研究課題名：新しい重力理論の探求と修正重力理論の観測的検証に関する理論研究 (公募研究)

Extended vector-tensor theories

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in collaboration with

Rampei KIMURA & Daisuke YOSHIDA (TiTech)

based on : 1608.07066 (published in JCAP)

vector field theory
on **curved** spacetime
with **degenerate** kinetic matrix

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goal

✓ Recently, in the field of cosmology or gravitation, **degenerate** theories have been intensively studied.

- **what** is **degenerate** kinetic matrix ?
- **why** do we need to impose such **degeneracy** ?
- **how** can **degenerate** theories be constructed ?

✓ Without introducing field theory on curved spacetime, we can (mostly) understand the essential part of degenerate theory in terms of analytical mechanics.

goal (in this talk..)

15:50 - 16:30 poster session + break

16:30 -18:10 A03 Dark Energy (Chair: M.Sasaki)

16:30-17:05 Shin'ichi Nojiri "Updates in A03 group & Simple Model of Cosmological Constant"

17:05–17:15 Jean-Baptiste Durrive "Constraining scalar field dark energy models and its future prospects"

17:15-17:30 Atsushi Nishizawa "重力波伝播による重力理論の検証"

17:30-17:50 Chulmoon Yoo "On observational effects of spherical inhomogeneity around us"

17:50-18:10 Hayato Motohashi "Degenerate theories with higher derivatives"

18:30−20:30 懇親会

⇒ in the meantime, we would like to focus on explaining

why we apply **that idea** to **vector field** ???

motivation ①

✓ to explain primordial/current accelerated expansion of the universe

- Λ ?? (why so small ? why that value ?)
- ϕ ?? = **scalar**-tensor theory

  e.g. canonical, k-essence, Horndeski, Beyond Horn, …

- $g_{\mu\nu}$?? (change of gravity law) = **tensor** theory

  e.g. (dRGT) massive gravity, bi-gravity…

$\Rightarrow$ decoupling limit of massive gravity (or bi-gravity)

= described by **scalar** & **vector** fields

motivation ②

✓ unique prediction from vector-field during inflation

$$\mathcal{L} = (GR) + (scalar) - \frac{1}{4} f^2(\phi) F_{\mu\nu} F^{\mu\nu}$$

Watanabe, Kanno Soda. (2009)

★ first (**healthy**) counter example for cosmic no-hair conjecture

★ predict **statistically anisotropic** power spectrum :

$$P(\vec{k}) = P(k)\left[1 + g_*(\hat{k}\cdot\hat{v})^2\right]$$

v : privileged direction

$$g_* = 0.002^{+0.016}_{-0.016}$$

Kim & Komatsu (2013)

degenerate theory

or

degenerate kinetic matrix

$\Leftrightarrow$ **magic** to introduce the kinetic term

for ***non-dynamical*** d.o.f.(s)

Maxwell & Proca

✓ Aμ has 4 components = (in maximum) **4 d.o.f.s**

✓ In Maxwell theory, **A0 is non-dynamical** (no kinetic term)

$$-F_{\mu\nu}F^{\mu\nu} \sim \vec{E}^2 - \vec{B}^2 \sim \dot{A}_i^2 - A_{[i\ ,j]}^2$$

(gauge sym. kills longitudinal mode → 2 d.o.f.s)

✓ **Proca theory** (+m^2A^2, no gauge sym.) ⇔ 3 d.o.f.s

In Maxwell & Proca, **A0 is non-dynamical = no kinetic term**

→ **with magic (degeneracy)**, kinetic term for **A0 ??**

Kimura, **AN**, Yoshida [1608.07066 (2016)]

Extended vector-tensor

✓ action with two first derivative of A_μ **& 4D general covariance**

$$\mathcal{L} = f(Y)\,R + C^{\mu\nu\rho\sigma}\,(\nabla_\mu A_\nu)\,(\nabla_\rho A_\sigma)$$

sym.

asym.

$$\begin{aligned} C^{\mu\nu\rho\sigma} = {} & \alpha_1 g^{\mu(\rho}g^{\sigma)\nu} + \alpha_2 g^{\mu\nu}g^{\rho\sigma} + \frac{1}{2}\alpha_3(A^\mu A^\nu g^{\rho\sigma} + A^\rho A^\sigma g^{\mu\nu}) \\ & + \frac{1}{2}\alpha_4(A^\mu A^{(\rho}g^{\sigma)\nu} + A^\nu A^{(\rho}g^{\sigma)\mu}) + \alpha_5 A^\mu A^\nu A^\rho A^\sigma + \alpha_6 g^{\mu[\rho}g^{\sigma]\nu} \\ & + \frac{1}{2}\alpha_7(A^\mu A^{[\rho}g^{\sigma]\nu} - A^\nu A^{[\rho}g^{\sigma]\mu}) + \frac{1}{4}\alpha_8(A^\mu A^\rho g^{\nu\sigma} - A^\nu A^\sigma g^{\mu\rho}) + \frac{1}{2}\alpha_9 \varepsilon^{\mu\nu\rho\sigma}. \end{aligned}$$

f, α_i : functions of $Y = A_\mu A^\mu$

☝ C is ***not symmetric*** under $\mu \leftrightarrow \nu$ & $\rho \leftrightarrow \sigma$ (cf. $\nabla_\mu\nabla_\nu\phi$)

degeneracy condition

✓ Action after ADM decomposition (separate time & space) :

$$\mathcal{L}_{\rm kin} = \underbrace{\mathcal{A}\dot{A}_*^2 + 2\mathcal{B}^i \dot{A}_* \dot{\hat{A}}_\mu + 2\mathcal{C}^{\mu\nu}\dot{A}_* K_{\mu\nu}}_{\text{kinetic term for A0}} + \mathcal{D}^{\mu\nu}\dot{\hat{A}}_\mu \dot{\hat{A}}_\nu + 2\mathcal{E}^{\mu\nu\rho}\dot{\hat{A}}_\mu K_{\nu\rho} + \mathcal{F}^{\mu\nu\rho\sigma}K_{\mu\nu}K_{\rho\sigma},$$

$$A_* \, (= n^\mu A_\mu) \sim A_0$$

☝ in general, 6 d.o.f.s = 4 (A_μ) + 2 (GW)

✓ **degeneracy** cond. ⇔ making **A0 non-dynamical** :

$$0 = |\mathcal{M}_{\rm kin}| = \mathcal{D}_0 + \mathcal{D}_2\, A_*^2 + \mathcal{D}_4\, A_*^4$$

$$0 = \mathcal{D}_0 \propto (\alpha_1 + \alpha_2)\, F(\alpha_i\,, f)$$

case A : $\alpha_1 + \alpha_2 = 0$

case B : $F = 0$ $(f \neq 0)$

case C : $F = 0$ $(f = 0)$

$$\mathcal{L}_{\rm kin} \to (\dot{A}_* + \dot{\hat{A}}_\mu)^2 + (\dot{A}_* + K_{\mu\nu})^2$$

example of degenerate theory

$$\mathcal{L} = f(Y)\,R + C^{\mu\nu\rho\sigma}\,(\nabla_\mu A_\nu)\,(\nabla_\rho A_\sigma)$$

$$\begin{aligned}C^{\mu\nu\rho\sigma} &= \alpha_1 g^{\mu(\rho}g^{\sigma)\nu} + \alpha_2 g^{\mu\nu}g^{\rho\sigma} + \frac{1}{2}\alpha_3(A^\mu A^\nu g^{\rho\sigma} + A^\rho A^\sigma g^{\mu\nu}) \\ &+\frac{1}{2}\alpha_4(A^\mu A^{(\rho}g^{\sigma)\nu} + A^\nu A^{(\rho}g^{\sigma)\mu}) + \alpha_5 A^\mu A^\nu A^\rho A^\sigma + \alpha_6 g^{\mu[\rho}g^{\sigma]\nu} \\ &+\frac{1}{2}\alpha_7(A^\mu A^{[\rho}g^{\sigma]\nu} - A^\nu A^{[\rho}g^{\sigma]\mu}) + \frac{1}{4}\alpha_8(A^\mu A^\rho g^{\nu\sigma} - A^\nu A^\sigma g^{\mu\rho}) + \frac{1}{2}\alpha_9\varepsilon^{\mu\nu\rho\sigma}.\end{aligned}$$

- ✓ an example :

$$f=1 \quad 2\alpha_6 + Y\alpha_7 = 0 \quad \alpha_1 = \frac{-8(2\alpha_2+Y\alpha_3) - Y(4+4Y\alpha_2 - Y^2\alpha_3)\alpha_8}{2Y^2\alpha_8}$$

$$\alpha_4 = \frac{4(1+Y\alpha_2)}{Y^2} - \alpha_3 + \frac{8(2\alpha_2+Y\alpha_3)}{Y^3\alpha_8} + \alpha_8 - \frac{Y^2\alpha_8^2}{8}$$

$$\alpha_5 = \frac{-2+Y^2\alpha_3}{Y^3} - \frac{4(2\alpha_2+Y\alpha_3)}{Y^4\alpha_8} - \frac{\alpha_8}{Y} + \frac{Y\alpha_8^2}{8} + \frac{12(2\alpha_2+Y\alpha_3)}{Y^2(Y^2\alpha_8-8)}$$

$g_{\mu\nu}$ & A_μ transformations

- ✓ consider metric & vector field transformations

$$g_{\mu\nu} \to \Omega(Y) g_{\mu\nu} + \Gamma(Y) A_\mu A_\nu \quad \textbf{\&} \quad A_\mu \to \Upsilon(Y) A_\mu$$

☝ classification is stable : case A ➞ case A

(as far as the tr. is regular) $\alpha_1+\alpha_2=0$

- ✓ Rewriting Maxwell theory [$g_{\mu\nu} \to g_{\mu\nu} - 2A_\mu A_\nu$] :

$$R - F_{\mu\nu}^2 \to \sqrt{1-2Y}\,R - \frac{1}{\sqrt{1-2Y}}\left[(\nabla_\mu A^\mu)^2 - (\nabla_\mu A_\nu)^2\right]$$

- ✓ no U(1) gauge symmetry : $A_\mu \to A_\mu + \nabla_\mu \psi$
- ✓ same # of d.o.f.s ➞ new gauge symmetry ??

summary

- ✓ We have constructed degenerate vector-tensor theory that includes two first derivative of vector field.

- ✓ New theory for massive vector field includes 5 d.o.f.s = 3 massive vector in A_μ & 2 GW in $g_{\mu\nu}$

- ✓ Applying transformations of metric & vector field, we have investigated the stability of classification

- ✓ no instabilities in vector theory (?) ⇔ scalar theory

Thank you
for your attention