# Formation of Primordial Black Holes in Double Inflation

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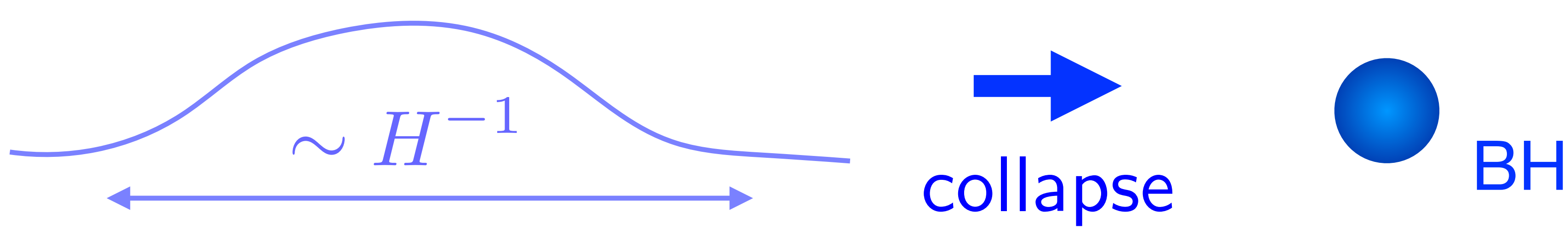
Based on MK Mukaida Yanagida, arXiv:1605.04974
MK Kusenko Tada Yanagida arXiv:1606.07631
Inomate MK Mukaida Tada Yanagida,
arXiv:1611.06130, 1701.02544

# 1. Introduction

- Primordial Black Holes (PBHs) introduced about 50 years ago
  Zeldovich-Novikov (1967) Hawking (1971)
- PBHs with $M_{PBH} > 10^{15}$ g survive today without evaporating
  - Give a significant contribution to dark matter
  - Account for GW events detected LIGO recently
- PBHs can be formed by gravitational collapse of over-density region with Hubble radius in the early universe



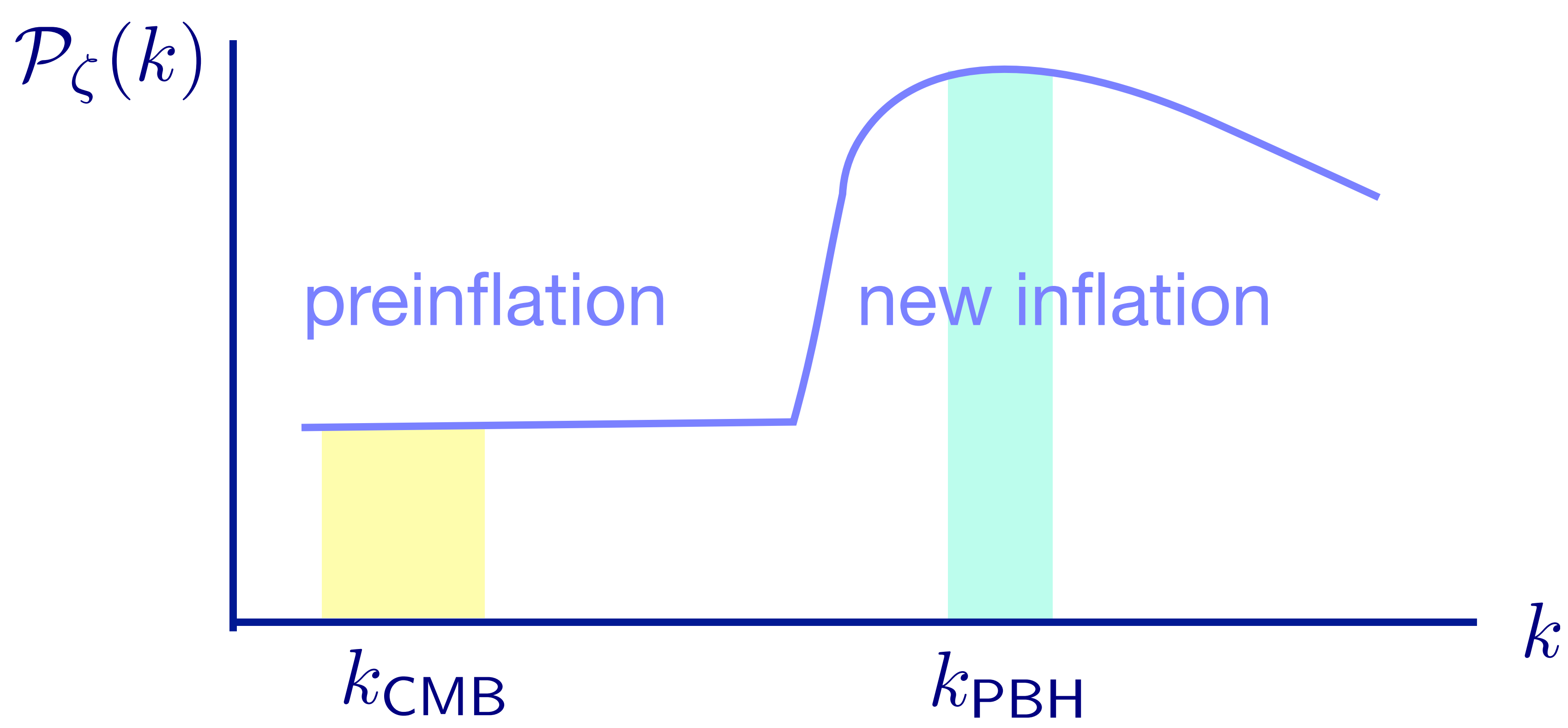

- Large density fluctuations δ with O(0.1) are required for PBH formation but δ ~ O($10^{-5}$ ) on CMB scale

→ need to break scale invariance of spectrum of density fluctuations

- We consider double inflation (preinflation+new inflation)

MK, Sugiyama, Yanagida (1998)

  - Preinflation (no specific model is required) accounts for perturbations on large scales observed by Planck
  - New inflation ( after preinflation) with e-fold $N_{new} < 50$ produces large curvature perturbations on small scales



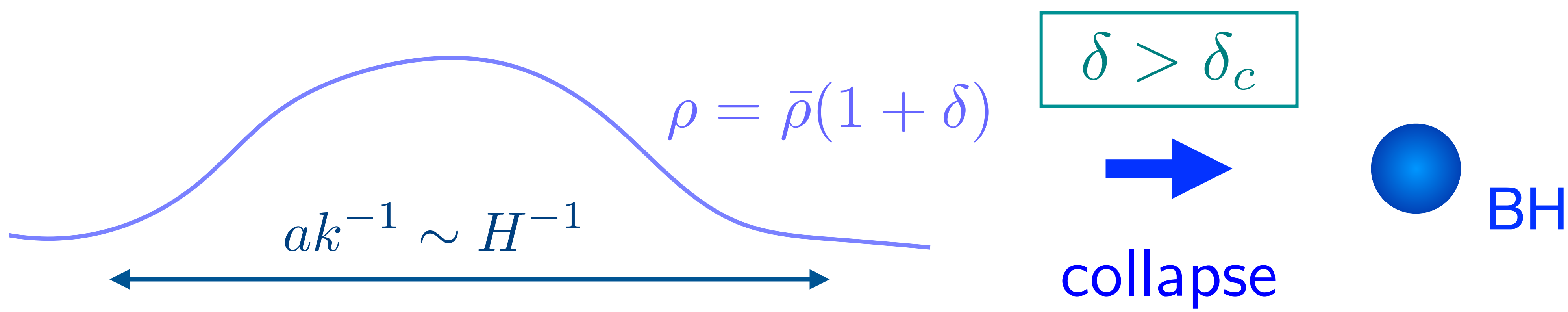

  - Large curvature perturbations lead to PBH formation when they re-enter the horizon after inflation

# Today's Talk

1. Introduction
2. PBH formation in radiation dominated universe
3. New inflation model and PBH formation
4. Observational constraints
5. PBHs as dark matter
6. PBHs for LIGO gravitational wave events
7. Conclusion

## 2. PBH formation in radiation dominated universe

- After inflation, when density fluctuations reenter the horizon, region with Hubble radius collapses to form a PBH if its over-density is higher than $\delta_c$ ($\approx$0.3)




- PBH mass (~ Horizon mass)

$$\boxed{M_{\mathrm{PBH}} \simeq 3.6 M_\odot \left(\frac{\gamma}{0.2}\right)\left(\frac{k}{10^6\mathrm{Mpc}^{-1}}\right)^{-2} \simeq 4.5 M_\odot \left(\frac{\gamma}{0.2}\right)\left(\frac{T}{0.1\mathrm{GeV}}\right)^{-2}}$$

$M_{\mathrm{PBH}} = \gamma M_H$ (horizon mass)

[ $\gamma = 0.2$ Carr (1975) ]

- PBH abundance is estimated by Press-Schechter formalism
- PBH mass fraction β=$\rho_{\mathrm{PBH}}$(M)/ρ (assuming Gaussian perturbations )

$$\boxed{\beta(M) = \int_{\delta_c} d\delta \frac{1}{\sqrt{2\pi\sigma^2(k)}} \exp\left(-\frac{\delta^2}{2\sigma^2(k)}\right)}$$

$$\boxed{\mathcal{P}_\zeta(k)}$$

$\mathcal{P}_\zeta(k) \sim O(10^{-2})$ for PBH formation

$$\sigma^2(k) = \int d\log k' \, W^2(k'/k) \frac{16}{81} (k'/k)^4 \mathcal{P}_\zeta(k')$$

$\sigma^2(k)$ : variance of the comoving density perturbation coarse-grained on $k^{-1}$

- Present PBH fraction to DM

$$\boxed{f_{\mathrm{PBH}}(M) = \frac{\Omega_{\mathrm{PBH}}(M)}{\Omega_{\mathrm{DM}}} \simeq 1.3 \times 10^8 \, \beta(M) \left(\frac{M_{\mathrm{PBH}}}{M_\odot}\right)^{-1/2}}$$

[ fraction per log M ]

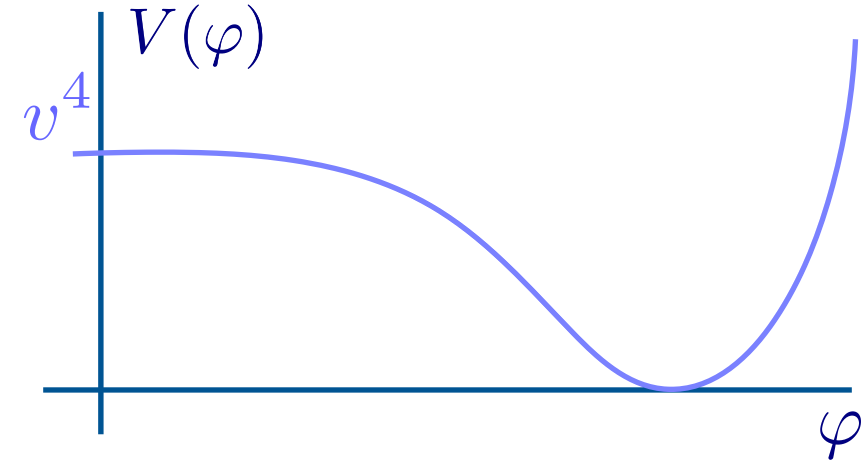
$$\boxed{f_{\mathrm{PBH,tot}} = \int d\log M \, f_{\mathrm{PBH}}(M)}$$

# 3. New Inflation after Preinflation

- Potential for new inflation

$$\boxed{V(\varphi) = (v^2 - g\varphi^n)^2 - \varepsilon v^4 \varphi - \frac{1}{2}\kappa v^4 \varphi^2}$$

Hubble $\boxed{H_{\mathrm{inf}} \simeq \frac{v^2}{\sqrt{3}}}$ $n = 3, 4, \cdots$ $M_p = 1$




- Linear term $\varepsilon \ll 1$
  - amplitude of curvature perturbations
  - determining initial value (next slide)
- Quadratic term $\kappa \sim O(0.1)$
  - spectrum index (shape of power spectrum)
- Slow-roll parameters

$$\epsilon = \frac{1}{2}\left(\frac{V'}{V}\right)^2 = \frac{1}{2}\left(-\varepsilon - \kappa\varphi - 2ng\frac{\varphi^{n-1}}{v^2} + \cdots\right)^2$$

$$\eta = \frac{V''}{V} = -\kappa - 2n(n-1)g\frac{\varphi^{n-2}}{v^2} + \cdots$$

$\epsilon \ll 1 \quad \eta \ll 1 \quad \Rightarrow$ inflation

- Curvature perturbation

$$\boxed{\mathcal{P}_\zeta^{1/2} = \frac{H_{\rm inf}}{2\pi}\frac{1}{\sqrt{2\epsilon}} \simeq \frac{1}{2\sqrt{3}\pi}\frac{v^2}{\varepsilon + \kappa\varphi}} \sim \frac{1}{2\sqrt{3}\pi}\frac{v^2}{\varepsilon} \qquad (\varphi \lesssim \varepsilon)$$

$\mathcal{P}_\zeta \sim O(0.01)$ for $\varepsilon \sim v^2$ → PBH formation

- Initial value for the inflaton $\varphi$ ?

  - Interaction with the inflaton $\chi$ of preinflation

→ Hubble induced mass term

$$\Delta V = \frac{1}{2} c_{\rm pot} V_\chi \varphi^2$$

$$\mathcal{L}_{\rm kin} = \frac{1}{2}\left(1 - \frac{1}{2}c_{\rm kin}\varphi^2\right)\partial_\mu\chi\partial^\mu\chi$$

→ $\boxed{V_H = cH^2\varphi^2}$

→ $V(\varphi) = cH^2\varphi^2 - \varepsilon v^4\varphi + \cdots \Rightarrow \varphi \simeq \dfrac{\varepsilon v^4}{cH^2}$

  - Just before new inflation $H \sim v^2$

$\boxed{\varphi_{\rm ini} \sim \varepsilon}$ → $\mathcal{P}_\zeta^{1/2} \sim \dfrac{1}{2\sqrt{3}\pi}\dfrac{v^2}{\varepsilon} \sim O(0.1)$ for $\varepsilon \sim v^2$

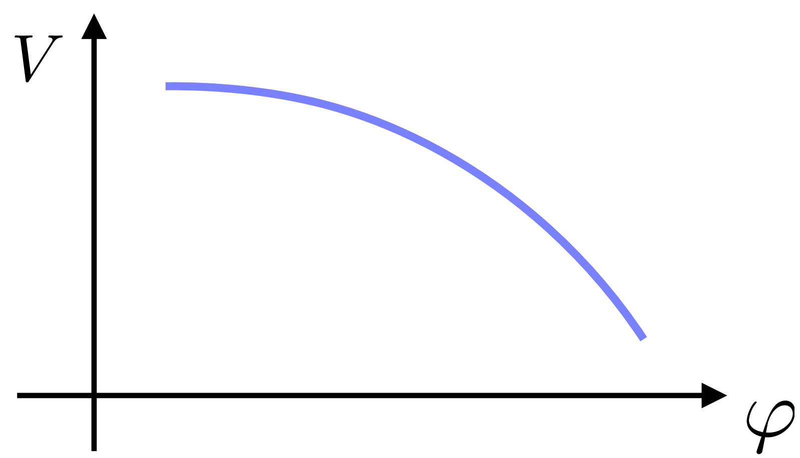
# Shape of mass spectrum of PBHs

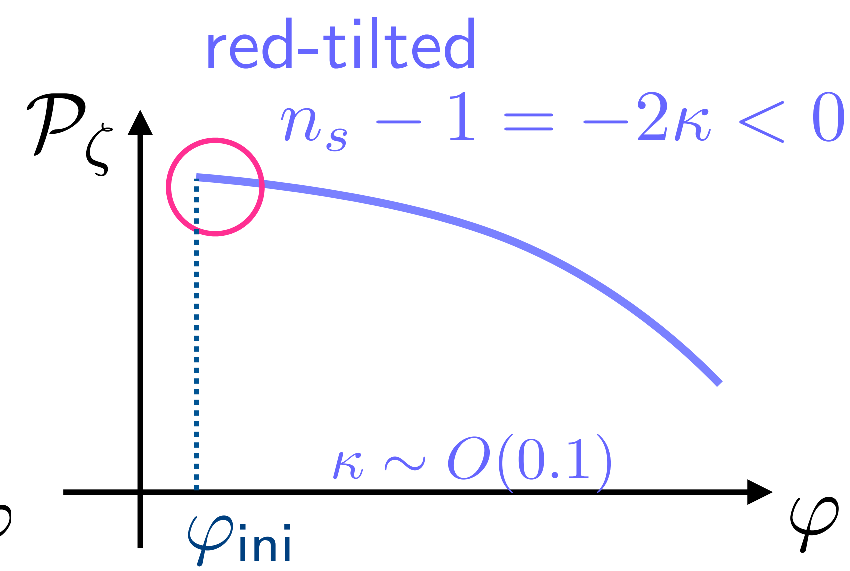
- Spectral index of curvature perturbations

$$\boxed{n_s - 1 \simeq -3\varepsilon^2 - 2\kappa - 4n(n-1)g\frac{\varphi^{n-2}}{v^2}}$$

$$\varepsilon^2 \sim v^4 \ll 1$$

- κ > 0
  - power spectrum takes the largest value at $\varphi \sim \varphi_{\rm ini}$

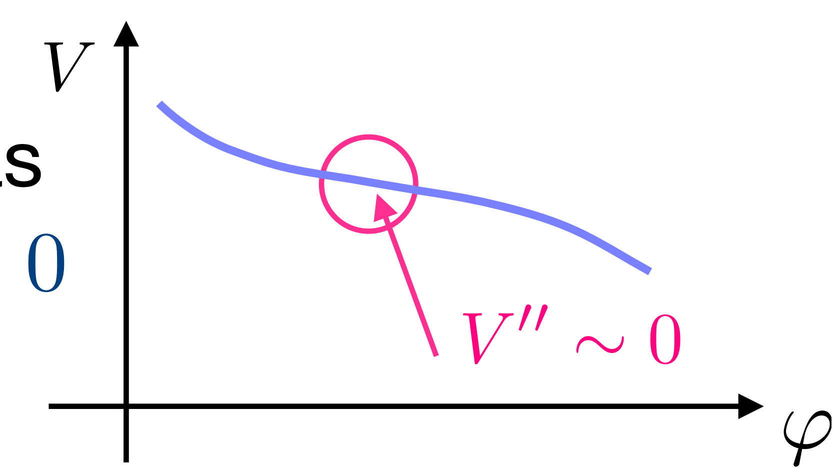


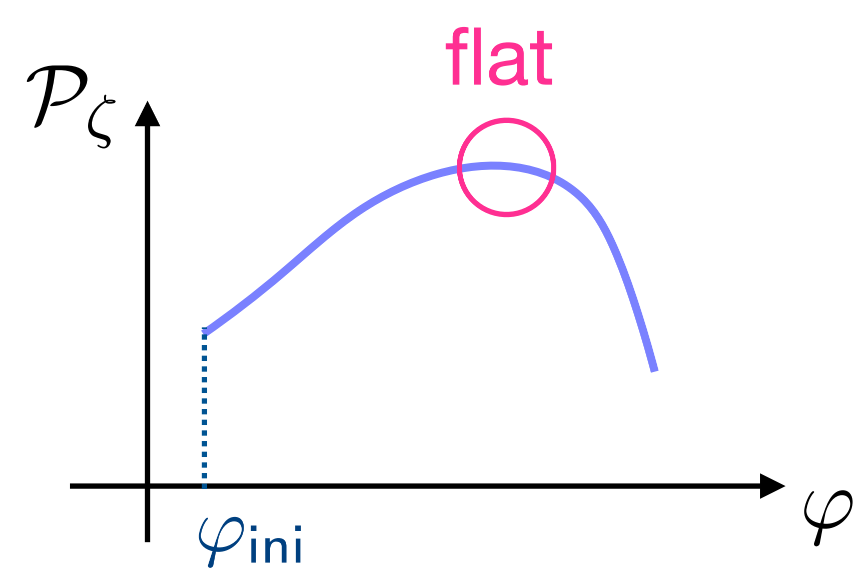




➡ PBHs with sharp mass spectrum

- κ < 0
  - power spectrum has a peak when $V'' \sim 0$

$\Rightarrow \ n_s - 1 = 0$




➡ PBHs with broad mass spectrum

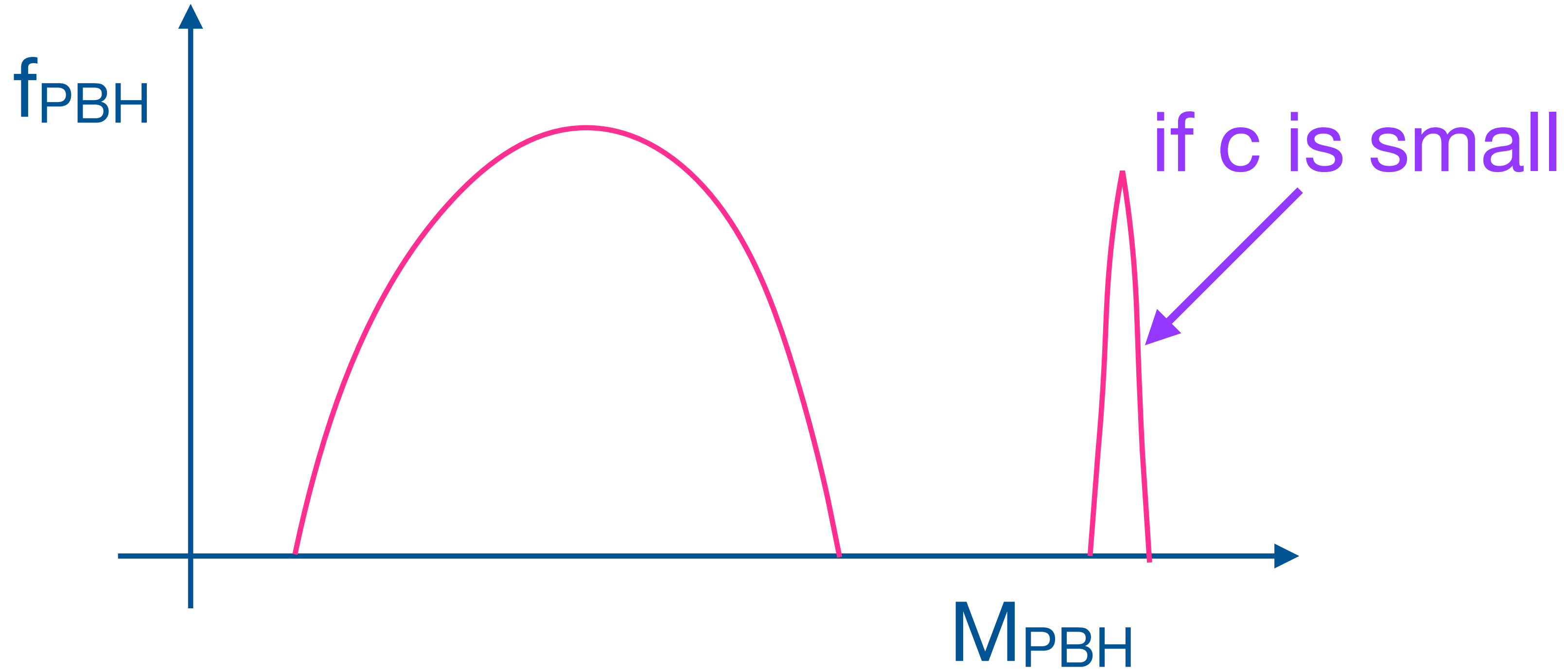
## Remark

- Perturbation modes that exit around the end of preinflation and reenter horizon near the beginning of new inflation

  $\delta\varphi$ during preinflation ➡ decreases due to $cH^2\varphi^2$

- Curvature perturbations generated at beginning of new inflation are affected by Hubble induced mass

- If the Hubble induced mass is suppressed after preinflation the curvature perturbations are relatively enhanced and this produces double peaks in PBH mass spectrum for $\kappa < 0$



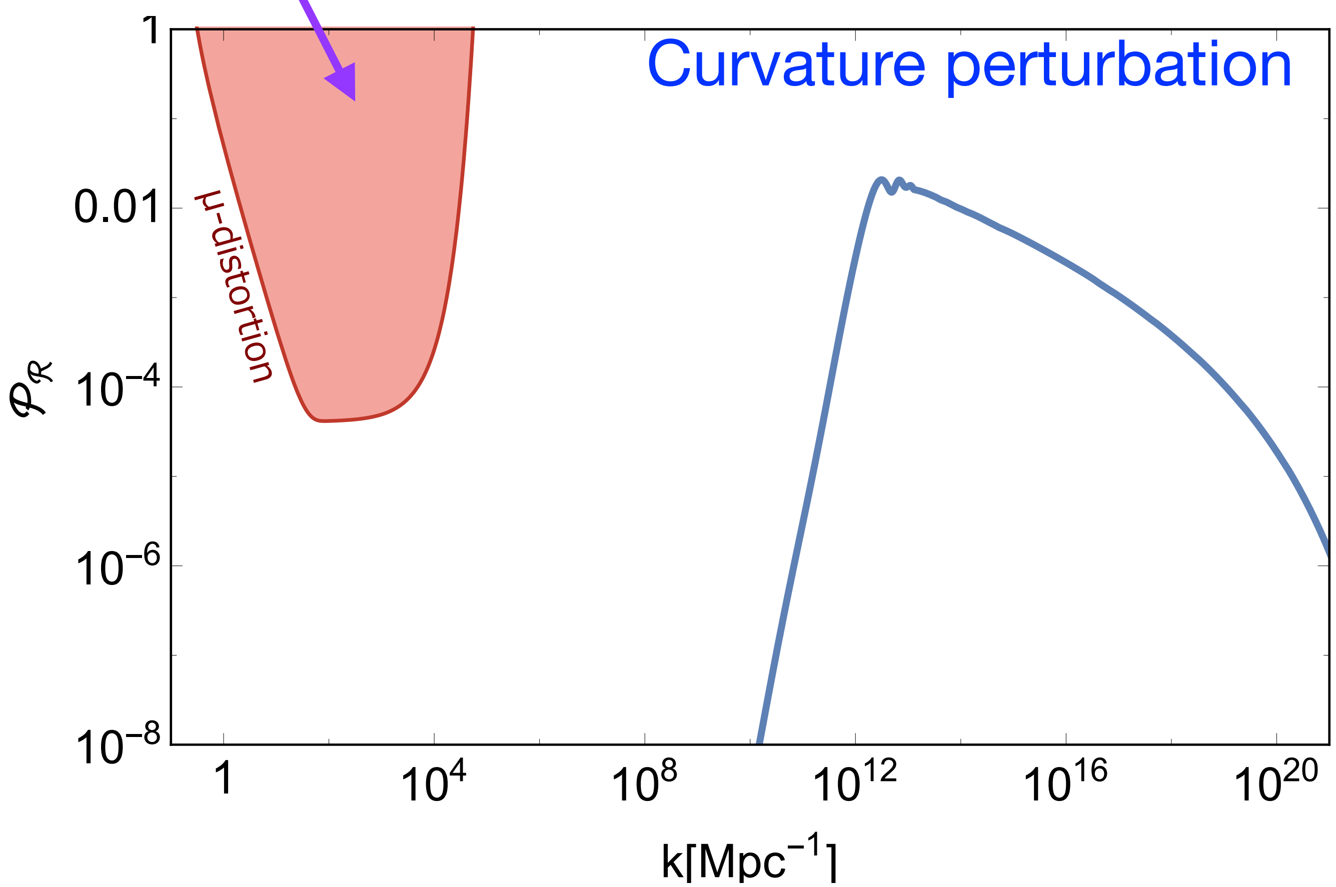
# Example 1: κ > 0

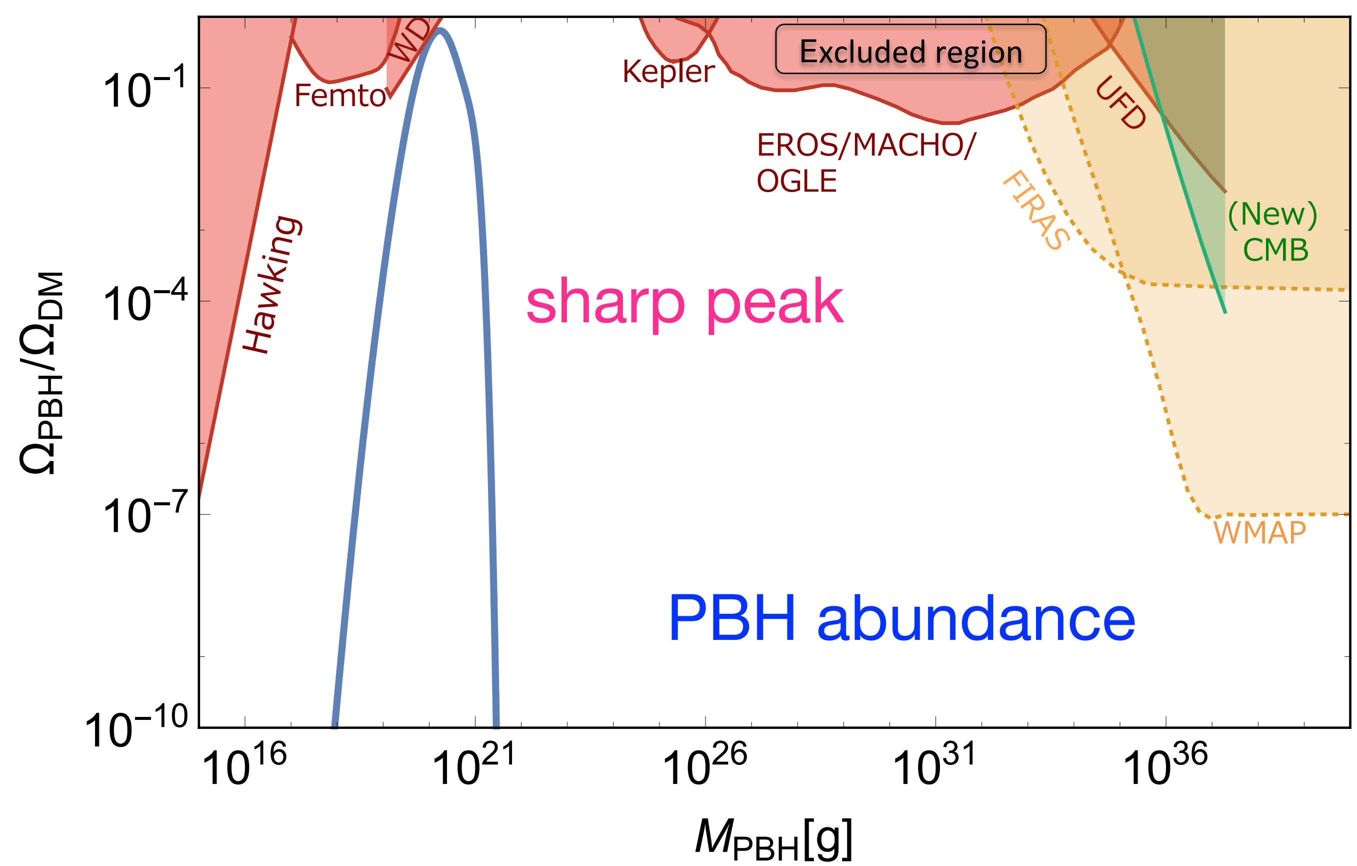
- We made full numerical calculation including 1st order perturbations of inflaton fields and metrics
- We adopt chaotic inflation as pre-inflation
- Sharp mass spectrum

$$\Omega_{\rm PBH}/\Omega_{\rm DM} = \int d\log M\, f_{\rm PBH}(M) \simeq 1$$

μ distortion of CMB due to Silk damping

courtesy of K. Inomata

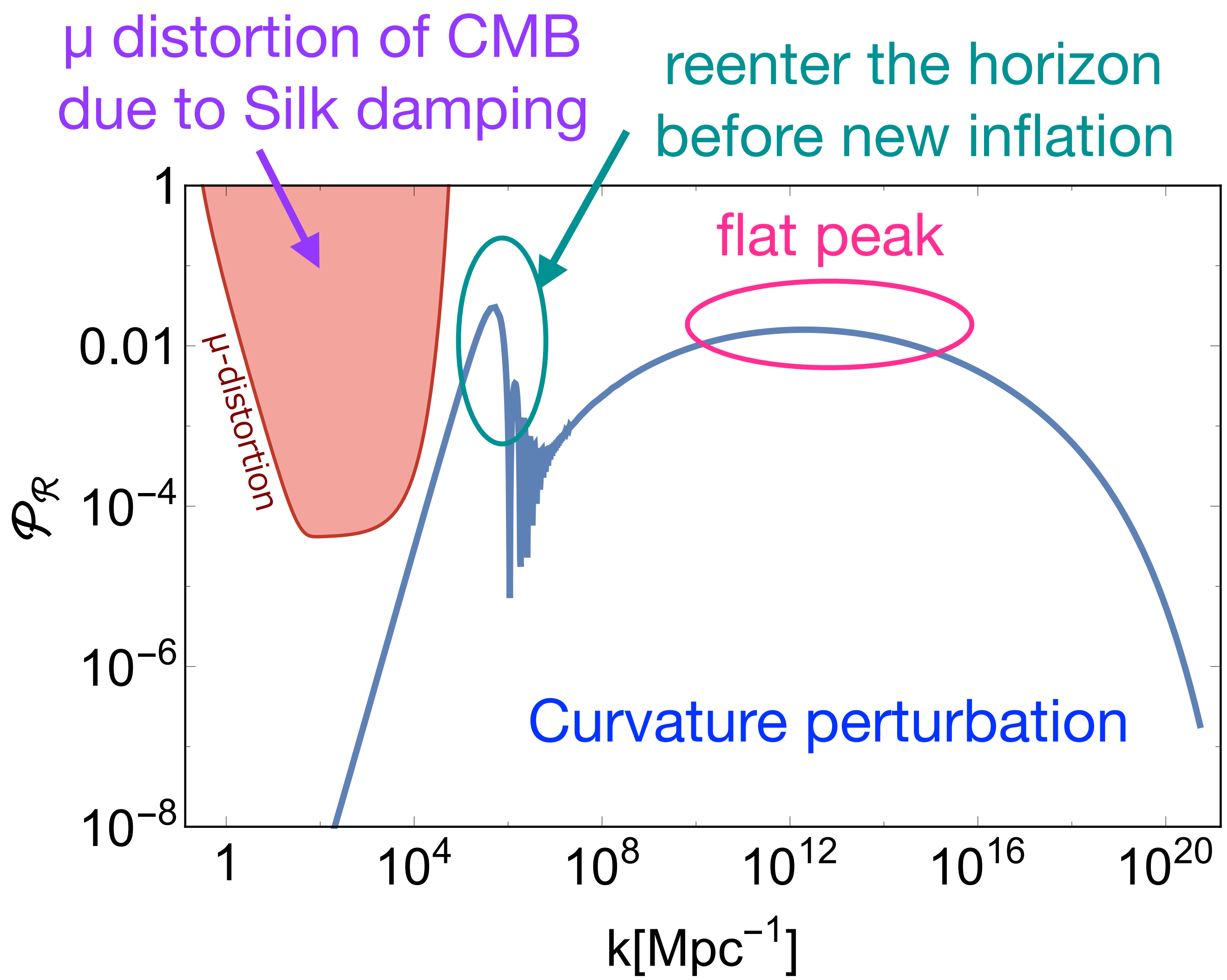


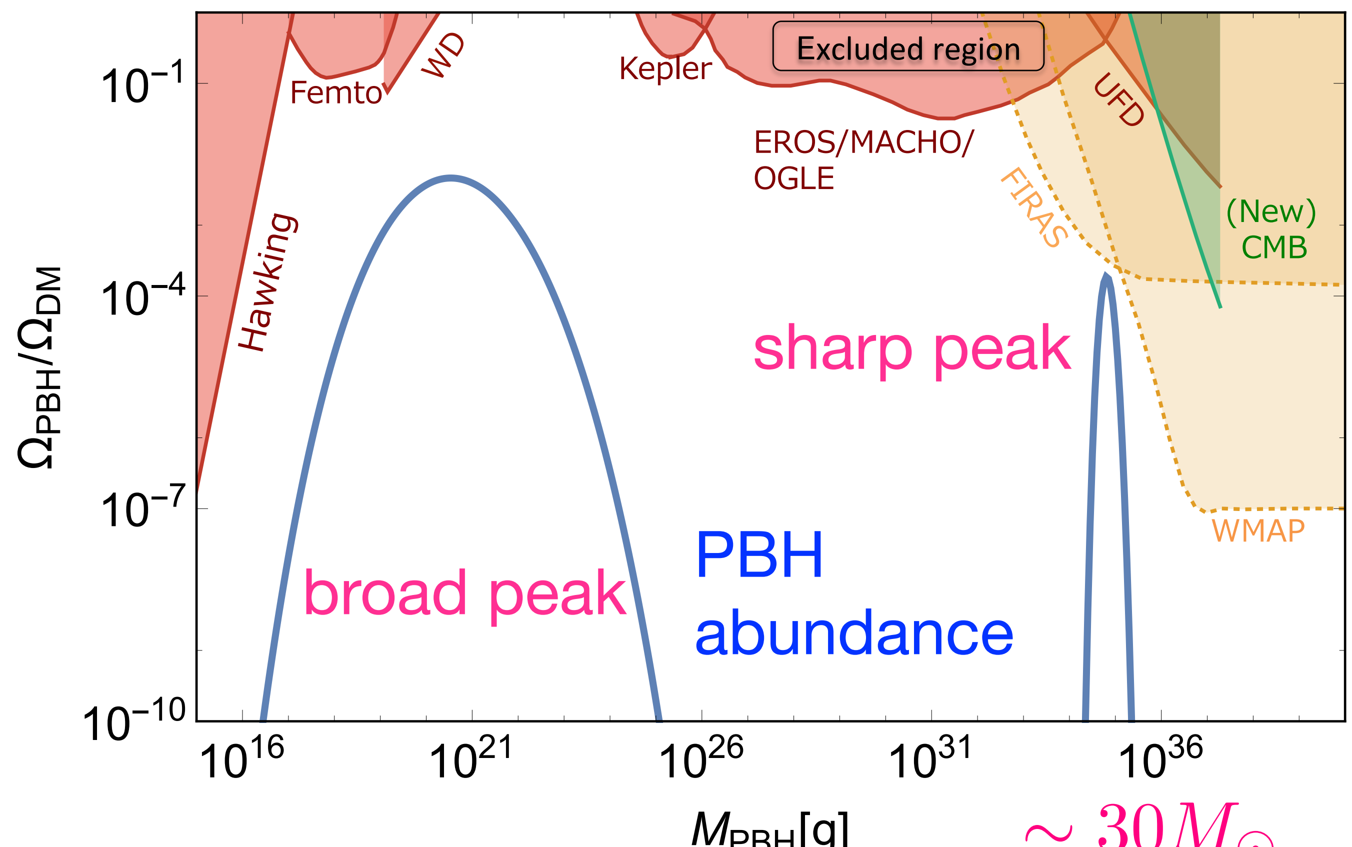

$v = 10^{-3}$ $\kappa = 0.13$ $\varepsilon = 6.6\times10^{-7}$ $g = 5.6\times10^{-4}$ $c_{\rm pot} = 1$ $c_{\rm kin} = 0$

# Example 2: κ < 0

- Broad mass spectrum
- Another sharp peak due to modes that reenter the horizon just before new inflation
( for parameters we take Hubble induced mass suppressed after preinflation ) $V_H \sim (c_{\text{pot}} + c_{\text{kin}})\varphi^2 \sim 0$




courtesy of K. Inomata

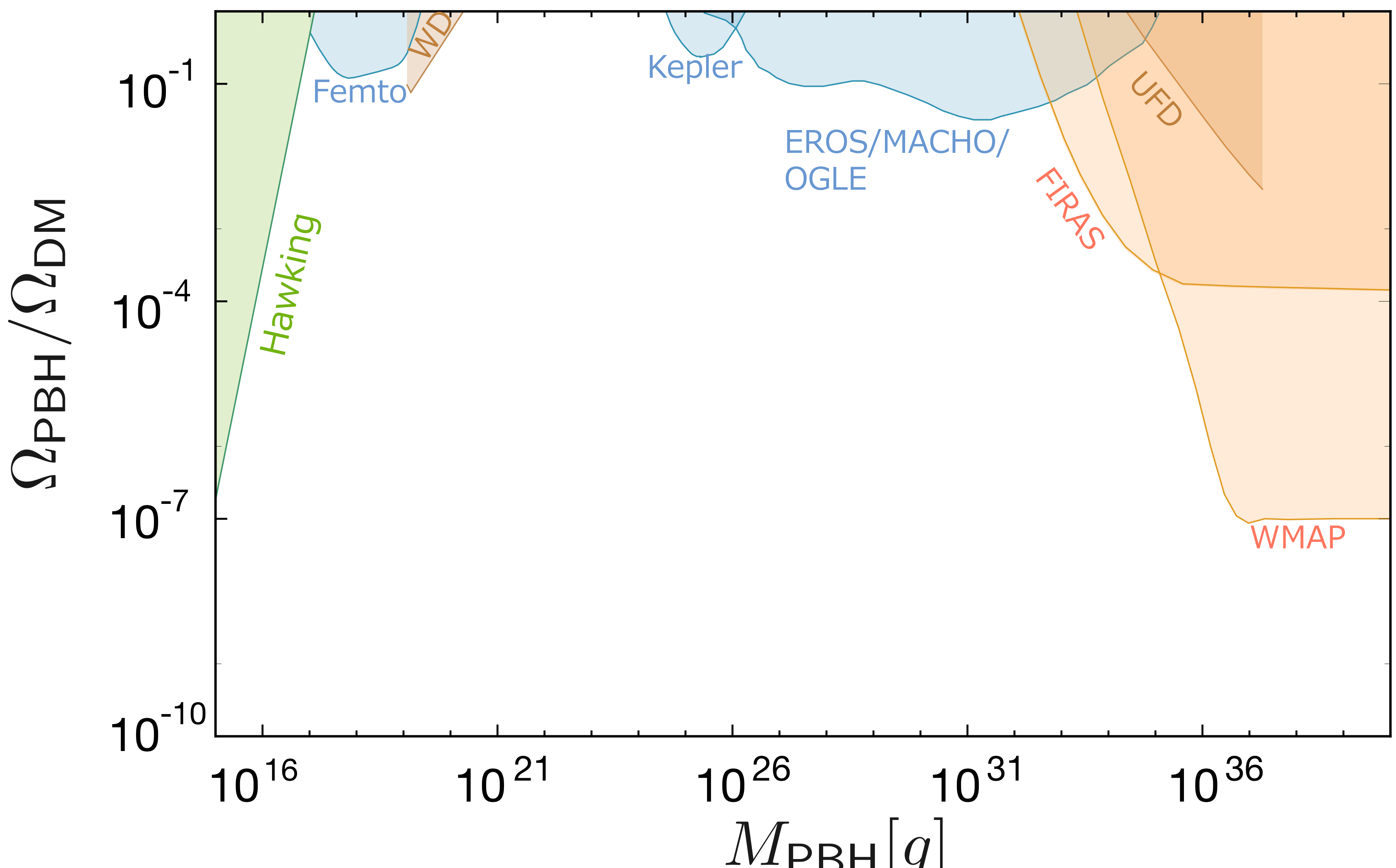



$\sim 30 M_\odot$

$v = 10^{-4}$ $\kappa = -0.64$ $\varepsilon = 10^{-7}$ $g = 1.8 \times 10^{-3}$ $c_{\text{pot}} = 0.68$ $c_{\text{kin}} = -0.69$

# 4. Observational Constraints

- Gamma rays emission from evaporating PBHs
  Carr Kohri Sendouda Yokoyama (2010)
- Gravitational lensing
  Barnacka Glicenstein Moderski (2012)
  Griest Cieplak Lehner (2013) Tisserand et al. (2006)
- Dynamical constraints ( Heating by PBH collisions)
  Graham Rajendran Varela (2015) Brandt (2016)
- CMB ( accretion of gas during recombination ) Ricotti Ostriker Mack (2008)

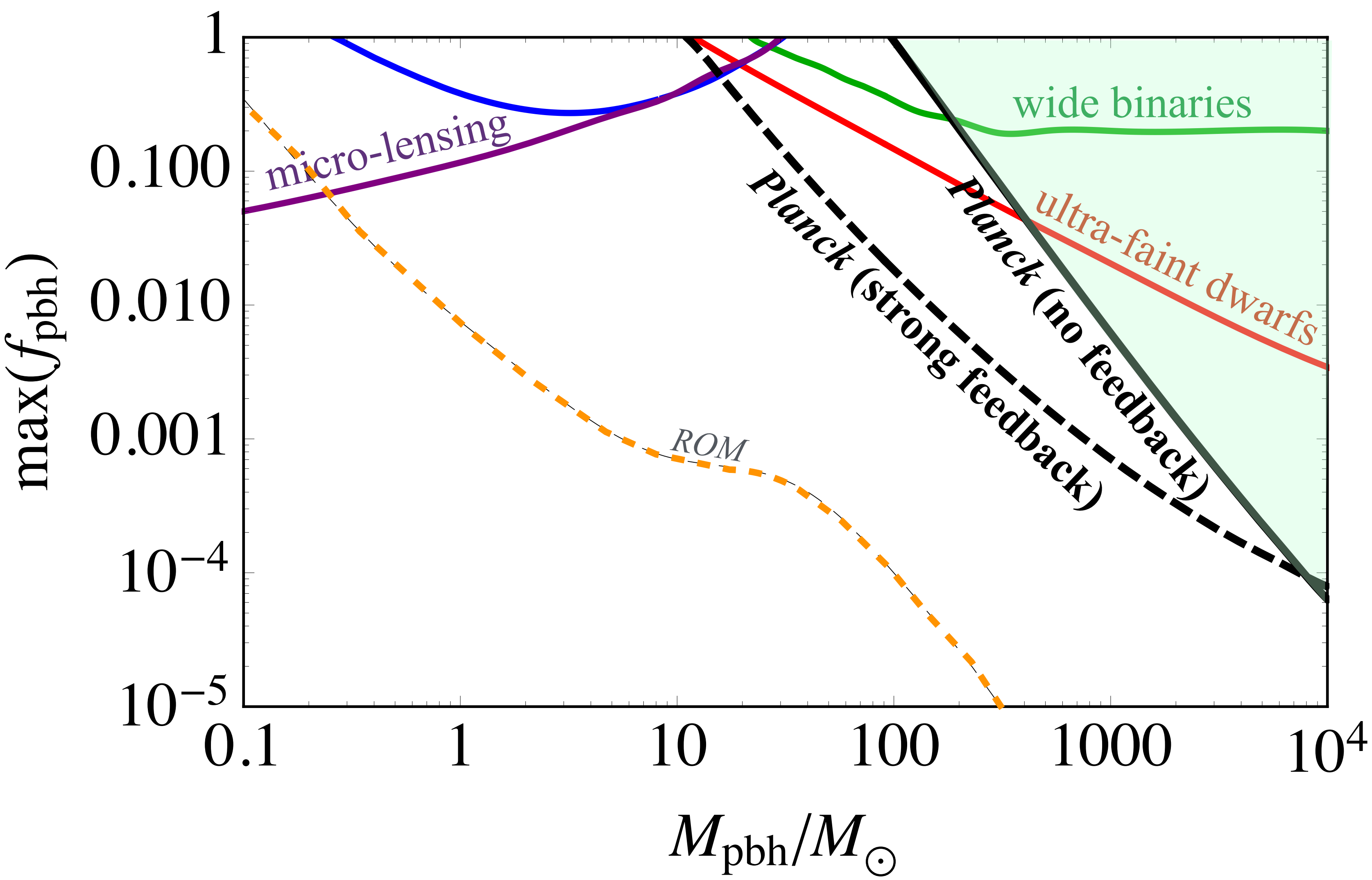


# Recent updates — CMB constraint

- Stringent constraint for PBHs with mass larger than $10^{33}$ g

  Riccotti Ostriker Mack (2007)

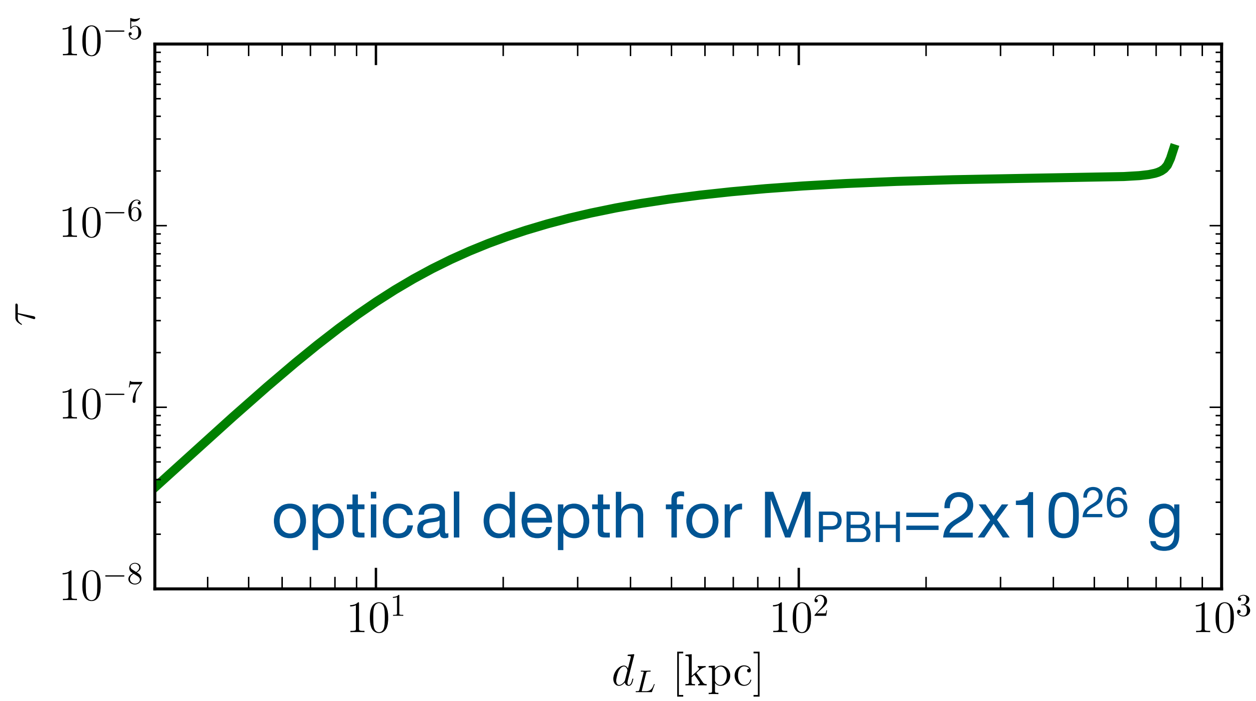
- Reexamined by Ali-Hamimoud-Kamionkowski and others (2016)
- Accounting for Compton cooling by CMB and ionization cooling
  - much smaller radiative efficiency $\epsilon \equiv L/\dot{M}$
  - smaller accretion rate $\dot{M}$

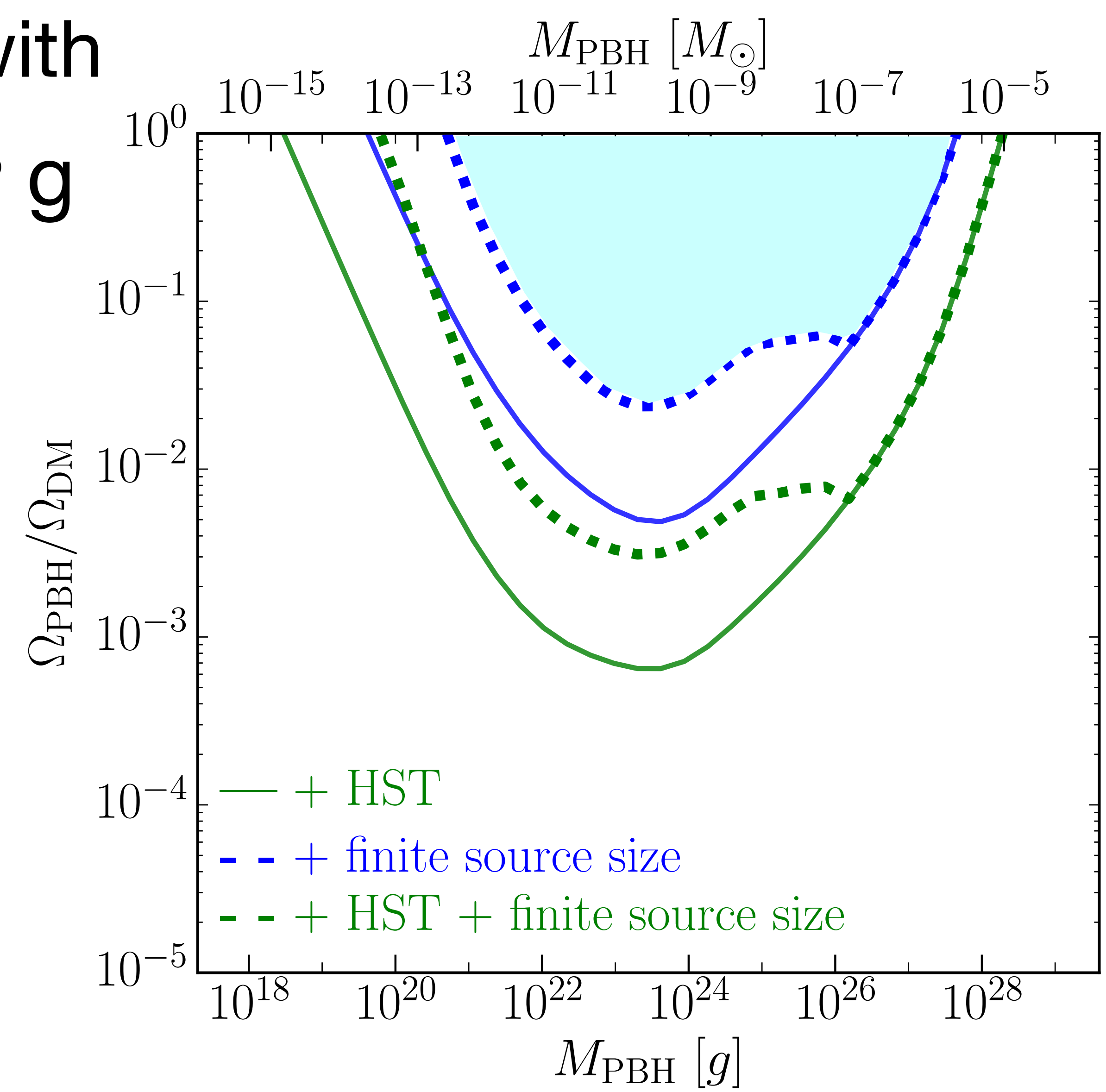


# Recent updates — Microlensing

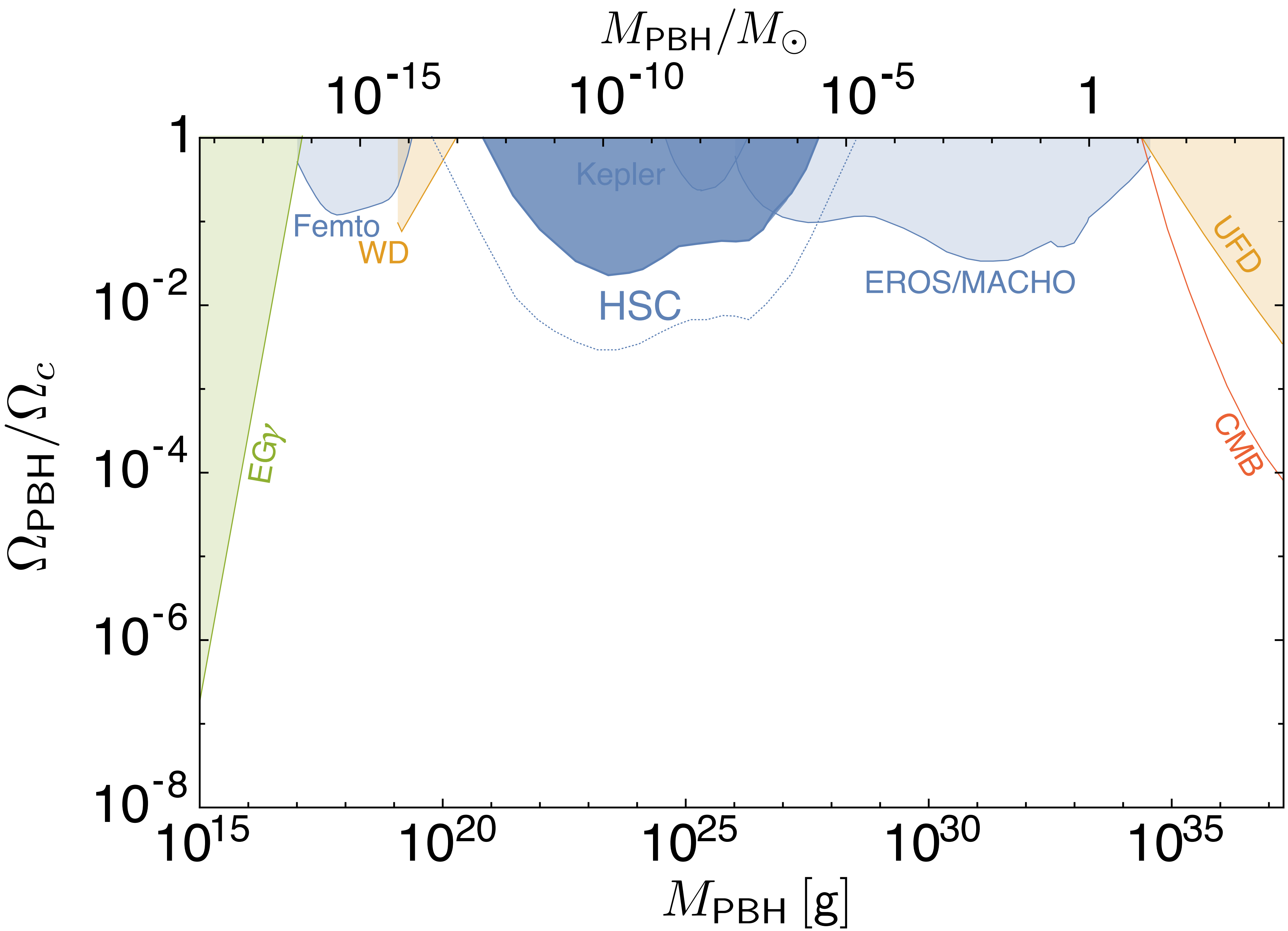
Niikura Takada et al (2017)

- Using Subaru Hyper-Suprime-Cam (HSC)
- Sources are stars in M31( Andromeda galaxy ~ 770kpc)
- Microlensing due to PBHs in M31 and our galaxy
- No secure event
- Stringent constraint on PBHs with mass between $10^{20}$ g and $10^{28}$ g



# Current observational Constraints

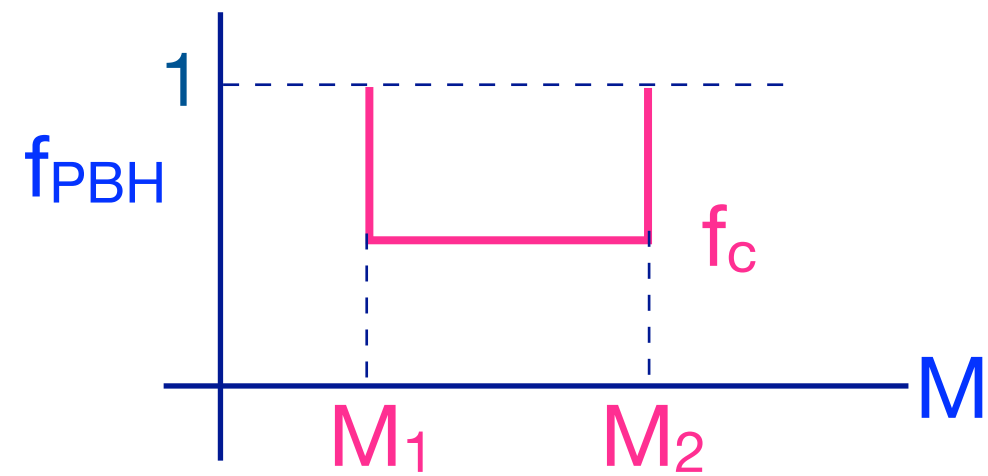



- New microlensing constraint almost closed DM PBH window
- Only mass region~ $10^{20}$g remains

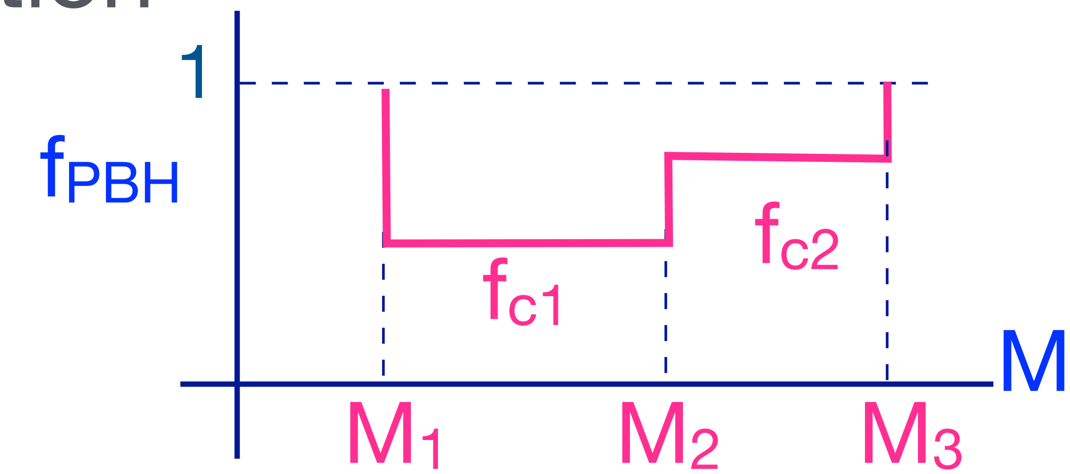
# Constraints on extended mass function

- Observational constraints are obtained assuming all PBHs have the same mass $f_{\mathrm{PBH}}(M) \propto \delta(M - M_*)$
- How we apply the constraints to the predicted extended mass function?
- For the simplest case the constraint is independent of M
  - expected (lensing) event number $\propto \int_{M_1}^{M_2} f_{\mathrm{PBH}}(M) d\log M$
  - Maximum allowed event number

    $N_{\mathrm{max}}$ is obtained for $\int_{M_1}^{M_2} f_{\mathrm{PBH}}(M) d\log M = f_c$
  - Expected event number = $\dfrac{N_{\mathrm{max}}}{f_c} \displaystyle\int_{M_1}^{M_2} f_{\mathrm{PBH}}(M) d\log M < N_{\mathrm{max}}$

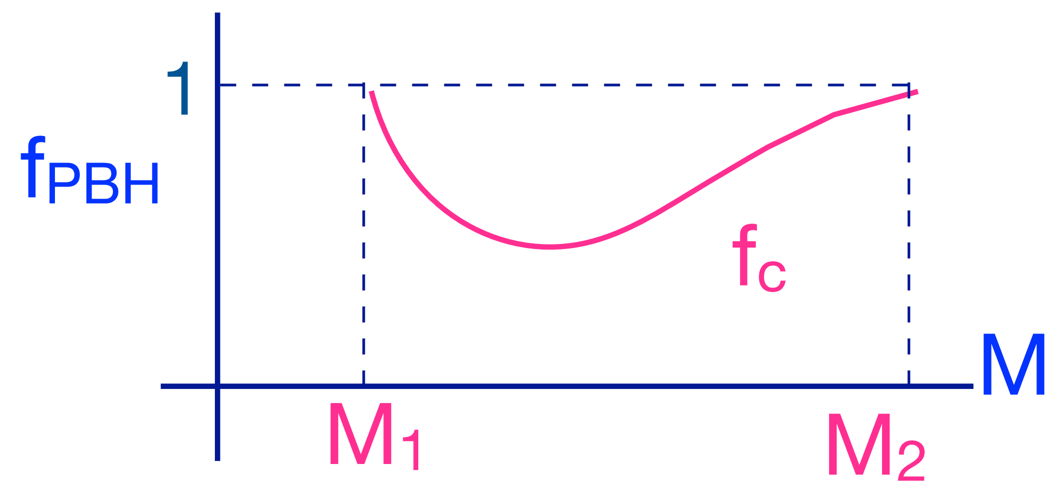


# Constraints on extended mass function




- For the next simplest case
  - expected event number

$$\simeq N_{\rm max}\left[\frac{1}{f_{c1}}\int_{M_1}^{M_2} f_{\rm PBH}(M)d\log M + \frac{1}{f_{c2}}\int_{M_2}^{M_3} f_{\rm PBH}(M)d\log M\right] < N_{\rm max}$$




- For general case
  - event number

$$\simeq N_{\rm max}\int_{M_1}^{M_2}\frac{f_{\rm PBH}(M)}{f_c(M)}d\log M < N_{\rm max}$$

- For a given $f_c(M)$

$$\boxed{\int_{M_1}^{M_2}\frac{f_{\rm PBH}(M)}{f_c(M)}d\log M < 1}$$

# 5. Black Hole as Dark Matter

- PBHs can account for all dark matter of the universe?
- Observational constraints on wide range of PBH mass
- In particular the new microlensing constraint

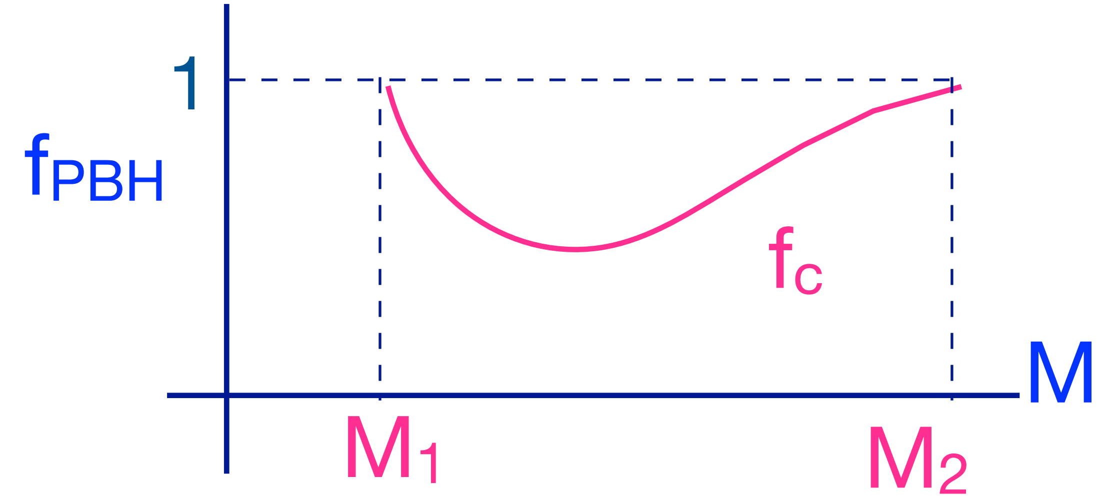
  → only mass region~ $10^{20}$g remains

- We apply the method of constraining on extended mass function of PBHs

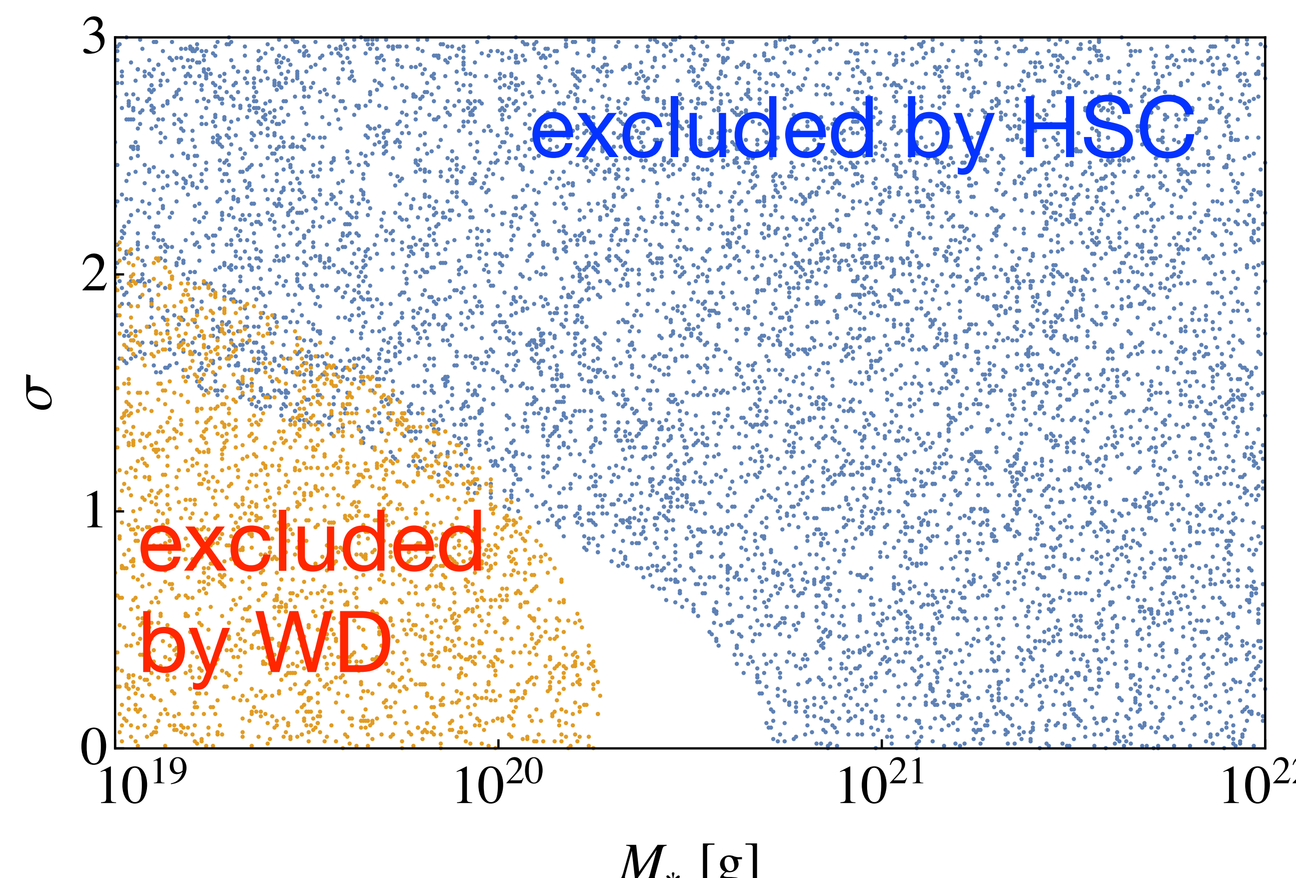
# Constraints on extended mass function

- We require

$$\boxed{\int_{M_1}^{M_2} \frac{f_{\mathrm{PBH}}(M)}{f_c(M)} d\log M < 1}$$




- Mass function $f_{\mathrm{PBH}} \propto M \exp\left[\frac{-(\log M - \log M_*)^2}{2\sigma^2}\right]$
  - Account for all DM $\int f_{\mathrm{PBH}}\, d\log M = 1$
  - Observational constraints



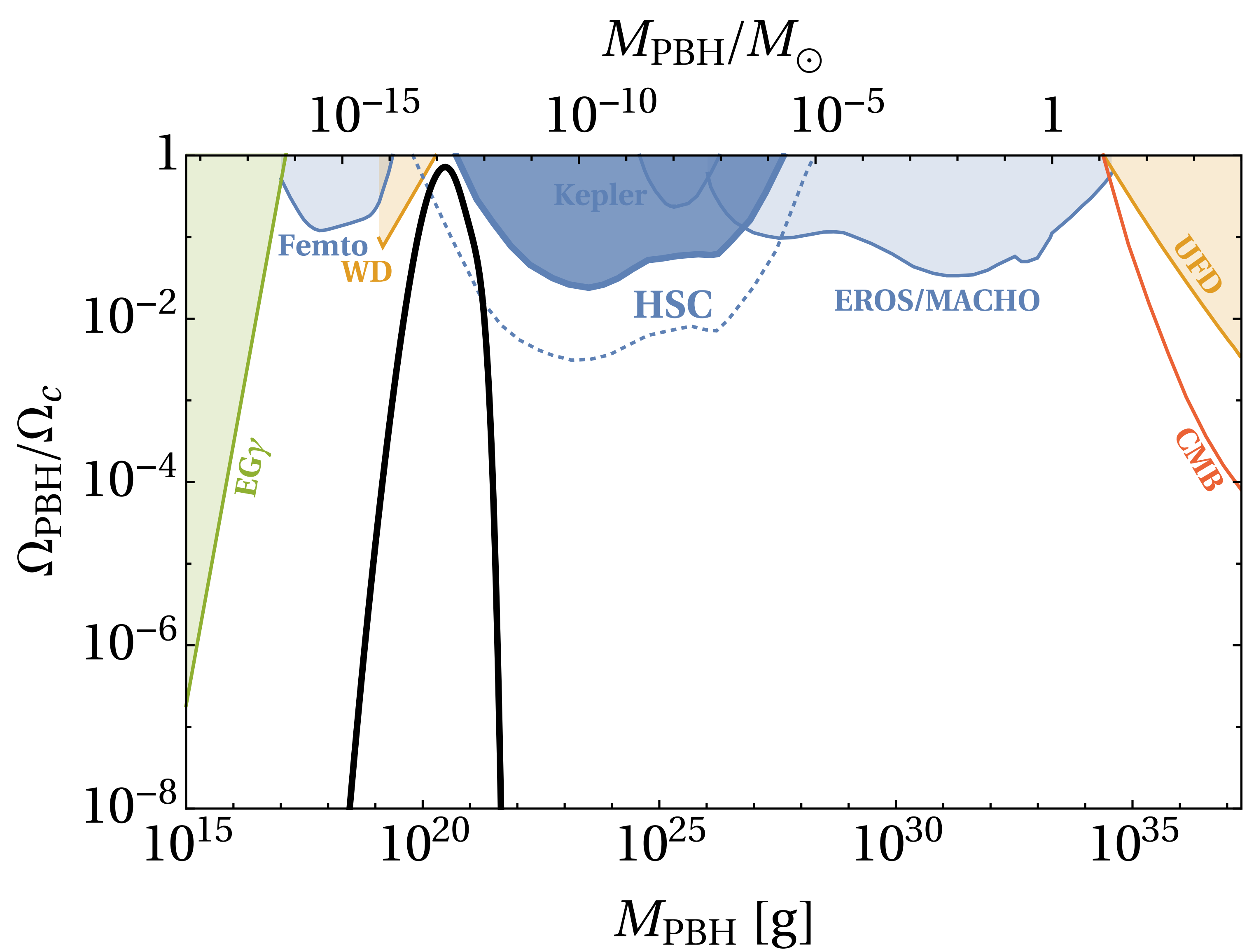

- Sharp PBH mass function can account for all DM

# Black Hole as Dark Matter

- PBHs can account for all dark matter of the universe?
- Observational constraints on wide range of PBH mass
- In particular the new microlensing constraint

  → only mass region~ $10^{20}$g remains

- Double inflation model can produce such DM PBHs




Inomata MK Mukaida Tada Yanagida (2017)

$v = 10^{-3}$ $c_{\rm pot} = 1$

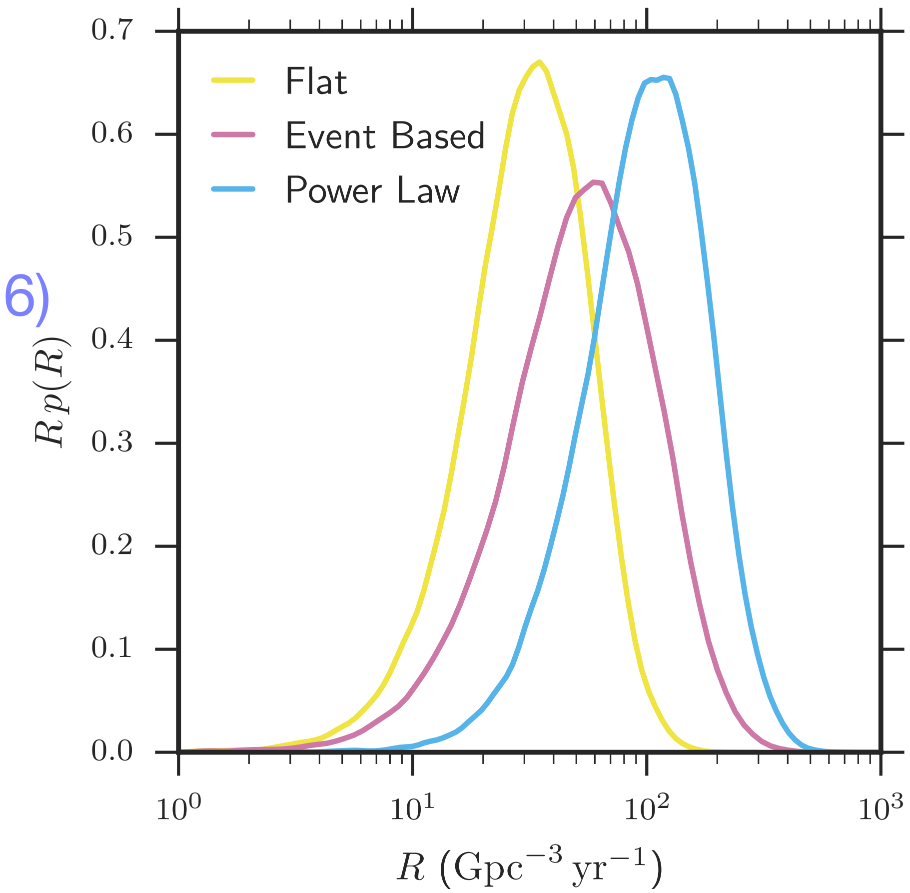
$\kappa = 0.13$

# 6. PBHs for LIGO gravitational wave events

- First detection of GW event by LIGO-Virgo collaboration which comes from BH-BH binary

$$M_1 = 36^{+5}_{-4} M_\odot \quad M_2 = 29^{+4}_{-4} M_\odot$$

Abbott et al (2016)

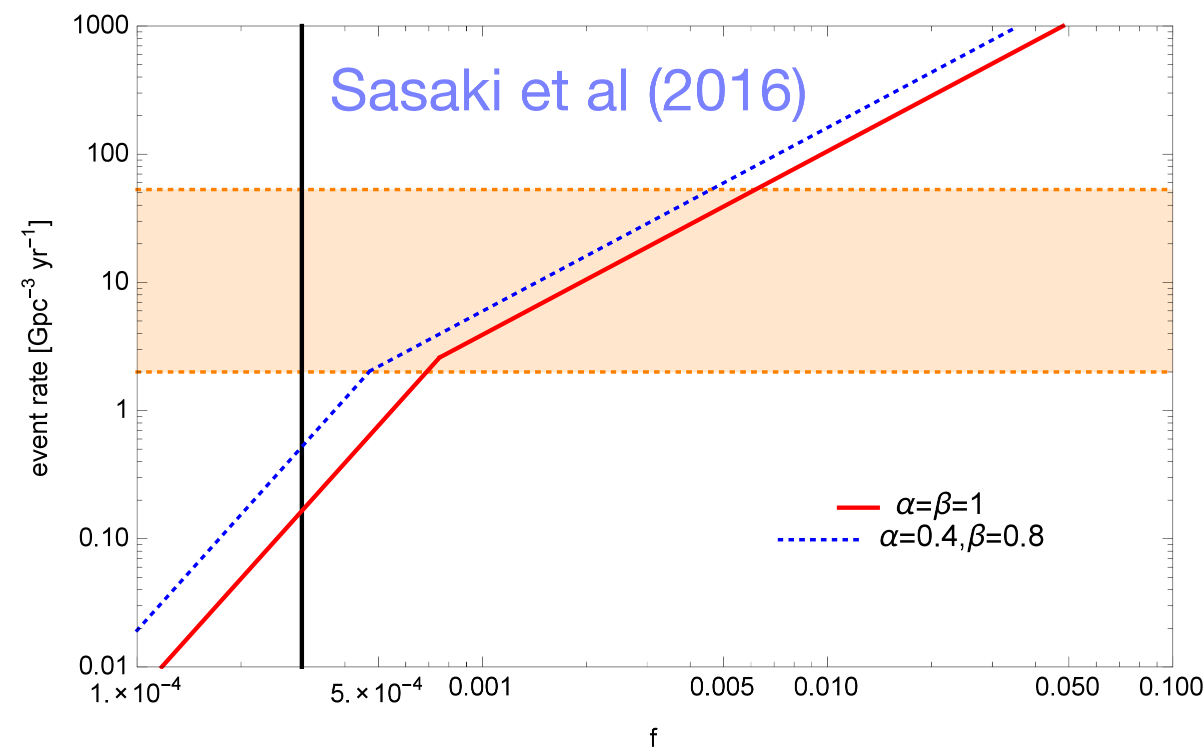



- Origin of BHs?

  → PBHs are one of candidates

- Double inflation can produce such PBHs
- Event rate of BH-mergers is estimated as

$$R = 9 - 240\ \mathrm{Gpc}^{-3}\mathrm{yr}^{-1}$$




- Required fraction of PBHs

$$\Omega_{\mathrm{PBH}}/\Omega_c \sim 10^{-3} - 10^{-2}$$

(now consistent with CMB const.)

- However, stringent constraint from pulsar timing experiments

# Production of gravitational waves

- PBHs form from large curvature perturbations

- 2nd order perturbations ~ $O(\zeta_{\vec{k}}\zeta_{\vec{k}-\vec{k}'})$ induce a source term of tensor perturbations

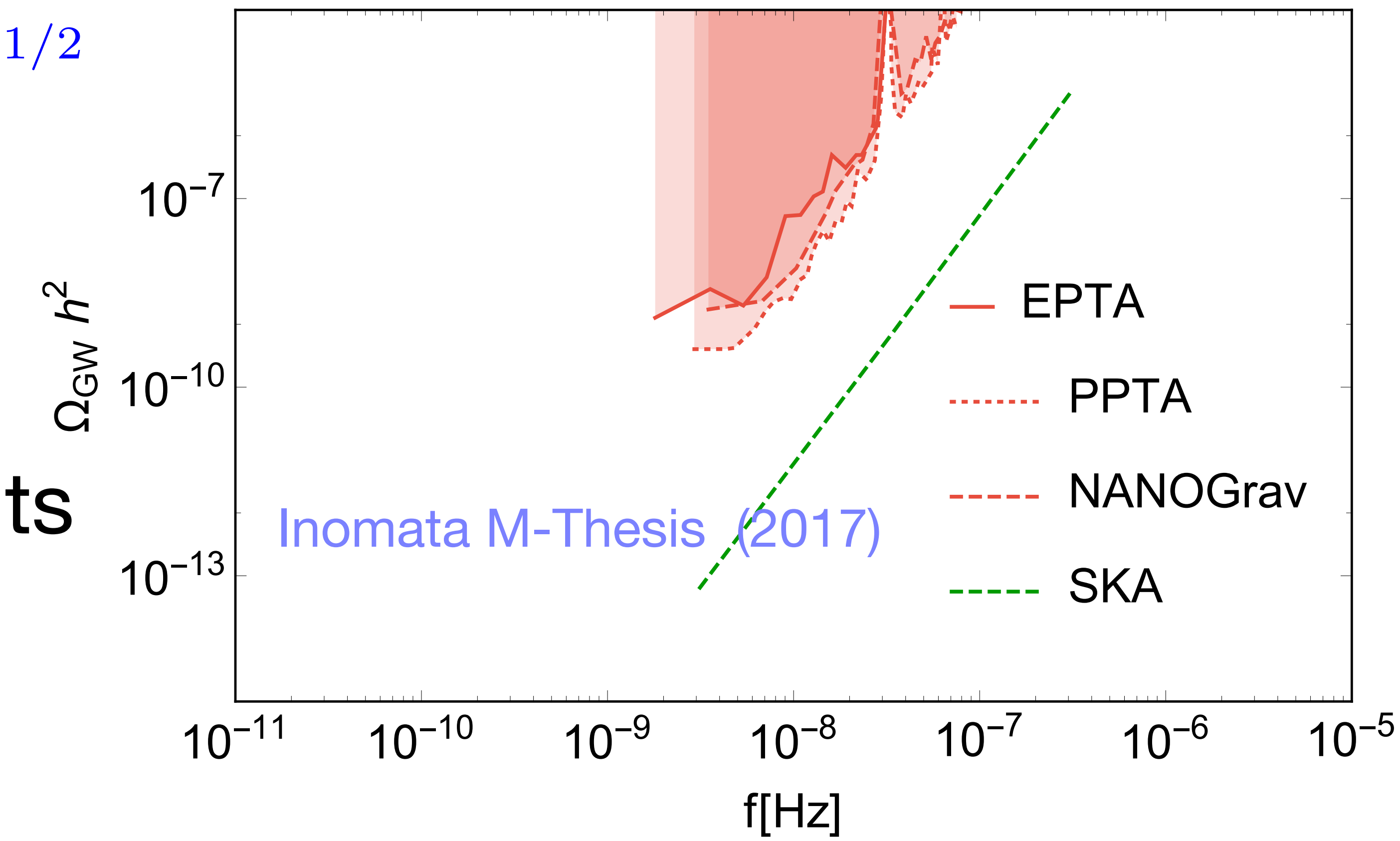
Saito Yokoyama (2009) Bugaev Kulimai (2010)

$$\boxed{h''_{\vec{k}} + 2\mathcal{H}h'_{\vec{k}} + k^2 h_{\vec{k}} = \mathcal{S}(\vec{k},t)}$$

$$\Rightarrow \quad \Omega_{\mathrm{GW}}h^2 \sim 10^{-8}(\mathcal{P}_\zeta/10^{-2})^2 \quad \sim (\underbrace{\Omega_R h^2}_{10^{-4}})\, \mathcal{P}_\zeta^2$$

$$f_{\mathrm{GW}} \sim 2\times 10^{-9}\mathrm{Hz}\left(\frac{\gamma}{0.2}\right)^{1/2}\left(\frac{M_{\mathrm{PBH}}}{M_\odot}\right)^{-1/2}$$

$$M_{\mathrm{PBH}} = \gamma M_H \ \text{(horizon mass)}$$

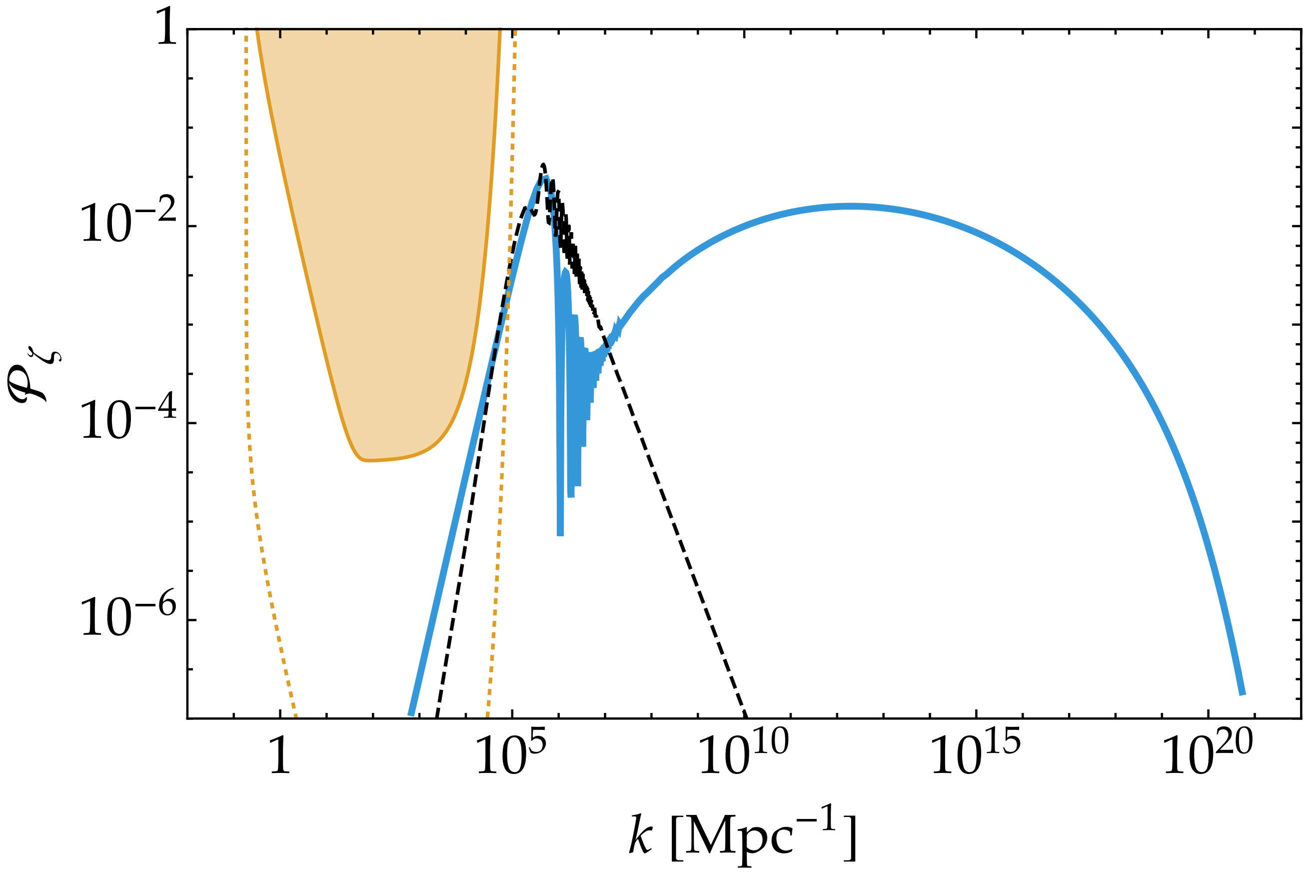


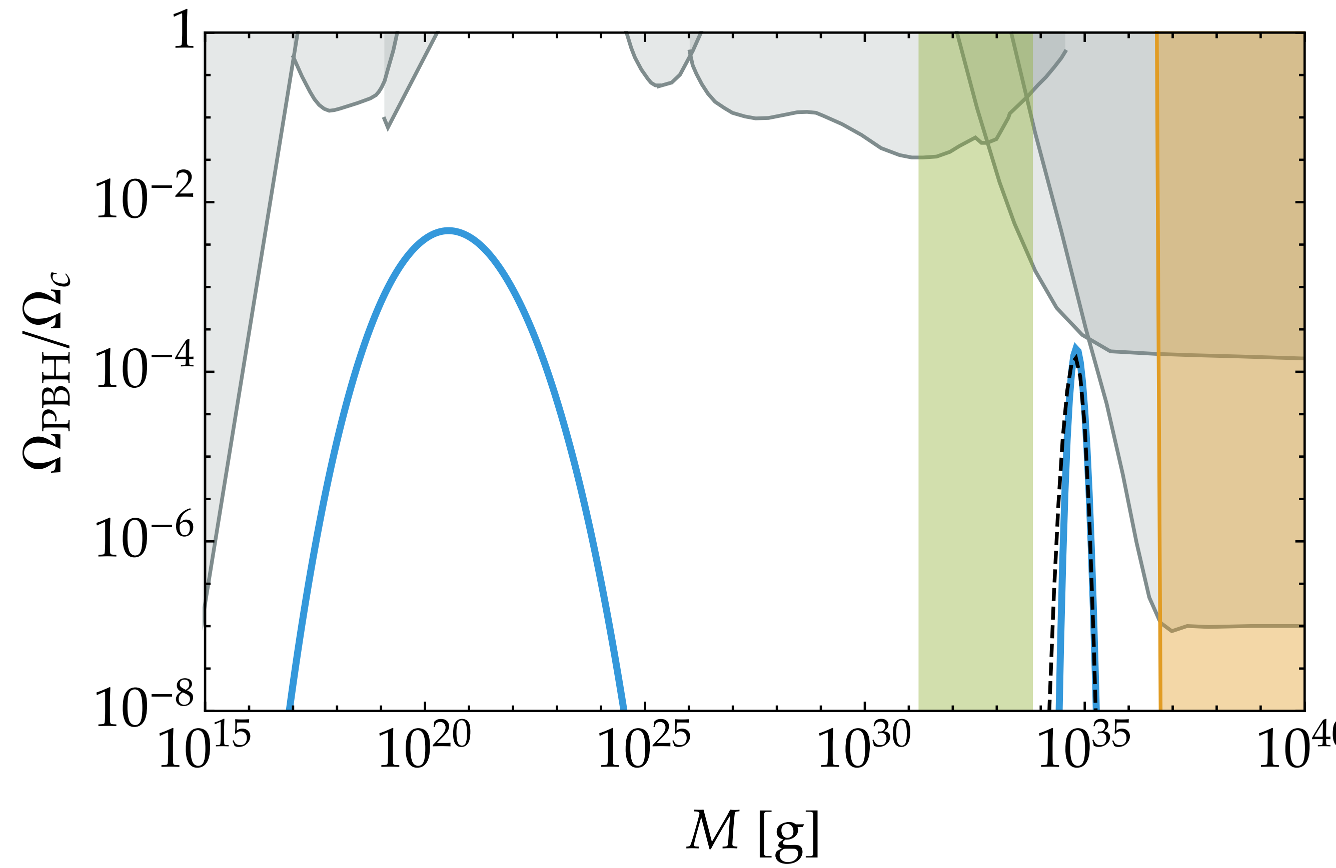

- Pulsar timing array experiments already give a stringent constraint on PBHs with

$$M_{\mathrm{PBH}} \sim 0.1 M_\odot - \text{a few } M_\odot$$

# Double inflation model

- We need a sharp peak to avoid PTA constraints
- This is possible to choose appropriate sets of parameters
  - Small Hubble induced mass after pre-inflation leads to a very sharp peak $V_H \sim (c_{\mathsf{pot}} + c_{\mathsf{kin}}) H^2 \varphi^2$



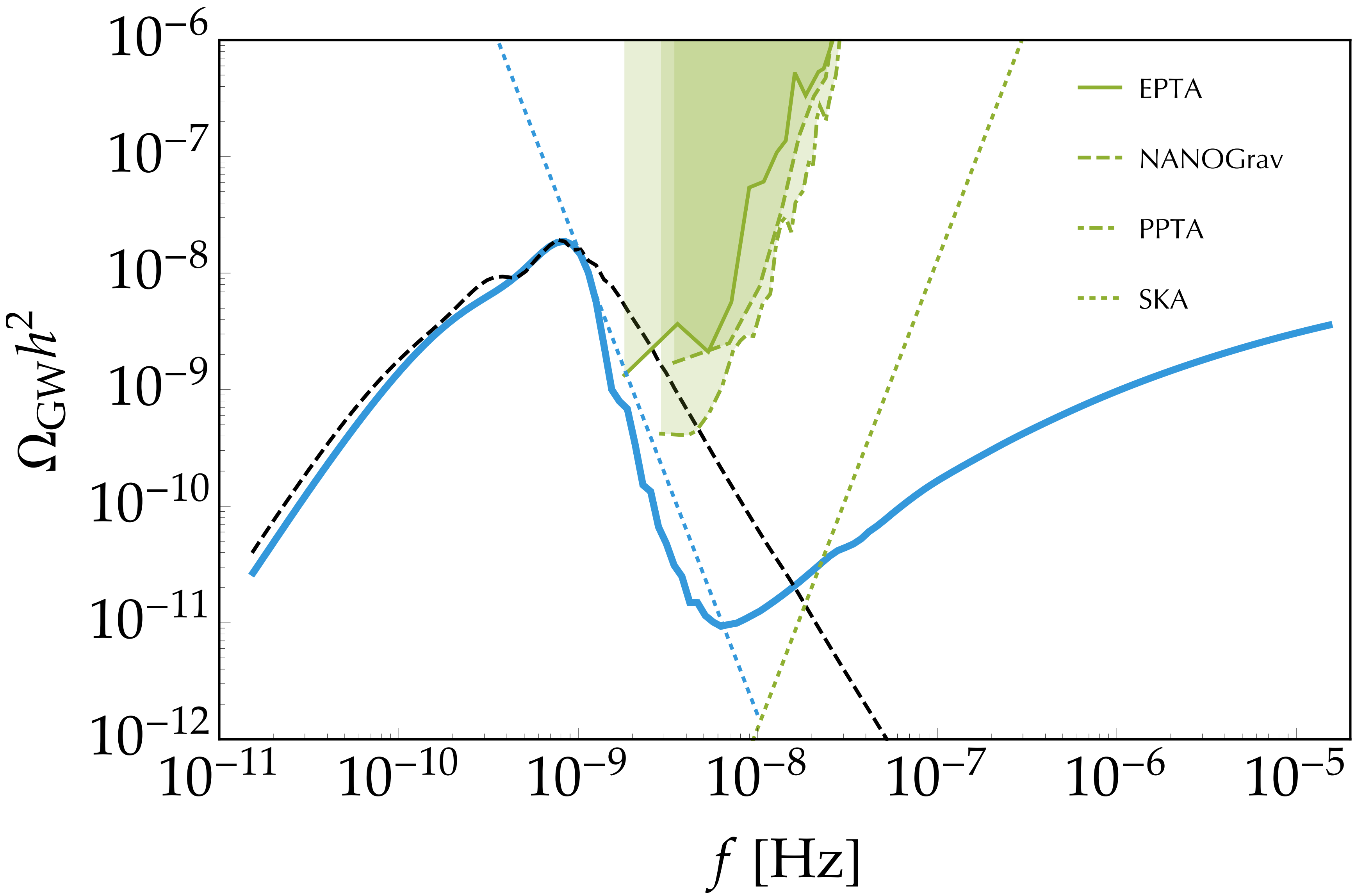

(black) $c_{\mathsf{pot}} = 1 \quad c_{\mathsf{kin}} = 0.1 \quad \kappa = 0.76$

(cyan) $c_{\mathsf{pot}} = 0.681 \quad c_{\mathsf{kin}} = -0.676 \quad \kappa = -0.61$

Inomata MK Mukaida Tada Yanagida (2016)

Hubble induced mass becomes small after pre-inflation

- Current PTA constraints can be (marginally) avoided
- Notice that there is uncertainty in $\gamma$




Inomata MK Mukaida
Tada Yanagida (2016)

- Future PTA experiment such as SKA will probe GWs as small as $\Omega_{\rm GW}h^2 \sim 10^{-15}$

# 6. Conclusion

- PBHs can be formed in double inflation (=preinflation + new inflation)
- By appropriate choice of model parameters the model can produce PBHs with various masses and abundances
- Although observational constraints are stringent, this model can produce PBHs that account for all DM of the universe
- The model also can produce PBHs for LIGO events and (marginally) evade constraints from PTA experiments on gravitational waves