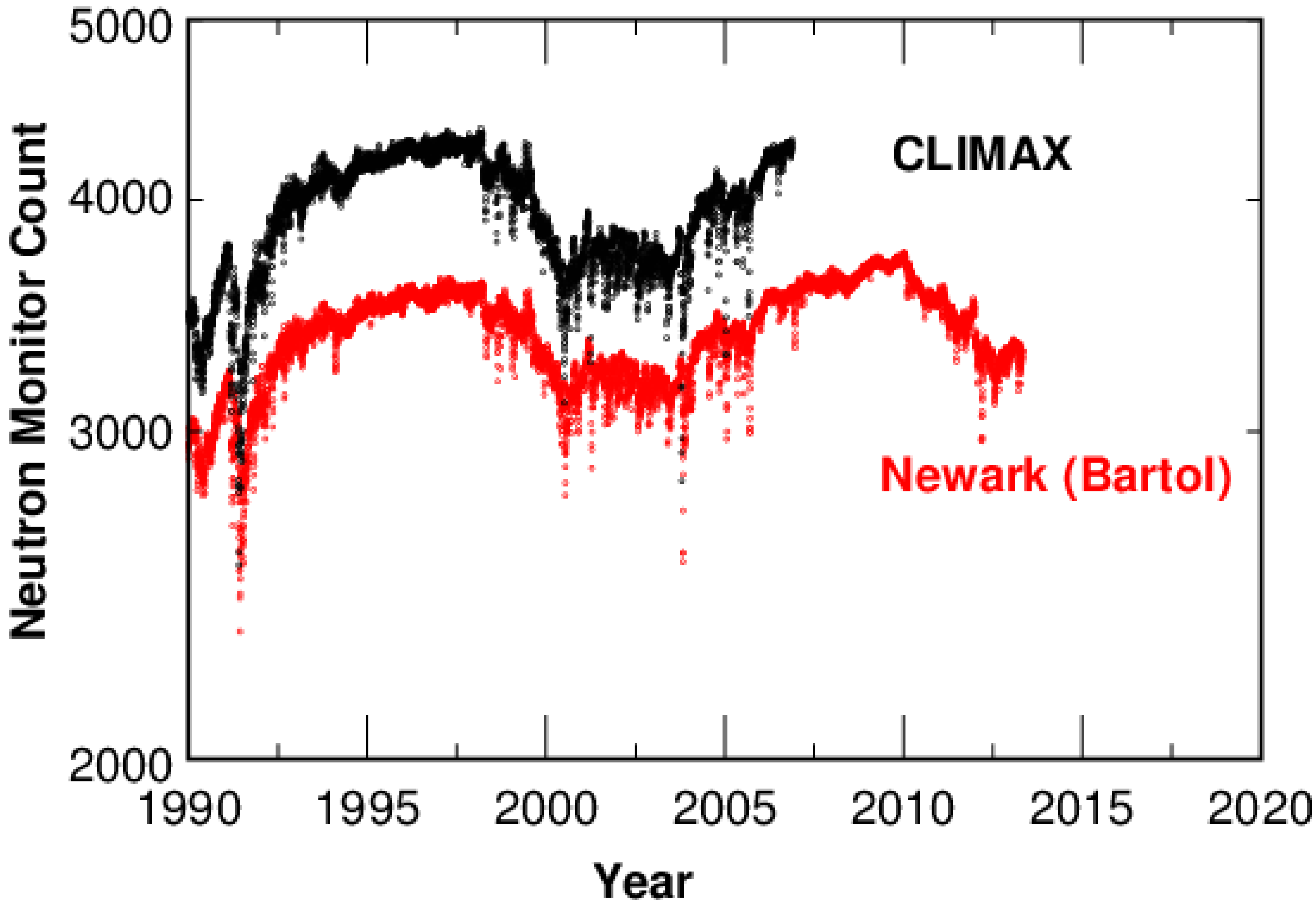


Discarded some data from model construction.

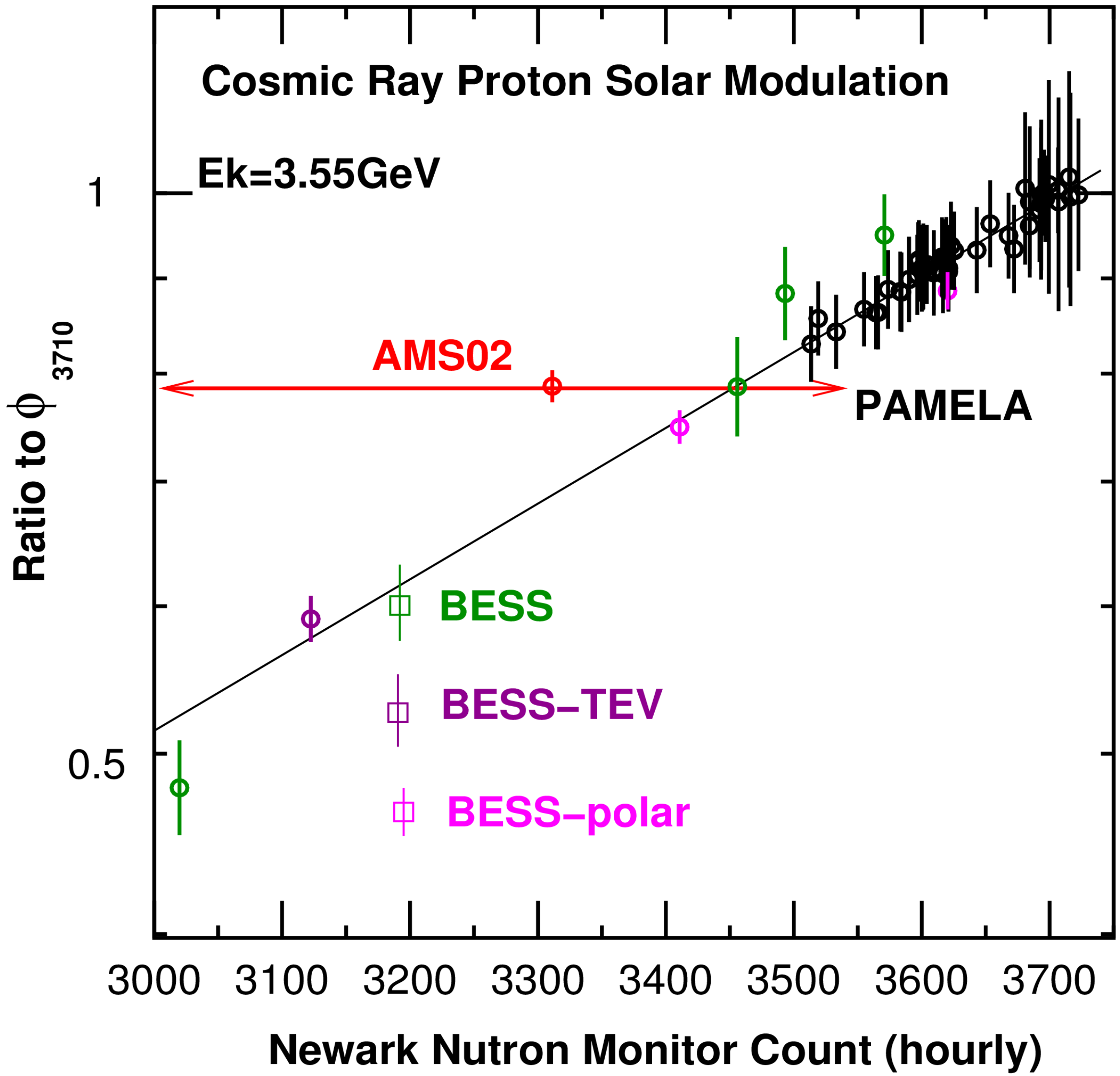
μ–data Observed by BESS (2001)



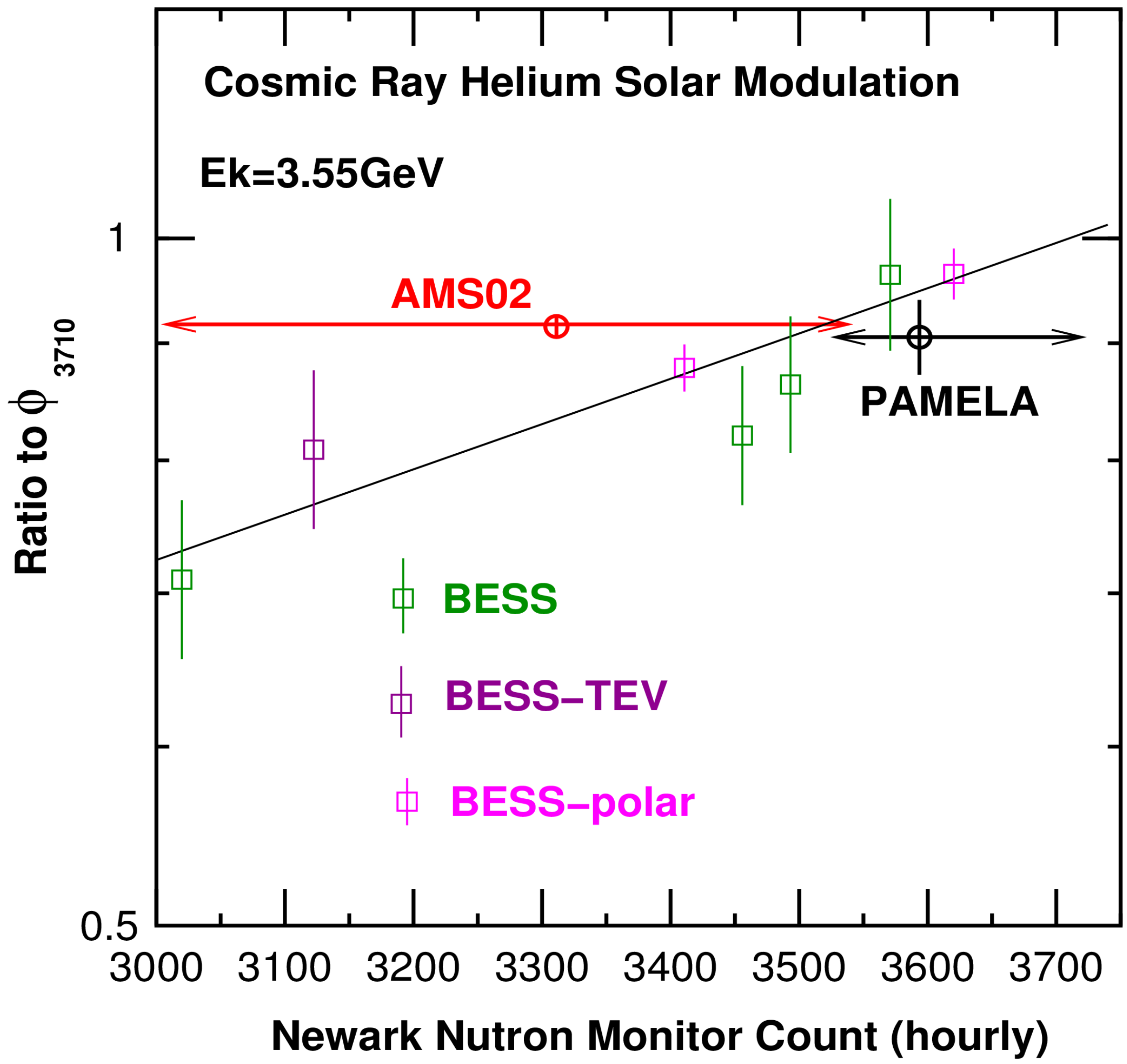
Solar Modulation by Neutron Observed by Monitor



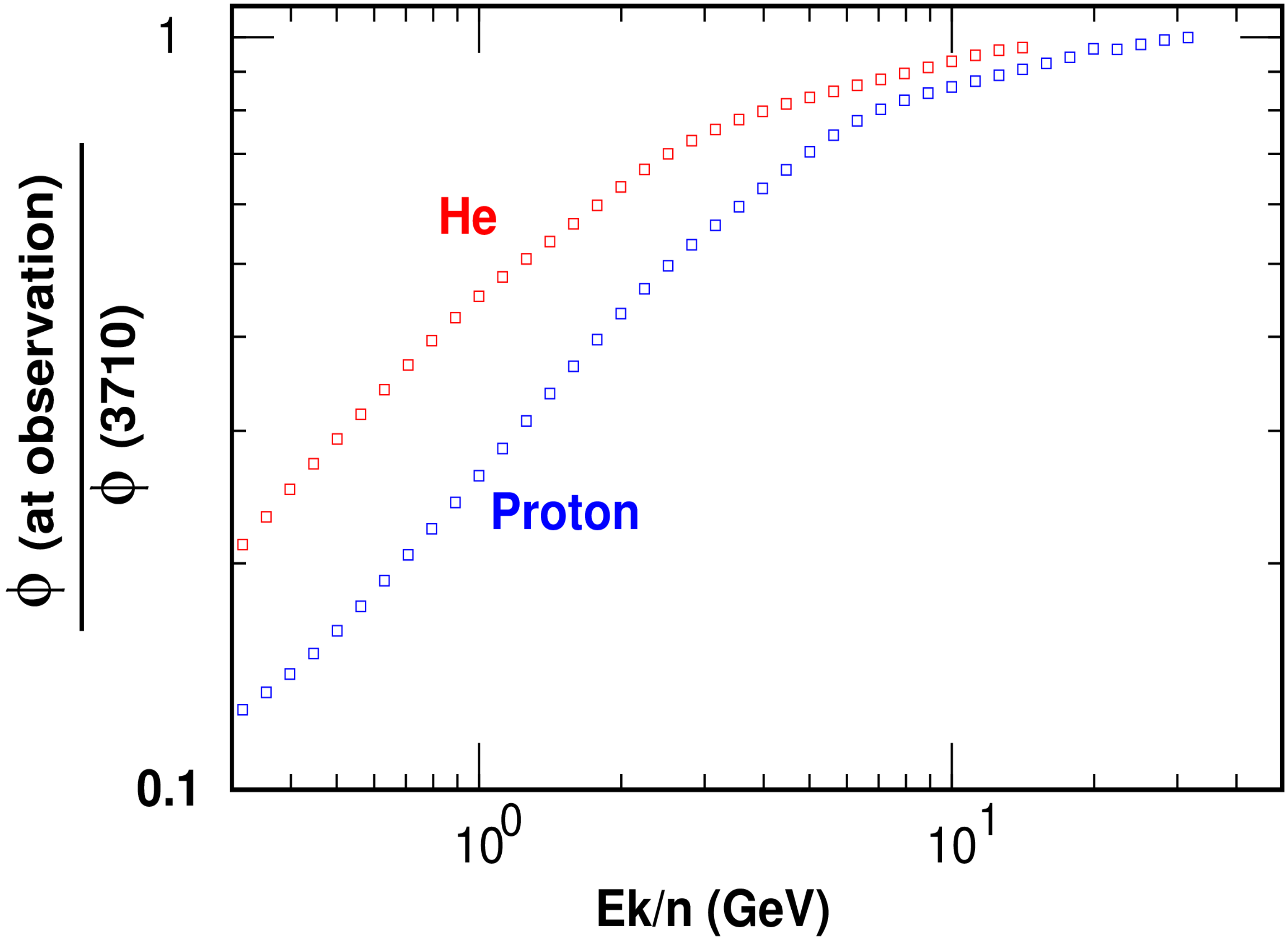
Solar Modulation of Cosmic Ray Proton



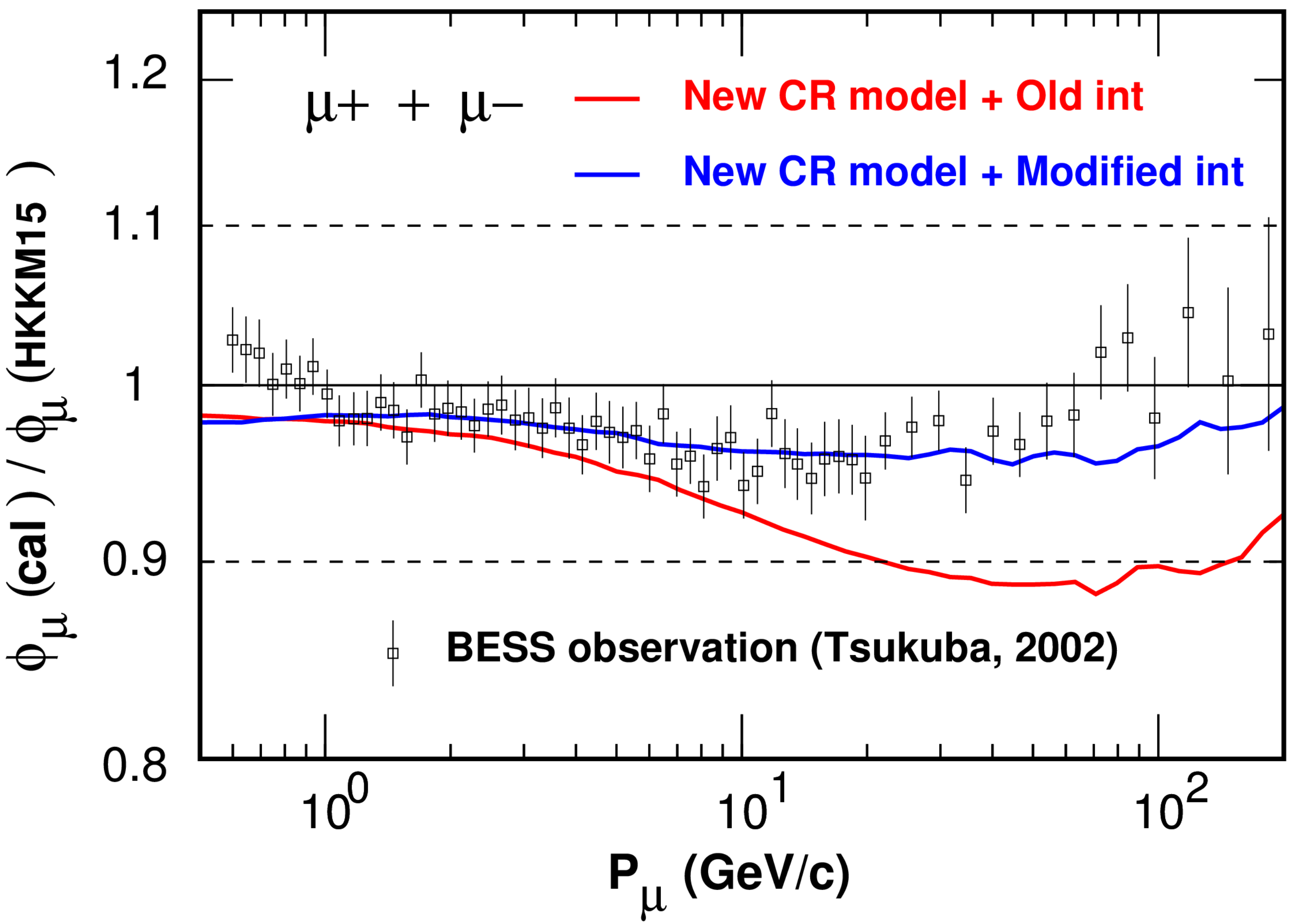
Solar Modulation of Cosmic Ray Helium



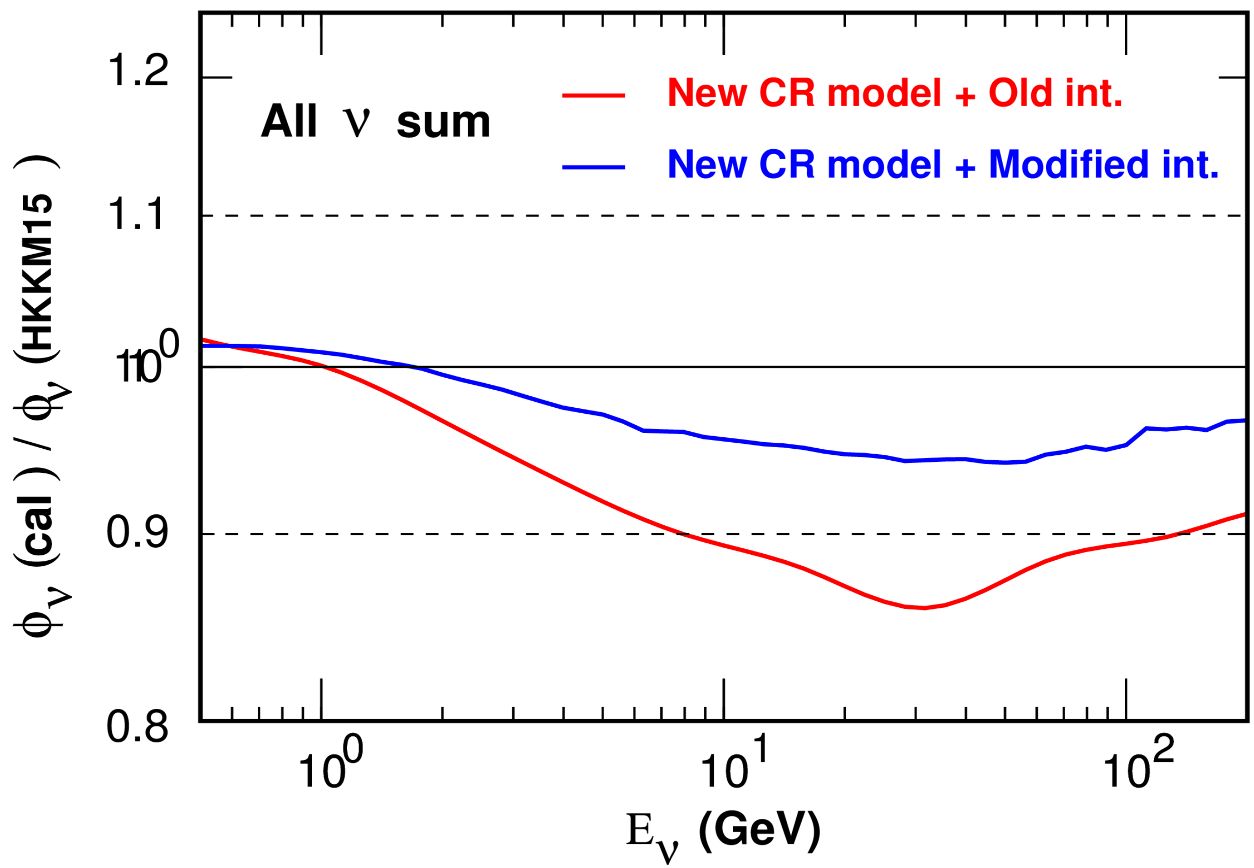
Primary flux ratio to solar minimum at BESS μ-observation



Muon Calibration of Interaction Model with New Cosmic Ray Model

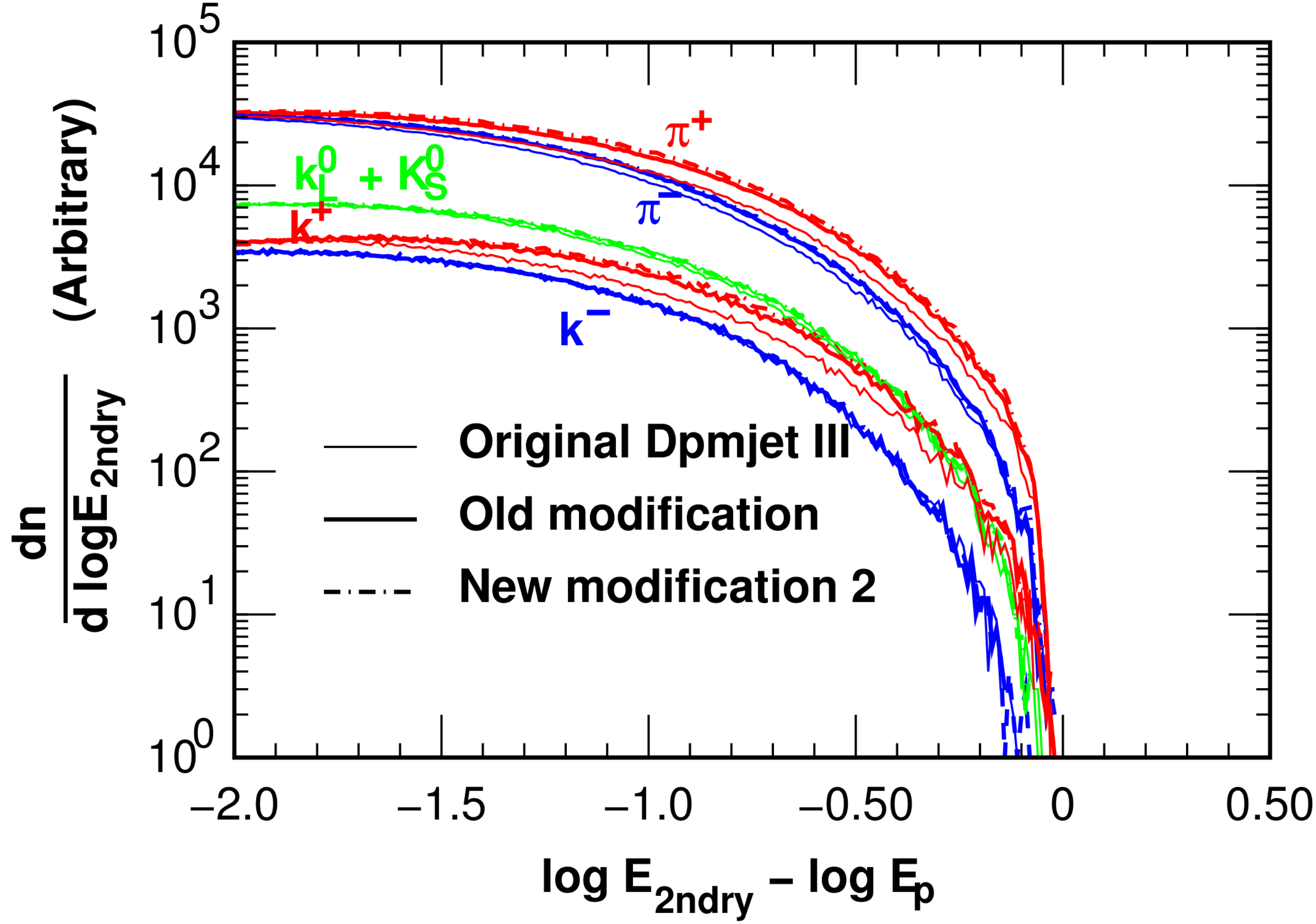


Resulting Neutrino Flux (all ν sum)

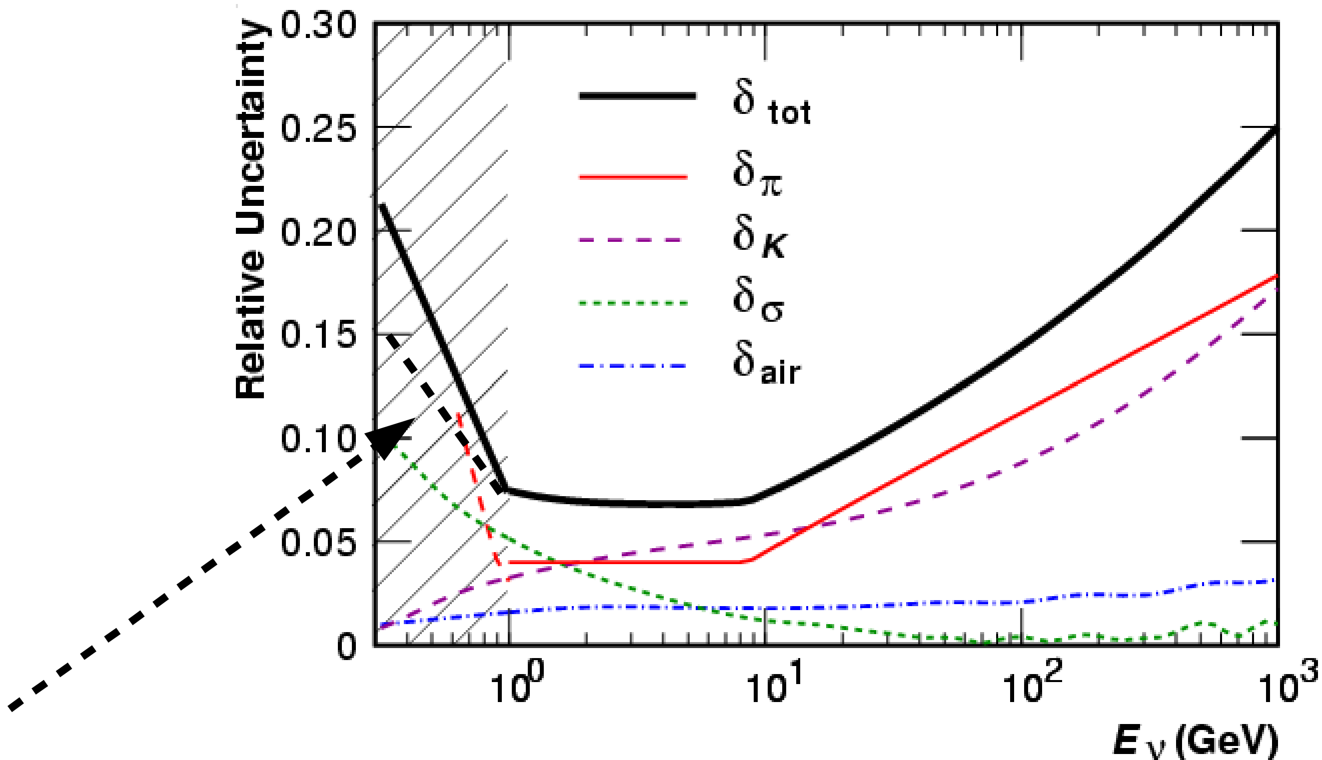


Muon calibration works !

Comparison of secondary spectra of interaction models at 1 TeV



Estimated Error in Atmospheric ν-flux Calculation (HKKMS07)



Possible Error with JAM (HKKM11)

δ_π μ -observation error + Residual of reconstruction

δ_K Kaon production uncertainty

δ_σ Mean free path (interaction crossection) uncertainty

δ_{air} Atmosphere density profule uncertainty

Summary

- We overviewed the calculation of atmospheric neutrino flux in HKKM.

- With NRLMSISE-00 atmosphere model, we find a large seasonal variation of neutrino flux at polar region. This also cause a variation in $\frac{\nu_\mu + \bar{\nu}_\mu}{\nu_e + \bar{\nu}_e}$ ratio.

- We presented preliminary study based on AMS02 and BESS-polar. However, with the muon calibration, resulted atmospheric neutrino flux is very similar to the one with our (old) primary flux model.

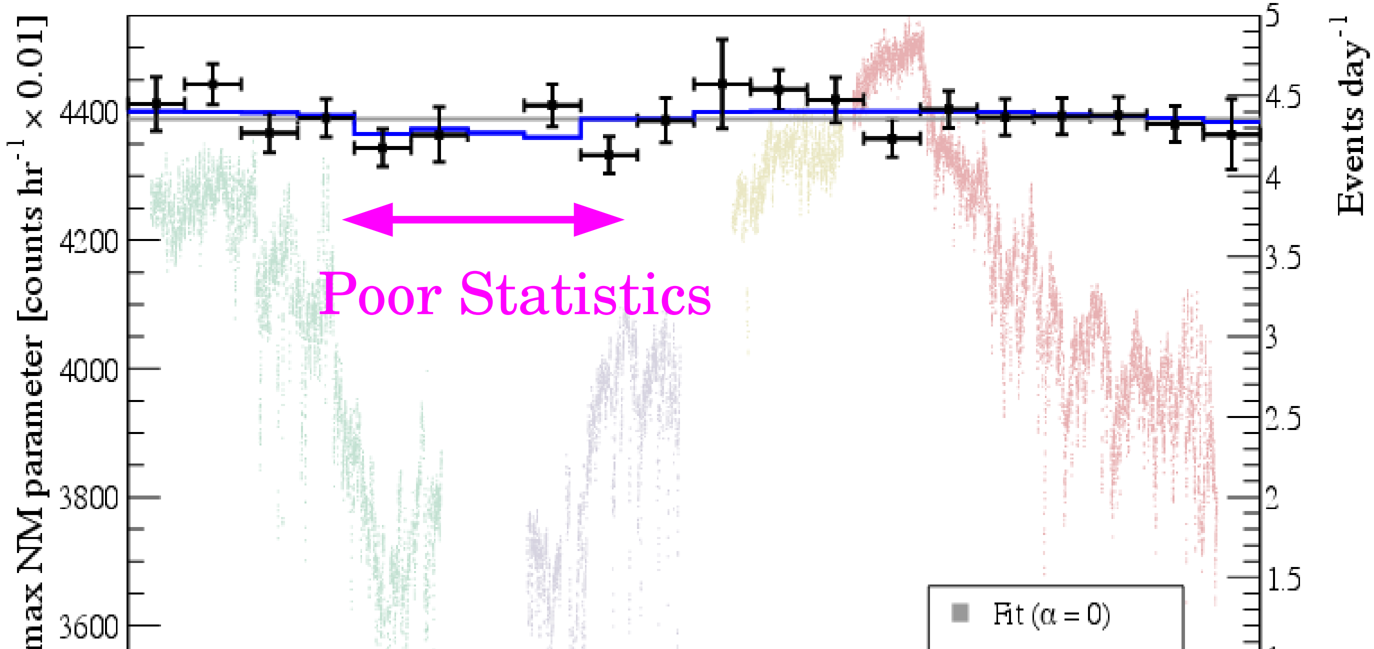
- SK started to observe the predicted features of atmospheric neutrino flux.
- Advertisement: We are planning to record all the atmospheric neutrino on the earth. Then, we will be able to provide the atmospheric neutrino flux at any site on the Earth in a shorter period without re-calculation.

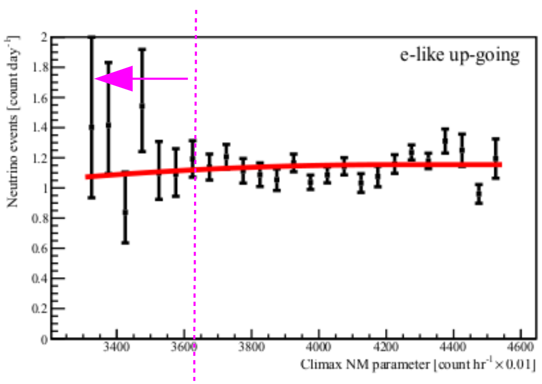
Atmospheric neutrino observed by SK

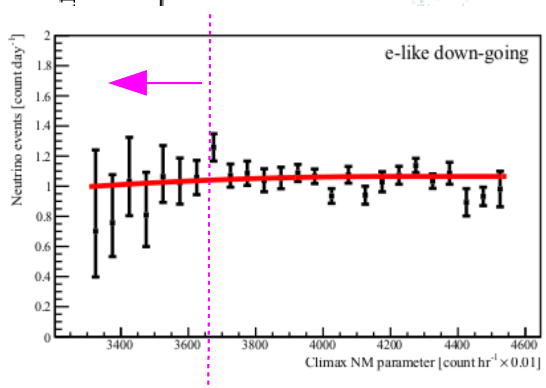
(Advertisement of the talk of Okumura-san)

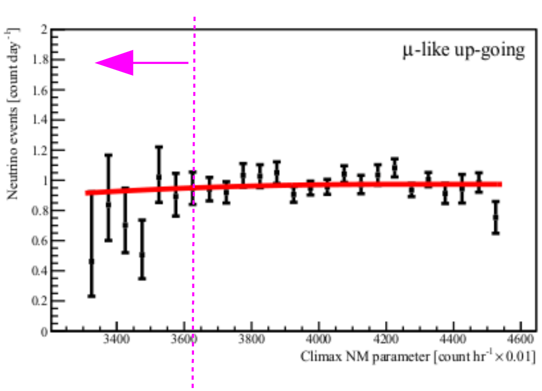
Solar Modulation of Atmospheric Neutrinos

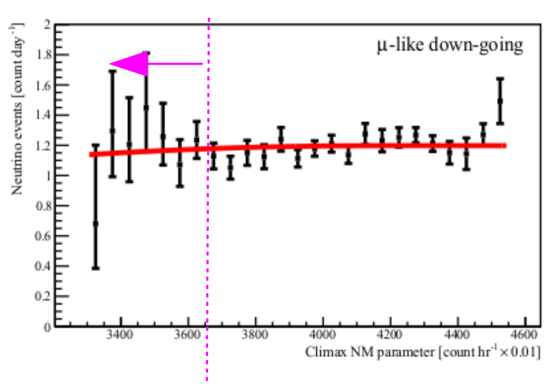
From PHD thesis of
E. Richard







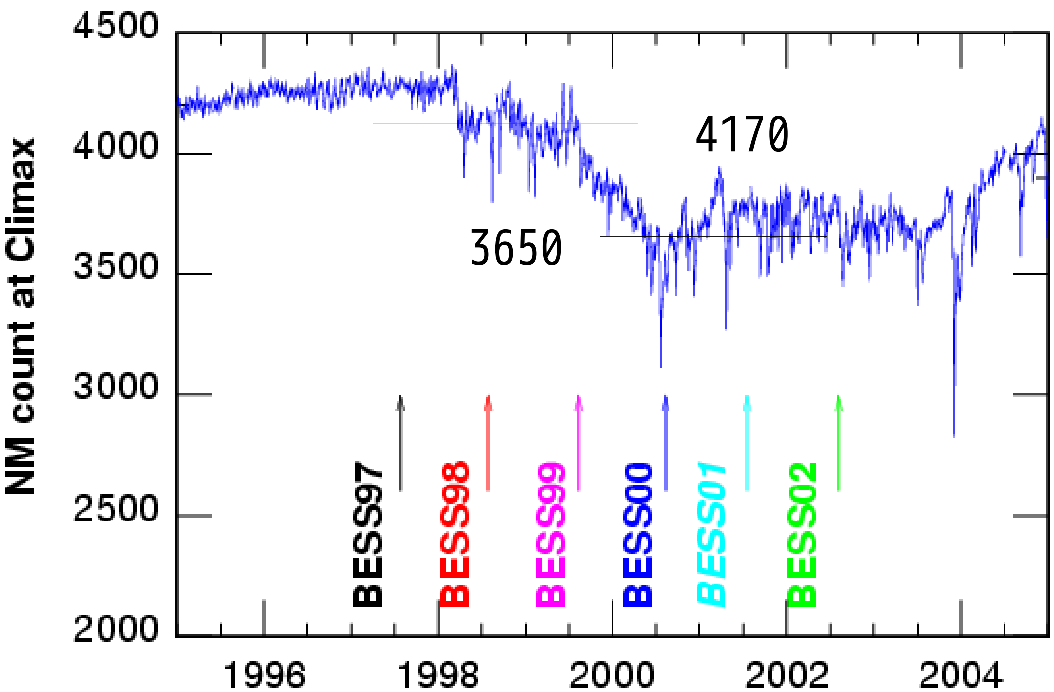




Best fit corresponds to 62 % of the predicted variations

Picked up mainly the forbush decrease ?

Solar Modulation of Primary Cosmic Rays and Atmospheric Neutrino

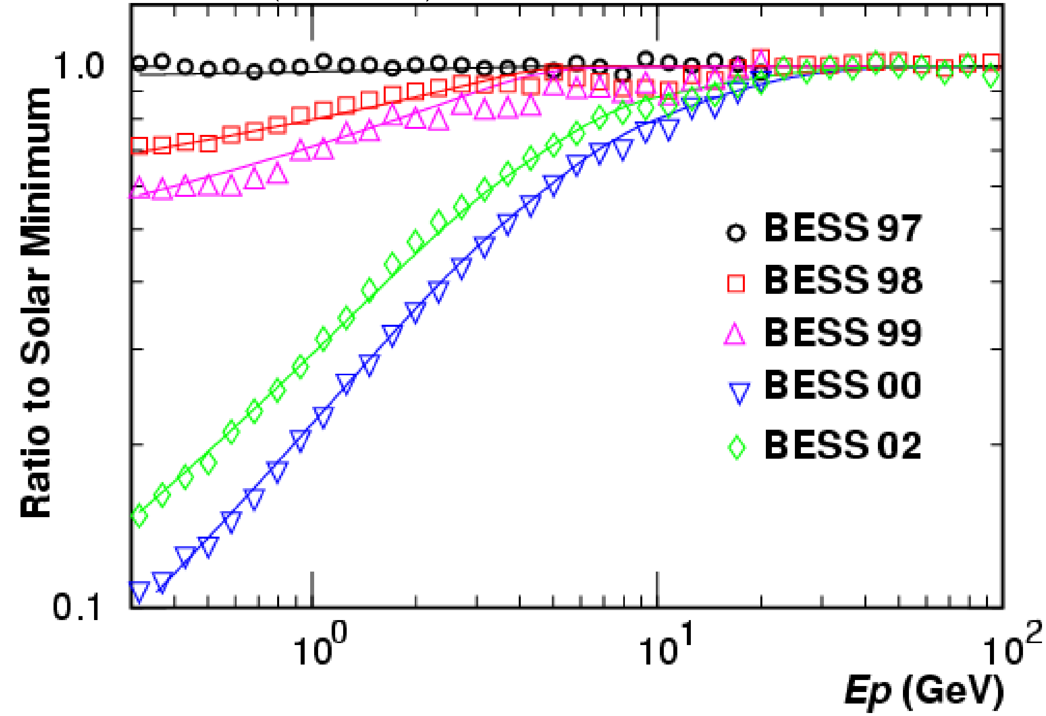


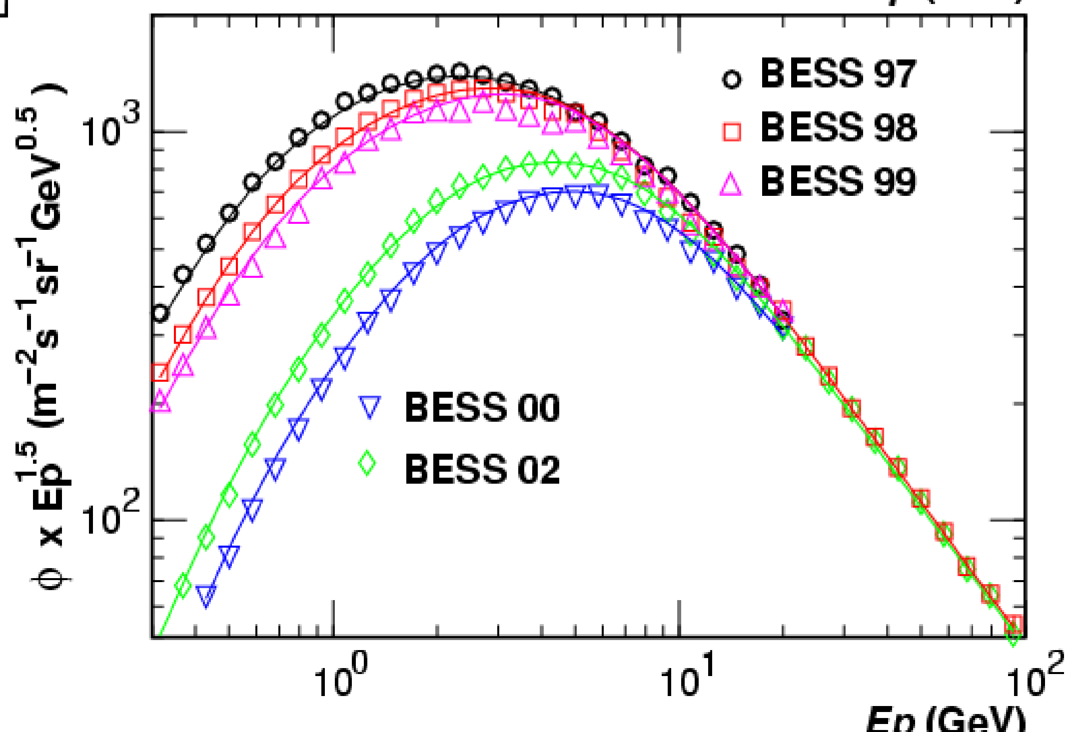
BESS flight

$$\phi_i(N, E_k) = \phi_i^{min}(E_k) \cdot M(N, r)$$

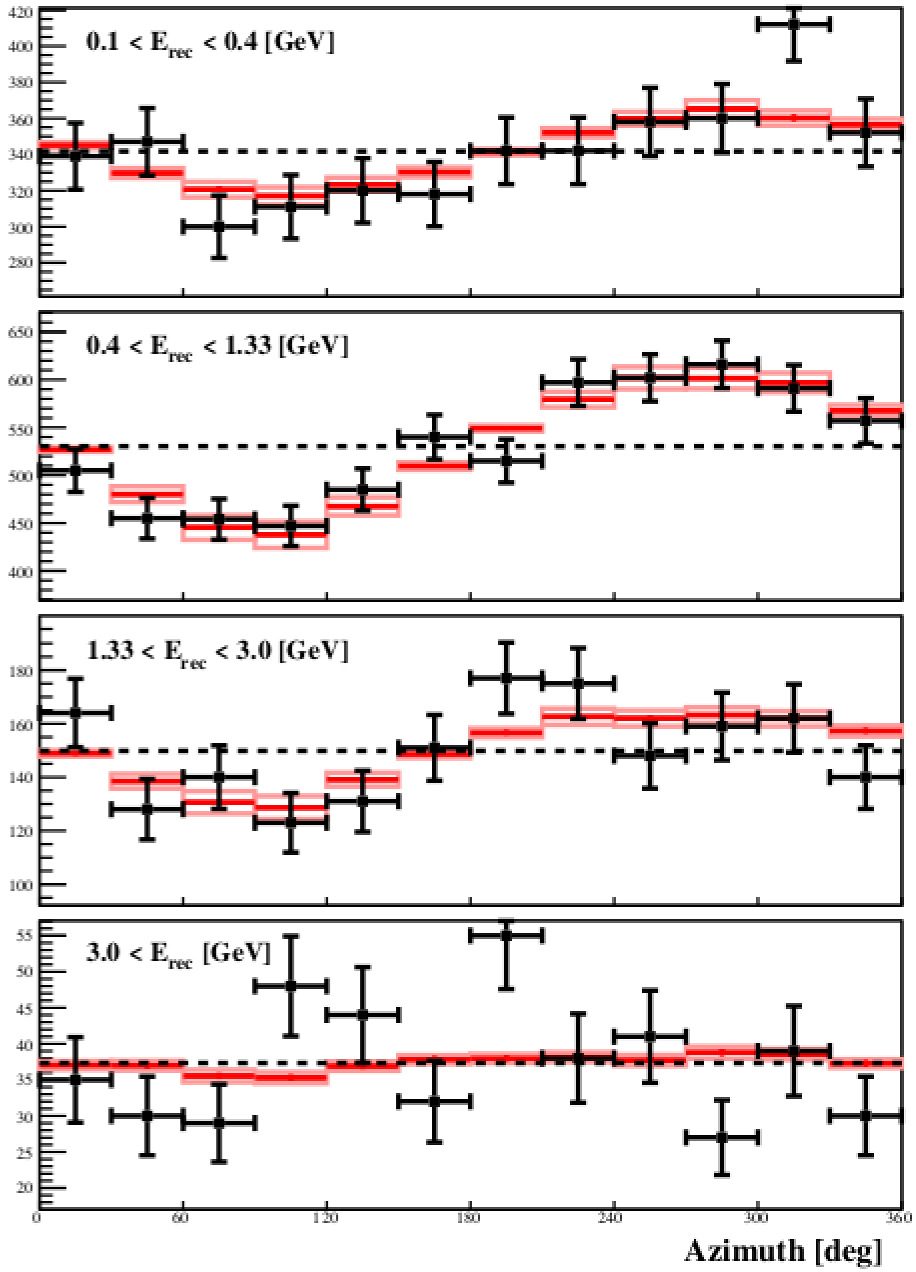
$$\phi_i^{min}(E_k) \equiv \phi_i^{1997}(E_k)$$

$M(N, r)$: modulation function

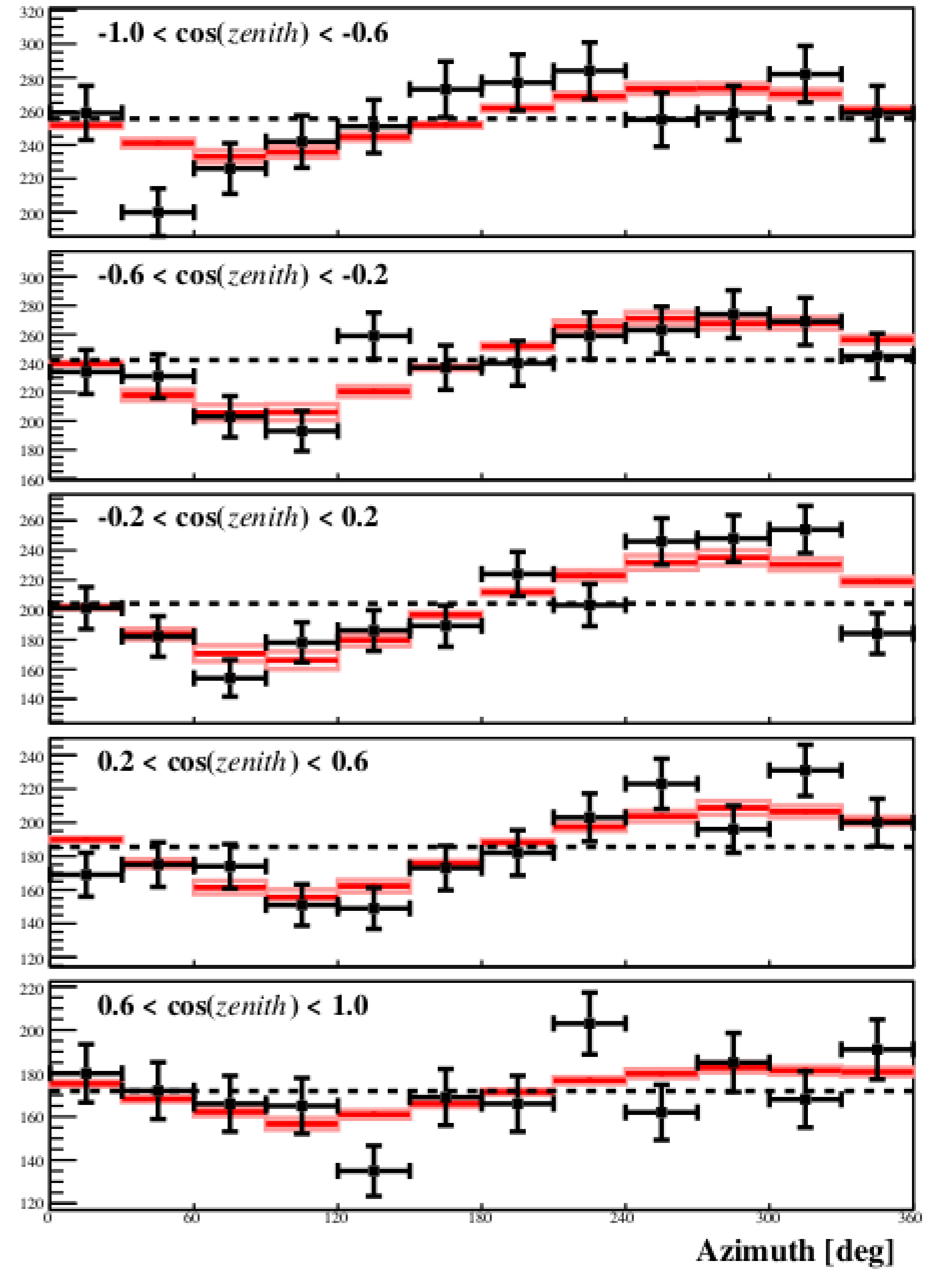




Observed Azimuthal Variation of ν_e flux (from PHD thesis of E.Richard)

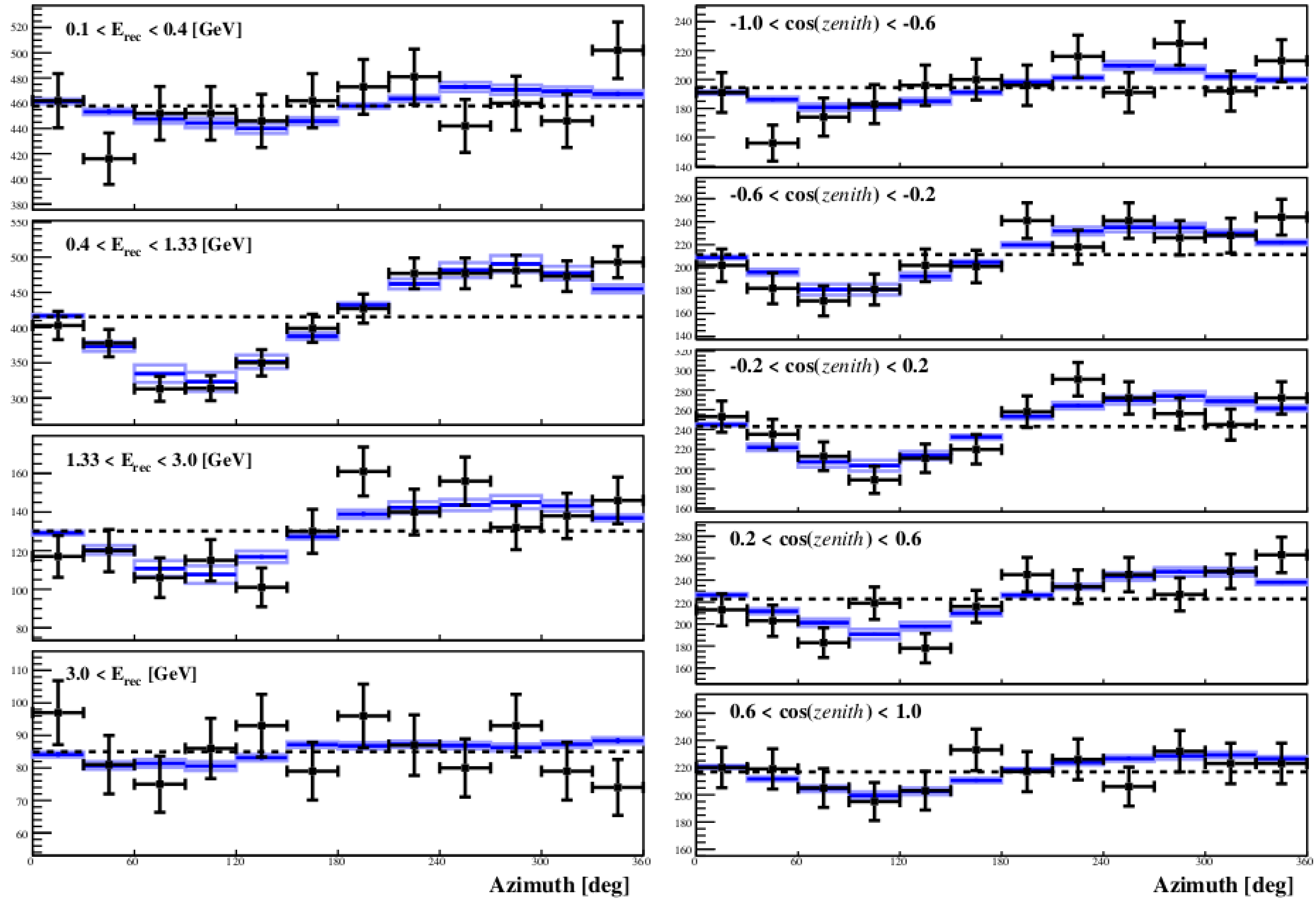


Energy Binned All Azimuth angles



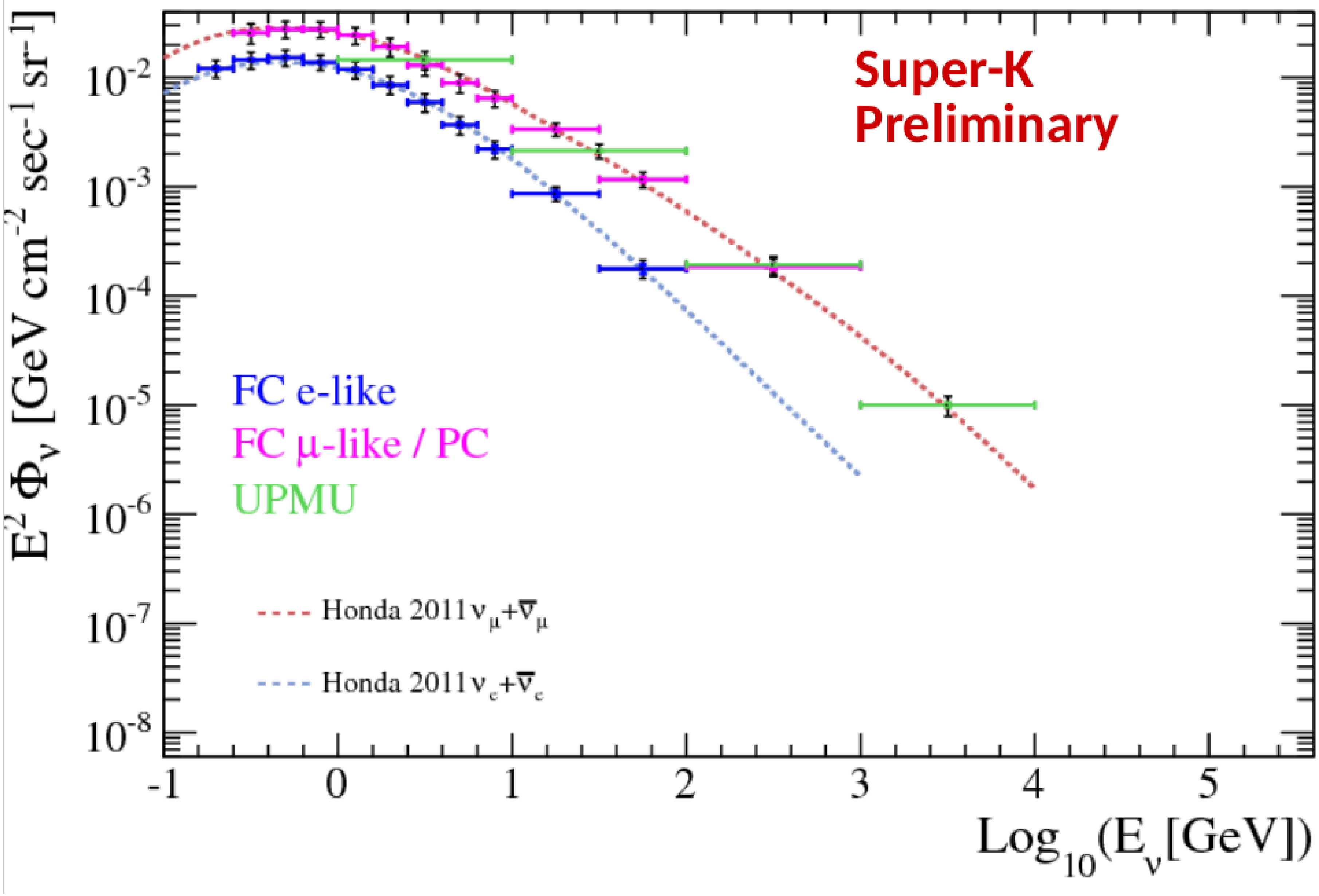
Zenith Angle Binned All Energies

Observed Azimuthal Variation of ν_μ flux (from PHD thesis of E.Richard)



Energy Binned All Azimuth angles

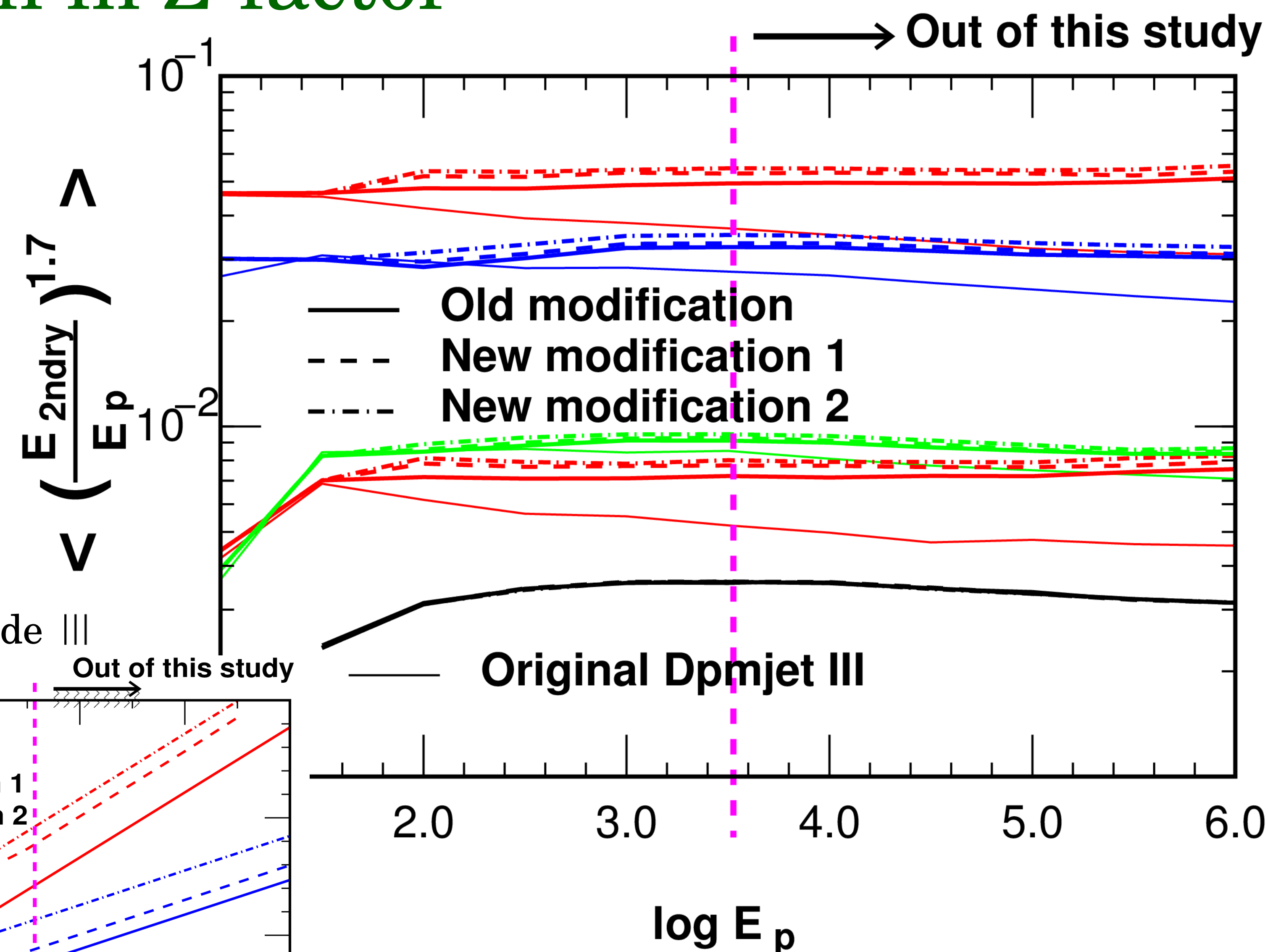
Zenith Angle Binned All Energies



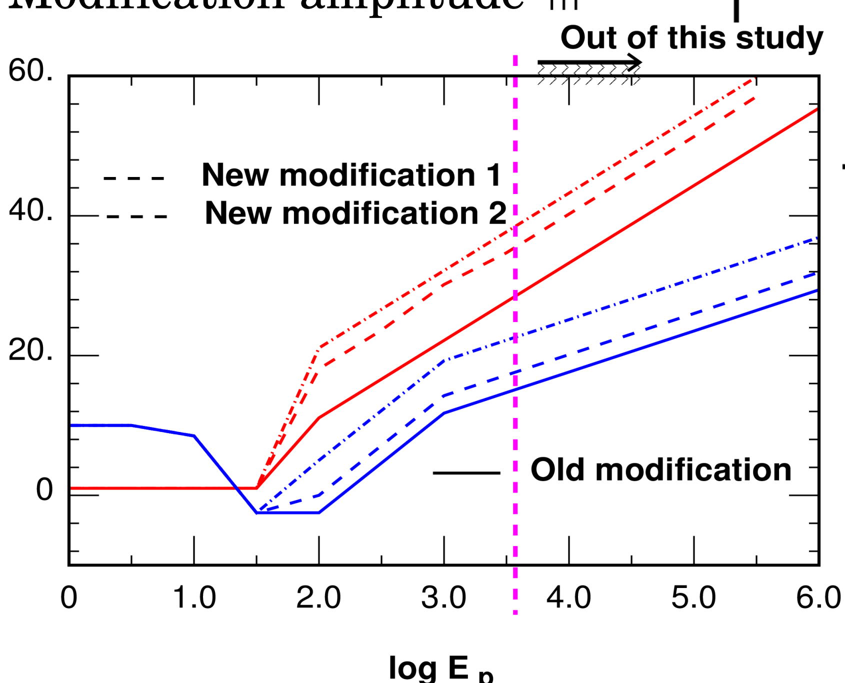
From K.Okumura in ICRC2015

Back up

Comparison in Z-factor



Modification amplitude |||



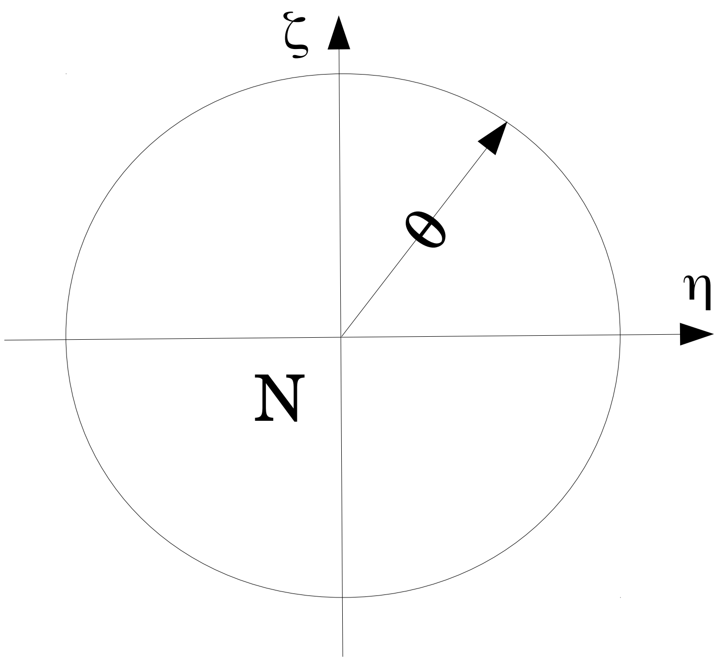
Assume the atmospheric neutrino flux is expanded as

$$\varphi(\zeta,\eta)=\varphi(0,0)+\frac{\partial\varphi}{\partial\zeta}\zeta+\frac{\partial\varphi}{\partial\eta}\eta+\frac{1}{2}\frac{\partial^2\varphi}{\partial^2\zeta}\zeta^2+\frac{\partial^2\varphi}{\partial\eta\partial\zeta}\zeta\eta+\frac{1}{2}\frac{\partial^2\varphi}{\partial^2\eta}\eta^2+....$$

Average in a virtual detector with radius θ is given as

$$\varphi_\theta\equiv\frac{1}{\pi\theta^2}\int_{\sqrt{\eta^2+\zeta^2}<\theta}\phi(\eta,\zeta)d\eta d\zeta=\frac{1}{\pi\theta^2}\int_{-\theta}^{+\theta}\int_{-\sqrt{\theta^2-\eta^2}}^{+\sqrt{\theta^2-\eta^2}}\phi(\eta,\zeta')d\zeta' d\eta$$

$$=\frac{1}{\pi\theta^2}\int_{-\theta}^{+\theta}\int_{-\sqrt{\theta^2-\zeta^2}}^{+\sqrt{\theta^2-\zeta^2}}\phi(\eta',\zeta)d\eta' d\zeta$$



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$$\varphi(\zeta,\eta)=\varphi(0,0)+\cancel{\frac{\partial\varphi}{\partial\zeta}}\zeta+\cancel{\frac{\partial\varphi}{\partial\eta}}\eta+\frac{1}{2}\frac{\partial^2\varphi}{\partial^2\zeta}\zeta^2+\cancel{\frac{\partial^2\varphi}{\partial\eta\partial\zeta}}\zeta\eta+\frac{1}{2}\frac{\partial^2\varphi}{\partial^2\eta}\eta^2+....$$

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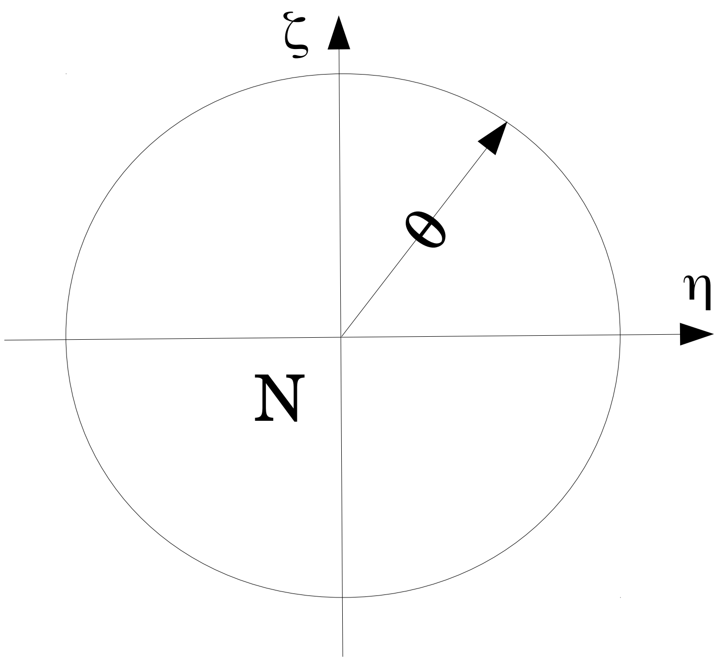
$$\varphi_\theta\equiv\frac{1}{\pi\theta^2}\int_{\sqrt{\eta^2+\zeta^2}<\theta}\phi(\eta,\zeta)\,d\eta\,d\zeta=\frac{1}{\pi\theta^2}\int_{-\theta}^{+\theta}\int_{-\sqrt{\theta^2-\eta^2}}^{+\sqrt{\theta^2-\eta^2}}\phi(\eta,\zeta')\,d\zeta'\,d\eta$$

$$=\frac{1}{\pi\theta^2}\int_{-\theta}^{+\theta}\int_{-\sqrt{\theta^2-\zeta^2}}^{+\sqrt{\theta^2-\zeta^2}}\phi(\eta',\zeta)\,d\eta'\,d\zeta$$



$$\int_{\sqrt{\eta^2+\zeta^2}<\theta}\eta\,d\eta\,d\zeta=\int_{\sqrt{\eta^2+\zeta^2}<\theta}\zeta\,d\eta\,d\zeta=0$$

$$\int_{\sqrt{\eta^2+\zeta^2}<\theta}\eta\zeta\,d\eta\,d\zeta=0$$

(continued)

$$\int_{\sqrt{\eta^2+\zeta^2}<\theta} \eta^2 d\eta d\zeta = \int_{\sqrt{\eta^2+\zeta^2}<\theta} \zeta^2 d\eta d\zeta = \int_{-\theta}^{+\theta} \int_{-\sqrt{\theta^2-\zeta^2}}^{+\sqrt{\theta^2-\zeta^2}} \eta'^2 d\eta' d\zeta$$

$$= \frac{2}{3} \int_{-\theta}^{+\theta} \sqrt{\theta^2-\zeta^2}^3 d\zeta$$

$$= \frac{2}{3} \theta^4 \int_{-1}^{+1} \sqrt{1-t^2}^3 dt$$

$$= \frac{1}{4} \pi \theta^4$$

Then we get

$$\varphi_\theta \equiv \frac{1}{\pi \theta^2} \int_{\sqrt{\eta^2+\zeta^2}<\theta} \phi(\eta, \zeta) d\eta d\zeta = \phi(0,0) + \frac{1}{8} \left(\frac{\partial^2 \varphi}{\partial^2 \zeta} + \frac{\partial^2 \varphi}{\partial^2 \eta} \right) \theta^2 + \ldots.$$

Note, the factor before θ^2 would be a little different, due to the Jacobian for the integration on a sphere.

Cosmic rays in atmosphere

$$p_{CR} + [Air] \rightarrow \begin{pmatrix} n^{\pm} \cdot \pi^{\pm} \\ m \cdot \pi^{0} \end{pmatrix} + X(p, n, K, ...)$$

$$\pi^{0} \rightarrow 2\gamma$$

$$\pi^{\pm} \rightarrow \mu^{\pm} + \nu_{\mu}(\bar{\nu}_{\mu})$$

$$\mu^{\pm} \rightarrow \nu_{e}(\bar{\nu}_{e}) + \bar{\nu}_{\mu}(\nu_{\mu}) + e^{\pm}$$

Atmospheric Neutrino

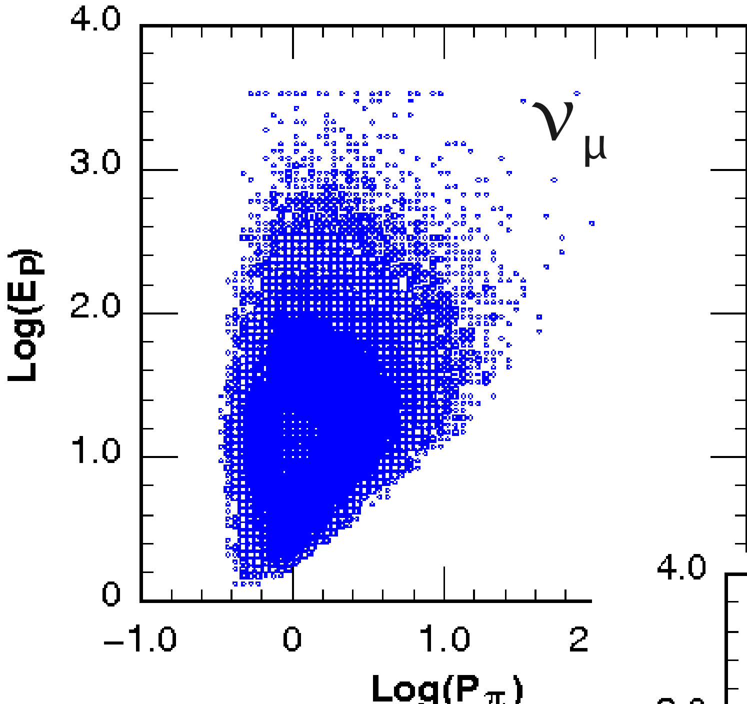
$$\nu_{\mu} : \nu_{e} \approx 2:1$$

$\gamma, e^{\pm} \rightarrow$ EM-cascade → Air Shower

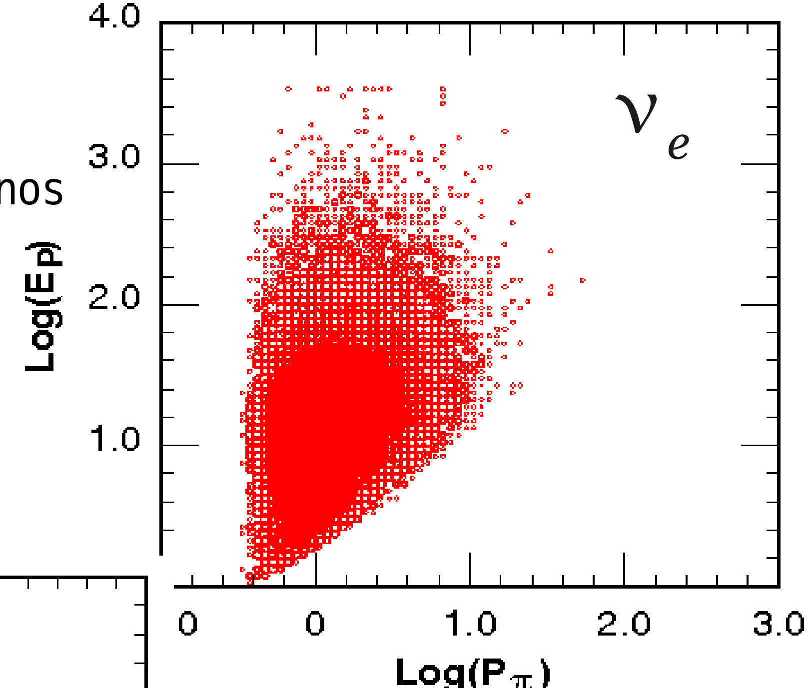
Other p's, n's, and sometimes π's repeat above interactions.

Analysis of calculation error:

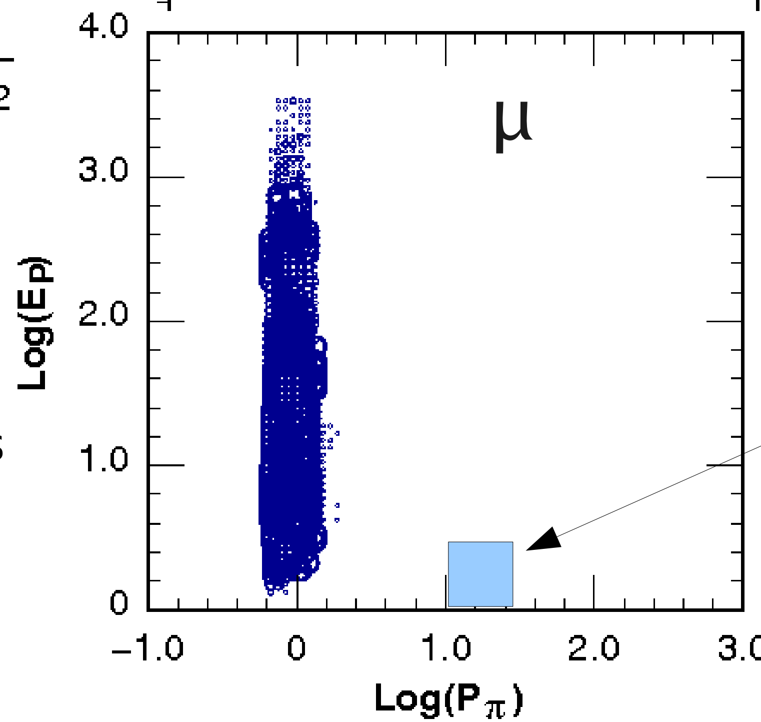
Give Variations in the phase space and compare the variation of neutrino flux and the Maximum variation of muon flux in 0.5 ~ 2 GeV/c (μ+) and 0.5 ~ 4 GeV/c (μ-), where BESS Balloon observation was available.



Phase space for 0.32 GeV neutrinos

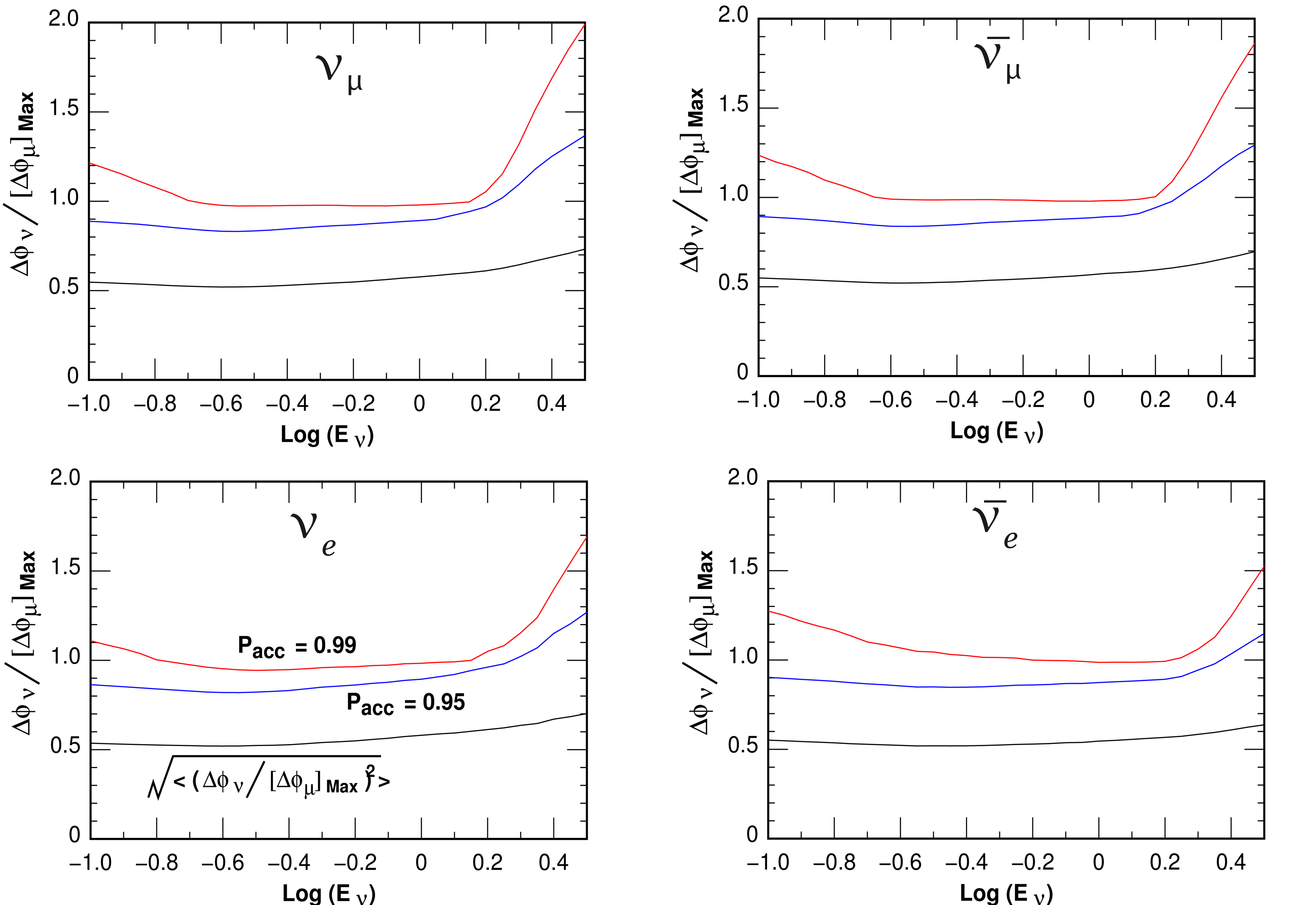


Phase space for corresponding muons

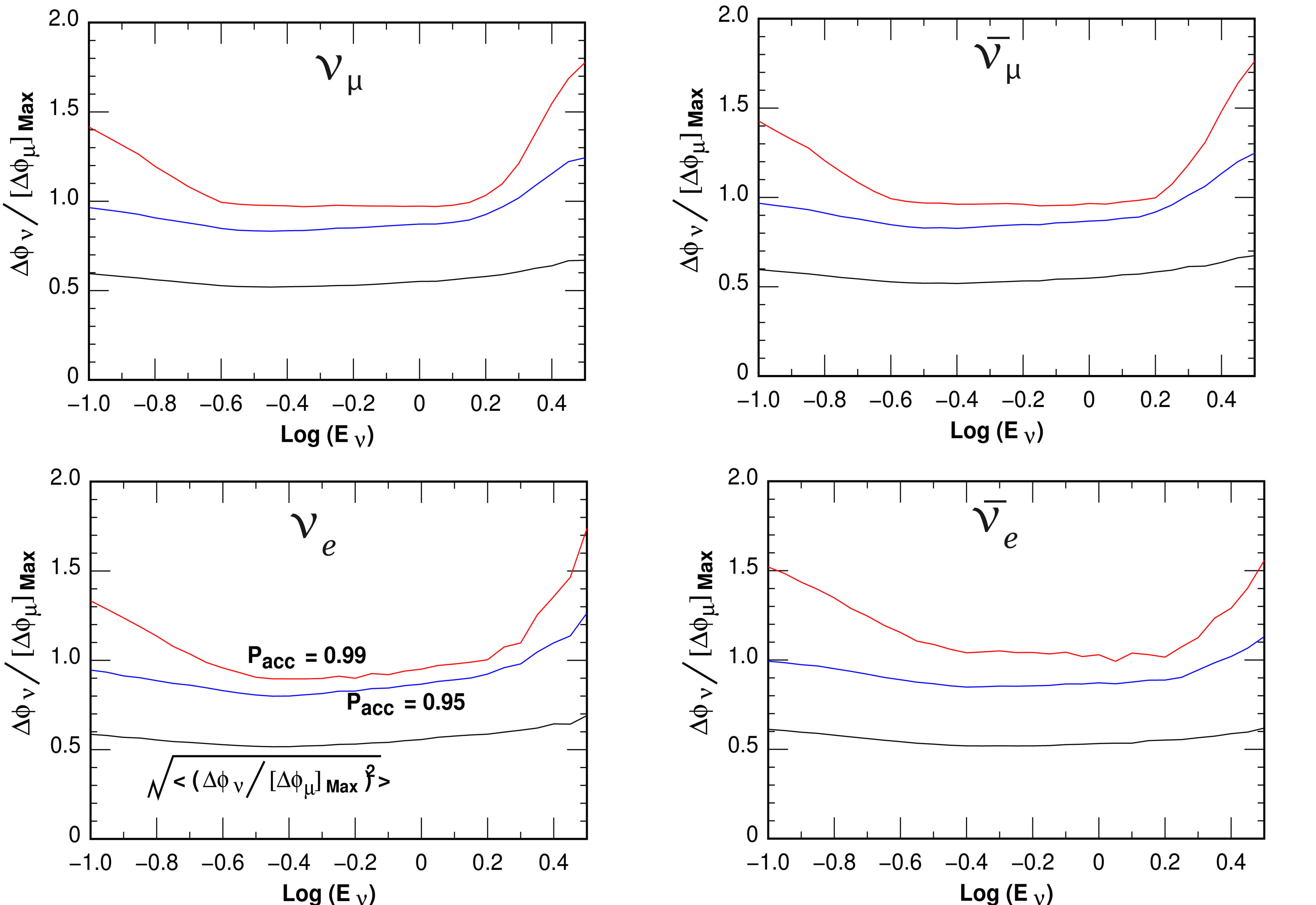


Size of Phase space to give the variation

Vertical neutrino flux

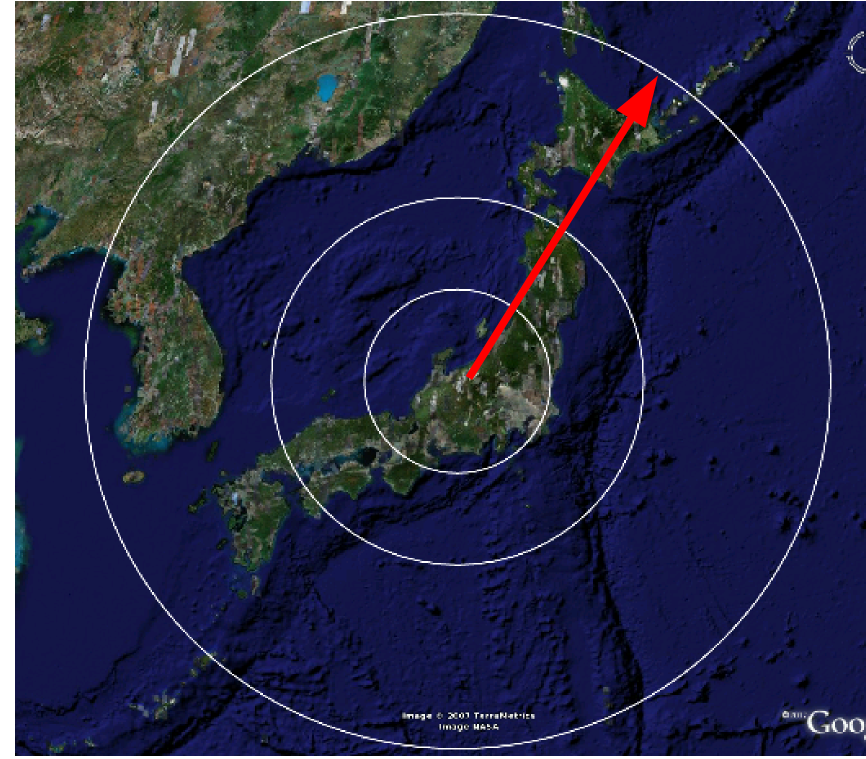


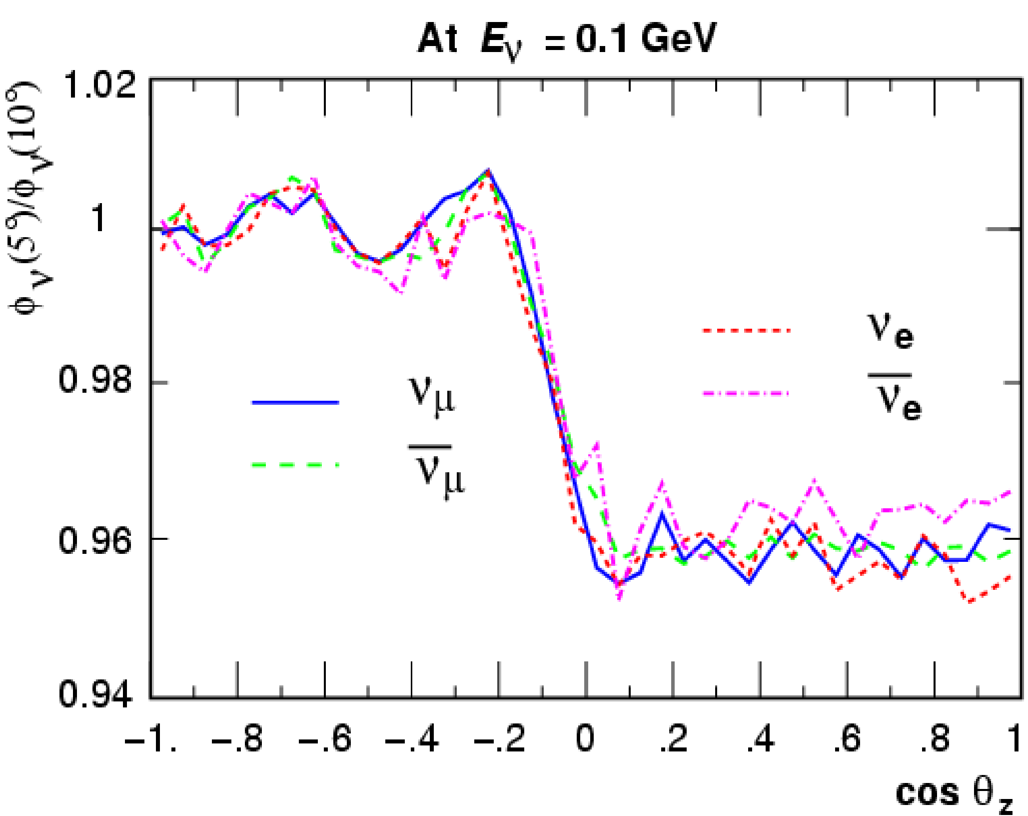
Horizontal neutrino flux

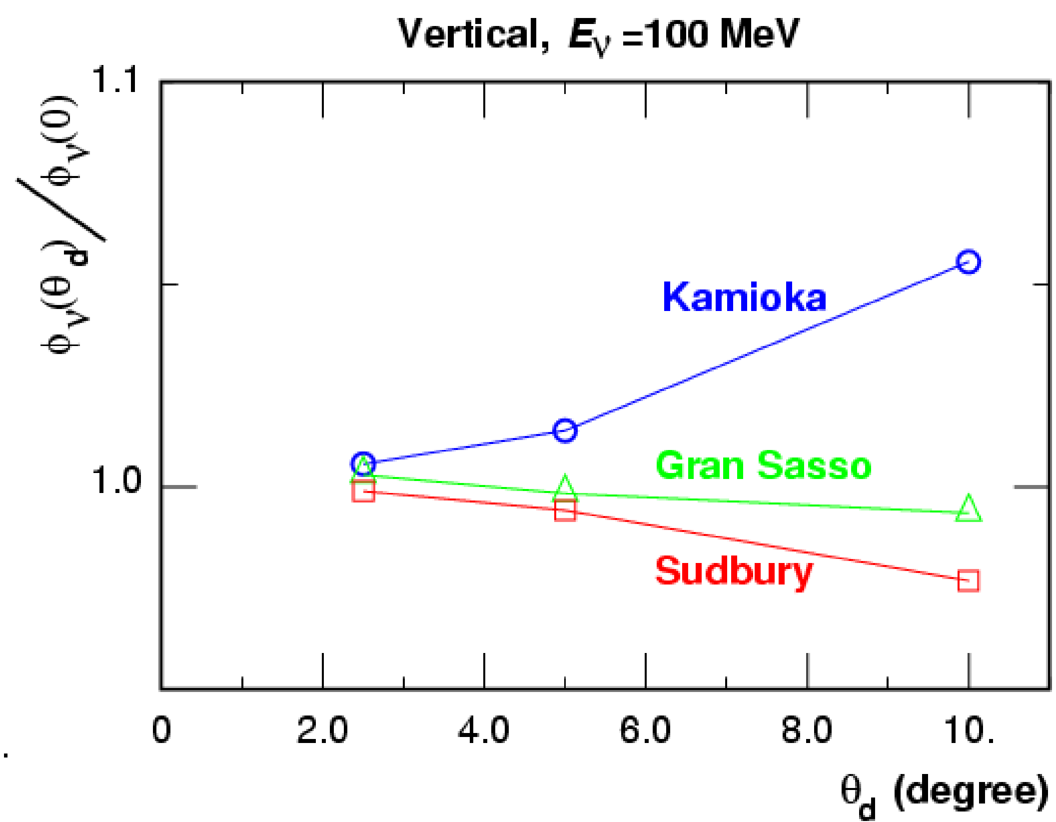


Example in HKKM06 (PRD 2007) with

$$\phi_\nu(0) \simeq -\frac{1}{3}\phi_\nu(10) + \frac{4}{3}\phi_\nu(5)$$

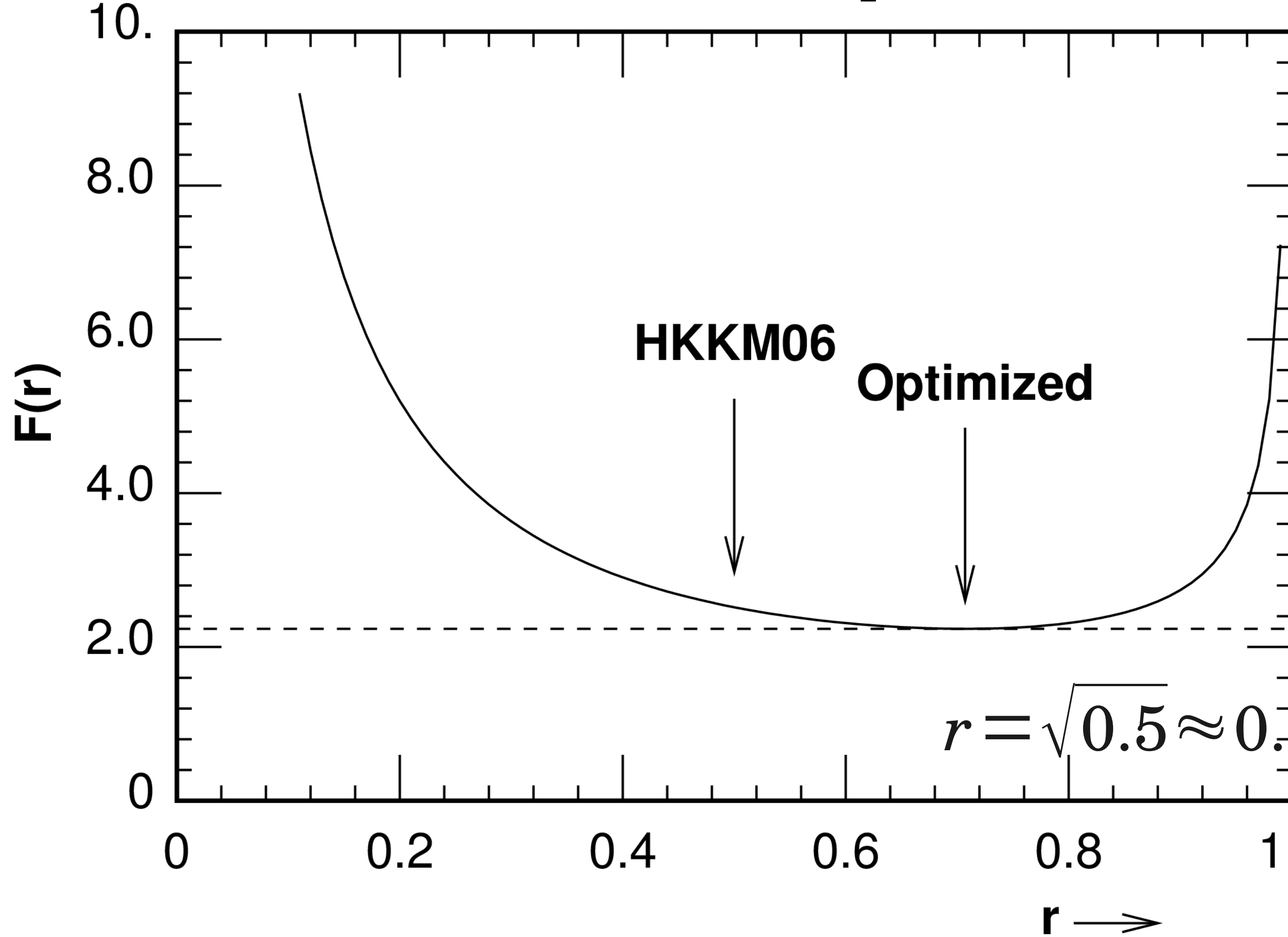






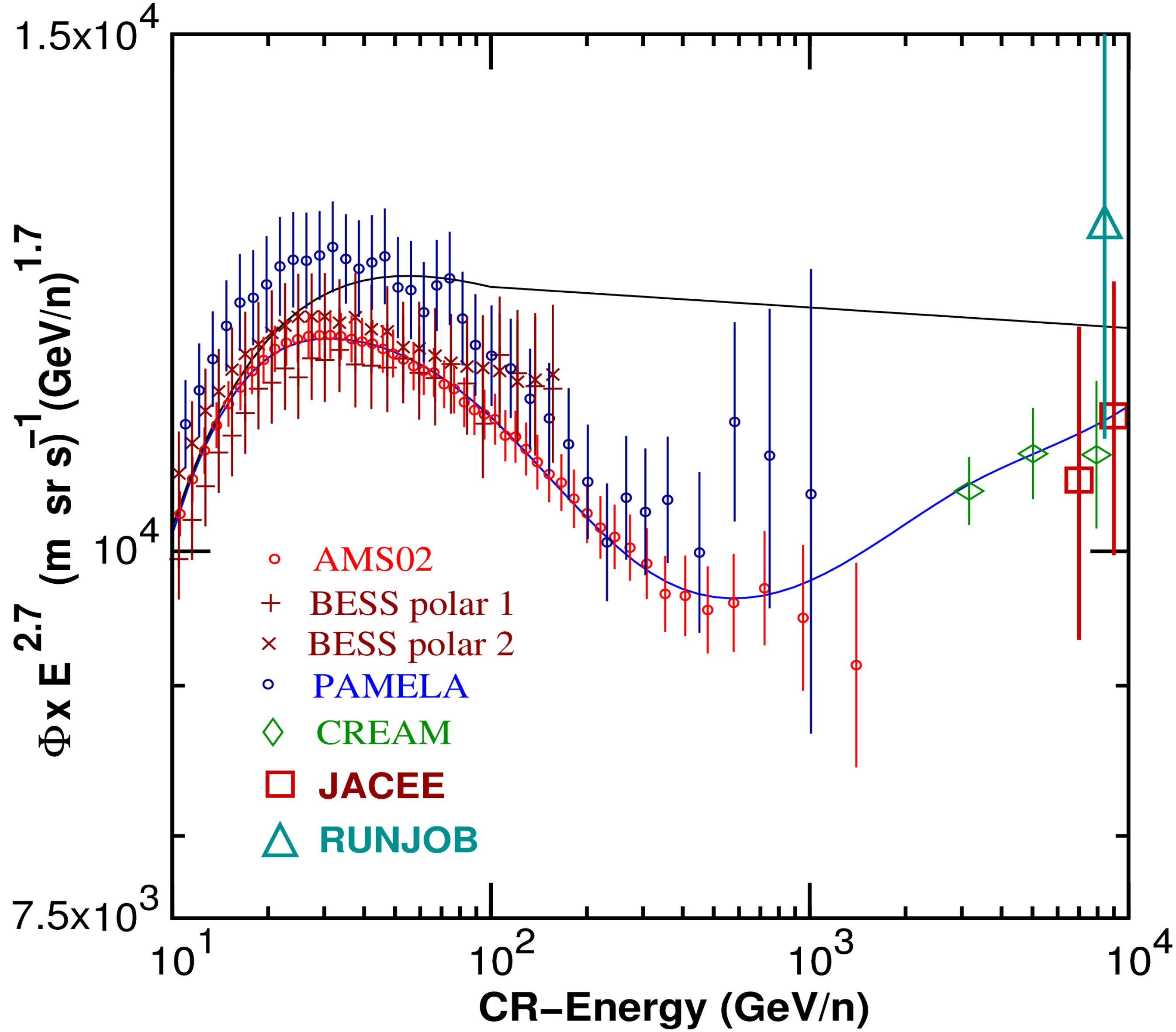
Optimization of $r = \left(\frac{\theta_2}{\theta_1}\right)$ to minimize the statistical error.

$$\frac{\Delta \varphi_0}{\varphi_0} = F(r) \cdot \frac{\Delta \phi_1}{\phi_1}$$

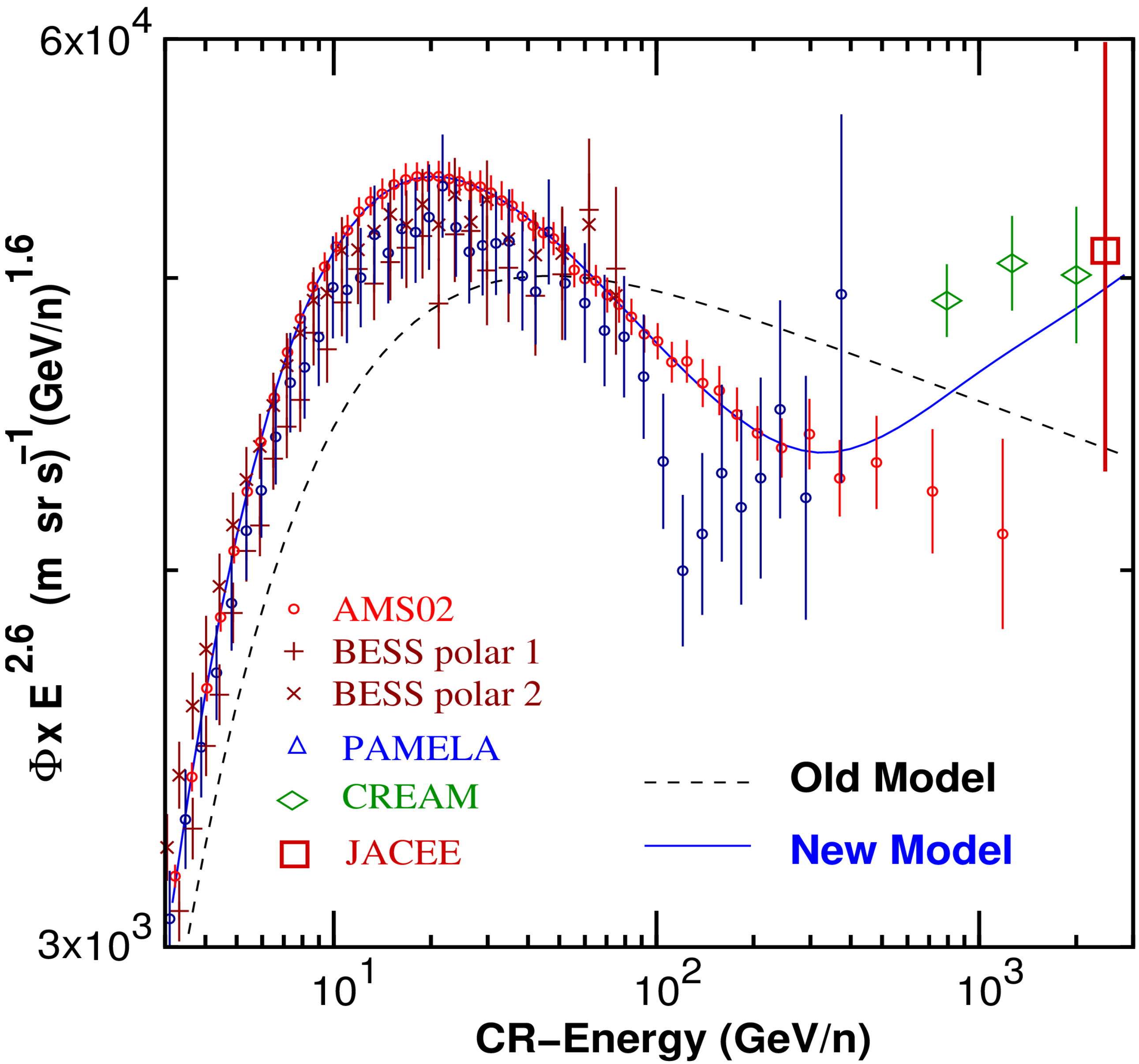


$r = \sqrt{0.5} \approx 0.7071$

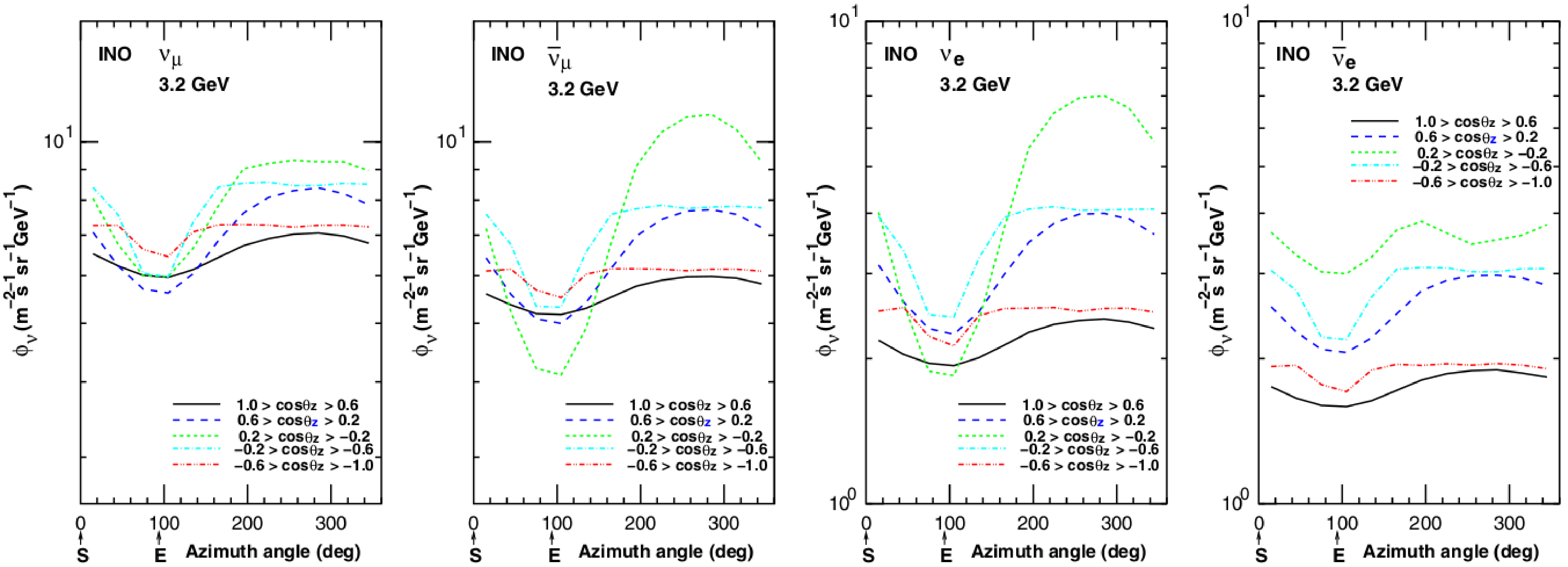
Proton closeup



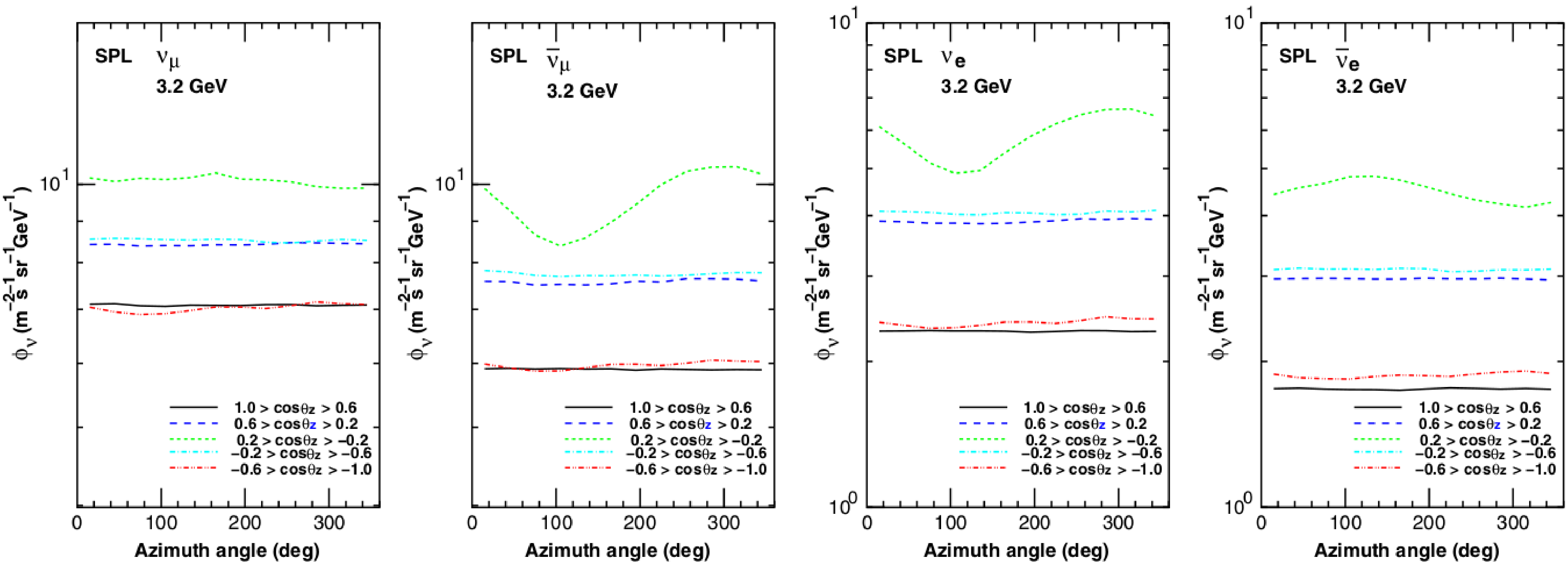
Helium closeup



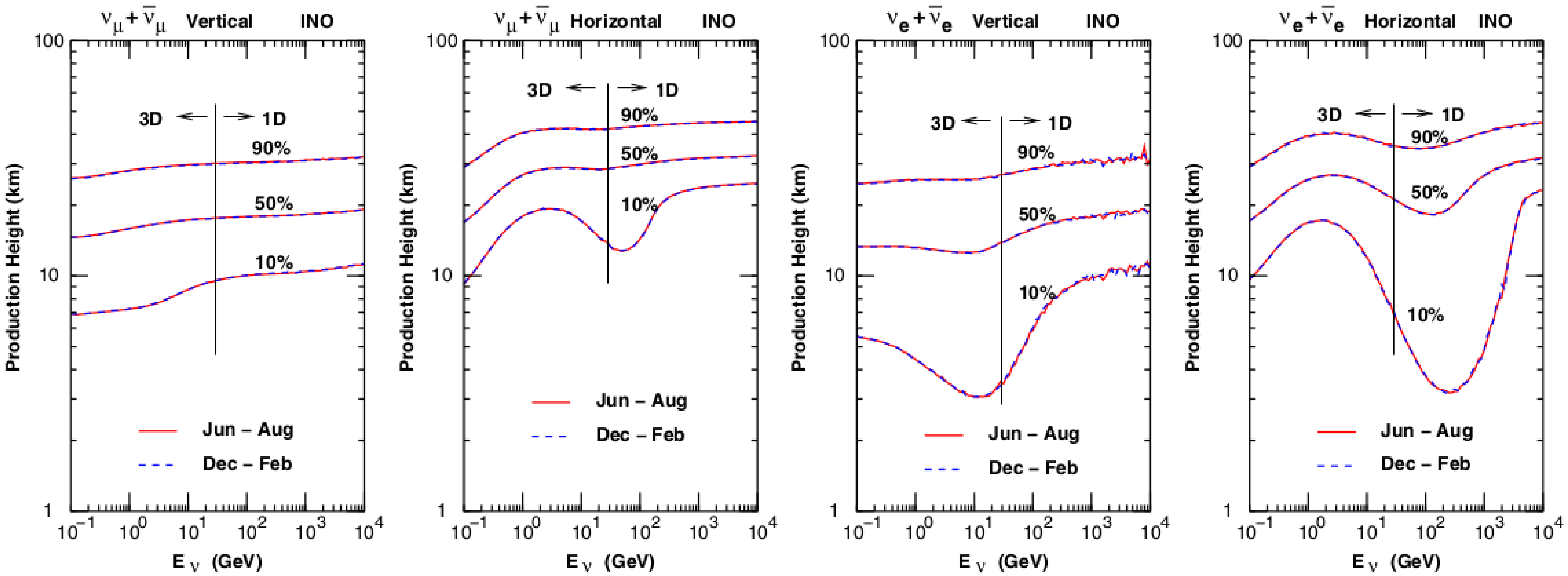
Azimuth Angle Variation of Neutrino Fluxes at 3.2 GeV at INO site



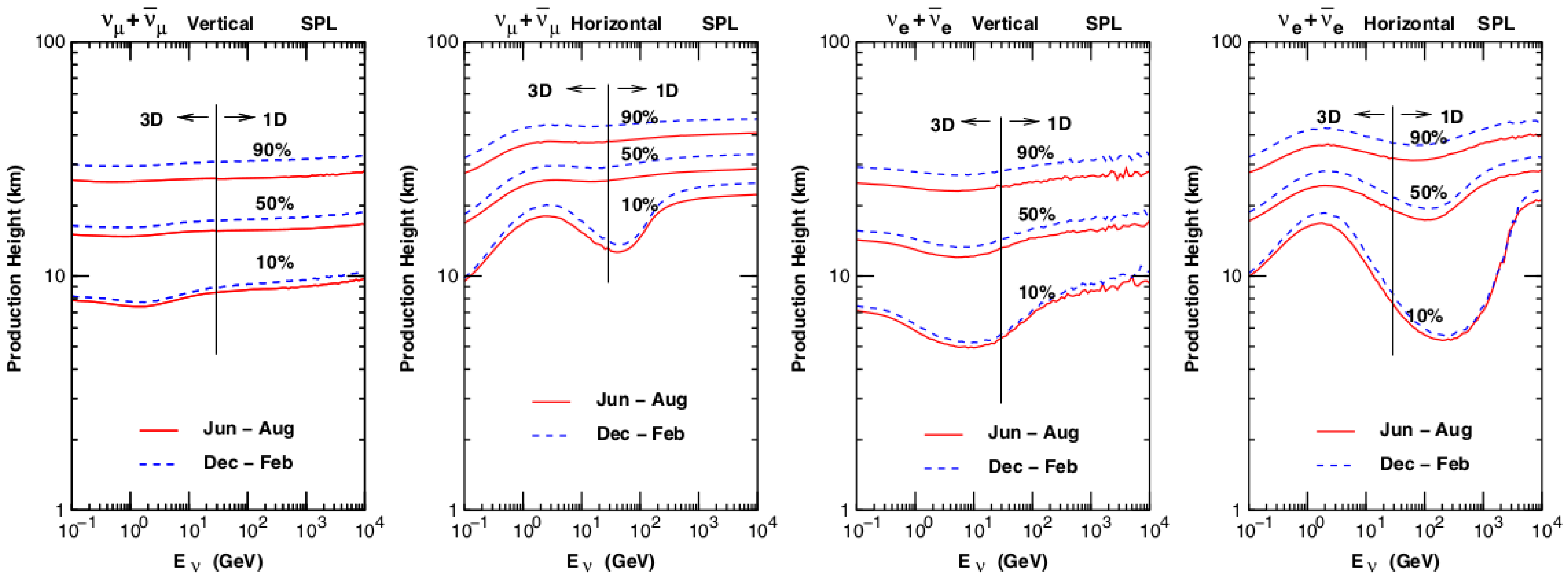
Azimuth Angle Variation of Neutrino Fluxes at 3.2 GeV at Suth Pole



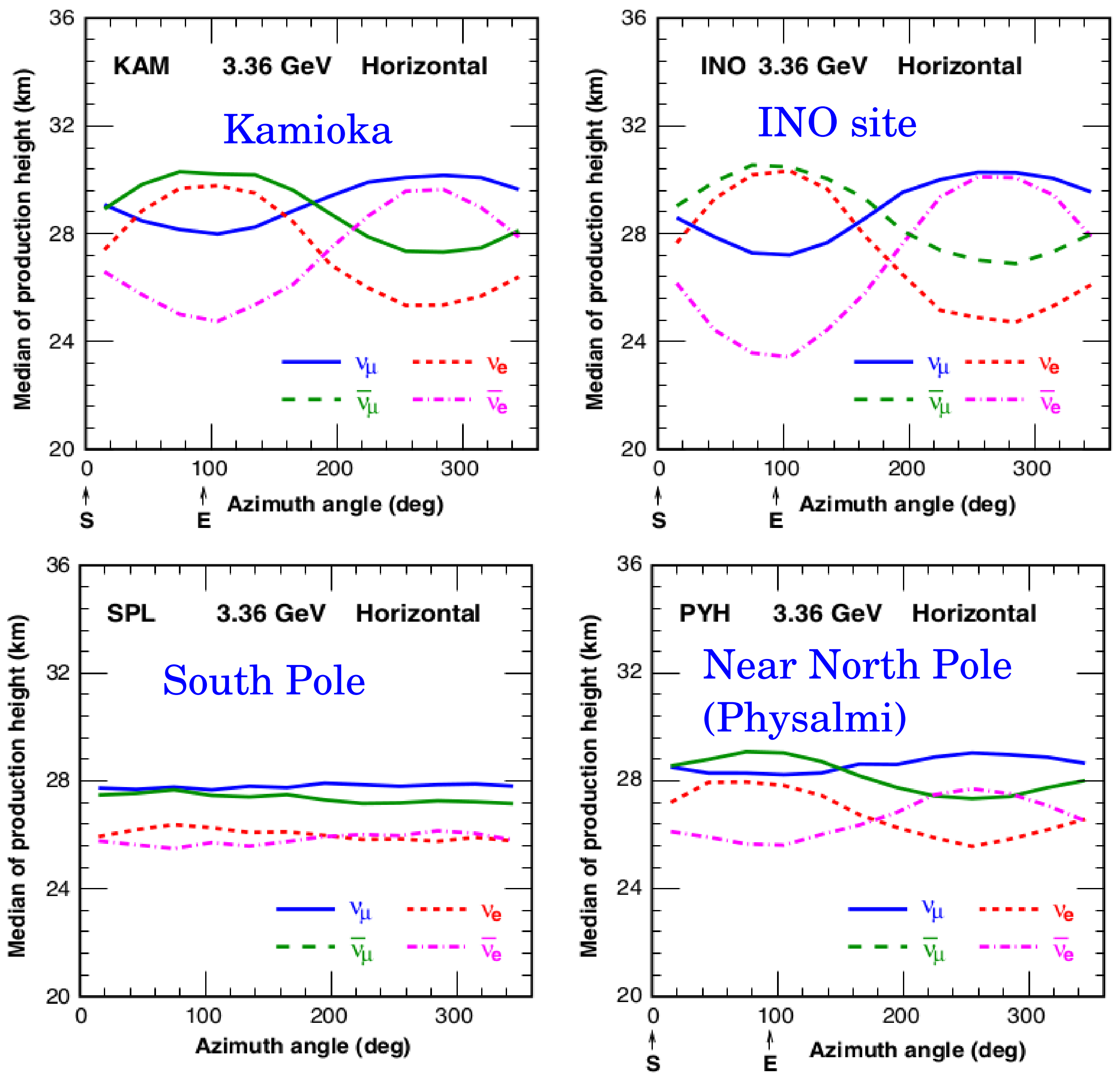
Cumulative Neutrino Production Height at INO site (Summed over all azimuth angles)



Cumulative Neutrino Production Height at South Pole (Summed over all azimuth angles)

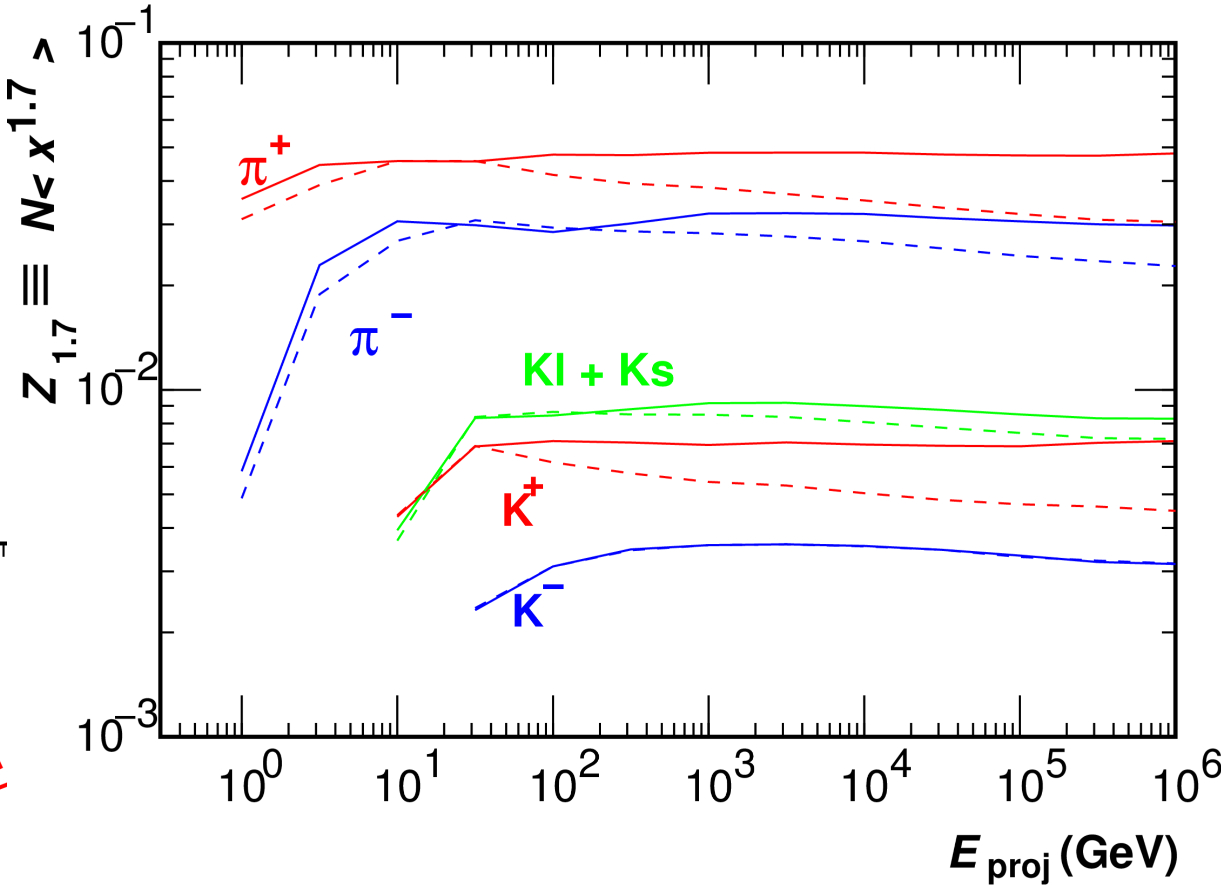


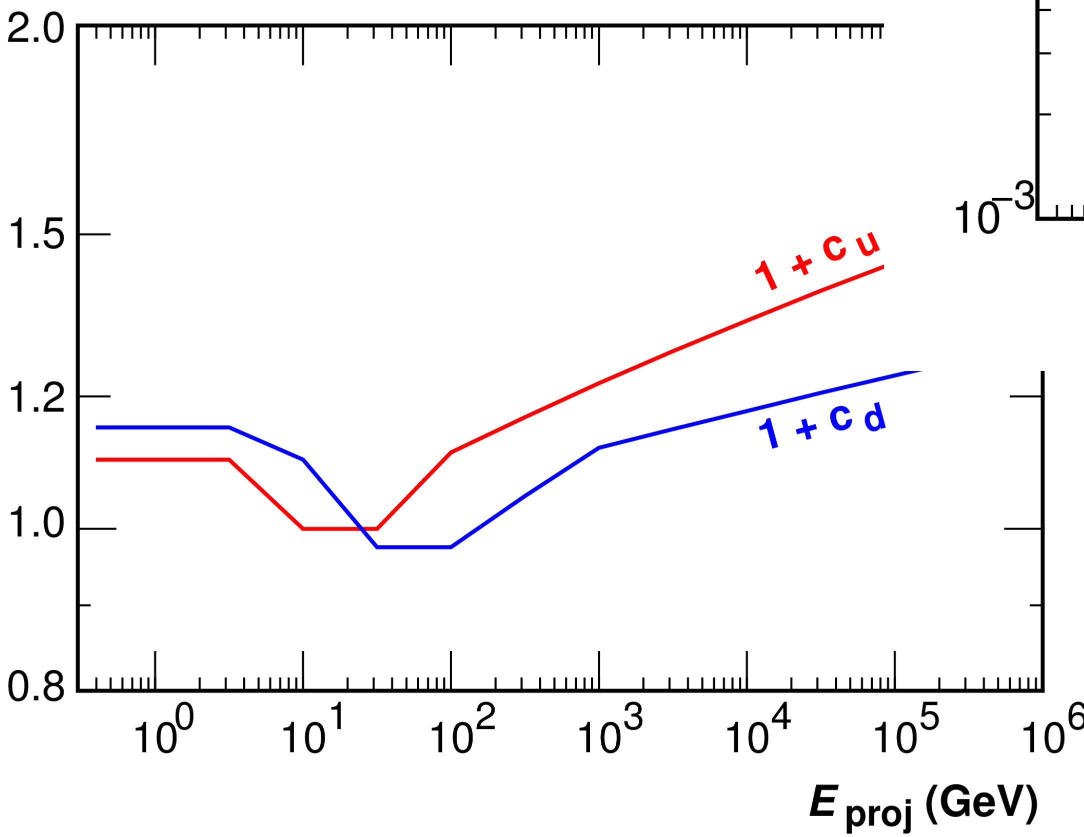
Azimuth Angle Variaiton of Neutrino Production Height



Amplitude of Modification (SHKKM 2006)

Z-Factor →





<--- Modification parameter in code