D-modules and Hodge theory

Abstracts

Yalong Cao: Donaldson-Thomas theory for Calabi-Yau 4-folds

Donaldson-Thomas theory on Calabi-Yau 3-folds is a complexification of Chern-Simons-Floer theory on 3-manifolds. In this talk, we will discuss a complexification of Donaldson theory on complex oriented 4-folds (i.e. CY4). This is based on my PhD thesis under supervision of Naichung Conan Leung.

Andrea D'Agnolo: On the Riemann-Hilbert correspondence for irregular holonomic D-modules I, II (2 talks)

The problem of describing irregular ordinary differential equations in geometrical terms has been standing for a long time. In a joint work with Masaki Kashiwara, we proved a Riemann-Hilbert correspondence for holonomic D-modules which are not necessarily regular. The construction of our target category is based on the theory of ind-sheaves by Kashiwara-Schapira and is influenced by Tamarkin's work on symplectic topology. Among the main ingredients of our proof is the description of the structure of flat meromorphic connections due to Mochizuki and Kedlaya. In these talks, I will present the irregular Riemann-Hilbert correspondence and show how Stokes phenomena get decribed in a topological way.

Hélène Esnault: On a question of Deligne on ramification

(joint work in progress with Lars Kindler and V. Srinivas)
Deligne (end of February 2016) asked me 2 questions:
Let X be smooth of finite type over $k = \bar{k}$ of char. $p > 0$. Fix (r, D) D Cartier effective with support $\bar{X} \setminus X$, $\bar{X}$ normal compactification.
1) Is there a curve $C \to X$, such that any irreducible $\bar{Q}_\ell$ lisse sheaf V of rank r and ramification bounded by D, $V|_C$ is irreducible?
2) Does there is a R such that for any V as before, $f : Y \to X$ finite étale surjective such that f^*V has tame ramification, the fierce ramification of f at the codimension 1 points at infinity is bounded by R?
We can answer (hopefully) Question 2, not (yet?) Question 1.

Mikhail Kapranov: Perverse schobers and topological Fukaya categories with coefficients

Perverse schobers are conjectural categorical generalizations of perverse sheaves, in which vector spaces are replaced by triangulated categories. We give a precise definition of perverse schobers on a Riemann surface as well as the analog of the complex (category in our case) of derived global sections. This is given using the approach of Fukaya categories. In particular, our construction includes the interpretation of the Fukaya-Seidel categories as coming from appropriate perverse schobers. Joint with with T. Dyckerhoff, V. Schechtman, Y. Soibelman.

Kiran Kedlaya: (2 talks)
1. Resolution of turning points in zero and mixed characteristic

We describe the close analogy between two results. One of these is the fact that an irregular meromorphic connection on a complex manifold admits a good formal structure after a suitable (local) blowup of the ambient space. The other is that an overconvergent F-isocrystal on a smooth variety over a perfect field of characteristic p admits a good formal structure after a suitable alteration (blowup plus finite cover) of the ambient space.

2. Nonarchimedean differential equations and turning points

We give an overview of how the close study of differential equations over nonarchimedean fields can be used to prove the two theorems described in the previous lecture.

Toshiro Kuwabara: Deformation-quantization of jet bundles over symplectic manifolds and affine W-algebras

We discuss deformation-quantization of infinite-dimensional vector bundles, called jet bundles, over certain symplectic manifolds. While the jet bundles equipped with a structure of sheaves of vertex Poisson algebra, their deformation-quantization has a structure of sheaves of (h-adic) vertex algebras. We discuss a construction of such sheaves given by a certain variation of (quantum) Hamiltonian reduction, which is analogous to quantum Drinfeld-Sokolov reduction, (semi-infinite) Hamiltonian reduction giving affine W-algebras. We also discuss Zhu algebras of the vertex algebra of global sections of such sheaves, associative algebras corresponding to vertex algebras.

Tatsuki Kuwagaki: On the coherent-constructible correspondence

Kapstin-Witten observed that the connection between A-brane categories and the category of $\mathcal{D}$-modules in their study of geometric Langlands program. Their observation and Riemann-Hilbert correspondence together imply the connection between Fukaya categories and constructible sheaves. In this talk, I'll give an introduction to the coherent-constructible correspondence which is homological mirror symmetry for toric varieties using constructible sheaves as A-branes instead of Fukaya categories.

Kevin McGerty: Kirwan surjectivity for quiver varieties

A classical result of Kirwan proves that cohomology ring of a quotient stack surjects onto the cohomology of an associated GIT quotient via the natural restriction map. In many cases the cohomology of the quotient stack is easy to compute so this often yields, for example, generators for the cohomology ring of the GIT quotient. In the symplectic case, it is natural to ask whether a similar result holds for (algebraic) symplectic quotients. Although this surjectivity is thought to fail in general, it is expected to hold in many cases of interest. In recent work with Tom Nevins (UIUC) we establish this surjectivity result for Nakajima's quiver varieties. An important role is played by a new compactification of quiver varieties which arises from the study of graded representations of the preprojective algebra.

Takuro Mochizuki: Mixed twistor D-modules and some examples I, II (2 talks)

Mixed twistor $\mathcal{D}$-modules are roughly holonomic $\mathcal{D}$-modules with mixed twistor structure. Because of the functoriality, we can observe that many interesting holonomic $\mathcal{D}$-modules are naturally enhanced to mixed twistor $\mathcal{D}$-modules.

In this talk, we shall give a brief review of the theory of mixed twistor $\mathcal{D}$-modules, and discuss some examples.

Luis Narvaez-Macarro: Around the symmetry of the roots of Bernstein-Sato polynomials

The symmetry of the roots of the Bernstein-Sato polynomials of quasi-homogeneous isolated singularities is strongly related with the symmetry of the Hodge spectrum of the singularity, but for other singularities this relationship is unclear. However, symmetry properties of the roots of Bernstein-Sato polynomials occur for many other examples of non-isolated singularities. In the case of free divisors of linear jacobian type (e.g. free hyperplane arrangements), this symmetry is well understood by means of a convenient application of the duality in $\mathcal{D}$-module theory. In this talk we will review on these results and we will present some open questions.

Yoshiki Oshima: Tropical geometric compactifications and Satake compactifications

Odaka arXiv:1406.7772 studied compactifications of moduli varieties for Riemann surfaces and Abelian varieties by using the Gromov-Hausdorff collapse, which he called tropical geometric compactifications. For Abelian varieties, this is isomorphic to one of Satake compactifications of the Siegel modular variety. We propose a similar picture for the moduli of K3 surfaces. Joint work with Yuji Odaka.

Takahiro Saito: Limit mixed Hodge structures of families of algebraic varieties and their applications

We study the monodromies and the limit mixed Hodge structures of families of complete intersection varieties over a punctured disk in the complex plane. For this purpose, we express their motivic nearby fibers in terms of the geometric data of some Newton polyhedra. In particular, the limit mixed Hodge numbers and some part of the Jordan normal forms of the monodromies of such a family will be described very explicitly. This is a joint work with Kiyoshi Takeuchi.

Jörg Schürmann: Characteristic classes of mixed Hodge modules

We give an introduction to the theory of characteristic classes of mixed Hodge modules, which capture information about the graded pieces of the filtered de Rham complex associated to the filtered $\mathcal{D}$-module underlying a mixed Hodge module. For global hypersurfaces, we focus on a corresponding specialization as well as Thom-Sebastiani result.
This is joint work with S. Cappell, L. Maxim, J. Shaneson and M. Saito.

Yota Shamoto: An analog of the Dubrovin conjecture

B. Dubrovin conjectured the equivalence between the semi-simplicity of the quantum $\mathcal{D}$-module of a Fano manifold and the existence of full exceptional collection in the derived category of coherent sheaves on it. He also conjectured the Stokes matrix of the semi-simple quantum D-module can be described by the Euler pairings of the full exceptional collection. Recently, the latter statement is refined as "Gamma conjecture" by Galkin-Golyshev-Iritani. In this talk, I will speak about an analog of the Dubrovin conjecture for the case that the quantum $\mathcal{D}$-module is not necessarily semi-simple. This is a joint work with F. Sanda.

Jean-Baptiste Teyssier: Skeletons and moduli of Stokes torsors

In the local classification of differential equations of one complex variable, a certain sheaf of complex unipotent groups plays a central role. This is the Stokes sheaf.
On the other hand, Deligne defined in arithmetic a notion of skeleton for l-adic local systems, constructed a coarse moduli space for skeletons with bounded ramification and conjectured that every such skeleton comes from an actual l-adic local system.
In this talk, we will explain how to use a variant of Deligne's skeleton conjecture for torsors under the Stokes sheaf to prove the representability of the functor of relative Stokes torsors by an affine scheme of finite type. We will show how the geometry of this moduli can be used to prove new finiteness results on differential equations.