Symposium "Why does the Universe accelerate ?", 08-10 Mar 2017 @ KEK
公募研究「宇宙マイクロ波背景放射における揺らぎの非線形進化」

A new code for non-linear evolution of CMB anisotropy

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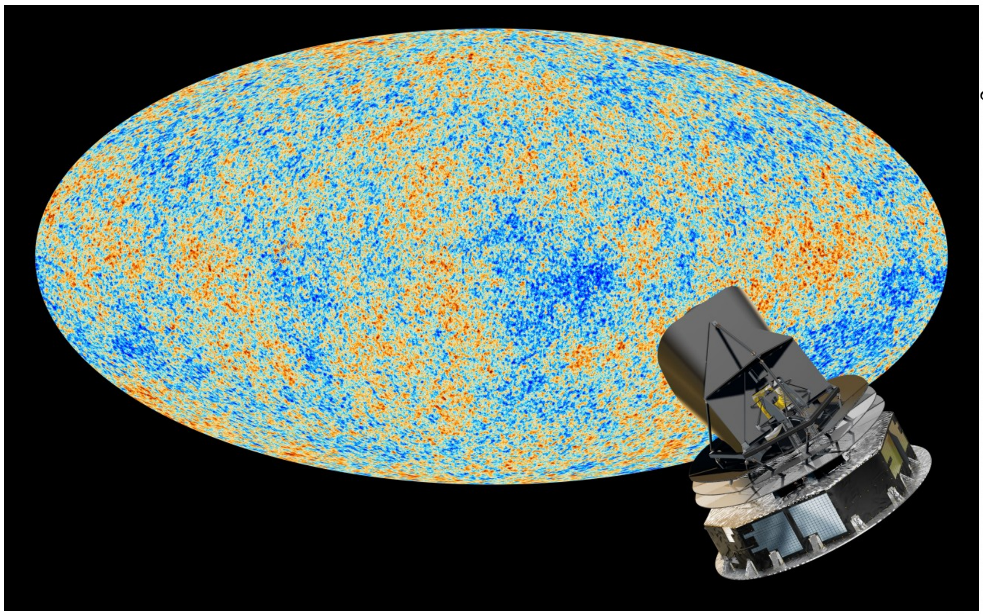
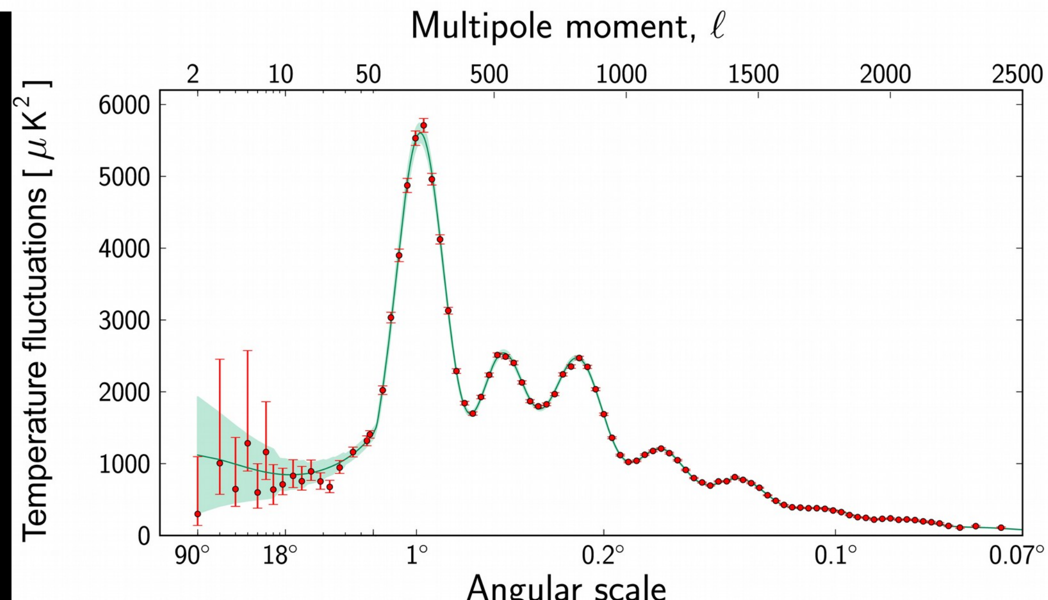
Temperature anisotropy with $\mathcal{O}(10)\mu\mathrm{K}$





http://www.sciops.esa.int

Many kinds of information on inflation, for example,

$$\langle\zeta(\mathbf{k}_1)\zeta(\mathbf{k}_2)\rangle = (2\pi)^3 P_\zeta(k_1)\delta^{(3)}(\mathbf{k}_1+\mathbf{k}_2)$$

Introduction : Non-Gaussianity

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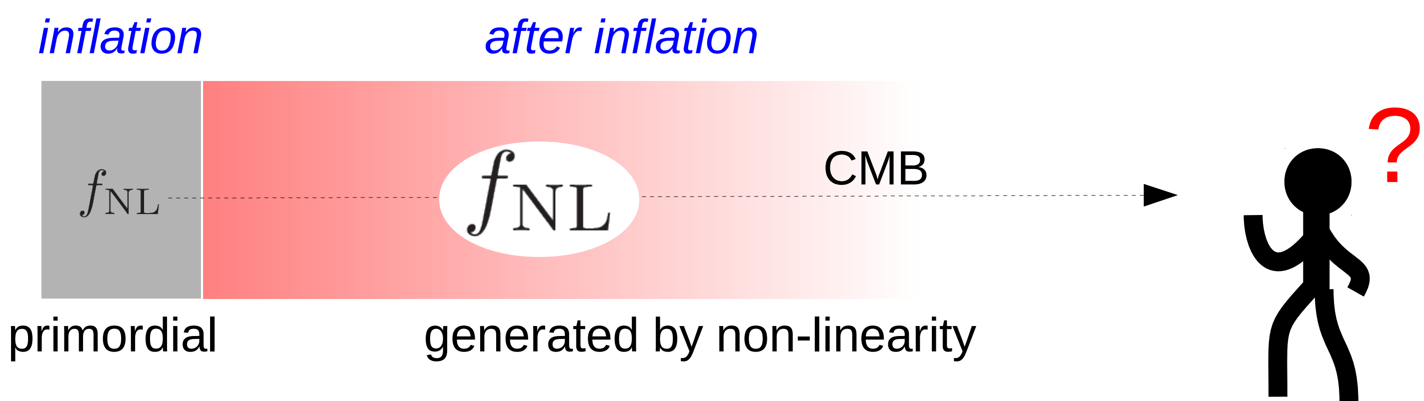
3-point function (Bispectrum)

$$\langle \zeta(\mathbf{k}_1)\zeta(\mathbf{k}_2)\zeta(\mathbf{k}_3)\rangle = (2\pi)^3 B_\zeta(k_1,k_2,k_3)\delta^{(3)}(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3)$$

... parameterised by $f_{\rm NL}$

New inflation
Chaotic inflation
Power-law inflation
DBI inflation
K-inflation
Hybrid inflation
MSSM inflation
Brane inflation
⋮



Unfortunately dominant ...

How large ?

Basic equations

Dodelson, "Modern cosmology", (Academic press)
Matsubara, "Uchuron no Butsuri" (Tokyo Univ.)

Photon/Neutrino

$$f_{\gamma,\nu}(\mathbf{x},\mathbf{p},\eta) = \left[\exp\left\{\frac{p}{T_{\gamma,\nu}(\eta)\left[1+\Theta_{\mathrm{T,N}}(\mathbf{x},\hat{p},\eta)\right]}\right\} \pm 1\right]^{-1}$$

Boltzmann eqs. $\dfrac{df_{\gamma,\nu}}{d\eta} = \dfrac{\mathcal{C}[f_{\gamma,\nu}]}{E}$

CDM/Baryon

$$T_{\mu\nu} = (\rho + p)u_\mu u_\nu + g_{\mu\nu}p \qquad \rho = \overline{\rho}(1+\delta) \quad u^\mu = \frac{1}{a}(u^0, \hat{k}^i v)$$

Continuity/Euler eqs. $\nabla^\mu \delta T_{\mu\nu} = 0$

Gravity

$$ds^2 = a^2\left[-(1+2\Psi)d\eta^2 + (1+2\Phi)\delta_{ij}dx^i dx^j\right]$$

Perturbed Einstein eqs. $\delta G_{\mu\nu} = 8\pi G \delta T_{\mu\nu}$

linear perturbation equations

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Photon temperature

$$\begin{cases} \dot{\Theta}_0^{\mathrm{T}} = -k\Theta_1^{\mathrm{T}} - \dot{\Phi} \\ \dot{\Theta}_1^{\mathrm{T}} = \frac{1}{3}k(-2\Theta_2^{\mathrm{T}} + \Theta_0^{\mathrm{T}}) + \dot{\tau}\left(\Theta_1^{\mathrm{T}} + \frac{1}{3}v_b\right) + \frac{1}{3}k\Psi \\ \dot{\Theta}_2^{\mathrm{T}} = \frac{1}{5}k(-3\Theta_3^{\mathrm{T}} + 2\Theta_1^{\mathrm{T}}) + \frac{\dot{\tau}}{10}\left(9\Theta_2^{\mathrm{T}} - \Theta_0^{\mathrm{P}} - \Theta_2^{\mathrm{P}}\right) \\ \dot{\Theta}_\ell^{\mathrm{T}} = \frac{1}{2\ell+1}k\left[-(\ell+1)\Theta_{\ell+1}^{\mathrm{T}} + \ell\Theta_{\ell-1}^{\mathrm{T}}\right] + \dot{\tau}\Theta_\ell^{\mathrm{T}} \end{cases}$$

Photon polarisation

$$\begin{cases} \dot{\Theta}_0^{\mathrm{P}} = -k\Theta_1^{\mathrm{P}} + \frac{\dot{\tau}}{2}\left(\Theta_0^{\mathrm{P}} - \Theta_2^{\mathrm{T}} - \Theta_2^{\mathrm{P}}\right) \\ \dot{\Theta}_1^{\mathrm{P}} = \frac{1}{3}k(-2\Theta_2^{\mathrm{P}} + \Theta_0^{\mathrm{P}}) + \dot{\tau}\Theta_1^{\mathrm{P}} \\ \dot{\Theta}_2^{\mathrm{P}} = \frac{1}{5}k(-3\Theta_3^{\mathrm{P}} + 2\Theta_1^{\mathrm{P}}) + \frac{\dot{\tau}}{10}\left(9\Theta_2^{\mathrm{P}} - \Theta_2^{\mathrm{T}} - \Theta_0^{\mathrm{P}}\right) \\ \dot{\Theta}_\ell^{\mathrm{P}} = \frac{1}{2\ell+1}k\left[-(\ell+1)\Theta_{\ell+1}^{\mathrm{P}} + \ell\Theta_{\ell-1}^{\mathrm{P}}\right] + \dot{\tau}\Theta_\ell^{\mathrm{P}} \end{cases}$$

Massless neutrino temperature

$$\begin{cases} \dot{\Theta}_0^{\mathrm{N}} = -k\Theta_1^{\mathrm{N}} - \dot{\Phi} \\ \dot{\Theta}_1^{\mathrm{N}} = \frac{1}{3}k(-2\Theta_2^{\mathrm{N}} + \Theta_0^{\mathrm{N}}) + \frac{1}{3}k\Psi \\ \dot{\Theta}_2^{\mathrm{N}} = \frac{1}{5}k(-3\Theta_3^{\mathrm{N}} + 2\Theta_1^{\mathrm{N}}) \\ \dot{\Theta}_\ell^{\mathrm{N}} = \frac{1}{2\ell+1}k\left[-(\ell+1)\Theta_{\ell+1}^{\mathrm{N}} + \ell\Theta_{\ell-1}^{\mathrm{N}}\right] \end{cases}$$

CDM, baryon

$$\begin{cases} \dot{\delta}_{\mathrm{c}} = -ikv_{\mathrm{c}} - 3\dot{\Phi} \\ \dot{\delta}_{\mathrm{b}} = -ikv_{\mathrm{b}} - 3\dot{\Phi} \\ \dot{v}_{\mathrm{c}} = -\mathcal{H}v_{\mathrm{c}} - ik\Psi \\ \dot{v}_{\mathrm{b}} = -\mathcal{H}v_{\mathrm{b}} - ik\Psi + \frac{\dot{\tau}}{R}(v_{\mathrm{b}} + 3i\Theta_{\mathrm{T}1}) \end{cases}$$

Gravity
(conformal Newtonian gauge)

$$\dot{\Phi} = -\frac{k^2}{3\mathcal{H}}\Phi + \mathcal{H}\Psi + \frac{a^2\mathcal{H}_0^2}{2\mathcal{H}}\delta\Omega_0$$

$$\left(\Psi = -\Phi - 12\frac{a^2\mathcal{H}_0^2}{k^2}\Omega_r\Theta_{r,2}\right)$$

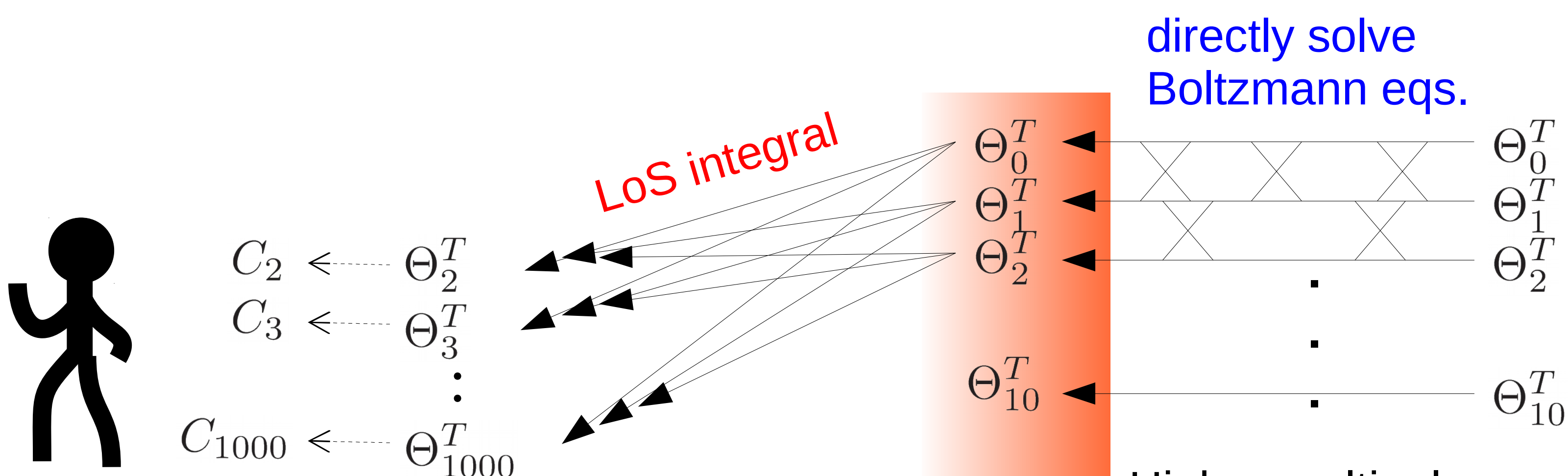
Line-of-sight formula

Integral form of Boltzmann equation

$$\dot{\Theta}^{\mathrm{T}} + ik\mu\Theta^{\mathrm{T}} = -\dot{\Phi} - ik\mu\Psi$$
$$-\dot{\tau}\left[\Theta_0^{\mathrm{T}} - \Theta^{\mathrm{T}} + \mu v_{\mathrm{b}} - \frac{1}{2}\mathcal{P}_2(\mu)(\Theta_2^{\mathrm{T}} + \Theta_0^{\mathrm{P}} + \Theta_2^{\mathrm{P}})\right]$$

$$\Theta_\ell^{\mathrm{T}}(k,\eta_0) = \int_0^{\eta_0} d\eta\, S(k,\eta) j_\ell[k(\eta_0-\eta)]$$

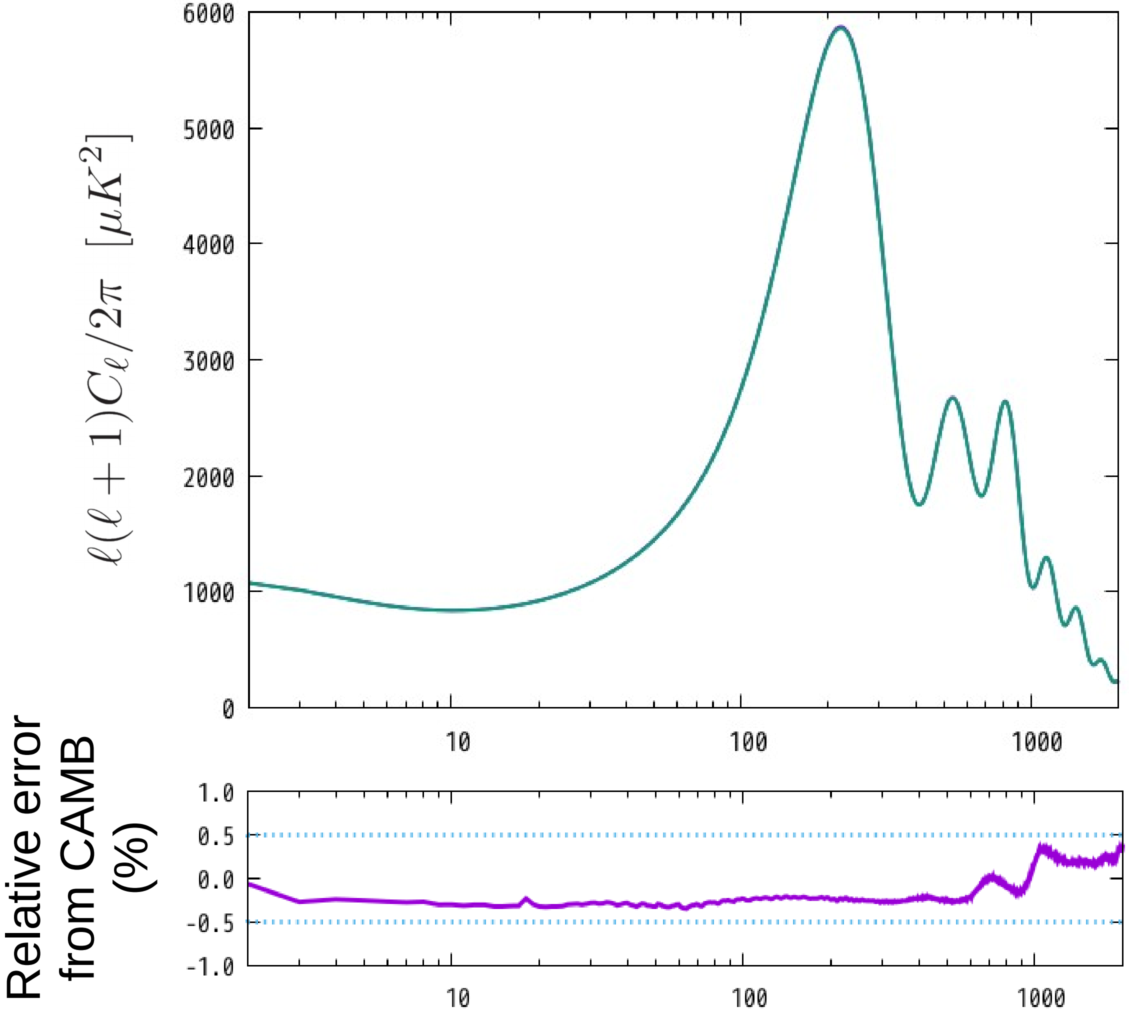
Seljak, Zaldarriaga, APJ 469 (1996) 437



Higher multipoles are well suppressed by tight-coupling between baryons-photons

Angular power spectrum (1st-order temp)

$$C_\ell^{\mathrm{TT}} = \frac{2}{\pi}\int_0^\infty dk\, k^2 \mathcal{T}_{\Theta_\ell^{\mathrm{T}}}(k,\eta)^2 P_\zeta(k,\eta_{\mathrm{in}})$$



parameters

$$\Delta_\zeta^2(k_{\mathrm{pivot}}) = 2.44\times 10^{-9}$$
$$k_{\mathrm{pivot}} = 0.002\ \mathrm{Mpc}^{-1}$$
$$n_s = 0.9667$$
$$h^2\Omega_{\mathrm{CDM}} = 0.1188$$
$$h^2\Omega_{\mathrm{B}} = 0.02230$$
$$h = 0.6774$$
$$\tau = 0.066$$
$$Y_p = 0.24667$$
$$N_{\mathrm{eff}} = 3.046$$
$$T_0 = 2.7255\ \mathrm{K}$$

existing codes

CMBFAST : Seljak, Zaldarriaga, APJ469 (1996) 437
CAMB : Lewis, Challinor, APJ538 (2000) 473
CLASS II : Blas, Lesgourgues, Tram, JCAP 1107 (2011) 034
CosmoLib : Huang, JCAP 1206 (2012) 012

Angular power spectrum (1st-order temp)

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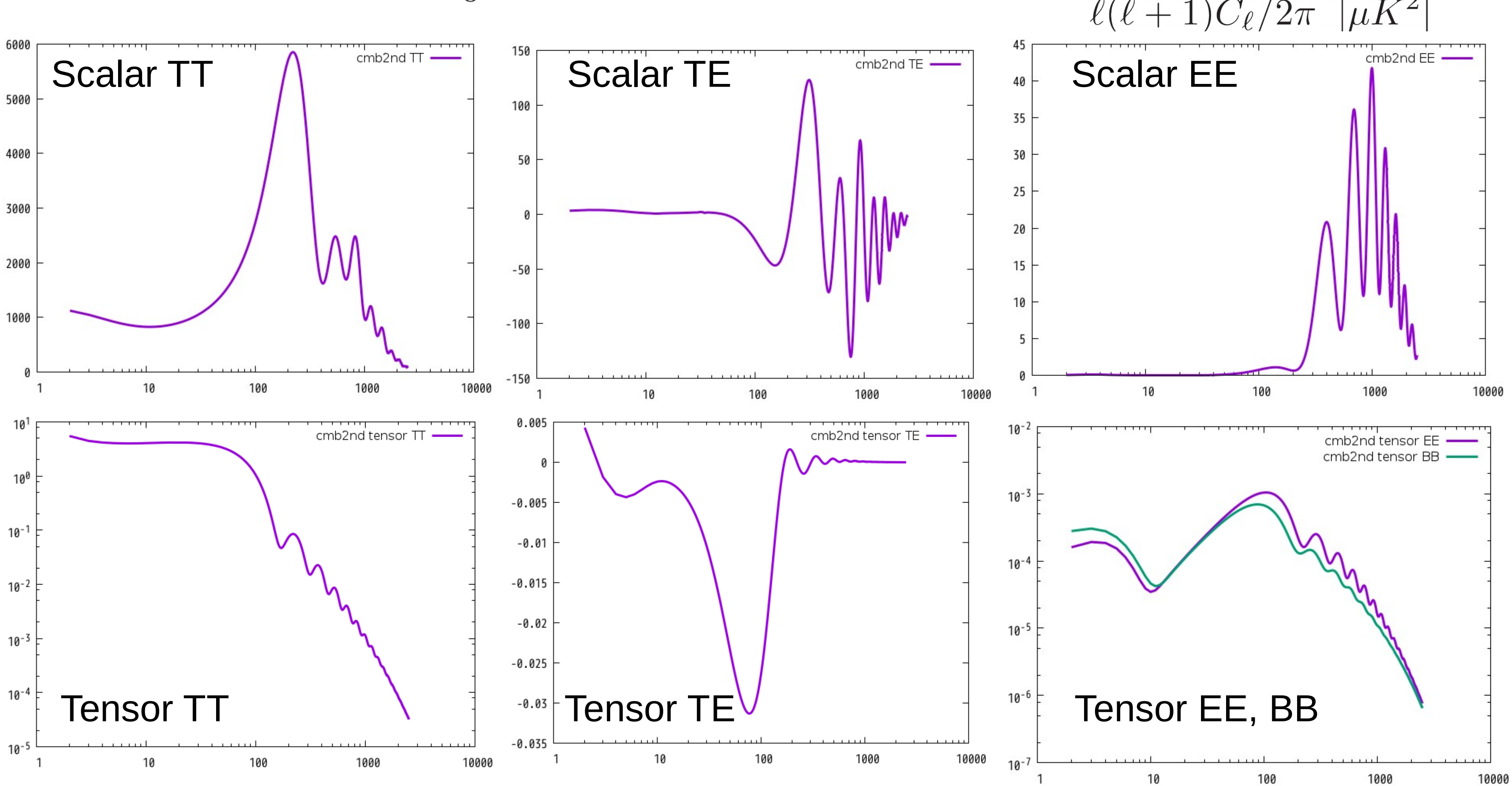
$$C_\ell^{\rm AB} = \frac{2}{\pi}\int_0^\infty dk\, k^2 \mathcal{T}_{A_\ell}(k,\eta)\mathcal{T}_{B_\ell}(k,\eta)P_\zeta(k,\eta_{\rm in})$$



All kinds of spectra are consistent to those computed by CAMB with ~1%

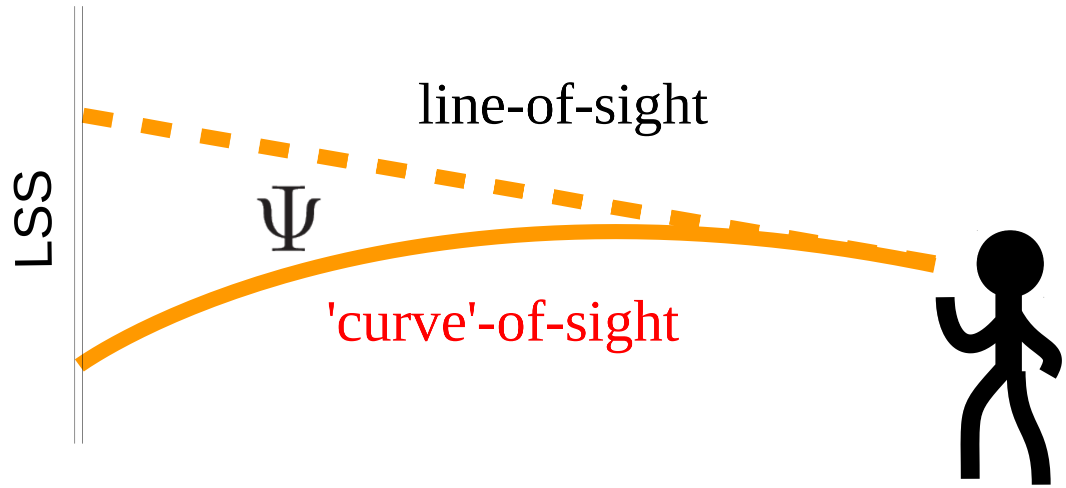
Go to 2nd-order

	Linear Boltzmann eqs.	2nd-order Boltzmann eqs.
Line-of-sight formula	CAMB, CMBfast, Class, CosmoLib,...	CMBquick, SONG, CosmoLib2
2nd-order line-of-sight formula	cmb2nd	

$f_{\rm NL} \sim 0.5$

CMBquick : Pitrou, Uzan, Bernardeau, JCAP 07 (2010) 003]
SONG : Petinarri et al., JCAP 1304 (2013) 003
CosmoLib2 : Huang, Vernizzi, PRL 110 (2013) 101303

Go to 2nd-order



$$\Theta_\ell^{T(1)}(k,\eta_0) = \int_0^{\eta_0} S(k,\eta) j_\ell[k(\eta_0-\eta)]\, d\eta$$

$$\Theta_\ell^{T(2)}(k,\eta_0) = \int S(\Theta_T^{(1)}, \Psi^{(1)}, \Phi^{(1)}, \cdots) \times T(\Psi^{(1)}, \Phi^{(1)}, \cdots)\, d\eta$$

Fluctuations on LSS + ISW

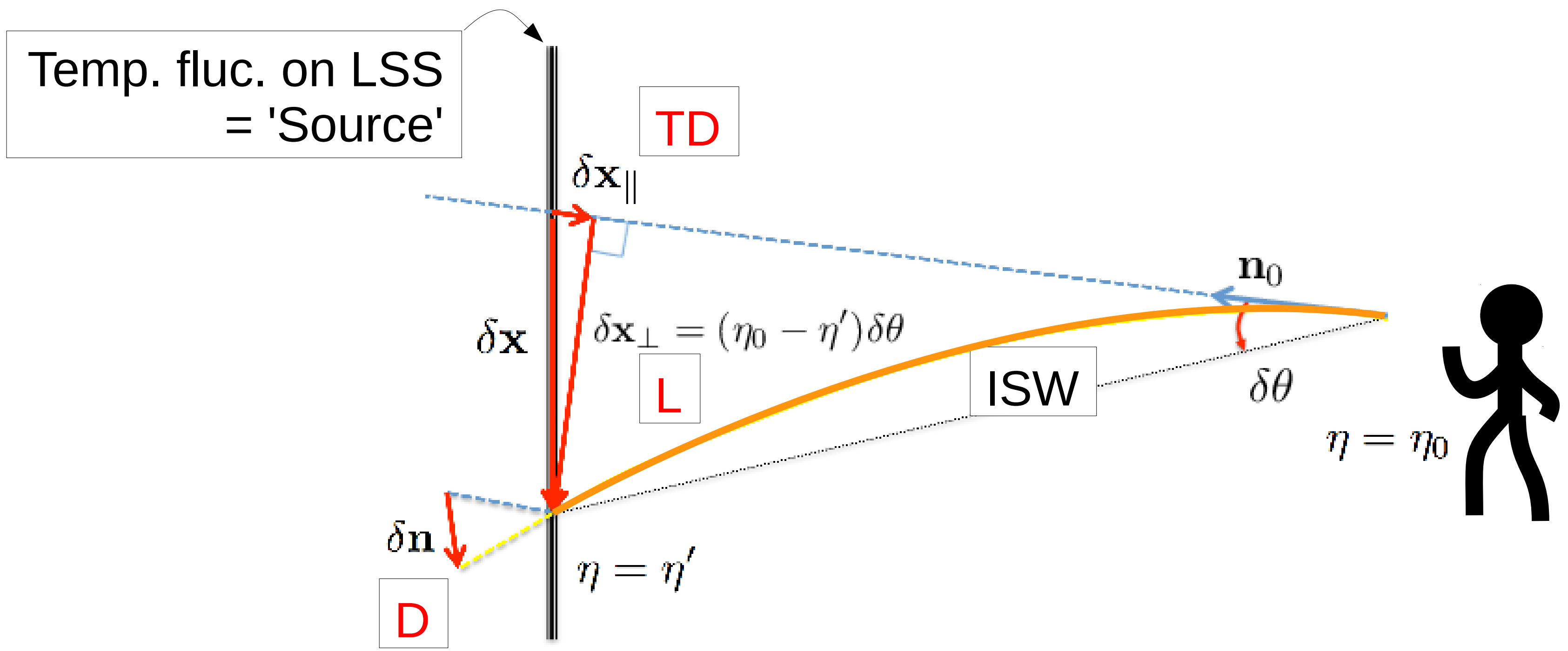
Bending effect on photon's geodesics by gravity potentials

R.Saito, Naruko, Hiramatsu, Sasaki, JCAP10(2014)051 [arXiv:1409.2464]

(cf. Fidler, Koyama, Pettinari, JCAP 04 (2015) 037)

'Curve'-of-sight formula : concept

R.Saito, Naruko, Hiramatsu, Sasaki, JCAP10(2014)051 [arXiv:1409.2464]



$\Theta_{\ell}^{\mathrm{T}(2)}$ is sourced by

- 1st -order x ISW
- 1st -order x Lensing (= ISW-Lensing)
- 1st -order x Time-delay
- 1st -order x Deflection (new effect)

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$$\langle\Theta_{\mathrm{T}}(\mathbf{k}_1)\Theta_{\mathrm{T}}(\mathbf{k}_2)\Theta_{\mathrm{T}}(\mathbf{k}_3)\rangle \sim \langle\Theta_{\mathrm{T}}^{(2)}(\mathbf{k}_1)\Theta_{\mathrm{T}}^{(1)}(\mathbf{k}_2)\Theta_{\mathrm{T}}^{(1)}(\mathbf{k}_3)\rangle \longrightarrow b_{\ell_1\ell_2\ell_3}$$

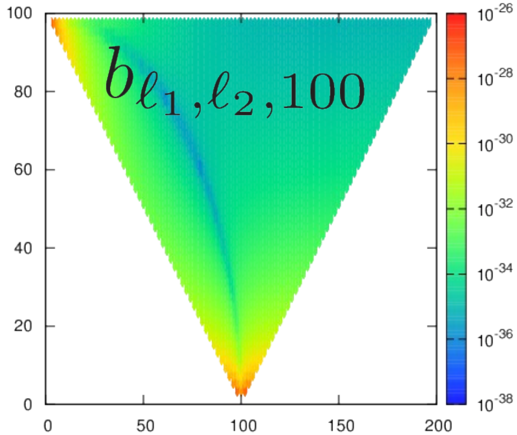
Bispectrum

$$b^{S\times T}_{\ell_1\ell_2\ell_3} = F_{\ell_1\ell_2\ell_3}\int_0^{\eta_0} d\eta'\, b^S_{\ell_1}(\eta')b^T_{\ell_2}(\eta') + 5\text{ perms.}$$
$$b^S_{\ell_1}(\eta') = \frac{2}{\pi}\int dk_1\, k_1^2 P_\zeta(k_1)\mathcal{T}_{\ell_1}(k_1)S(k_1,\eta')j_{\ell_1}[k_1(\eta_0-\eta')]$$
$$b^T_{\ell_2}(\eta') = \frac{2}{\pi}\int_{\eta'}^{\eta_0} d\eta_1 \int dk_2\, k_2^2 P_\zeta(k_2)\mathcal{T}_{\ell_2}(k_2)T(k_2,\eta_1,\eta')j_{\ell_2}[k_2(\eta_0-\eta_1)]$$

Quantify the magnitude of NG

$$B_\Theta(k_1,k_2,k_3) = \sum_i f^{(i)}_{\mathrm{NL}} B^{(i)}(k_1,k_2,k_3)$$

signals templates



$f_{\rm NL}$ from curve-of-sight formula

(Single-template fitting)

$$a(p) \equiv a \times 10^p$$

	Local	Equilateral	Orthogonal	Folded
1^{st}-order x ISW	-1.12(-3)	1.30(0)	5.24(-2)	3.95(-1)
1^{St}-order x Lensing	8.93(0)	-2.97(-1)	-2.89(+1)	4.45(+1)
1^{st}-order x Time-delay	9.82(-2)	2.87(-1)	-2.13(-1)	4.34(-1)
1^{st}-order x Deflection	1.82(-2)	1.76(-1)	-3.00(-1)	5.27(-1)

m309e

Lensing effect dominates as expected.

Past results on CMB lensing

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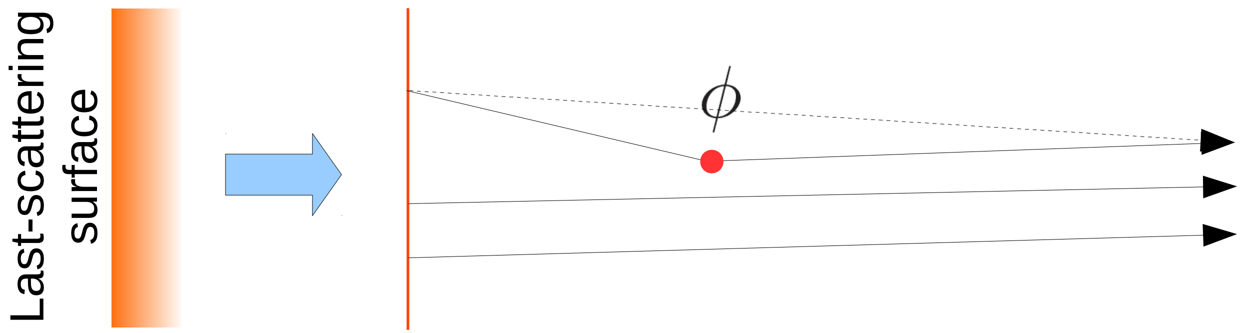
Remapping formula

Neglecting the thickness of LSS



$$\widetilde{\Theta}_T(\mathbf{n}) \equiv \Theta_T(\mathbf{n}+\nabla\phi)$$
$$\Theta_T(\mathbf{n})$$
$$\Theta_T(\mathbf{n})$$

$$B_{\ell_1\ell_2\ell_3} \approx \langle \widetilde{\Theta}^T_{\ell_1}\Theta^T_{\ell_2}\Theta^T_{\ell_3}\rangle = C^{T\phi}_{\ell_1}C^{TT}_{\ell_2} + \text{ 5 perms.}$$

Goldberg, Spergel, PRD 59 (1999) 103002
Hu, PRD 62 (2000) 043007
Zaldarriaga, PRD 62 (2000) 063510

Review : Lewis, Challinor, PR 429 (2006) 1

$$a(p) \equiv a \times 10^p$$

	Local	Equilateral	Orthogonal	Folded
Remapping	8.94(0)	-2.40(-1)	-2.91(+1)	4.48(+1)

m309e

	Local	Equilateral	Orthogonal	Folded
COS (Lensing)	8.93(0)	-2.97(-1)	-2.89(+1)	4.45(+1)

m309e

Thickness of LSS doesn't affect so much.

Leading contributions

$$\langle\Theta_{\rm T}(\mathbf{k}_1)\Theta_{\rm T}(\mathbf{k}_2)\Theta_{\rm T}(\mathbf{k}_3)\rangle$$

$$\left.\begin{array}{l}\langle\Theta_{\rm T}^{(Scal\times Tens)}(\mathbf{k}_1)\Theta_{\rm T}^{(Scal)}(\mathbf{k}_2)\Theta_{\rm T}^{(Tens)}(\mathbf{k}_3)\rangle \\ \langle\Theta_{\rm T}^{(Tens\times Scal)}(\mathbf{k}_1)\Theta_{\rm T}^{(Scal)}(\mathbf{k}_2)\Theta_{\rm T}^{(Tens)}(\mathbf{k}_3)\rangle\end{array}\right\} \propto \mathcal{P}_\zeta\mathcal{P}_h = r\mathcal{P}_\zeta^2$$

$$\langle\Theta_{\rm T}^{(Tens\times Tens)}(\mathbf{k}_1)\Theta_{\rm T}^{(Tens)}(\mathbf{k}_2)\Theta_{\rm T}^{(Tens)}(\mathbf{k}_3)\rangle \quad \propto \mathcal{P}_h^2 = r^2\mathcal{P}_\zeta^2$$

$$\Theta_{\rm T}^{(A\times B)}(\mathbf{k})$$

A : Source or ISW
B : Gravitational

Bispectra by tensor Curve-of-sight formula

$$b_{\ell_1\ell_2\ell_3}^{S(A)\times T(B)} = F_{\ell_1\ell_2\ell_3}\int_0^{\eta_0} d\eta'\, b_{\ell_1}^{S(A)}(\eta')b_{\ell_2}^{T(B)}(\eta') + 5\text{ perms.}$$

$$b_{\ell_1}^{S(A)}(\eta') = \frac{2}{\pi}\int dk_1\, k_1^2 P^{(A)}(k_1)\mathcal{T}_{\ell_1}^{(A)}(k_1)S(k_1,\eta')j_{\ell_1}[k_1(\eta_0-\eta')]$$

$$b_{\ell_2}^{T(B)}(\eta') = \frac{2}{\pi}\int_{\eta'}^{\eta_0} d\eta_1 \int dk_2\, k_2^2 P^{(B)}(k_2)\mathcal{T}_{\ell_2}^{(B)}(k_2)T(k_2,\eta_1,\eta')j_{\ell_2}[k_2(\eta_0-\eta_1)]$$

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PRELIMINARY

$r = 1$

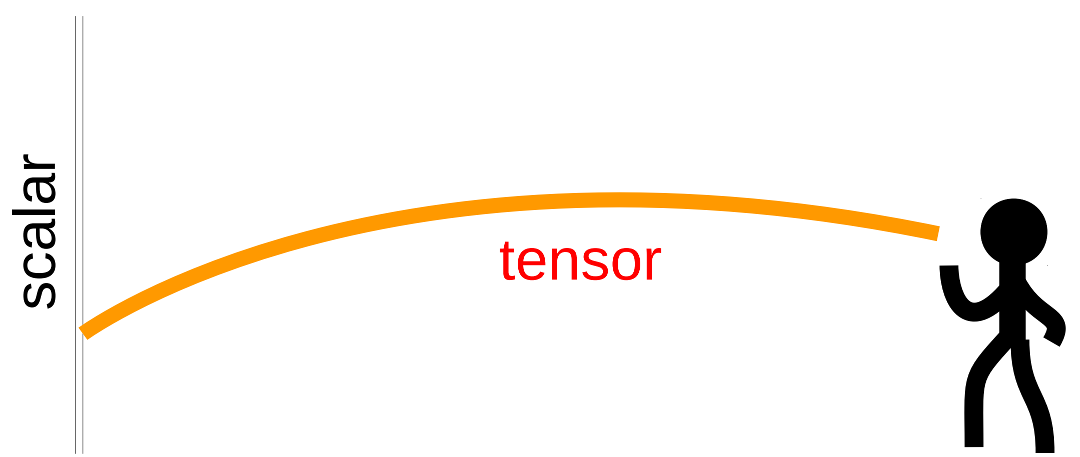
(Single-template fitting)

$a(p) \equiv a \times 10^p$

	Local	Equilateral	Orthogonal	Folded
$1^{\rm st}$-order x ISW	-2.16(-1)	1.58(-2)	3.78(-1)	-5.78(-1)
$1^{\rm st}$-order x Lensing	-6.70(-1)	1.17(-1)	1.69(0)	-2.57(0)
$1^{\rm st}$-order x Time-delay	1.21(-2)	1.07(-3)	2.70(-3)	-3.78(-3)
$1^{\rm st}$-order x Deflection	-1.53(-2)	-5.50(-2)	1.39(-1)	-2.34(-1)

m320c

scalar tensor



$f_{\rm NL}$ of $\langle\Theta_{\rm T}^{(Tens\times Scal)}(\mathbf{k}_1)\Theta_{\rm T}^{(Scal)}(\mathbf{k}_2)\Theta_{\rm T}^{(Tens)}(\mathbf{k}_3)\rangle$

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PRELIMINARY

(Single-template fitting)

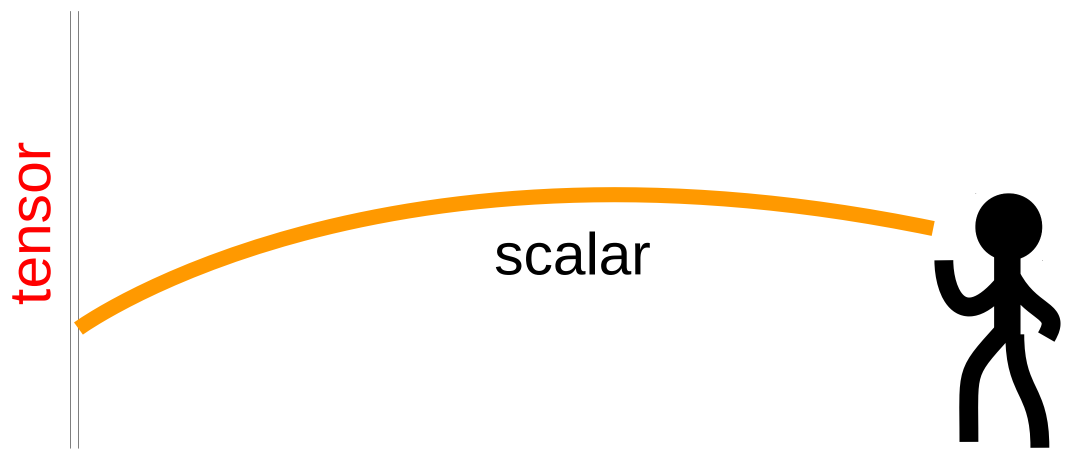
$r = 1$

$a(p) \equiv a \times 10^p$

	Local	Equilateral	Orthogonal	Folded
1st-order x ISW	3.38(-7)	-3.32(-5)	-1.34(-5)	8.46(-6)
1st-order x Lensing	1.33(-3)	-3.06(-2)	-2.45(-2)	2.66(-2)
1st-order x Time-delay	-5.12(-5)	-3.13(-4)	1.18(-4)	-2.96(-4)
1st-order x Deflection	-9.25(-5)	3.58(-3)	2.96(-3)	-3.25(-3)

m320c

tensor scalar



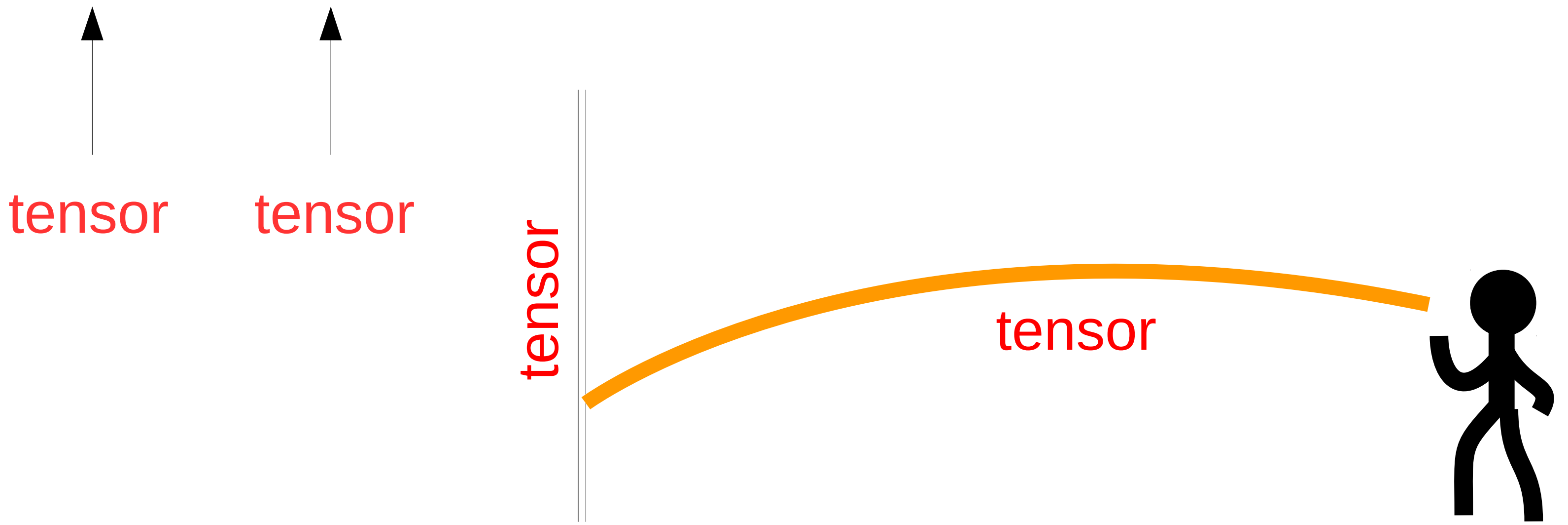
$f_{\rm NL}$ of $\langle \Theta_{\rm T}^{Tens\times Tens}(\mathbf{k}_1)\Theta_{\rm T}^{(Tens)}(\mathbf{k}_2)\Theta_{\rm T}^{(Tens)}(\mathbf{k}_3)\rangle$

PRELIMINARY

$r=1$ (Single-template fitting) $a(p)\equiv a\times 10^p$

	Local	Equilateral	Orthogonal	Folded
1st-order x ISW	-6.38(-5)	-6.39(-4)	7.58(-4)	-1.40(-3)
1st-order x Lensing	-7.72(-5)	1.34(-3)	1.02(-3)	-1.08(-3)
1st-order x Time-delay	-2.16(-6)	5.75(-5)	-1.63(-4)	2.72(-4)
1st-order x Deflection	1.83(-5)	-3.54(-3)	-1.82(-3)	1.50(-3)

m320c



f_{NL} including both scalar and tensor

PRELIMINARY

(Single-template fitting)

$r = 1$ $a(p) \equiv a \times 10^{p}$

	Local	Equilateral	Orthogonal	Folded
Scalar x Scalar	9.05(0)	1.46(0)	-2.94(+1)	4.59(+1)
Scalar x Tensor	-8.89(-1)	7.83(-2)	2.21(0)	-3.39(0)
Tensor x Scalar	1.18(-3)	-2.74(-2)	-2.14(-2)	2.30(-2)
Tensor x Tensor	-1.25(-4)	-2.78(-3)	-2.04(-4)	-7.07(-4)

m320c

On LSS + ISW

Geodesics

- [Scalar] x [Scalar] is dominated.
- tensor contribution is at most 10% even if $r = 1$

Summary : current status of cmb2nd

New CMB Boltzmann code implemeting 'curve'-of-sight formulas

* 2nd-order Implicit Runge-Kutta method
* 1st-order scalar and tensor are completed. (TT, TE, EE, BB)
* Different schemes from CAMB, but consistent within O(1)%
* Implemented “curve”-of-sight formulas (2nd-order line-of-sight) for scalar and tensor temperature fluctuations.
* Implemented Komatsu-Spergel bispectrum estimator.
* Implemented remapping approximation.

Summary : current status of cmb2nd

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Our findings :

* Computed new geodesic contribution, “Deflection”,
  but yields no significant contributions to the scalar perturbations.

* Confirmed that the lensing contribution is dominated.

* Justified the remapping formula. (for [Scalar] x [Scalar])

* Confirmed that the tensor contribution can be neglected
  in the analysis of CMB bispectrum

Summary : To-do and application ?

To-do

- Implement the curve-of-sight formulas for polarisation

- Implement the 2nd-order Boltzmann equations
  (cf. **SONG, CosmoLib2**)

SONG : Petinarri et al., JCAP 1304 (2013) 003
CosmoLib2 : Huang, Vernizzi, PRL 110 (2013) 101303

Applications ?

- 2nd-order gravitational waves, magnetic field from $[\text{1st-order}]^2$

e.g. Saga et al., PRD 91 (2015) 024030
Saga et al., PRD 91 (2015) 123510

- Spectral distortion at 2^{nd}-order level

- COS including large-scale structure
  (non-linear evolution of density perturbations)

e.g. Morazzi et al., arXiv:1612.07263
Morazzi et al., arXiv:1612.07650