Simple Model of Cosmological Constant

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Old Cosmological Constant Problem

If the dark energy is the vacuum energy,
the quantum corrections from the matter diverge $\sim \Lambda_{\rm cutoff}^4$.

$$\rho_{\rm vacuum} = \frac{1}{(2\pi)^3} \int d^3k \frac{1}{2} \sqrt{k^2 + m^2} \sim \Lambda_{\rm cutoff}^4$$

$\Lambda_{\rm cutoff}$: cutoff scale
If the supersymmetry is restored in the high energy, the situation is not so changed.
If we use the counter term in order to obtain the very small vacuum energy $\left(10^{-3}\,\mathrm{eV}\right)^4$, we need very very fine-tuning and extremely unnatural.

Maybe we do not understand quantum gravity?

The above problems could be clues to understand the gravity.

Simple Topological Model

- S. Nojiri, "Some solutions for one of the cosmological constant problems," Mod. Phys. Lett. A **31** (2016) no.37, 1650213, [arXiv:1601.02203 [hep-th]].
- T. Mori, D. Nitta and S. Nojiri, "BRS structure of Simple Model of Cosmological Constant and Cosmology," arXiv:1702.07063 [hep-th].

$$S = \int d^4x \sqrt{-g} \left\{ \mathcal{L}_{\rm gravity} - \lambda + \partial_\mu \lambda \partial^\mu \varphi \right\} + S_{\rm matter} \, .$$

No parameter theory.

$\mathcal{L}_{\rm gravity} = \mathcal{L}^{(0)}_{\rm gravity} - \Lambda$, $\lambda \to \lambda - \Lambda \Rightarrow$

$$S = \int d^4x \sqrt{-g} \left\{ \mathcal{L}^{(0)}_{\rm gravity} - \lambda + \partial_\mu \lambda \partial^\mu \varphi \right\} + S_{\rm matter} \, .$$

Cosmological constant Λ does not affect the dynamics.
Cosmological constant may include the large quantum corrections from matters to the vacuum energy.

$\Rightarrow$ Large quantum corrections can be tuned to vanish.

The term $\partial_\mu \lambda \partial^\mu \varphi \Rightarrow$ this model may include a ghost.
Redefinition of scalar fields φ and λ,

$$\varphi = \frac{1}{\sqrt{2}}(\eta + \xi) \, , \quad \lambda = \frac{1}{\sqrt{2}}(\eta - \xi) \, ,$$

$\Rightarrow$

$$S = \int d^4x \sqrt{-g} \left\{ \mathcal{L}_{\rm gravity} - \frac{1}{2}\partial_\mu \xi \partial^\mu \xi + \frac{1}{2}\partial_\mu \eta \partial^\mu \eta + \frac{1}{\sqrt{2}}(\eta - \xi) \right\} + S_{\rm matter}$$

η generates the negative norm state and therefore η is a ghost.

$\Rightarrow$ Introducing the fermionic (Grassmann odd) ghosts b and c,

$$S' = \int d^4x \sqrt{-g} \left\{ \mathcal{L}_{\rm gravity} - \lambda + \partial_\mu \lambda \partial^\mu \varphi - \partial_\mu b \partial^\mu c \right\} + S_{\rm matter} \, .$$

Invariant under BRS transformation

$$\delta\lambda = \delta c = 0 \, , \quad \delta\varphi = \epsilon c \, , \quad \delta b = \epsilon \lambda$$

ϵ: fermionic parameter.

Defining the physical states as the states invariant under the BRS transformation, the negative norm states can be consistently removed.

- T. Kugo and I. Ojima, "Manifestly Covariant Canonical Formulation of Yang-Mills Field Theories: Physical State Subsidiary Conditions and Physical S Matrix Unitarity," Phys. Lett. B **73** (1978) 459. doi:10.1016/0370-2693(78)90765-7
- T. Kugo and I. Ojima, "Local Covariant Operator Formalism of Nonabelian Gauge Theories and Quark Confinement Problem," Prog. Theor. Phys. Suppl. **66** (1979) 1. doi:10.1143/PTPS.66.1

Conserved ghost number, 1 for c and -1 for b and ϵ
(λ, φ, b, c): a quartet in Kugo-Ojima's quartet mechanism.

Topological Field Theory

Lagrangian density,

$$\mathcal{L} = -\lambda + \partial_\mu \lambda \partial^\mu \varphi - \partial_\mu b \partial^\mu c$$

regarded as the Lagrangian density of a topological field theory

- E. Witten, "Topological Quantum Field Theory," Commun. Math. Phys. **117** (1988) 353. doi:10.1007/BF01223371

Lagrangian density is BRS exact, that is, given by the BRS transformation of some quantity.

Start with field theory of φ but $\mathcal{L}_\varphi = 0$
$\Rightarrow$ Under any transformation of φ, the Lagrangian density
is trivially invariant
$\Rightarrow$ gauge theory.

Gauge condition,

$$1 + \nabla_\mu \partial^\mu \varphi = 0$$

Gauge-fixing Lagrangian + ghost Lagrangian
= BRS transformation of $-b\left(1 + \nabla_\mu \partial^\mu \varphi\right)$.

$$\begin{aligned}\delta\left(-b\left(1 + \nabla_\mu \partial^\mu \varphi\right)\right) &= \epsilon\left(-\lambda\left(1 + \nabla_\mu \partial^\mu \varphi\right) + b \nabla_\mu \partial^\mu c\right) \\ &= \epsilon\left(\mathcal{L} + (\text{total derivative terms})\right)\end{aligned}$$

- T. Kugo and S. Uehara, Nucl. Phys. B **197** (1982) 378. doi:10.1016/0550-3213(82)90449-7

Lagrangian density: BRS exact up to total derivative.

More about BRS symmetry

Nakanishi-Lautrup field

λ: Nakanishi-Lautrup field, BRS exact, $\delta b = \epsilon\lambda$
$\Rightarrow$ Vacuum expectation value of λ should vanish.

If does not vanish, **Spontaneous Breakdown of BRS symmetry**.
Cannot consistently impose the physical state condition?

Action is also invariant
under (infinite numbers of) deformed BRS transformation,

$$\delta\lambda = \delta c = 0\,, \quad \delta\varphi = \epsilon c\,, \quad \delta b = \epsilon\left(\lambda - \lambda_0\right)\,.$$

λ_0 satisfies the condition $\nabla_\mu \partial^\mu \lambda_0 = 0 \Leftarrow$ Field equation for λ.

$$\mathcal{L} = -\lambda + \partial_\mu \lambda \partial^\mu \varphi - \partial_\mu b \partial^\mu c$$

One and Only One Unbroken BRS symmetry

where λ_0 is equal to λ in the real world.

If λ_0 may include large quantum correction to the vacuum energy

$$\Rightarrow \lambda - \lambda_0\text{: BRS exact.}$$

λ exactly cancells the large cosmological constant etc. in the physical states.

For the BRS transformation with $\lambda_0 \neq 0$, $\mathcal{L}$ is NOT BRS exact.
This might be a reason why this model gives non-trivial and physically relevant contributions.

A Remark

BRS symmetry $\Rightarrow$
In the physical states, no quantum fluctuation up to zero norm states.

However

λ_0 can include fluctuations from a background like a inhomogeneous and isotropic universe as long as λ_0 is a solution of the classical field equation.
$\Rightarrow$
Although quantum fluctuations are prohibited, classical fluctuations can appear.

$$\lambda = \lambda_0 \, \text{(including classical flucutuation)} + \cancel{\delta\lambda_{\mathrm{quantum}}} \, .$$

Cosmology

The problem of the large vacuum energy might be solved.

The value of $(\Lambda +)\,\lambda$ could be determined by some initial conditions.

$\Rightarrow$ The fine-tuning problem in quantum theory reduces to the fine-tuning problem in the initial conditions in classical theory.

What could be the initial conditions consistent with the current universe?

$\Rightarrow$ **Cosmology**

Cosmological evolution

$$\mathcal{L}_{\rm gravity} = \frac{R}{2\kappa^2} - \Lambda \qquad \text{(Einstein gravity)}$$

R: scalar curvature, κ: gravitational coupling constant.

FRW metric with flat spacial part,

$$ds^2 = -dt^2 + a(t)^2 \sum_{i=1}^{3} \left(dx^i\right)^2 \ ,$$

Assume λ, φ only depend on t. $a(t)$: scale factor.

Eq. for φ

$$0 = 1 - \left(\frac{d^2\varphi}{dt^2} + 3H\frac{d\varphi}{dt} \right)$$

Hubble rate $H \equiv \frac{1}{a}\frac{da}{dt}$.

Eq. for λ

$$0 = \frac{d^2\lambda}{dt^2} + 3H\frac{d\lambda}{dt}$$

Neglect contributions from matters.
(t, t) and (i, j) components of Einstein equation,

$$\begin{aligned} \frac{3}{\kappa^2}H^2 =& \Lambda + \lambda - \frac{d\lambda}{dt}\frac{d\varphi}{dt}, \\ -\frac{1}{\kappa^2}\left(3H^2 + 2\frac{dH}{dt}\right) =& -\Lambda - \lambda - \frac{d\lambda}{dt}\frac{d\varphi}{dt} \end{aligned}$$

Deleting Λ,

$$\frac{1}{\kappa^2}\frac{dH}{dt} = \frac{d\lambda}{dt}\frac{d\varphi}{dt} \,.$$

A solution for λ: $\lambda = \lambda_0$ (constant) $\Rightarrow H = H_0$ (constant)

$$\Rightarrow \quad \lambda_0 = -\Lambda + \frac{3H_0^2}{\kappa^2} \,, \quad \varphi = \frac{t}{3H_0}$$

$H = H_0 \Rightarrow$ de Sitter space-time but H_0 does not depend on Λ.
H_0 could be determined by initial condition or something else.
Value of Λ is irrelevant for the cosmology.

The obtained de Sitter space-time solution is attractor.

Stability for de Sitter space-time solution

Stability of solution expressing the de Sitter space-time
Pertubation from the solution,

$$H = H_0 + \delta H\,, \quad \lambda = -\Lambda + \frac{3H_0^2}{\kappa^2} + \delta\lambda\,, \quad \varphi = \frac{t}{3H_0} + \delta\varphi\,,$$

$$\Rightarrow \quad 0 = \delta\ddot{\varphi} + 3H_0\delta\dot{\varphi} - \frac{1}{3H_0}\delta H\,,$$

$$0 = \delta\ddot{\lambda} + 3H_0\delta\dot{\lambda}\,, \quad \frac{6}{\kappa^2}H_0\delta H = \delta\lambda + \frac{1}{3H_0}\delta\dot{\lambda}\,.$$

By deleting δH,

$$0 = \delta\ddot{\varphi} + 3H_0\delta\dot{\varphi} - \frac{\kappa^2}{18H_0^2}\left(\delta\lambda + \frac{1}{3H_0}\delta\eta\right)\,.$$

Defined a new variable $\delta\eta \equiv \delta\dot{\lambda} \Rightarrow 0 = \delta\dot{\eta} + 3H_0\delta\eta$.

$\Rightarrow$

$$\begin{pmatrix} \delta\dot{\lambda} \\ \delta\dot{\eta} \\ \delta\ddot{\varphi} \end{pmatrix} = A \begin{pmatrix} \delta\lambda \\ \delta\eta \\ \delta\dot{\varphi} \end{pmatrix} , \quad A \equiv \begin{pmatrix} 0 & 1 & 0 \\ 0 & -3H_0 & 0 \\ \frac{\kappa^2}{18H_0^2} & -\frac{\kappa^2}{54H_0^3} & -3H_0 \end{pmatrix} .$$

Eigenvalues: $-3H_0$, two 0's.
No positive eigenvalues $\Rightarrow$ the solution is stable or at least quasi-stable.

Constraints for Initial Conditions

Problem of the vacuum energy in the quantum theory $\Rightarrow$ the initial value problem in the classical theory.
What could be the initial condition corresponding to the value of the vacuum enegy in the present universe?

The solutions of the field equations

$$0 = 1 + \left(\frac{d^2\varphi}{dt^2} + 3H\frac{d\varphi}{dt} \right) , \quad 0 = \frac{d^2\lambda}{dt^2} + 3H\frac{d\lambda}{dt} ,$$

$$\Rightarrow$$

$$\varphi(t) = \int^t \frac{dt_1}{a(t_1)^3} \int^{t_1} dt_2 a(t_2)^3 , \quad \lambda(t) = \lambda_0 + \lambda_1 \int^t \frac{dt_1}{a(t_1)^3} .$$

After the inflation, the universe passed through the radiation-dominated era and the matter-dominated era, and entered into the dark energy-dominated era.

In the radiation-dominated era and the matter-dominated era, the contributions from λ and φ should be neglected.

In the future of the dark energy-dominated era, the universe is expected to be described by the asymptotically de Sitter space-time solution.

Radiation-dominated era $a(t) = a_{\rm rad} t^{1/2}$:

$$\begin{aligned}\varphi(t) =& \varphi_{\rm rad}(t) \equiv \varphi_{\rm rad\,2} - \frac{2\varphi_{\rm rad\,1}}{a_{\rm rad}^3} t^{-1/2} + \frac{t^2}{5} \,, \\ \lambda(t) =& \lambda_{\rm rad}(t) \equiv \lambda_{\rm rad\,1} - \frac{2\lambda_{\rm rad\,2}}{a_{\rm rad}^3} t^{-1/2} \,.\end{aligned}$$

Matter-dominated era $a(t) = a_{\rm mat} t^{2/3}$:

$$\begin{aligned}\varphi(t) =& \varphi_{\rm mat}(t) \equiv \varphi_{\rm mat\,2} - \frac{\varphi_{\rm mat\,1}}{a_{\rm mat}^3} t^{-1} + \frac{1}{6} t^2 \,, \\ \lambda(t) =& \lambda_{\rm mat}(t) \equiv \lambda_{\rm mat\,1} - \frac{\lambda_{\rm mat\,2}}{a_{\rm mat}^3} t^{-1} \,.\end{aligned}$$

Dark energy-dominated era $a(t) = a_\Lambda \mathrm{e}^{H_0\sqrt{\Omega_\Lambda} t}$:

$$\begin{aligned}\varphi(t) =& \varphi_\Lambda(t) \equiv \varphi_{\Lambda\,2} - \frac{\varphi_{\Lambda\,1}}{3H_0\sqrt{\Omega_\Lambda} a_\Lambda^3} \mathrm{e}^{-3H_0\sqrt{\Omega_\Lambda} t} + \frac{t}{3H_0\sqrt{\Omega_\Lambda}} \,, \\ \lambda(t) =& \lambda_\Lambda(t) \equiv \lambda_{\Lambda\,1} - \frac{\lambda_{\Lambda\,2}}{3H_0\sqrt{\Omega_\Lambda} a_\Lambda^3} \mathrm{e}^{-3H_0\sqrt{\Omega_\Lambda} t} \,.\end{aligned}$$

$a_{\rm rad}$, $a_{\rm rad}$, a_Λ: constants, H_0: H in the current universe, Ω_Λ: dark energy density parameter, $\varphi_{\rm rad\,1}$, $\varphi_{\rm rad\,2}$, $\lambda_{\rm rad\,1}$, $\lambda_{\rm rad\,2}$, $\varphi_{\rm mat\,1}$, $\varphi_{\rm mat\,2}$, $\lambda_{\rm mat\,1}$, $\lambda_{\rm mat\,2}$, $\varphi_{\Lambda\,1}$, $\varphi_{\Lambda\,2}$, $\lambda_{\Lambda\,1}$, $\lambda_{\Lambda\,2}$: constants.

Approximations:

The radiation-dominated era transited to the matter-dominated era at the time $t = t_1$ and the matter-dominated era to the dark-energy dominated era at $t = t_2$.

Connect the solutions by imposing the continuities of the values of φ, λ, $\dot{\varphi}$, and $\dot{\lambda}$ at the transit points. $\Rightarrow$

$$
\begin{aligned}
\lambda_0 + \Lambda =& \frac{3H_c^2}{\kappa^2} = \Lambda + \lambda_{\Lambda\,1} - \frac{\lambda_{\Lambda\,2}}{3H_0\sqrt{\Omega_\Lambda}a_\Lambda^3}\mathrm{e}^{-3H_0\sqrt{\Omega_\Lambda}t_2}\,, \\
\lambda_{\Lambda\,1} =& \lambda_{\mathrm{rad1}} - \frac{\lambda_{\mathrm{rad2}}}{a_{\mathrm{rad}}^3} t_1^{-1/2}\left[1 + t_1 t_2^{-1}\left(1 - \frac{t_2^{-1}}{3H_0\sqrt{\Omega_\Lambda}}\right)\right]\,, \\
\lambda_{\Lambda\,2} =& \lambda_{\mathrm{rad}\,2}\left(\frac{a_\Lambda}{a_{\mathrm{rad}}}\right)^3 \mathrm{e}^{3H_0\sqrt{\Omega_\Lambda}t_2} t_2^{-2} t_1^{1/2}\,, \\
& \mathrm{etc.}
\end{aligned}
$$

Constraints

1. In the present universe, $\Lambda + \lambda$ becomes a constant corresponding to the cosmological constant $\Lambda_0 \sim 10^{-11}\,[\mathrm{eV}^4]$.
2. In the matter-dominated era and the radiation-dominated era, the contribution to the energy density from the scalar fields λ and φ could be neglected.

$\Rightarrow$

$$
\begin{aligned}
\Lambda + \lambda_{\mathrm{rad1}} - \lambda_{\mathrm{rad2}} \times \left(3.1 \times 10^{65}\,[\mathrm{eV}^{-1}]\right) &\sim 10^{-11}\,[\mathrm{eV}^4]\,, \\
\left|\lambda_{\mathrm{rad2}}\right| &\ll 10^{-47}\,[\mathrm{eV}^5]\,, \\
\left|\lambda_{\mathrm{rad}\,2}\left(\varphi_{\mathrm{rad}\,1} + 1.4 \times 10^{16}\,\left[\mathrm{eV}^{-1}\right]\right)\right| &\ll 10^{53}\,\left[\mathrm{eV}^4\right]\,, \\
\left|\lambda_{\mathrm{rad}\,2}\varphi_{\mathrm{rad}\,1}\right| &\ll 10^{-62}\,\left[\mathrm{eV}^4\right]\,.
\end{aligned}
$$

The first constraint seems to tell that we need the fine tuning for the initial conditions.

More about the initial condition for λ.

$$\lambda(t) = \lambda_{\mathrm{rad}}(t) \equiv \lambda_{\mathrm{rad}\,1} - \frac{2\lambda_{\mathrm{rad}\,2}}{a_{\mathrm{rad}}^3} t^{-1/2} \,.$$

If we assume $\lambda_{\mathrm{rad}\,1} = 0$, which might be unnatural, we find the value of λ at the beginning of the radiation-dominated era $t \sim t_3$,

$$\lambda = \lambda_{\mathrm{rad}}(t_3) \sim (0.1\,[\mathrm{keV}])^4 \,.$$

The obtained value might be a little bit more reasonable.

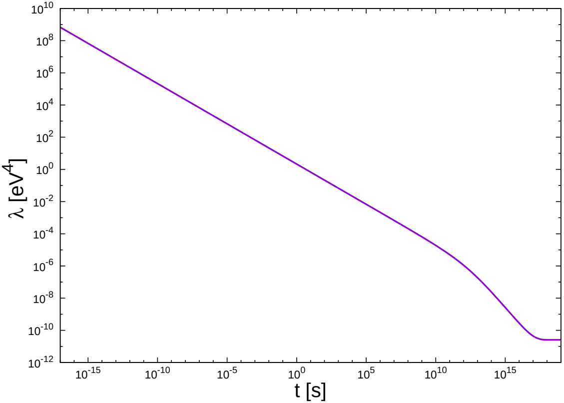
Even if $\lambda \sim 10^{-12}\,[\mathrm{eV}^4]$ in the present universe, $\lambda \sim (0.1\,[\mathrm{keV}])^4$ at the beginning of the radiation-dominated era.

The converse is not true because $\lambda_{\mathrm{rad}\,1} \neq 0$ in general:
Even if $\lambda \sim (0.1\,[\mathrm{keV}])^4$ at the beginning of the radiation-dominated era, we may find $\lambda \sim (0.1\,[\mathrm{keV}])^4$ in the present universe.

Numerical Calculations

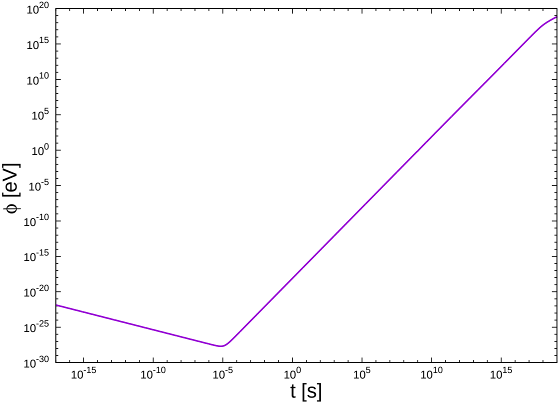
Numerical Calculations by Daisuke Nitta
Solving equations,

$$0 = 1 + \left(\frac{d^2\varphi}{dt^2} + 3H\frac{d\varphi}{dt} \right) , \quad 0 = \frac{d^2\lambda}{dt^2} + 3H\frac{d\lambda}{dt} ,$$
$$\frac{3}{\kappa^2} H^2 = \Lambda + \lambda - \frac{d\lambda}{dt}\frac{d\varphi}{dt} + \rho_{\mathrm{matter}} + \rho_{\mathrm{radiation}} .$$



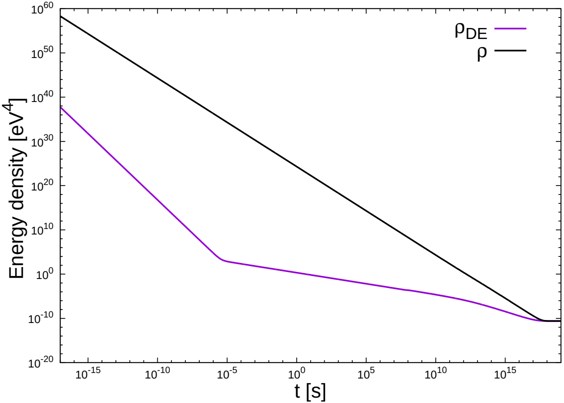
The development of λ.

Consistent with the analytic result.



The development of φ.

$$\phi \equiv M_{\rm Planck}^3 \varphi \, .$$



The development of energy density.

The parameters $\lambda_{\mathrm{rad}\,1}$ and $\lambda_{\mathrm{rad}\,2}$ are chosen to reproduce the value of the dark energy density in the current universe.
The dark energy density in the matter-dominated era or the radiation-dominated era is surely negligible.

Summary

- We have proposed a simple and totally covariant model, which is a topologcal field theory and may solve the problem of the quantum corrections from the matters to the vacuum energy.
- The mechanism is similar to that in the unimodular gravity but the variation of the scalar field λ does not give any constraint on the metric like the unimoduar constraint, but the variation of λ gives the equation for another scalar field φ.
- The problem of the quantum corrections reduce to the classical problem of the initial conditions.

Recently the cosmological perturbation based on the model in was investigated in
R. Saitou and Y. Gong,
"Cosmological perturbations from a Becchi-Rouet-Stora quartet,"
arXiv:1702.02806 [hep-th].

Further more generalization

Vacuum energy is not only the quantum corrections.
General quantum corrections from the matter,

$$\mathcal{L}_{\mathrm{qc}} = \alpha R + \beta R^2 + \gamma R_{\mu\nu} R^{\mu\nu} + \delta R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma} \,.$$

Coefficient α diverges quadrutically, β, γ, δ logarithmically.
$\Rightarrow$ Further more generation

$$\begin{aligned}
\mathcal{L} =& - \Lambda - \lambda_{(\Lambda)} + \left(\alpha + \lambda_{(\alpha)}\right) R + \left(\beta + \lambda_{(\beta)}\right) R^2 + \left(\gamma + \lambda_{(\gamma)}\right) R_{\mu\nu} R^{\mu\nu} \\
& + \left(\delta + \lambda_{(\delta)}\right) R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma} \\
& + \partial_\mu \lambda_{(\Lambda)} \partial^\mu \varphi_{(\Lambda)} - \partial_\mu b_{(\Lambda)} \partial^\mu c_{(\Lambda)} + \frac{1}{\mu} \partial_\mu \lambda_{(\alpha)} \partial^\mu \varphi_{(\alpha)} - \partial_\mu b_{(\alpha)} \partial^\mu c_{(\alpha)} \\
& + \mu \partial_\mu \lambda_{(\beta)} \partial^\mu \varphi_{(\beta)} - \partial_\mu b_{(\beta)} \partial^\mu c_{(\beta)} + \mu \partial_\mu \lambda_{(\gamma)} \partial^\mu \varphi_{(\gamma)} - \partial_\mu b_{(\gamma)} \partial^\mu c_{(\gamma)} \\
& + \mu \partial_\mu \lambda_{(\delta)} \partial^\mu \varphi_{(\delta)} - \partial_\mu b_{(\delta)} \partial^\mu c_{(\delta)} \,.
\end{aligned}$$

Λ, α, β, γ, and δ may include the divergences due to the quantum corrections from the matters.

The redefinitions of the parameters, $\lambda_{(\Lambda)}$, $\lambda_{(\alpha)}$, $\lambda_{(\beta)}$, $\lambda_{(\gamma)}$, and $\lambda_{(\delta)}$,

$$\lambda_{(\Lambda)} \to \lambda_{(\lambda)} - \Lambda \,, \quad \lambda_{(\alpha)} \to \lambda_{(\alpha)} - \alpha \,, \quad \lambda_{(\beta)} \to \lambda_{(\beta)} - \beta \,,$$
$$\lambda_{(\gamma)} \to \lambda_{(\gamma)} - \gamma \,, \quad \lambda_{(\delta)} \to \lambda_{(\delta)} - \delta \,,$$

$\Rightarrow$

$$\begin{aligned}
\mathcal{L} =& - \lambda_{(\Lambda)} + \lambda_{(\alpha)} R + \lambda_{(\beta)} R^2 + \lambda_{(\gamma)} R_{\mu\nu} R^{\mu\nu} + \lambda_{(\delta)} R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma} \\
& + \partial_\mu \lambda_{(\Lambda)} \partial^\mu \varphi_{(\Lambda)} - \partial_\mu b_{(\Lambda)} \partial^\mu c_{(\Lambda)} + \frac{1}{\mu} \partial_\mu \lambda_{(\alpha)} \partial^\mu \varphi_{(\alpha)} - \partial_\mu b_{(\alpha)} \partial^\mu c_{(\alpha)} \\
& + \mu \partial_\mu \lambda_{(\beta)} \partial^\mu \varphi_{(\beta)} - \partial_\mu b_{(\beta)} \partial^\mu c_{(\beta)} + \mu \partial_\mu \lambda_{(\gamma)} \partial^\mu \varphi_{(\gamma)} - \partial_\mu b_{(\gamma)} \partial^\mu c_{(\gamma)} \\
& + \mu \partial_\mu \lambda_{(\delta)} \partial^\mu \varphi_{(\delta)} - \partial_\mu b_{(\delta)} \partial^\mu c_{(\delta)} \,.
\end{aligned}$$

Divergences can be absorbed into the redefinition of $\lambda_{(i)}$, $(i = \Lambda, \alpha, \beta, \gamma, \delta)$
Divergences might become irrelevant for the dynamics.

Lagrangian density: BRS invariant

$$\delta\lambda_{(i)} = \delta c_{(i)} = 0\,, \quad \delta\varphi_{(i)} = \epsilon c\,, \quad \delta b_{(i)} = \epsilon\lambda_{(i)}\,, \quad (i = \Lambda, \alpha, \beta, \gamma, \delta)\ ,$$

Lagrangian density: BRS exact,

$$\delta\left(\sum_{i=0,\alpha,\beta,\gamma,\delta}\left(-b_{(i)}\left(\mathcal{O}_{(i)} + \nabla_\mu\partial^\mu\varphi_{(i)}\right)\right)\right) = \epsilon\left(\mathcal{L} + (\text{total derivative terms})\right)\ .$$

$\mathcal{O}_{(i)} = 1,\ R,\ R^2,\ R_{\mu\nu}R^{\mu\nu},\ R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}$.

Quantum corrections from the graviton

$\Rightarrow$ infinite numbers of quantum corrections which diverge.

$\mathcal{O}_{(i)}$: possible gravitational operators $\Rightarrow$ further generalization

$$\mathcal{L} = \sum_i \left(\lambda_{(i)} \mathcal{O}_{(i)} + \partial_\mu \lambda_{(i)} \partial^\mu \varphi_{(i)} - \partial_\mu b_{(i)} \partial^\mu c_{(i)} \right) .$$

All the divergence can be absorbed into the redefinition of λ_i.
The Lagrangian density: BRS invariant and BRS exact.

In order to determine the values of $\lambda_{(i)}$, however, we may need infinite numbers of the initial conditions, which might be physically irrelevant and the predictability of the theory could be lost.

Still understand the quantum gravity?
Might be any clue for the quantum gravity.