

# Probing physics behind the electroweak symmetry breaking at future gravitational wave interferometers and future collider experiments

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Why does the Universe accelerate?

-Exhaustive study and challenge for the future

March 8, 2017 @ KEK

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- References:
  - MK, Kanemura, Matsui, PRD92, no.11,115007 (2015), arXiv:1509.08394
  - Hashino, MK, Kanemura, Matsui, PRD94, no.1, 015005 (2016), arXiv:1604.02069
  - Hashino, MK, Kanemura, Ko, Matsui, PLB766, 49 (2017), arXiv:1609.00297

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2. Models with extended scalar sectors
  - A. Models with additional singlet scalars (without CSI)
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3. Gravitational waves from first order phase transition
4. Synergy between measurements of gravitational waves and various Higgs boson couplings
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# Motivation

## Discovery of the 125 GeV Higgs boson $h$ at the CERN LHC

- The Standard Model (SM) has been established as a low-energy effective theory below  $O(100)$  GeV

This is not the end of the story

## Puzzles in the Higgs sector

- Guiding principle?
- Shape of the Higgs potential (multiplets, symmetries, ...)?
- Dynamics behind the electroweak symmetry breaking (EWSB)?

## Phenomena beyond the SM (BSM)

- Baryon asymmetry of the Universe (BAU)
- Existence of dark matter
- Cosmic inflation
- Neutrino oscillations

## Idea: Higgs sector = Window to New Physics

- The structure of the Higgs sector is related to BSM models

Information on new physics can be obtained by investigating the properties of the Higgs sector

# Electroweak baryogenesis (EWBG) and Higgs boson couplings

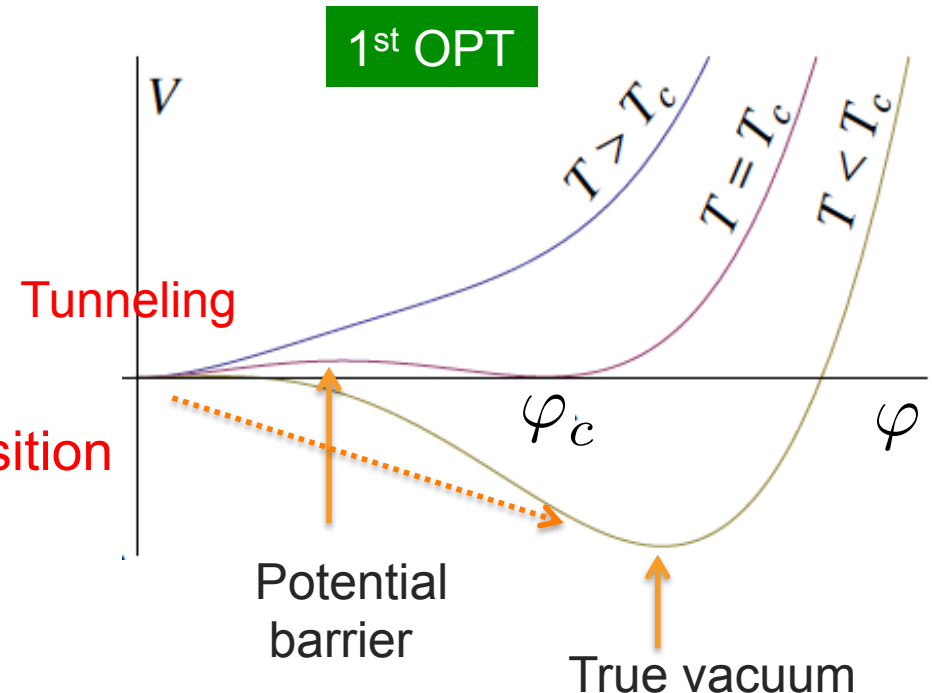
## Sakharov's conditions for BAU

1. Baryon number violation  
    ↑ Sphaleron process
2. Violation of C and CP  
    ↑ Extended Higgs sector
3. Departure from thermal equilibrium  
    ↑ Strongly first order phase transition  
    (1<sup>st</sup> OPT):  $\varphi_c/T_c \gtrsim 1$

## SM Higgs sector w/ one doublet:

- Electroweak phase transition (EWPT) is NOT of 1<sup>st</sup> order for  $m_h = 125$  GeV

EWBG is an important physics case relating the Higgs sector to BSM phenomena



# Strongly 1<sup>st</sup> OPT and Higgs boson couplings

Models with extended Higgs sector

- 1<sup>st</sup> OPT can be easily realized
- Signatures are testable at colliders

e.g. Two Higgs doublet model (2HDM)

- Condition for strongly 1<sup>st</sup> OPT:

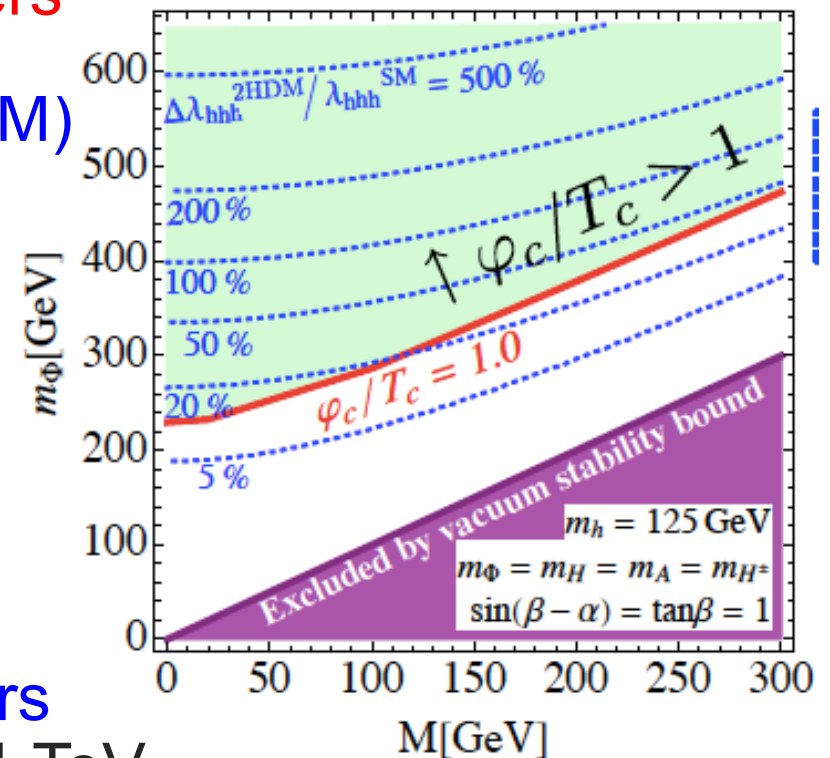
$$\varphi_c/T_c \gtrsim 1$$

➡ Large deviation in the triple Higgs boson coupling

$$(\Delta\lambda_{hhh}/\lambda_{hhh}^{\text{SM}} \gtrsim 10\%)$$

EWBG can be tested at future colliders

- International Linear Collider (ILC) 1 TeV w/  $L = 5 \text{ ab}^{-1}$  can measure  $\lambda_{hhh}$  at 10% accuracy [Fujii et al. (2015)]



[Kanemura, Okada, Senaha (2005)]

# Gravitational waves (GWs) as a probe of EWPT

## Ground-based interferometers (aLIGO, KAGRA, aVirgo)

- Targets: GWs from binary systems, supernovae, ...

- aLIGO made the first direct observation of GWs

➡ New era of GW astronomy

[LIGO and Virgo (2016)]

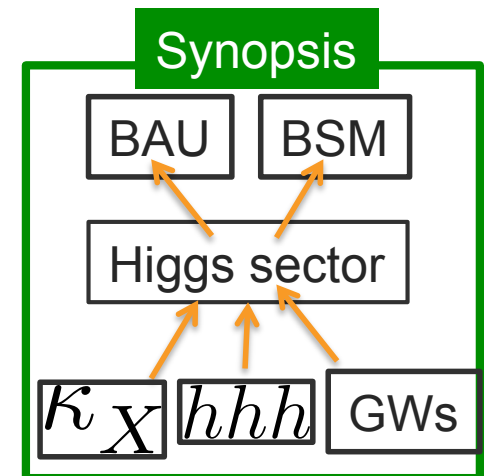
## Future space-based interferometers (LISA, DECIGO)

- Sensitive to GWs from the early Universe  
(Strongly 1<sup>st</sup> OPT, cosmic inflation, ...)

➡ New era for fundamental physics

## Goal:

- To investigate testability of models of EWSB using the synergy between the measurements of the GWs, Higgs boson couplings  $\kappa_X$  and the  $hhh$  coupling



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# Studies on the GWs from 1<sup>st</sup> order EWPT

## Model independent analysis

[Grojean, Servant (2007); Kikuta, Kohri, So (2014), ...]

## Higgs potential with higher order operators

[Delaunay, Grojean, Wells (2008); Huang, Wan, Wang, Cai, Zhang (2016), ...]

## Non-decoupling loop effects from hypothetical particles

- **Additional (Higgs) scalar loop effects**

[MK, Kanemura, Matsui (2015); Hashino, MK, Kanemura, Matsui (2016), Dorsch, Huber, Konstandin, No (2016), ...]

- **Light stop loop effects in the MSSM**

[Aprada, Maggiore, Nicolis, Riotto (2002), ...]

n.b. Light stops are excluded by LHC

## Non-thermal effects even at the tree level

- **Real singlet extension**

[Ashoorioon, Konstandin (2009); Huang, Long, Wang (2016); Hashino, MK, Kanemura Ko, Matsui (2016), ....]

- **Next-to-MSSM**

[Aprada, Maggiore, Nicolis, Riotto (2002), Huber, Konstandin (2007); Huber, Konstandin, Nardini, Rues (2015), ...]



# Model A: Models with additional singlet scalars (without CSI)

Idea: [MK, Kanemura, Matsui (2015)]

- To generally handle strongly 1<sup>st</sup> OPT via thermal loop,  $N$  isosinglet scalars  $S_i$  ( $i = 1, \dots, N$ ) are introduced
- For simplicity,  $O(N)$  symmetry is imposed

Tree-level scalar potential:  $V_0(\Phi, \vec{S}) = V_{\text{SM}}(\Phi) + \frac{\mu_S^2}{2} |\vec{S}|^2 + \frac{\lambda_S}{4} |\vec{S}|^4 + \frac{\lambda_{\Phi S}}{2} |\Phi|^2 |\vec{S}|^2$

Singlet scalar boson mass:  $m_S^2 = \mu_S^2 + \frac{\lambda_{\Phi S}}{2} v^2$

Triple Higgs boson coupling:

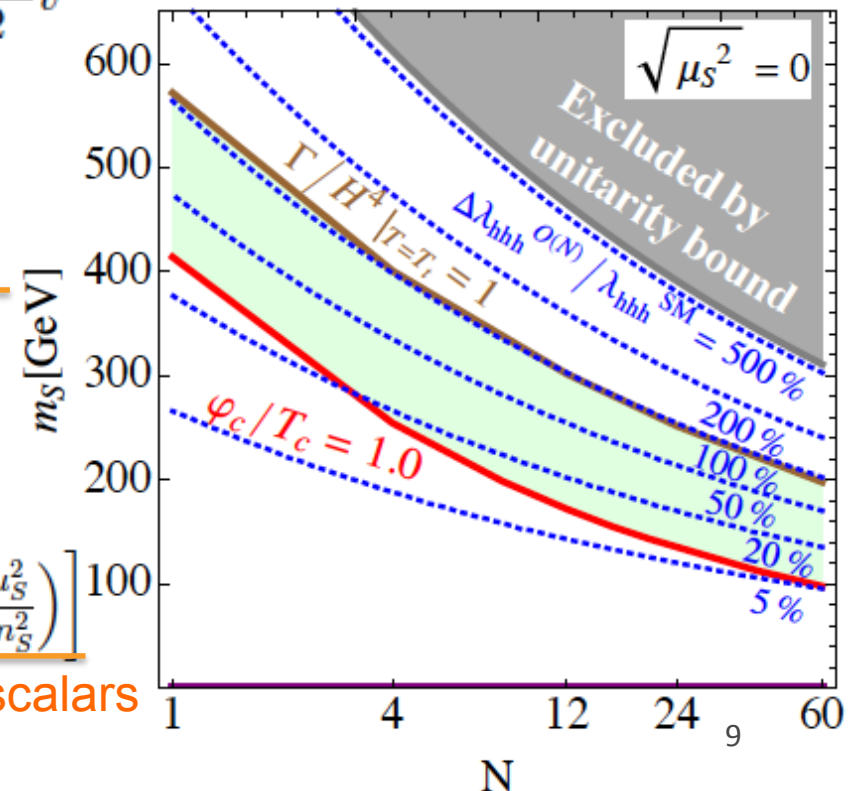
$$\lambda_{hhh}^{O(N)} = \frac{3m_h^2}{v} \left\{ 1 - \frac{1}{\pi^2} \frac{m_t^4}{v^2 m_h^2} + \frac{N}{12\pi^2} \frac{m_S^4}{v^2 m_h^2} \left( 1 - \frac{\mu_S^2}{m_S^2} \right)^3 \right\}$$

Finite temperature effective potential (high temperature expansion):

$$V_{\text{eff}}(\varphi, T) \simeq D(T^2 - T_0^2)\varphi^2 - ET\varphi^3 + \frac{\lambda_T}{4}\varphi^4 + \dots$$

$$\frac{\varphi_c}{T_c} \propto E = \frac{1}{12\pi v^3} \left[ 6m_W^3 + 3m_Z^3 + Nm_S^3 \left( 1 - \frac{\mu_S^2}{m_S^2} \right) \left( 1 + \frac{3\mu_S^2}{2m_S^2} \right) \right]$$

Non decoupling loop effect from additional scalars



# Model B: CSI models with additional singlet scalars

**Idea** [Hashino, Kanemura, Orikasa (2015)]

- Mass parameters are absent in the original Lagrangian due to Classical Scale Invariance (CSI) [Bardeen (1995)]
- EWSB is directly caused by thermal loop effects

**Tree-level scalar potential**

$$V_0(\Phi, \vec{S}) = \frac{\lambda}{2} |\Phi|^4 + \frac{\lambda_S}{4} |\vec{S}|^4 + \frac{\lambda_{\Phi S}}{2} |\Phi|^2 |\vec{S}|^2$$

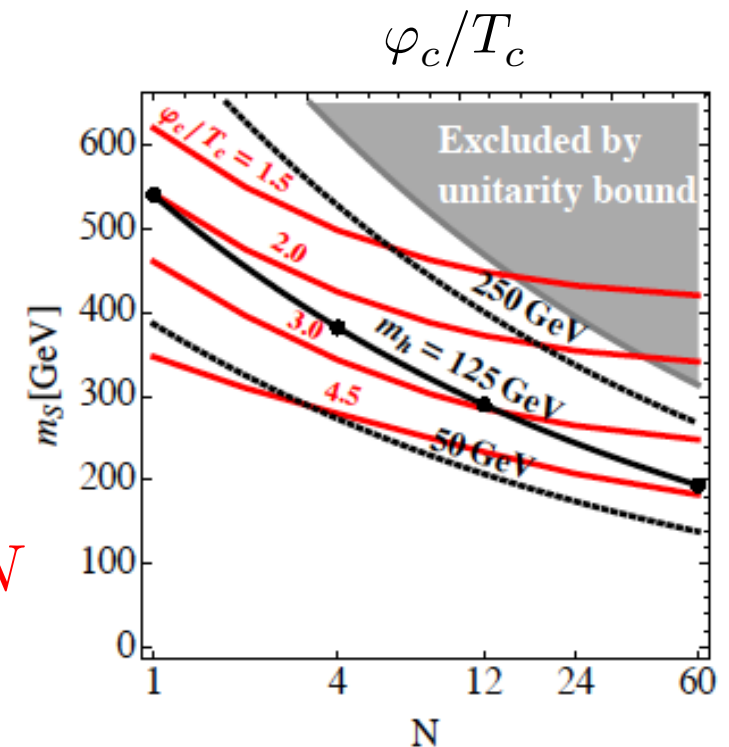
**Singlet scalar boson mass**

$$Nm_S^4 = 8\pi^2 v^2 m_h^2 - 6m_W^4 - 3m_Z^4 + 12m_t^4$$

**Triple Higgs boson coupling**

$$\frac{\Delta\lambda_{hhh}}{\lambda_{hhh}^{\text{SM(tree)}}} = \frac{\lambda_{hhh}}{\lambda_{hhh}^{\text{SM(tree)}}} - 1 = \frac{2}{3} \quad \text{independent of } N$$

[Hashino, Kanemura, Orikasa (2015)]



[Hashino, MK, Kanemura, Matsui (2015)]

# Model C: Higgs singlet model

## Idea

[Hashino, MK, Kanemura, Ko, Matsui (2016)]

- To investigate strongly 1<sup>st</sup> OPT and Higgs couplings induced through Higgs field mixing (at least 2 classical fields needed)
- For simplicity, we introduce a singlet Higgs field  $S$

## Tree-level Higgs potential

$$V_0 = -\mu_\Phi^2 |\Phi|^2 + \lambda_\Phi |\Phi|^4 + \mu_{\Phi S} |\Phi|^2 S + \frac{\lambda_{\Phi S}}{2} |\Phi|^2 S^2 + \mu_S^3 S + \frac{m_S^2}{2} S^2 + \frac{\mu'_S}{3} S^3 + \frac{\lambda_S}{4} S^4$$

$$\langle \Phi \rangle = \begin{pmatrix} 0 \\ \frac{1}{\sqrt{2}} \varphi_\Phi \end{pmatrix} : \text{SM Higgs doublet}$$

$$\langle S \rangle = \varphi_S : \text{Higgs singlet}$$

## Effective potential:

$$V_{\text{eff}, T=0}(\varphi_\Phi, \varphi_S) = V_0(\varphi_\Phi, \varphi_S) + \sum_i n_i \frac{M_i^4(\varphi_\Phi, \varphi_S)}{64\pi^2} \left( \ln \frac{M_i^2(\varphi_\Phi, \varphi_S)}{Q^2} - c_i \right) \quad \begin{array}{l} c_F = c_S = 5/6 \\ c_V = 5/6 \end{array}$$

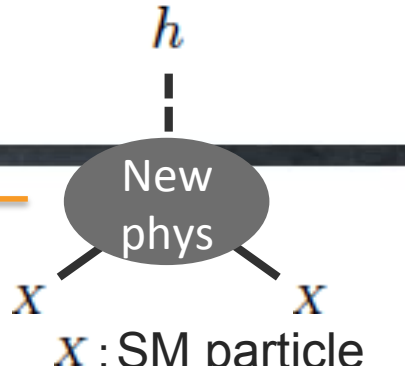
## Higgs boson masses and mixing

$m_h (= 125 \text{ GeV})$  : Mass of the discovered Higgs boson

$m_H$  : Mass of the additional Higgs boson       $\theta$  : Higgs mixing angle

## Model C: Higgs Singlet Model (contd.)

Higgs boson couplings to SM particles  $\kappa_X = \frac{g_{hXX}}{g_{hXX}|_{\text{SM}}}$



$\kappa = \kappa_V = \kappa_F = \cos \theta$

$X : \text{SM particle}$

Triple Higgs boson couplings (effective potential approach)

$$\Delta\lambda_{hhh} = \frac{\lambda_{hhh}^{\text{HSM}} - \lambda_{hhh}^{\text{SM}}}{\lambda_{hhh}^{\text{SM}}}$$

$$\lambda_{hhh}^{\text{SM}} = \frac{3m_h^2}{v_\Phi} \left[ 1 + \frac{9m_h^2}{32\pi^2 v_\Phi^2} + \sum_{i=W^\pm, Z, t, b} n_i \frac{m_i^4}{12\pi^2 m_h^2 v_\Phi^2} \right]$$

$$\lambda_{hhh}^{\text{HSM}} = c_\theta^3 \left\langle \frac{\partial^3 V_{\text{eff}, T=0}}{\partial \varphi_\Phi^3} \right\rangle + c_\theta^2 s_\theta \left\langle \frac{\partial^3 V_{\text{eff}, T=0}}{\partial \varphi_\Phi^2 \partial \varphi_S} \right\rangle + c_\theta s_\theta^2 \left\langle \frac{\partial^3 V_{\text{eff}, T=0}}{\partial \varphi_\Phi \partial \varphi_S^2} \right\rangle + s_\theta^3 \left\langle \frac{\partial^3 V_{\text{eff}, T=0}}{\partial \varphi_S^3} \right\rangle$$

Finite temperature effective potential in one direction  
(high temperature expansion)

$$V_{\text{eff}} = D(T^2 - T_0^2)\varphi^2 - (ET - e)\varphi^3 + \frac{\lambda(T)}{4}\varphi^4$$

$$\Rightarrow \frac{\varphi_c}{T_c} = \frac{2E}{\lambda} \left( 1 - \frac{e\lambda}{ET} \right)$$

Effects from the Higgs boson mixing

The Higgs boson mixing gives considerable contributions to deviation in the Higgs boson couplings and to phase transition

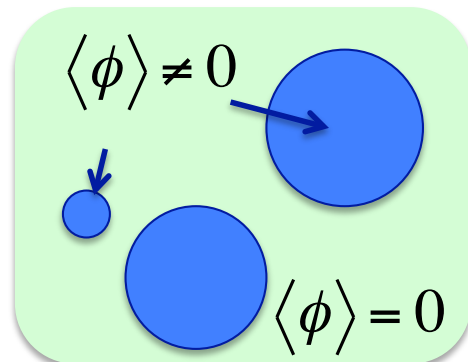
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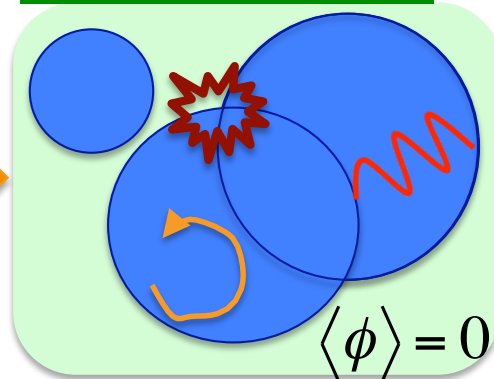
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# Important quantities for GW spectrum

## Bubble nucleation



## Bubbles collision



## Sources of GWs

1. Collision of bubble walls
  2. Compression wave of plasma
  3. Plasma turbulence
- GW spectrum is derived from finite temperature effective potential  $V_{\text{eff}}$

Bubble nucleation rate per unit volume per unit time:

$$\Gamma(t) = \Gamma_0(t) \exp[-S_E(t)] \quad S_E(T) = S_3(T)/T, \quad S_3 = \int d^3r \left[ \frac{1}{2} (\vec{\nabla} \varphi_b)^2 + V_{\text{eff}}(\varphi_b, T) \right]$$

Transition temperature  $T_t$  :  $\left. \frac{\Gamma}{H^4} \right|_{T=T_t} \simeq 1 \quad \Rightarrow \quad \frac{S_3(T_t)}{T_t} = 4 \ln(T_t/H_t) \simeq 140$

Released false vacuum energy (Latent heat)

$$\epsilon(T) = -V_{\text{eff}}(\varphi_B(T), T) + T \frac{\partial V_{\text{eff}}(\varphi_B(T), T)}{\partial T} \quad \text{Normalized parameter: } \alpha = \frac{\epsilon(T_t)}{\rho_{\text{rad}}(T_t)}$$

Inverse of the duration of phase transition

$$\beta = - \left. \frac{dS_E}{dt} \right|_{t=t_t} \simeq \frac{1}{\Gamma} \left. \frac{d\Gamma}{dt} \right|_{t=t_t} \quad \text{Normalized parameter: } \tilde{\beta} = \frac{\beta}{H_t}$$



# GW spectrum

- Complicated numerical simulations are necessary
- Approximate fitting formula are available [Caprini et al. (2015)]

Collision of bubble walls (Envelope approximation):

$$\tilde{\Omega}_{\text{env}} h^2 \simeq 1.67 \times 10^{-5} \times \left( \frac{0.11 v_b^3}{0.42 + v_b^2} \right) \tilde{\beta}^{-2} \left( \frac{\kappa_\phi \alpha}{1 + \alpha} \right)^2 \left( \frac{100}{g_*^t} \right)^{1/3}$$

$$\tilde{f}_{\text{env}} \simeq 1.65 \times 10^{-5} \text{ Hz} \times \left( \frac{0.62}{1.8 - 0.1 v_b + v_b^2} \right) \tilde{\beta} \left( \frac{T_t}{100 \text{ GeV}} \right)$$

Sound waves (Compression waves of thermal plasma):

$$\tilde{\Omega}_{\text{sw}} h^2 \simeq 2.65 \times 10^{-6} v_b \tilde{\beta}^{-1} \left( \frac{\kappa_v \alpha}{1 + \alpha} \right)^2 \left( \frac{100}{g_*^t} \right)^{1/3} \quad \tilde{f}_{\text{sw}} \simeq 1.9 \times 10^{-5} \text{ Hz} \frac{1}{v_b} \tilde{\beta} \left( \frac{T_t}{100 \text{ GeV}} \right)$$

Magnetohydrodynamic (MHD) turbulence:

$$\tilde{\Omega}_{\text{turb}} h^2 \simeq 3.35 \times 10^{-4} v_b \tilde{\beta}^{-1} \left( \frac{\epsilon \kappa_v \alpha}{1 + \alpha} \right)^{3/2} \left( \frac{100}{g_*^t} \right)^{1/3} \quad \tilde{f}_{\text{turb}} \simeq 2.7 \times 10^{-5} \text{ Hz} \frac{1}{v_b} \tilde{\beta} \left( \frac{T_t}{100 \text{ GeV}} \right)$$

- $v_b$  : wall velocity
- $\kappa_\phi$  ,  $\kappa_v$  and  $\epsilon = 0.05$  : efficiency factors

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# Model A: GW spectrum in $O(N)$ models without CSI

## Contribution to GWs

- Wall collision: ..... (dotted line)
- Sound wave: ——— (solid line)
- MHD Turbulence: - - - - (dashed line)

- Benchmark points:

$$\Delta\lambda_{hhh}/\lambda_{hhh}^{\text{SM}} = 2/3$$

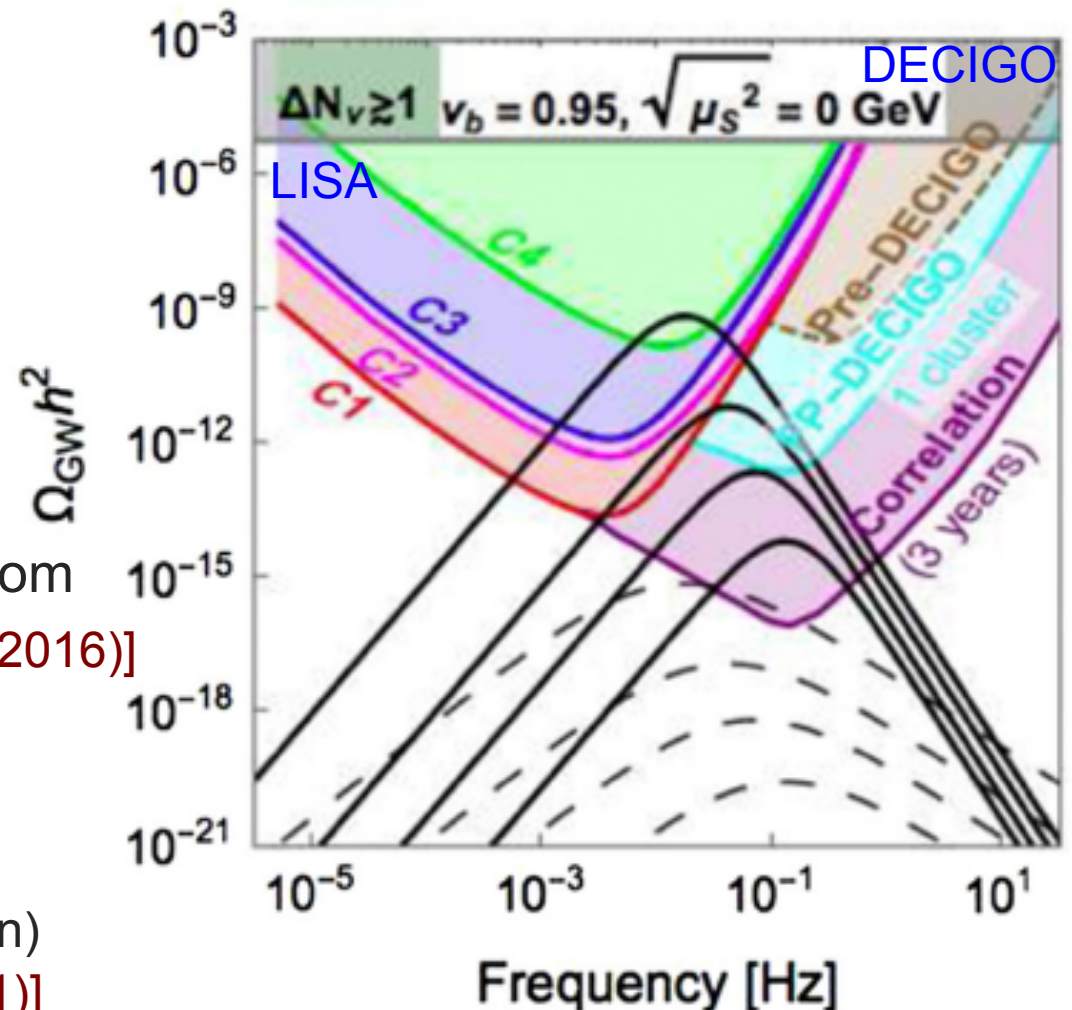
$N = 1, 4, 12, 60$  from the bottom

[Hashino, MK, Kanemura, Matsui (2016)]

## Experimental prospects:

- LISA (C1-C4)  
[Caprini et al. (2015)]
- DECIGO (Pre, FP, Correlation)  
[Kawamura et al. (2011)]

**GWs from 1st order EWPT in  $O(N)$  models are detectable at future space-based interferometers, LISA and DECIGO**



# Model A: Predicted values of $\alpha$ and $\tilde{\beta}$

- Condition for strongly 1<sup>st</sup> OPT

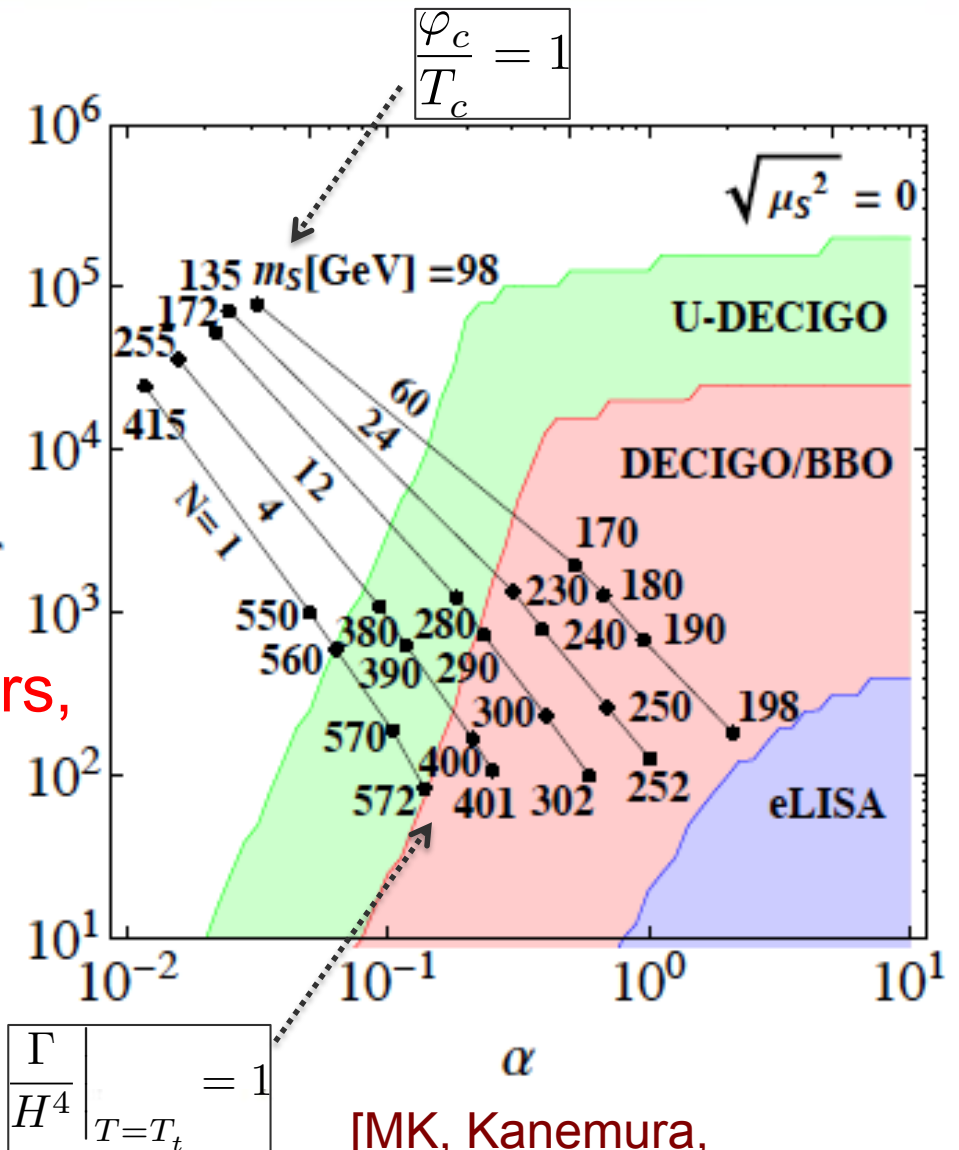
➔ Constraints on  $\alpha$  and  $\tilde{\beta}$  for each model

[MK, Kanemura, Matsui (2015)]

- $\alpha$  and  $\tilde{\beta}$  to be determined by GW observation are useful in determining model parameters, such as  $N$  and  $m_S$

n.b.: The experimental prospects are estimated based on the traditional GW spectrum

[See e.g. Grojean, Servant (2007)]



[MK, Kanemura, Matsui (2015)]

# Testability of models with additional singlet scalars with and without CSI

- What if the  $h h h$  coupling is found to be

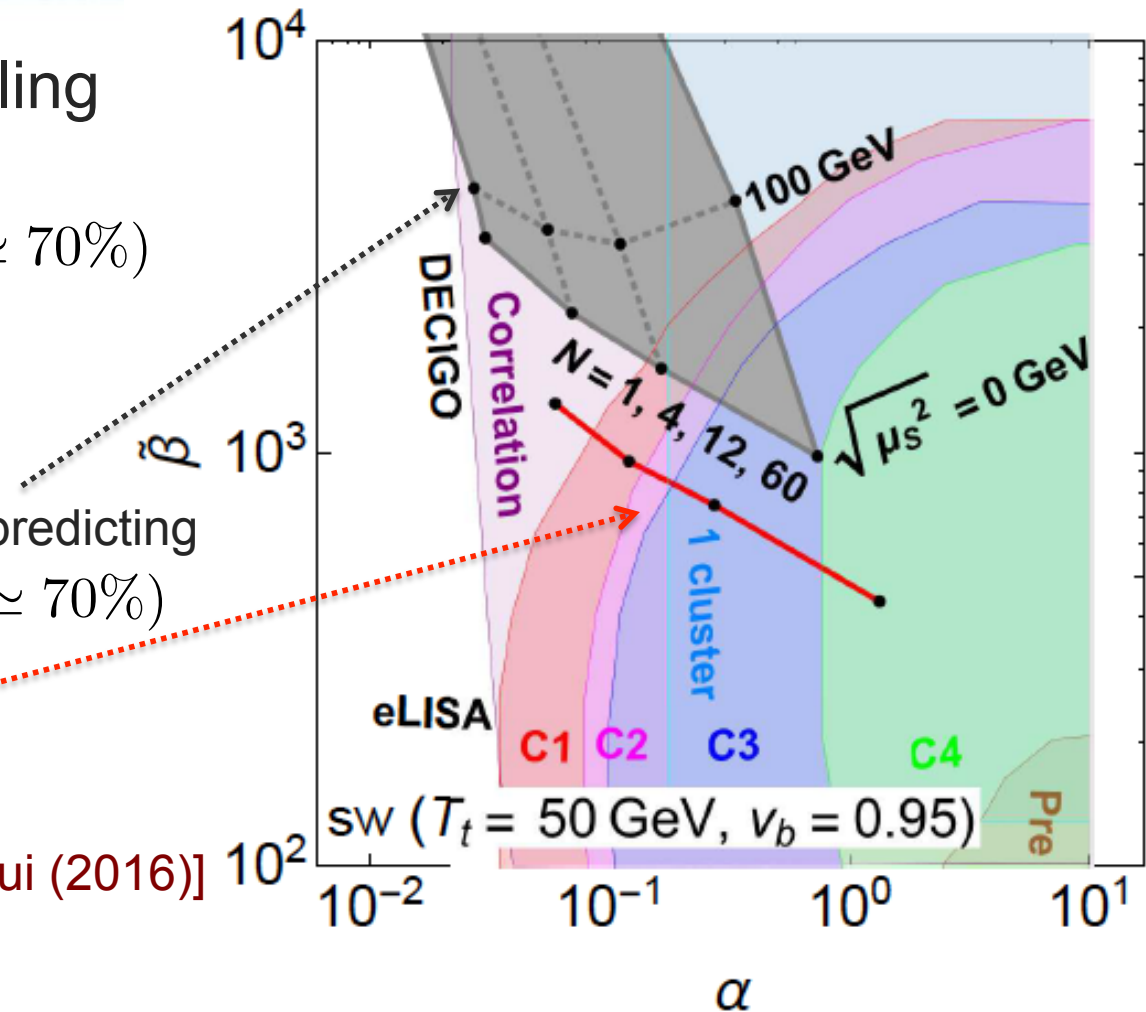
$$\Delta\lambda_{hhh}/\lambda_{hhh}^{\text{SM}} = 2/3 (\simeq 70\%)$$

at future colliders?

- $O(N)$  models without CSI predicting  $\Delta\lambda_{hhh}/\lambda_{hhh}^{\text{SM}} = 2/3 (\simeq 70\%)$

- CSI  $O(N)$  models

[Hashino, MK, Kanemura, Matsui (2016)]



Models with and without CSI can be distinguished at future GW interferometers even if they share common  $h h h$  coupling

# Model C: Predicted values of $\alpha$ and $\tilde{\beta}$ in the HSM

## Numerical results based on the two-field analysis

- Benchmark point

| $v_\Phi$ [GeV] | $v_S$ [GeV] | $m_h$ [GeV] | $\mu_{\Phi S}$ [GeV] | $\mu'_S$ [GeV] | $\mu_S$ [GeV] | $m_H$ [GeV] | $\theta$ [degrees] |
|----------------|-------------|-------------|----------------------|----------------|---------------|-------------|--------------------|
| 246.2          | 90          | 125.5       | -80                  | -30            | 0             | [160, 240]  | [-45, 0]           |

- Condition for strongly 1<sup>st</sup> OPT

➡ Constraints on  $\alpha$  &  $\tilde{\beta}$

[Hashino, MK, Kanemura, Ko, Matsui (2016)]

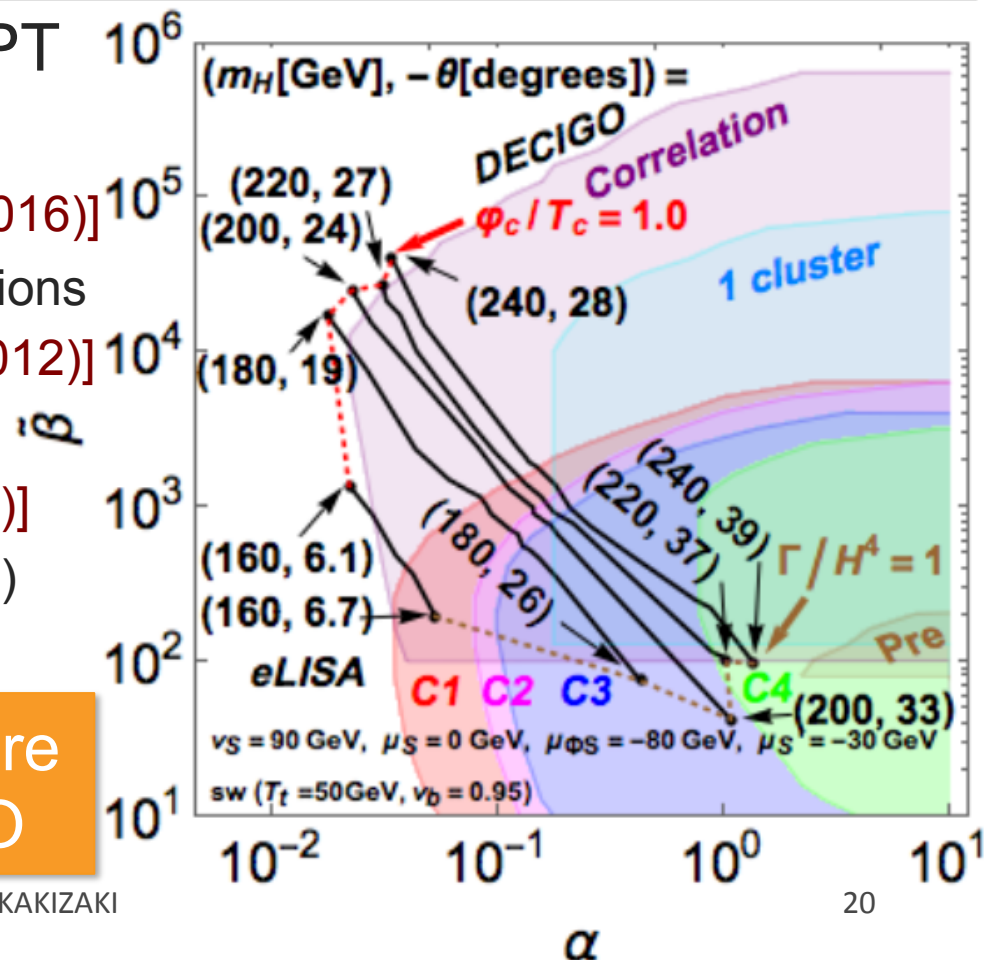
- Implemented into CosmoTransitions

[Wainwright (2012)]

## Prospects of future exps.

- eLISA (C1-C4): [Caprini et al. (2015)]
- DECIGO (Pre, 1 cluster, Correlation)  
[Kawamura et al. (2011)]

GWs from 1st OPT in the HSM are detectable at eLISA and DECIGO





# Model C: Synergy of measurements of various Higgs boson couplings and GWs in the HSM

[Hashino, MK, Kanemura,  
Ko, Matsui (2016)]

## Collider experiments

- LHC Run I results

$$\kappa_Z = 1.03^{+0.11}_{-0.11}, \kappa_W = 0.91^{+0.10}_{-0.10}$$

[ATLAS, CMS (2016)]

- HL-LHC

$$\kappa_V : 2\% \quad [\text{CMS (2013)}]$$

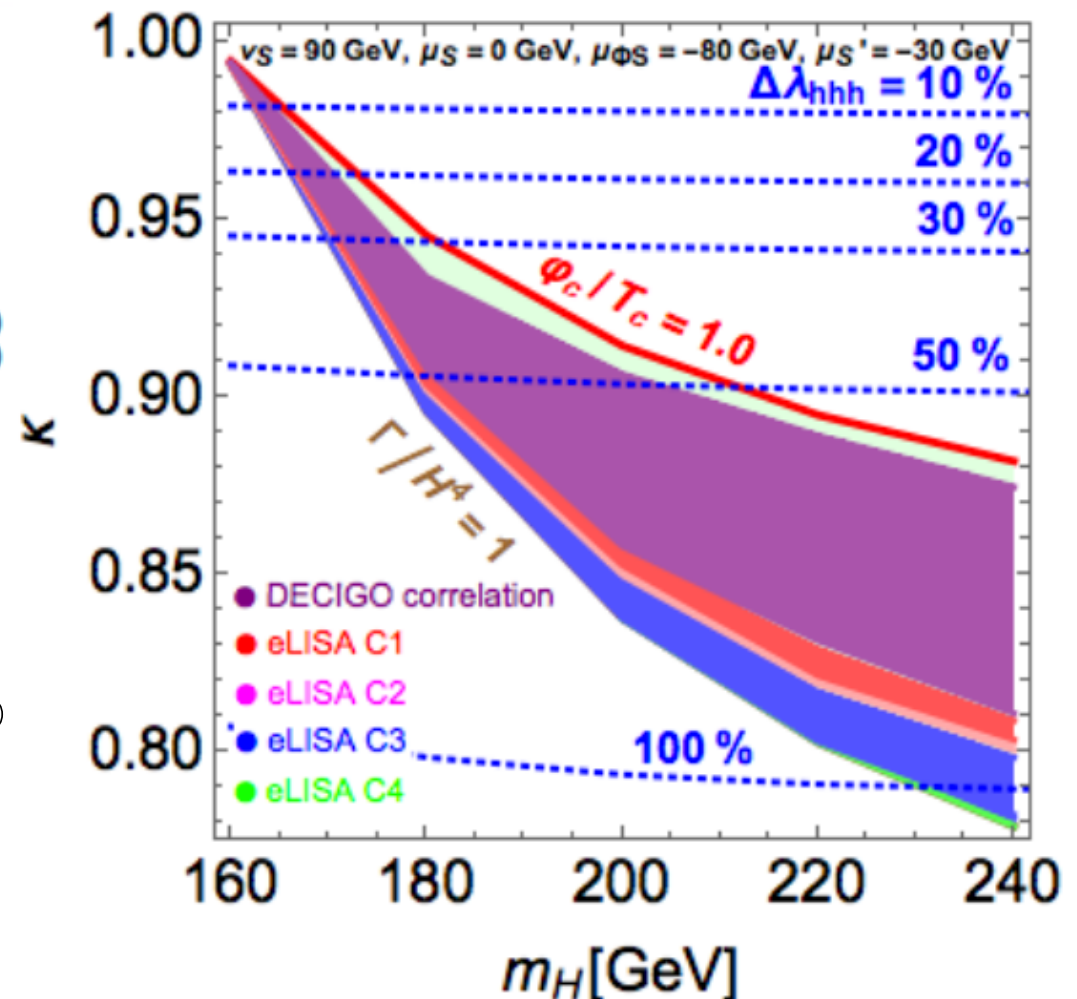
- ILC w/  $\sqrt{s} = 500$  GeV

$$\kappa_Z : 0.37\% \quad \kappa_W : 0.51\%$$

- ILC w/  $\sqrt{s} = 1$  TeV

$$\Delta\lambda_{hhh} : 10\%$$

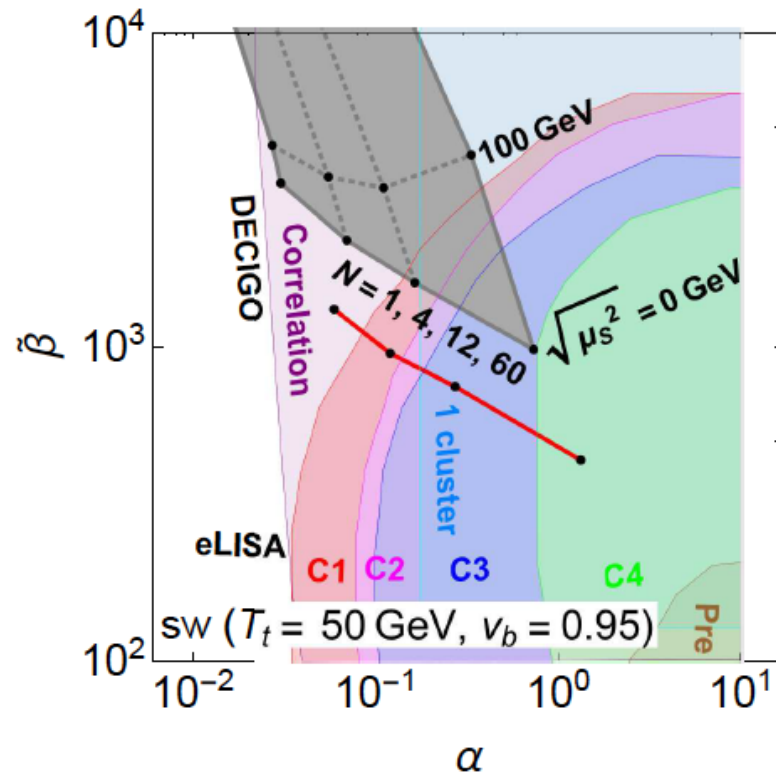
[Fujii et al. (2015)]



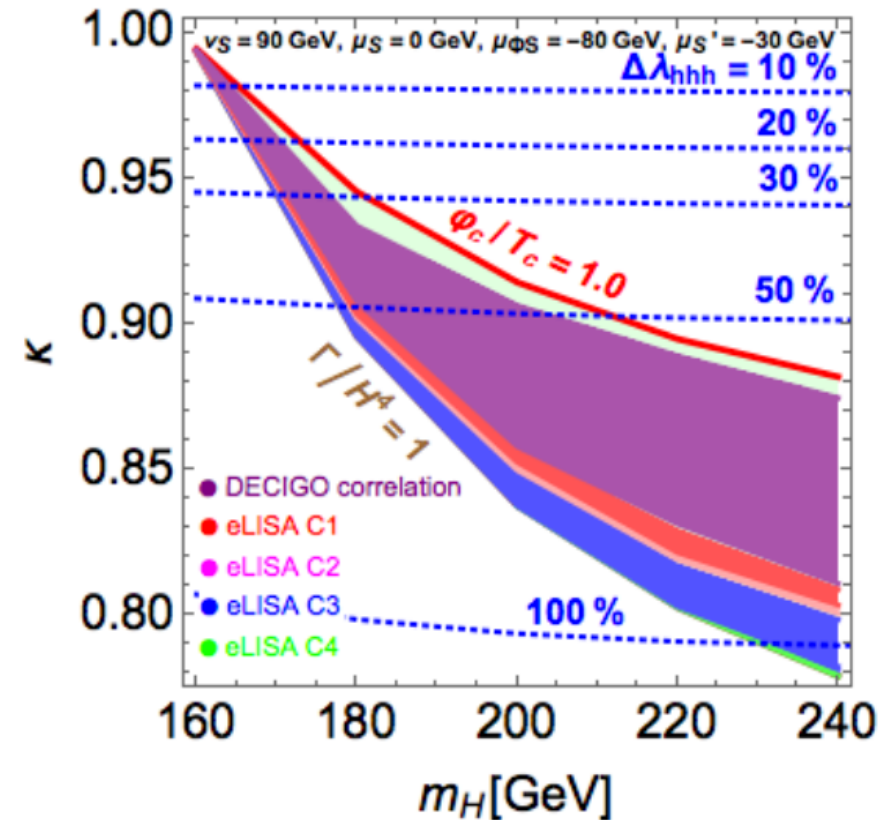
The synergy between the Higgs boson coupling measurements and GW observations is important for the HSM Higgs potential

# Summary

- Models with additional singlet scalars with and without CSI



- Higgs singlet model



- The strongly 1stOPT of the EWSB in extended Higgs sectors can be tested by the synergy of the measurements of Higgs boson couplings  $\kappa_X$  at the LHC, the  $hhh$  coupling at the ILC and GWs at future space-based interferometers

# Backup slides

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# Electroweak baryogenesis (EWBG) and Higgs boson couplings

## Sakharov's conditions for BAU

1<sup>st</sup> OPT

1. Baryon number violation ← Sphaleron
2. C and CP violation ← Extended Higgs sector
3. Departure from thermal equilibrium

Strongly first order phase transition  
(1<sup>st</sup> OPT):  $\varphi_c/T_c \gtrsim 1$

## SM Higgs sector w/ one doublet:

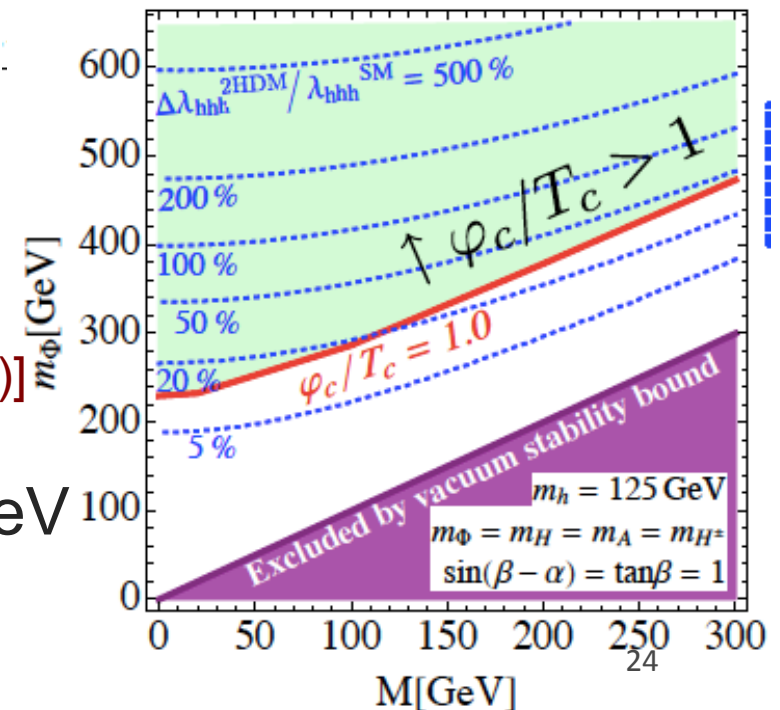
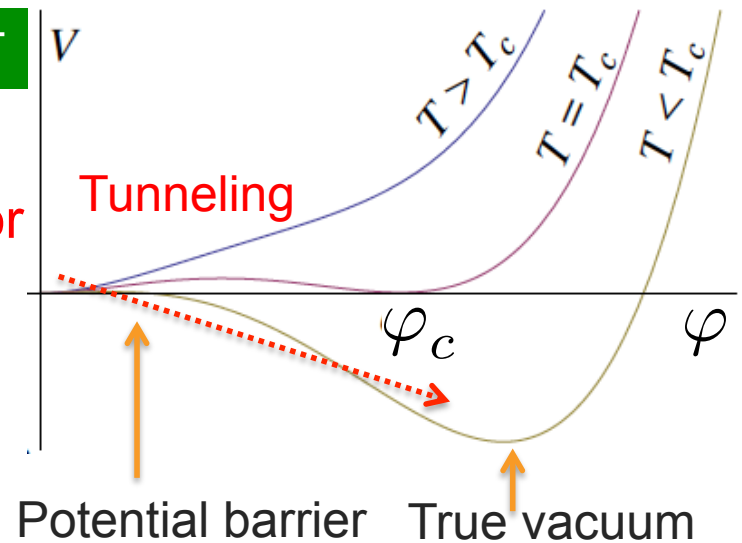
- Electroweak phase transition (EWPT) is NOT of 1<sup>st</sup> order for  $m_h = 125$  GeV  
e.g. Two Higgs doublet model (2HDM)

$$\varphi_c/T_c \gtrsim 1 \longrightarrow \Delta\lambda_{hhh}/\lambda_{hhh}^{\text{SM}} \gtrsim 10\%$$

[Kanemura, Okada, Senaha (2005)]

## EWBG can be tested at future colliders

- International Linear Collider (ILC) 1 TeV can measure  $\lambda_{hhh}$  at 10% accuracy



# Sketch of first order phase transition

Effective potential at one-loop level:

$$V_{\text{eff}}(\varphi) = -\frac{\mu^2}{2}\varphi^2 + \frac{\lambda}{4}\varphi^4 + \sum_i \frac{n_i}{64\pi^2} M_i^4(\varphi) \left( \ln \frac{M_i^2(\varphi)}{Q^2} - \frac{3}{2} \right)$$

Contribution at finite-temperatures:

$$\Delta V_T(\varphi, T) = \frac{T^4}{2\pi^2} \left[ \sum_{i=\text{bosons}} n_i I_B(a^2) + \sum_{i=\text{fermions}} n_i I_F(a^2) \right]$$

$$I_{B/F}(a^2) = \int_0^\infty dx x^2 \ln \left( 1 \mp e^{-\sqrt{x^2+a^2}} \right) \quad a^2 = \frac{M^2(\varphi, T)}{T^2}$$

High-temperature expansion:

$$V_{\text{eff}}(\varphi, T) \simeq D(T^2 - T_0^2)\varphi^2 - \underline{ET}\varphi^3 + \frac{\lambda_T}{4}\varphi^4 + \dots \quad \varphi_c/T_c \propto E$$

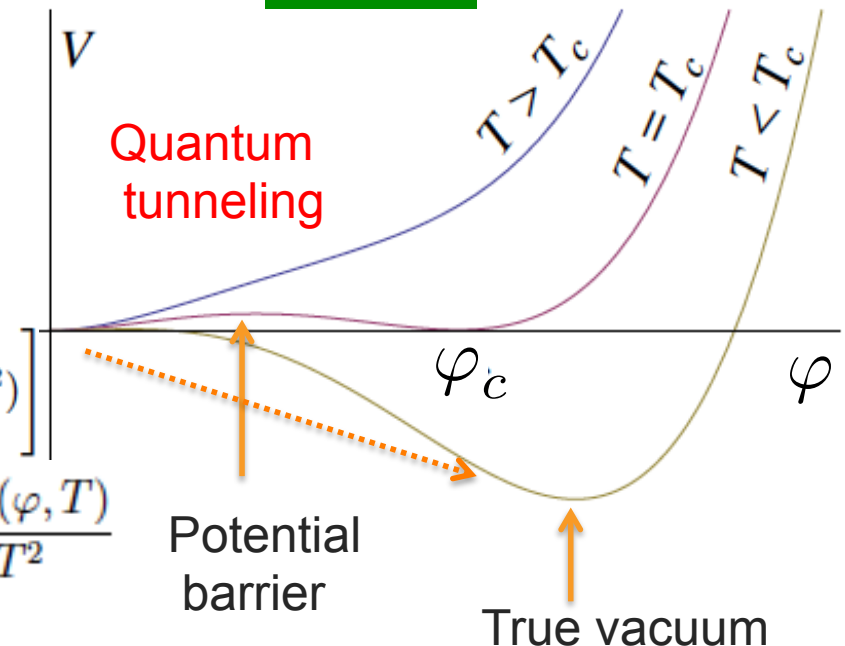
$$I_B(a^2) = -\frac{\pi^4}{45} + \frac{\pi^2}{12}a^2 - \frac{\pi}{6}(a^2)^{3/2} - \frac{a^4}{32} \left( \ln \frac{a^2}{\alpha_B} - 3/2 \right) + \mathcal{O}(a^6)$$

$$I_F(a^2) = \frac{7\pi^4}{360} - \frac{\pi^2}{24}a^2 - \frac{a^4}{32} \left( \ln \frac{a^2}{\alpha_F} - 3/2 \right) + \mathcal{O}(a^6)$$

Bosonic loop contribute to the cubic term

**Strongly 1<sup>st</sup> OPT ( $\varphi_c/T_c \gtrsim 1$ ) can be achieved by adding bosons**

1<sup>st</sup> OPT



Necessary for successful electroweak baryogenesis

# Gravitational waves

## Properties of gravitational waves

- GWs propagate at the speed of light
- GWs are transverse to the direction of propagation
- Spin 2
- 2 polarization modes: Plus mode  $h_+$  & Cross mode  $h_x$

## Weak-field approximation:

- Metric close to flat:  $g_{\mu\nu}(x) = \eta_{\mu\nu} + \underline{h_{\mu\nu}(x)}$   $|h_{\mu\nu}| \ll 1$

Linearized Einstein eq. in vacuum:  $\square h_{\mu\nu} = 0$  **Wave eq.!**

## Interactions of gravitational waves

- Reaction rate:  $\Gamma = n\sigma v$   $n \sim T^3$   $\sigma \sim G^2 T^2 = \frac{T^2}{M_{Pl}^4}$   $v \sim 1$
- Expansion rate of the Universe:  $H \sim \frac{T^2}{M_{Pl}}$

➡  $\frac{\Gamma}{H} \sim \frac{T^3}{M_{Pl}^3}$  : GWs decouple below the Planck scale  $M_{Pl}$

**GWs convey information on the time when produced**

# Relic gravitational waves

## Characteristics of relic gravitational waves

- Isotropic
- Static
- Unpolarized

➡ Relic GWs are characterized only by frequency

## Energy density of relic gravitational waves

$$\rho_{\text{GW}} = \frac{1}{32\pi G} \langle \dot{h}_{ij} \dot{h}_{ij} \rangle$$

- Normalized Energy density per unit logarithmic interval of frequency  $f$

$$\Omega_{\text{GW}}(f) \equiv \frac{1}{\rho_c} \frac{d\rho_{\text{GW}}}{d \ln f} \quad \rho_c = \frac{3H_0^2}{8\pi G} \quad : \text{Critical density}$$

# Typical frequency of relic gravitational waves

Gravitational waves produced with frequency  $f_t$ , abundance  $\Omega_{\text{GW}}^t$  are red-shifted due to the expansion of the Universe

- Frequency and energy density scale as  $f \propto \frac{1}{a}$   $\rho_{\text{GW}} \propto \frac{1}{a^4}$
- Conservation of entropy:  $sa^3 = \text{const.}$   $a$ : scale factor

➡ Red-shifted GW relic abundance observed today:

$$\Omega_{\text{GW}} h^2 \simeq 1.7 \times 10^{-5} \left( \frac{100}{g_*^t} \right)^{1/3} \Omega_{\text{GW}}^t$$

Red-shifted typical frequency observed today:

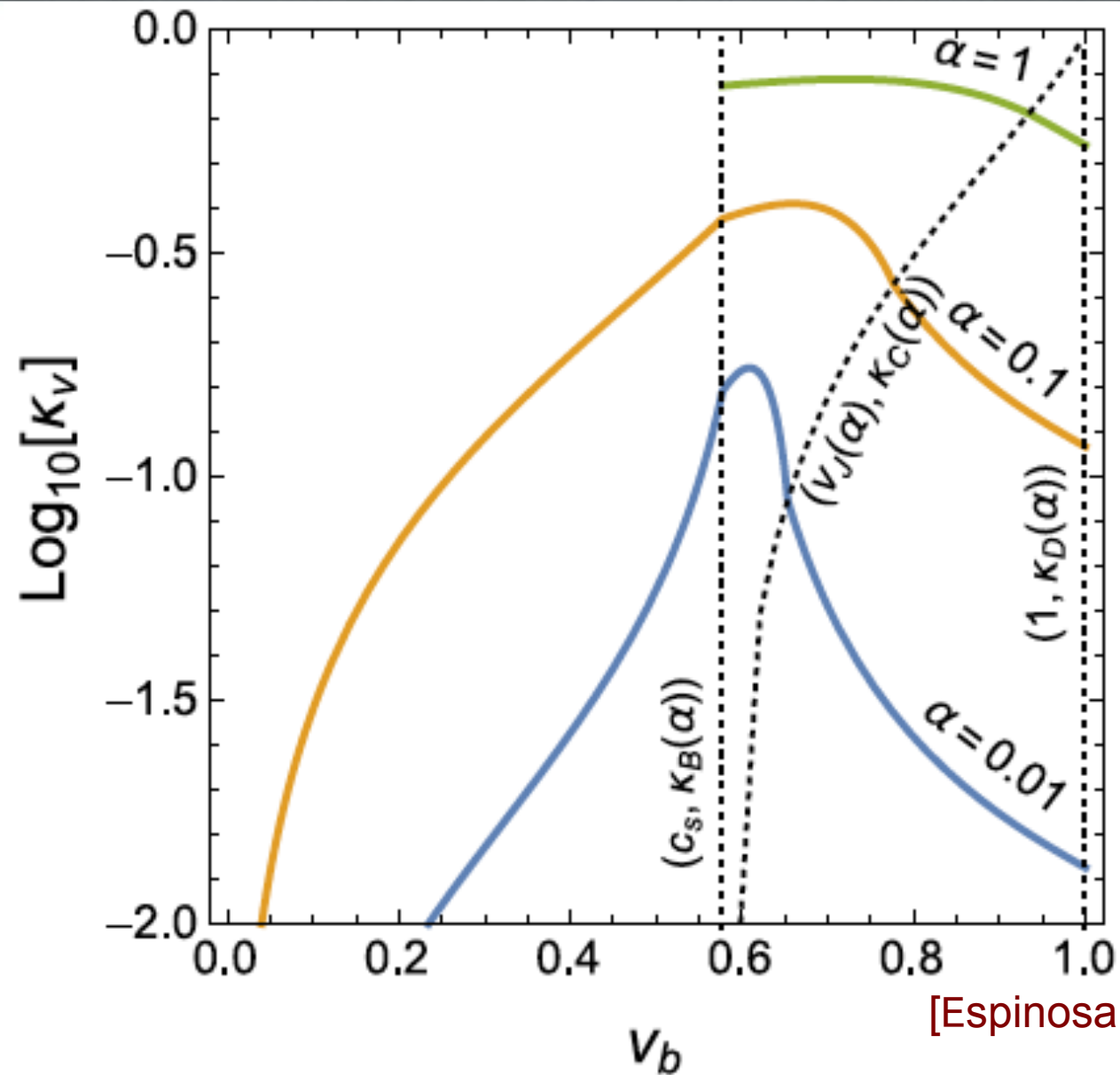
$$f_0 \simeq 1.7 \times 10^{-5} \left( \frac{g_*^t}{100} \right)^{1/6} \left( \frac{T_t}{100 \text{ GeV}} \right) \frac{f_t}{H_t} \text{ Hz}$$

For typical electroweak phase transition:

$$T_t \sim 100 \text{ GeV} \quad f_t/H_t \sim 10^2 - 10^4 \quad \Rightarrow \quad \underline{f_0 \sim 10^{-3} - 10^{-1} \text{ Hz}}$$

Range for future space-based interferometers

# Efficiency factor



[Espinosa et al. (2010)]



# Models with additional singlet scalars (without CSI) (contd.)

- Effective potential:

[MK, Kanemura, Matsui (2015)]

$$V_{\text{eff}}(\varphi) = -\frac{\mu^2}{2}\varphi^2 + \frac{\lambda}{4}\varphi^4 + \sum_i \frac{n_i}{64\pi^2} M_i^4(\varphi) \left( \ln \frac{M_i^2(\varphi)}{Q^2} - \frac{3}{2} \right)$$

$$\longrightarrow \lambda_{hhh}^{O(N)} = \frac{3m_h^2}{v} \left\{ 1 - \frac{1}{\pi^2} \frac{m_t^4}{v^2 m_h^2} + \frac{N}{12\pi^2} \frac{m_S^4}{v^2 m_h^2} \left( 1 - \frac{\mu_S^2}{m_S^2} \right)^3 \right\}$$

Non decoupling loop effect from additional scalars

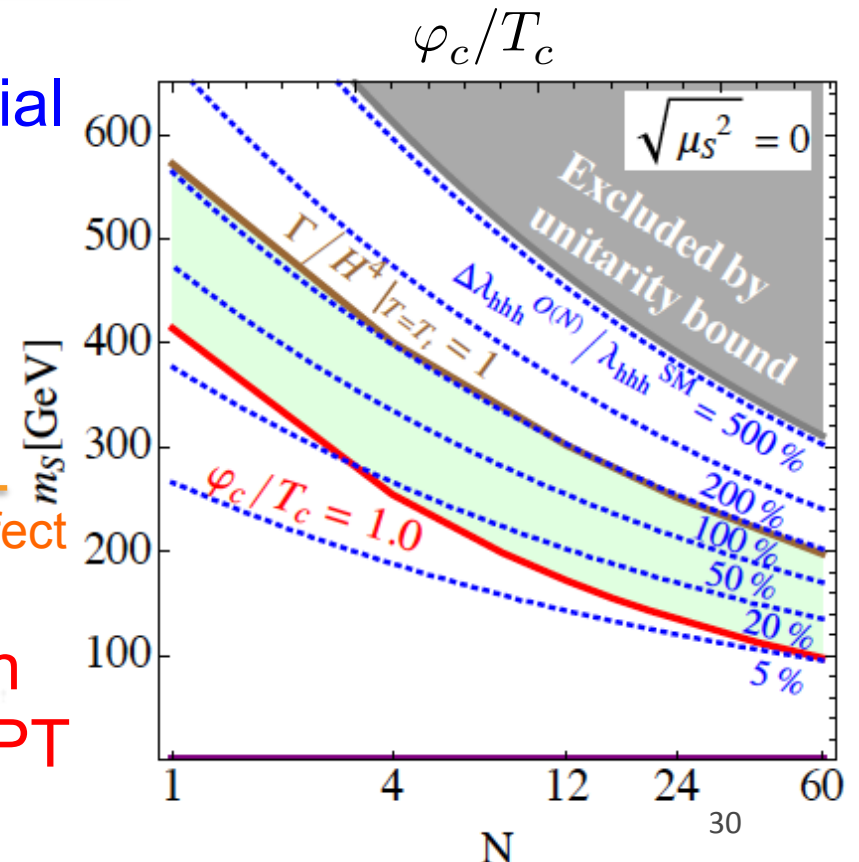
- Finite temperature effective potential (high temperature expansion):

$$V_{\text{eff}}(\varphi, T) \simeq D(T^2 - T_0^2)\varphi^2 - ET\varphi^3 + \frac{\lambda_T}{4}\varphi^4 + \dots$$

$$\frac{\varphi_c}{T_c} \propto E = \frac{1}{12\pi v^3} \left[ 6m_W^3 + 3m_Z^3 + Nm_S^3 \left( 1 - \frac{\mu_S^2}{m_S^2} \right)^3 \left( 1 + \frac{3\mu_S^2}{2m_S^2} \right) \right]$$

Non decoupling loop effect from additional scalars

$\longrightarrow$  Typically  $\mathcal{O}(10)\%$  deviation in  $\lambda_{hhh}$  for strongly 1<sup>st</sup> OPT



# Benchmark points

- $O(N)$  models without CSI:

$$\sqrt{\mu_S^2} = 0$$

| $N$                    | 1                        | 4                      | 12                     | 60                    |
|------------------------|--------------------------|------------------------|------------------------|-----------------------|
| $m_S$                  | 540 GeV                  | 382 GeV                | 290 GeV                | 194 GeV               |
| $\varphi_c/T_c, T_c$   | 2.01, 102 GeV            | 2.40, 90.1 GeV         | 2.91, 76.8 GeV         | 4.11, 56.1 GeV        |
| $(\alpha, \beta), T_t$ | (0.0593, 1320), 88.5 GeV | (0.120, 956), 74.3 GeV | (0.273, 705), 59.7 GeV | (1.33, 438), 38.4 GeV |

$$\sqrt{\mu_S^2} = 100 \text{ GeV}$$

| $(\sqrt{\mu_S^2}, N)$  | (0 GeV, 1)              | (0 GeV, 4)               | (0 GeV, 12)             | (0 GeV, 60)             |
|------------------------|-------------------------|--------------------------|-------------------------|-------------------------|
| $m_S$                  | 510 GeV                 | 361 GeV                  | 274 GeV                 | 183 GeV                 |
| $\varphi_c/T_c, T_c$   | 1.62, 119 GeV           | 2.03, 102 GeV            | 2.54, 85.6 GeV          | 3.65, 61.5 GeV          |
| $(\alpha, \beta), T_t$ | (0.0303, 3320), 111 GeV | (0.0695, 2180), 92.5 GeV | (0.164, 1600), 74.8 GeV | (0.739, 1090), 50.3 GeV |

- CSI  $O(N)$  models:

| $(\sqrt{\mu_S^2}, N)$  | (100 GeV, 1)            | (100 GeV, 4)             | (100 GeV, 12)           | (100 GeV, 60)           |
|------------------------|-------------------------|--------------------------|-------------------------|-------------------------|
| $m_S$                  | 524 GeV                 | 380 GeV                  | 299 GeV                 | 219 GeV                 |
| $\varphi_c/T_c, T_c$   | 1.56, 121 GeV           | 1.89, 106 GeV            | 2.25, 92.1 GeV          | 2.89, 71.6 GeV          |
| $(\alpha, \beta), T_t$ | (0.0272, 4380), 115 GeV | (0.0552, 3480), 99.5 GeV | (0.111, 3210), 85.7 GeV | (0.334, 4082), 67.2 GeV |

# GW spectrum in CSI $O(N)$ models

- Contribution to GWs:

- Collision: ..... (dotted line)
- Sound wave: ——— (solid line)
- MHD Turbulence: - - - - (dashed line)

- Benchmark points:

$$N = 1, 4, 12, 60$$

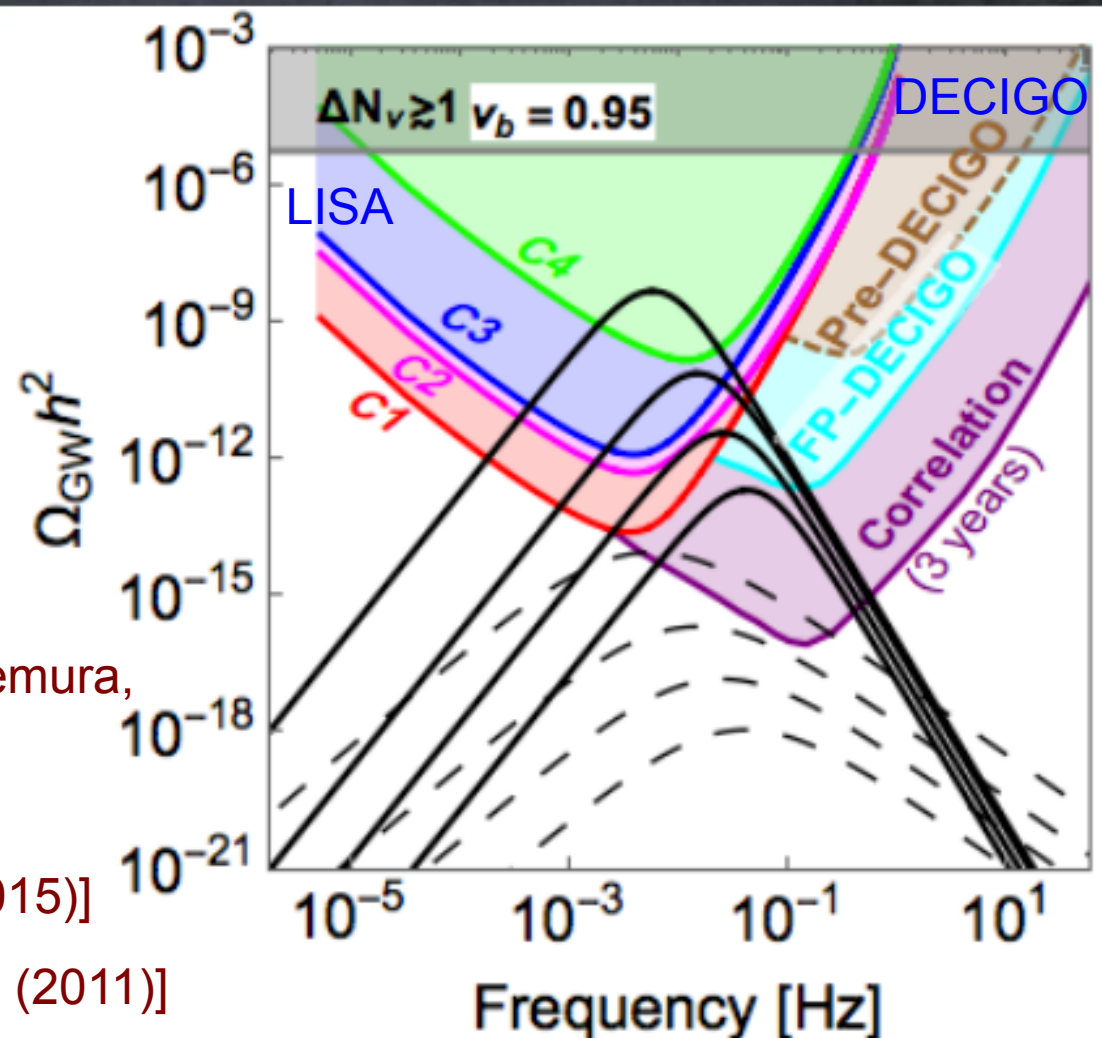
from the bottom

[Hashino, MK, Kanemura,  
Matsui (2016)]

- Experimental prospects:

- LISA: [Caprini et al. (2015)]
- DECIGO: [Kawamura et al. (2011)]

- Contribution from sound waves is dominant and detectable at future space-based interferometers, LISA and DECIGO



# Model A vs. Model B: Predicted values of $\alpha$ and $\tilde{\beta}$ in models with singlet scalars with and without CSI

- Condition for strongly 1<sup>st</sup> OPT

[See Hashino's poster]

→ Constraints on  $\alpha$  and  $\tilde{\beta}$  for each model

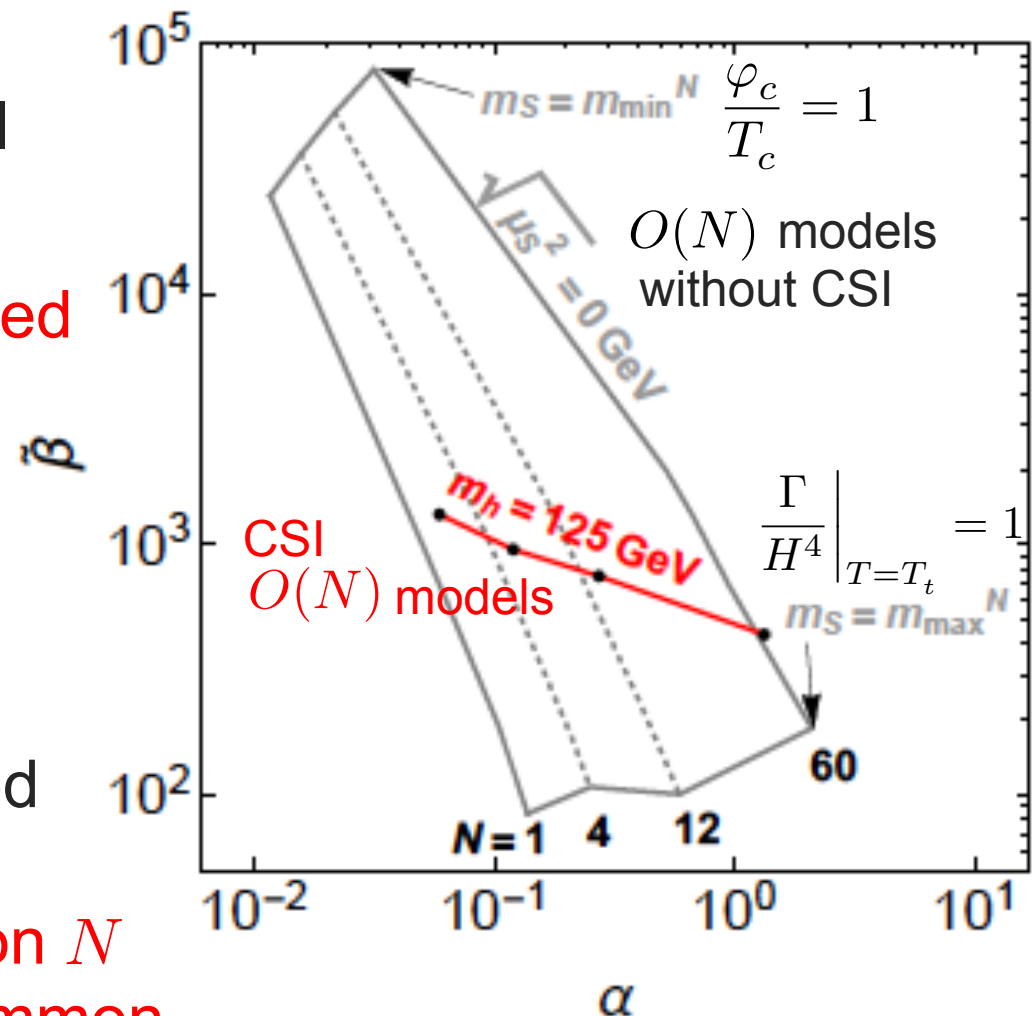
## $O(N)$ models without CSI

- $\alpha$  and  $\tilde{\beta}$  to be determined by GW observation are useful measures in determining  $N$  and  $m_S$

## CSI $O(N)$ models

- Scale invariance is violated at finite temperatures

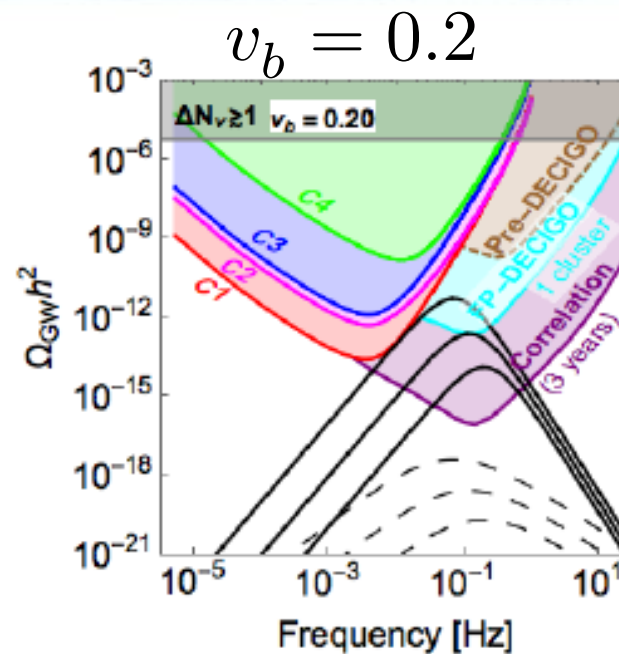
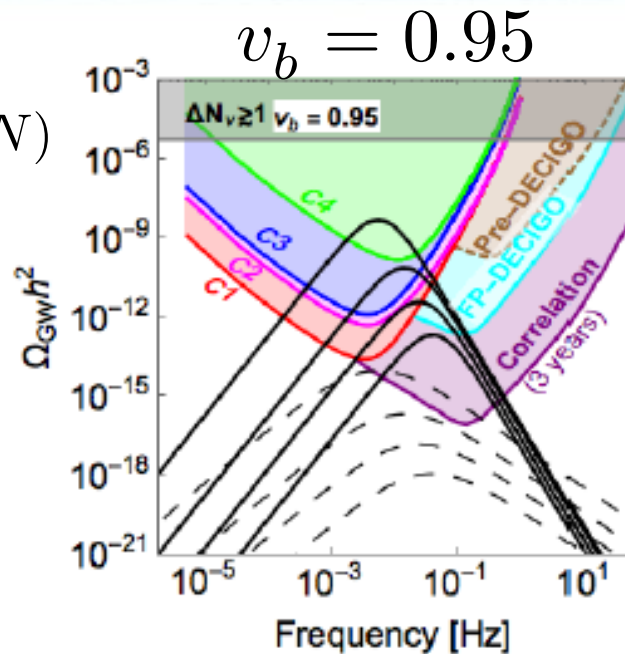
→  $\alpha$  and  $\tilde{\beta}$  depend on  $N$  though  $\lambda_{hhh}$  is common



[Hashino, MK, Kanemura, Matsui (2016)]

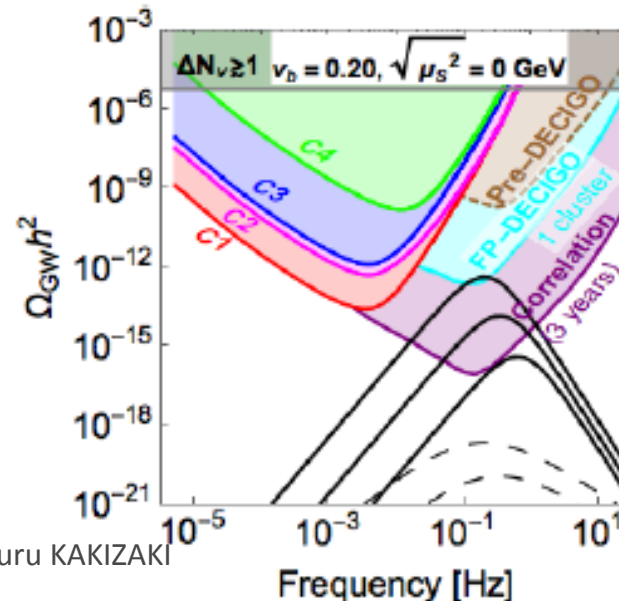
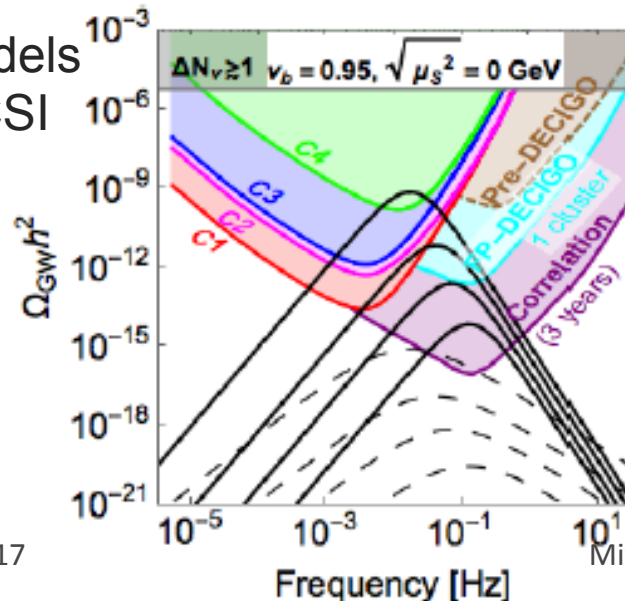
# Comparison of GW spectra

- CSI  $O(N)$  models



[Hashino, MK, Kanemura, Matsui (2016)]

- $O(N)$  models without CSI  
 $\sqrt{\mu_S^2} = 0$



- N.B. subsonic wall velocity is preferred for EWBG but not necessarily  
[No (2011)]



# GW spectrum for $v_b = 1$

## Case of accelerating wall:

- For large  $\alpha$ , the bubble wall can accelerate without reaching a terminal velocity

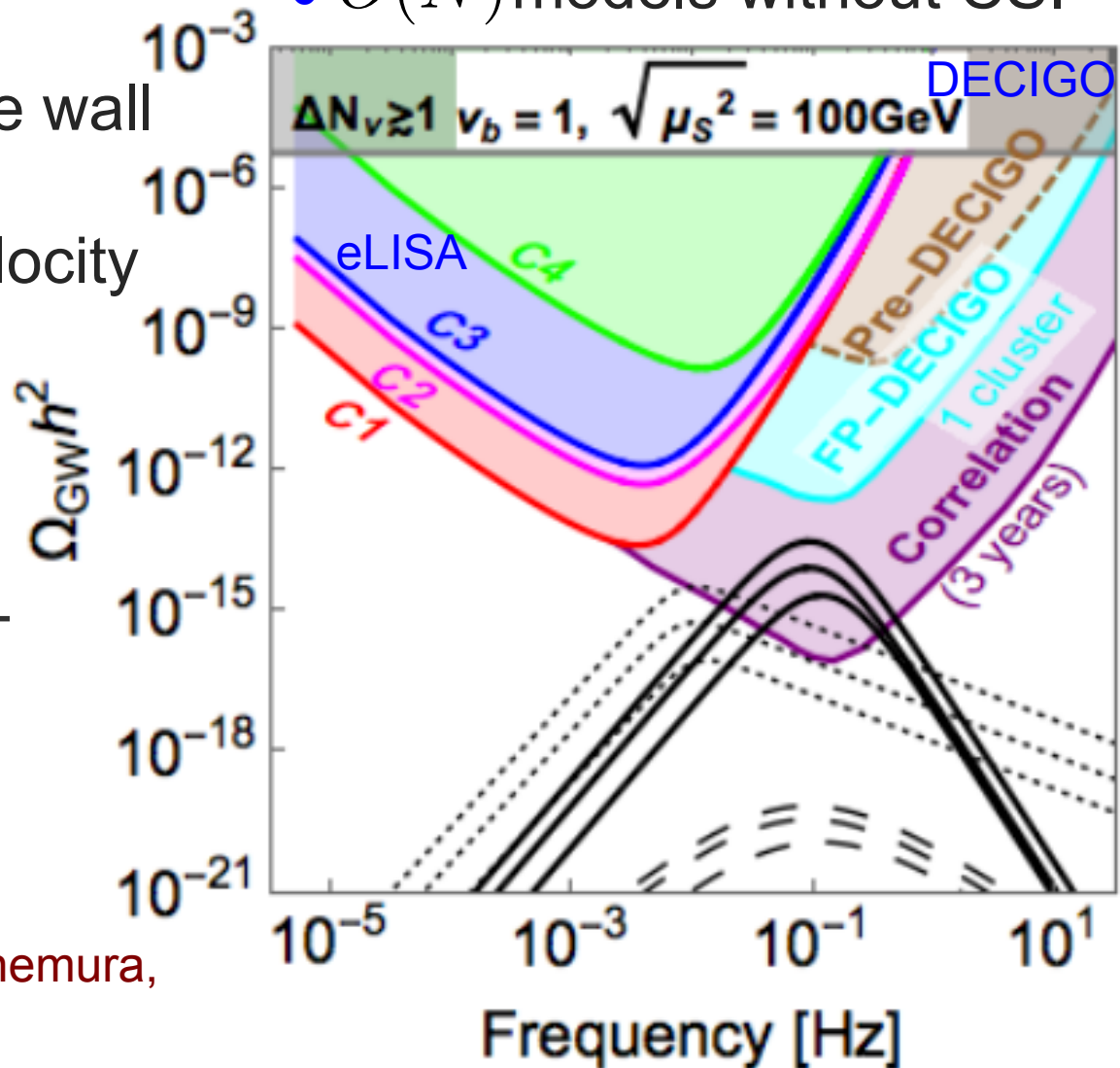
## Contribution to GWs:

- Collision: .....
- Sound wave: \_\_\_\_\_
- MHD Turbulence: - - - -

- Benchmark points:  
 $N = 4, 12, 60$   
from the bottom

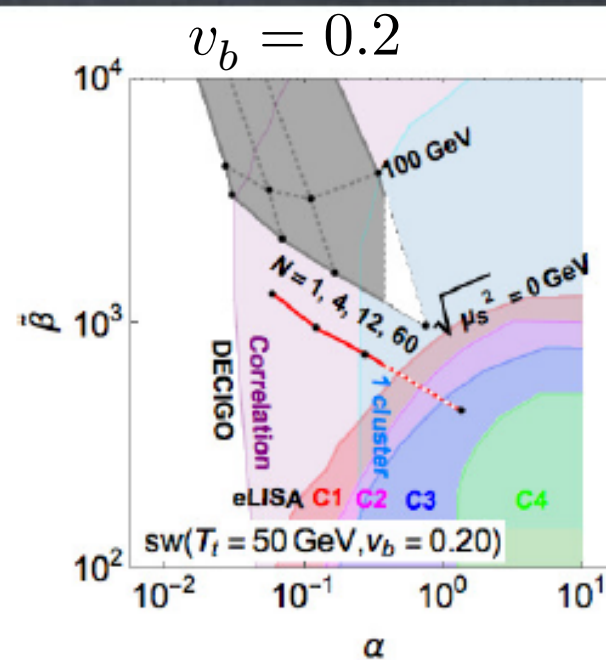
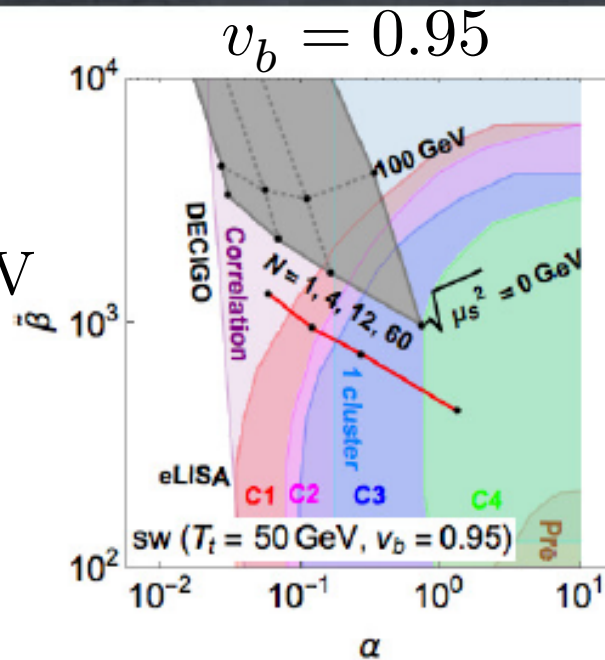
[Hashino, MK, Kanemura,  
Matsui (2016)]

- $O(N)$  models without CSI



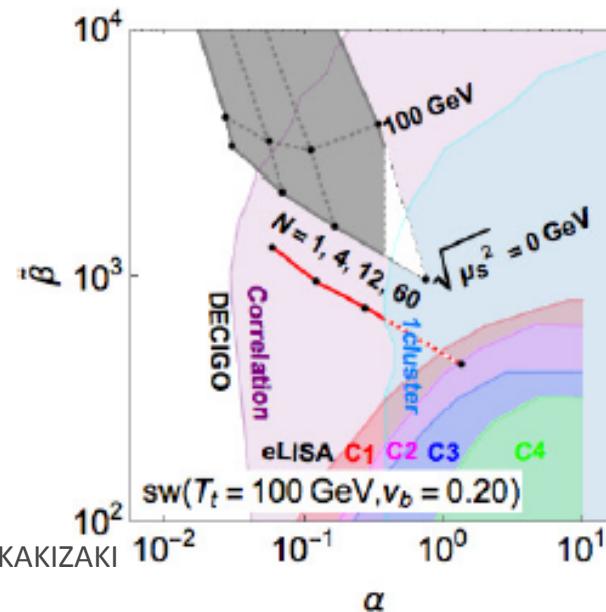
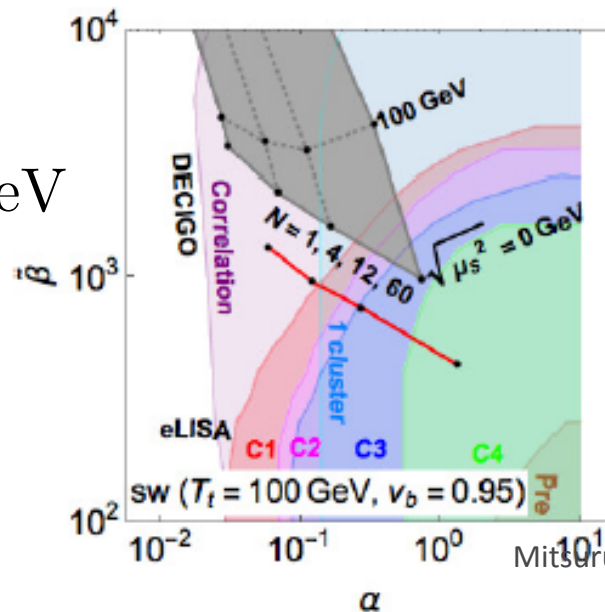
# Transition temperature and wall velocity dependence of detectability of GWs

$T_t = 50 \text{ GeV}$



[Hashino, MK,  
Kanemura,  
Matsui (2016)]

$T_t = 100 \text{ GeV}$





# Predicted values of $\alpha$ and $\tilde{\beta}$

## Numerical results based on the two-field analysis

- Benchmark point

| $v_\Phi$ [GeV] | $v_S$ [GeV] | $m_h$ [GeV] | $\mu_{\Phi S}$ [GeV] | $\mu'_S$ [GeV] | $\mu_S$ [GeV] | $m_H$ [GeV] | $\theta$ [degrees] |
|----------------|-------------|-------------|----------------------|----------------|---------------|-------------|--------------------|
| 246.2          | 90          | 125.5       | -80                  | -30            | 0             | [160, 240]  | [-45, 0]           |

- Condition for strongly 1<sup>st</sup> OPT

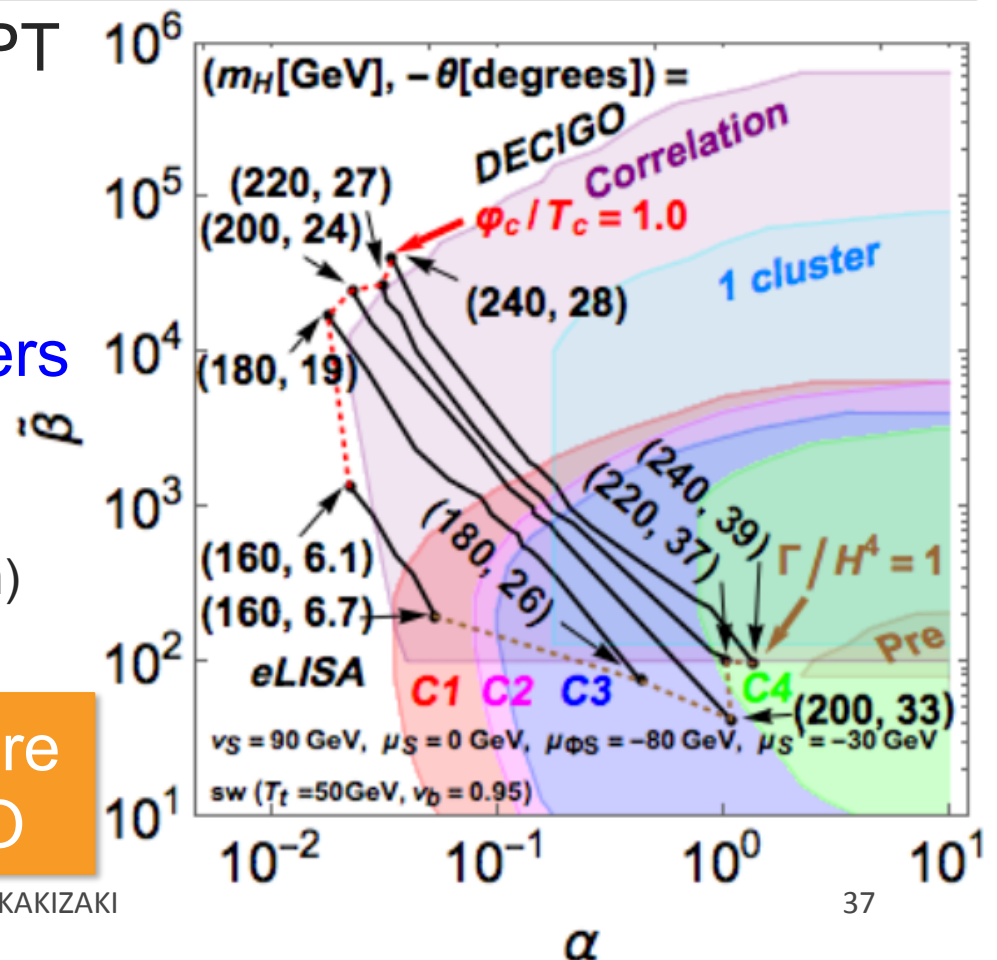
➔ Constraints on  $\alpha$  &  $\tilde{\beta}$

[Hashino, MK, Kanemura, Ko, Matsui (2016)]

## Prospects of future interferometers

- LISA (C1, C2, C3, C4):  
[Caprini et al. (2015)]
- DECIGO (Pre, 1 cluster, Correlation)  
[Kawamura et al. (2011)]

GWs from 1st OPT in the HSM are detectable at eLISA and DECIGO



# Landau pole in CSI model

- Large scalar couplings at the EW scale

➡ Landau pole near the EW scale

| $N$                        | 1       | 4       | 12      | 60      |
|----------------------------|---------|---------|---------|---------|
| $Q$                        | 381 GeV | 257 GeV | 188 GeV | 119 GeV |
| $\Lambda(\lambda_S = 0)$   | 5.4 TeV | 17 TeV  | 28 TeV  | 33 TeV  |
| $\Lambda(\lambda_S = 0.1)$ | 5.3 TeV | 16 TeV  | 23 TeV  | 13 TeV  |
| $\Lambda(\lambda_S = 0.2)$ | 5.2 TeV | 15 TeV  | 19 TeV  | 5.4 TeV |
| $\Lambda(\lambda_S = 0.3)$ | 5.0 TeV | 14 TeV  | 15 TeV  | 2.7 TeV |

[Hashino, MK, Kanemura, Matsui (2016)]

# LISA design

[Caprini et al (2015)]

| Name             | C1       | C2       | C3       | C4       |
|------------------|----------|----------|----------|----------|
| Full name        | N2A5M5L6 | N2A1M5L6 | N2A2M5L4 | N1A1M2L4 |
| # links          | 6        | 6        | 4        | 4        |
| Arm length [km]  | 5M       | 1M       | 2M       | 1M       |
| Duration [years] | 5        | 5        | 5        | 2        |
| Noise level      | N2       | N2       | N2       | N1       |

# Indirect detection of gravitational waves

- Effects of gravitational waves have been observed
- In 1976, Hulse and Taylor discovered the binary pulsar (neutron star) PSR 1913+16
- The orbit is contracting due to the emission of gravitational waves

➡ 1993 Nobel prize

