# Refined study of CDM isocurvature perturbations in curvaton

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新学術領域「なぜ宇宙は加速するのか？- 徹底的究明と将来への挑戦-」公募研究

課題名「複数場インフレーションモデルの検証に関する理論的研究」

In collab. with N. Kitajima (APCTP), D. Langlois (APC, Paris), T. Takahashi (Saga Univ.)

Kitajima, Langlois, Takesako, Takahashi, SY, arXiv:1407.5148
Kitajima, Langlois, Takahashi, SY, in preparation

# Curvaton scenario

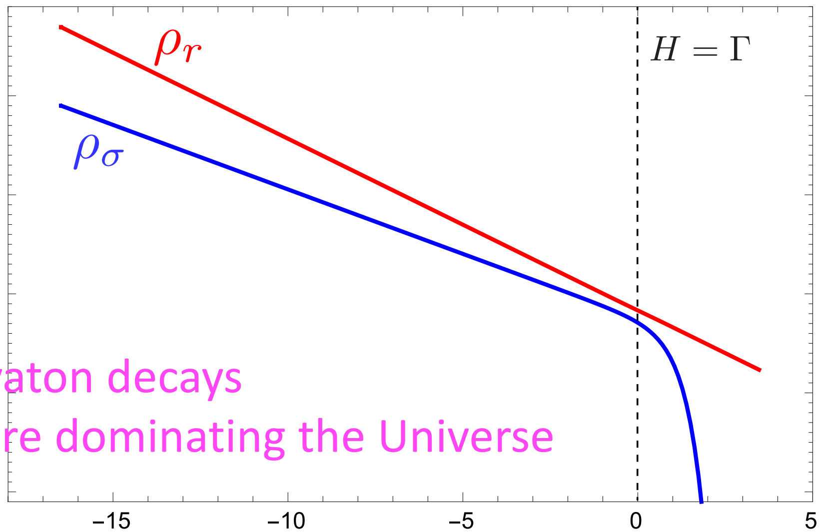
Linde and Mukhanov (1997), Moroi and Takahashi (2001), Lyth and Wands (2001), Enqvist and Sloth (2001), …

Inflaton + another scalar field (curvaton)

$\phi$ $\sigma$

oscillating scalar field (curvaton) in radiation-dominated Universe

radiation-dominated Universe after reheating




Curvaton decays before dominating the Universe

$N - N_{\rm dec}$ $(= \ln(a/a_{\rm dec}))$ ; e-folding number as a time coordinate

Evolution equations;

$$\frac{d\rho_r}{dt} + 4H\rho_r = \Gamma\rho_\sigma,$$
$$\frac{d\rho_\sigma}{dt} + 3H\rho_\sigma = -\Gamma\rho_\sigma,$$

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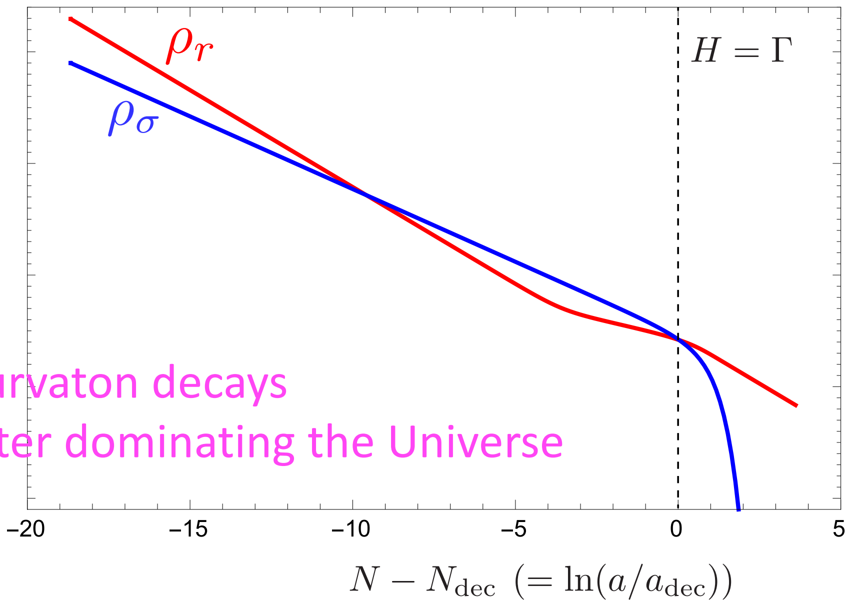


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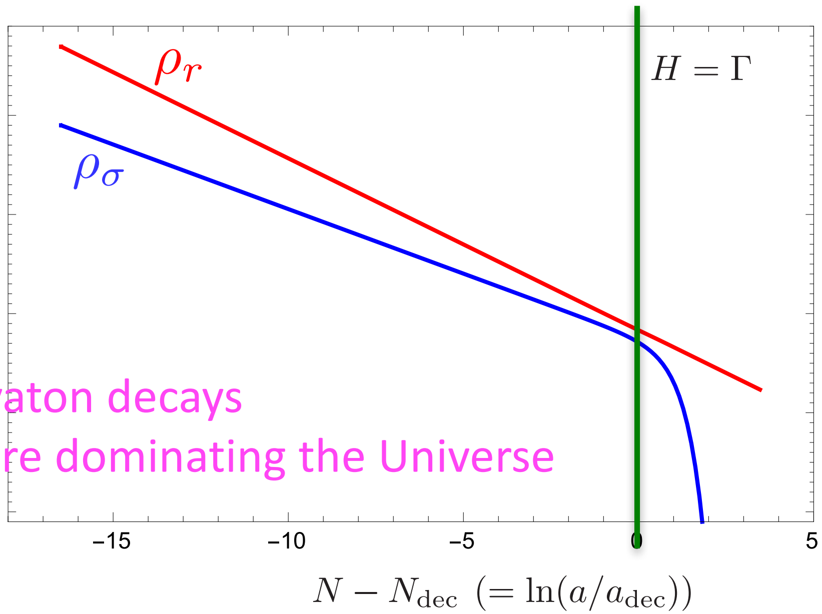


Curvaton decays
before dominating the Universe

primordial adiabatic curvature perturbations
generated from the curvaton fluctuations

$$\zeta = \frac{r_s}{3}\frac{\delta\sigma_{\rm i}}{\sigma_{\rm i}} = \frac{r_s}{3}\frac{H_{\rm i}}{2\pi\sigma_{\rm i}}$$

$$r_s := \left.\frac{3\rho_\sigma}{4\rho_r + 3\rho_\sigma}\right|_{N=N_{\rm dec}}$$

related with entropy production rate

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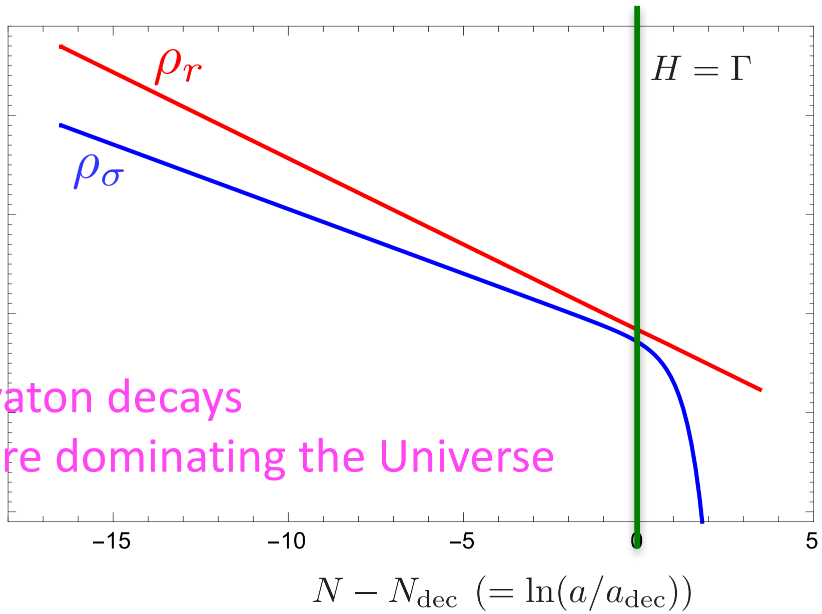



Curvaton decays
before dominating the Universe

primordial adiabatic curvature perturbations
generated from the curvaton fluctuations

$$\zeta \quad r_s \, \delta\sigma_{\rm i} \qquad r_s \, \frac{H_{\rm i}}{\tau\sigma_{\rm i}}$$

Key; decay time

$$r_s := \left.\frac{3\rho_\sigma}{4\rho_r + 3\rho_\sigma}\right|_{N=N_{\rm dec}} \qquad H = \Gamma$$

# Cold Dark Matter (thermally produced, WIMP)

$$\frac{dn_{\rm CDM}}{dt} + 3Hn_{\rm CDM} = -\lambda\left(n_{\rm CDM}^2 - \left(n_{\rm CDM}^{(\rm eq)}\right)^2\right)$$

$$\lambda = \langle \sigma v \rangle$$

; thermally averaged annihilation cross section

$$n_{\rm CDM}^{(\rm eq)} = g_* \left(\frac{mT}{2\pi}\right)^{3/2} e^{-m/T},$$

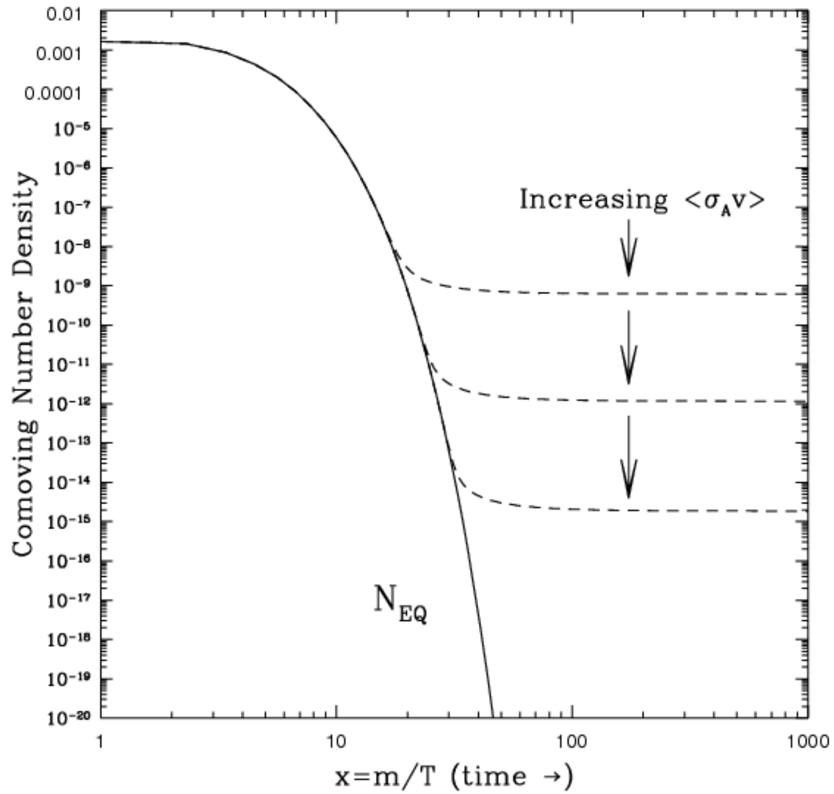
for non-relativistic particles

Relic abundance;

$$\Omega_{\rm CDM}h^2 \approx 0.1 \times \left(\frac{3\times10^{-26}\ [{\rm cm}^3/{\rm s}]}{\langle\sigma v\rangle}\right)$$

$$m/T_{\rm fr} \approx 20$$

$$\frac{n_{\rm CDM}}{s} \propto a^3\, n_{\rm CDM}$$ ; com. num. dens.




Kolb and Turner, ``The early Universe”

# Cold Dark Matter (thermally produced, WIMP)

$$\frac{n_{\rm CDM}}{s} \propto a^3\, n_{\rm CDM}$$ ; com. num. dens.

Key; freeze-out time

( ~ mass of the CDM particle)

$$H = \lambda\, n_{\rm CDM}$$

Relic abundance;

$$\Omega_{\rm CDM} h^2 \approx 0.1 \times \left( \frac{3 \times 10^{-26}\ [{\rm cm}^3/{\rm s}]}{\langle \sigma v \rangle} \right)$$

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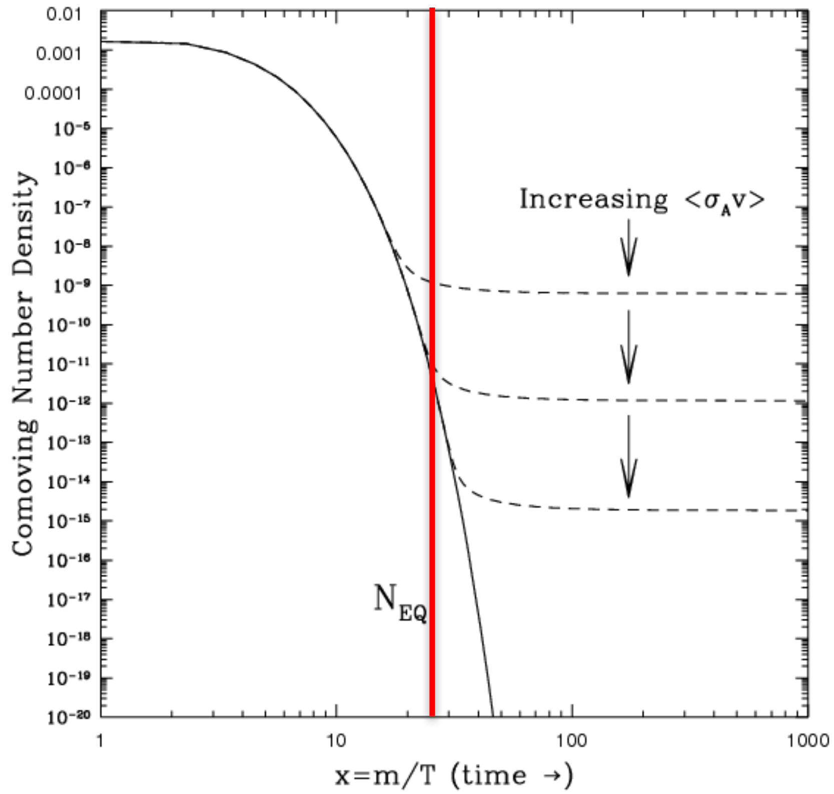



Kolb and Turner, ``The early Universe”

# CDM isocurvature perturbations

$\zeta_{\rm CDM}$ ; curvature perturbations on uniform CDM energy density hypersurface

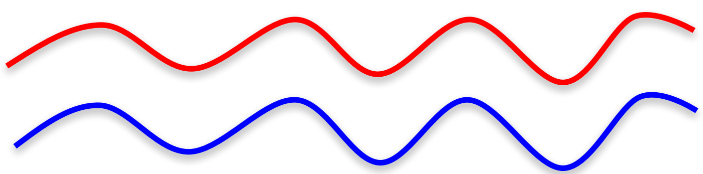
( or density fluctuations of CDM on flat hypersurface)

difference between $\zeta_{\rm CDM}$ and $\zeta$

$$S_{\rm CDM} \equiv 3(\zeta_{\rm CDM} - \zeta)$$

$\zeta_{\rm CDM}$

$\zeta$



$$S_{\rm CDM} = 0$$

pure adiabatic. standard single inflation

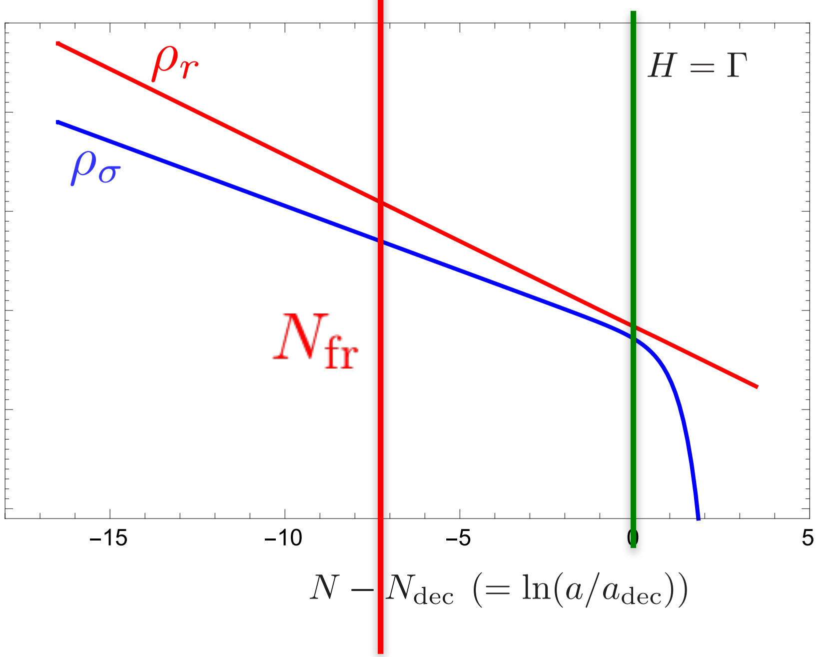
A probe of the existence of the multi-scalar in the early Universe!

*How about in the curvaton scenario ?*

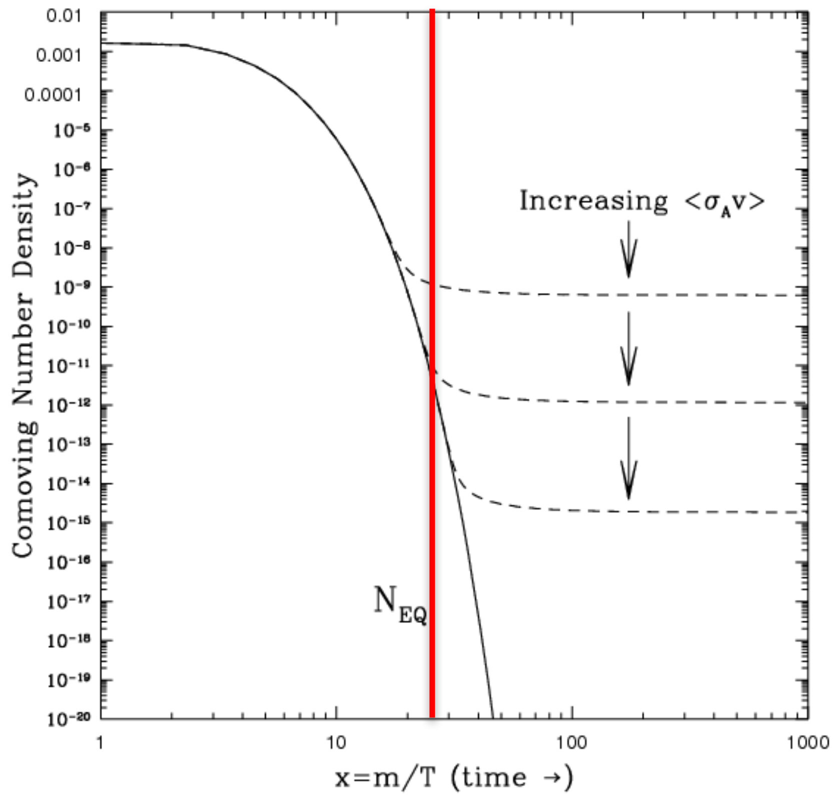
# Thermally produced CDM iso. in curvaton scenario

Radiation – curvaton sector

radiation-dominated Universe after reheating




n_CDM evolution




$$N_{\rm fr} < N_{\rm dec}$$

Before the curvaton decay, the radiation component does not fluctuate and hence

$$\zeta_{\rm CDM} = 0 \ \rightarrow \ S_{\rm CDM} = -3\zeta$$

# Thermally produced CDM iso. in curvaton scenario

Radiation – curvaton sector

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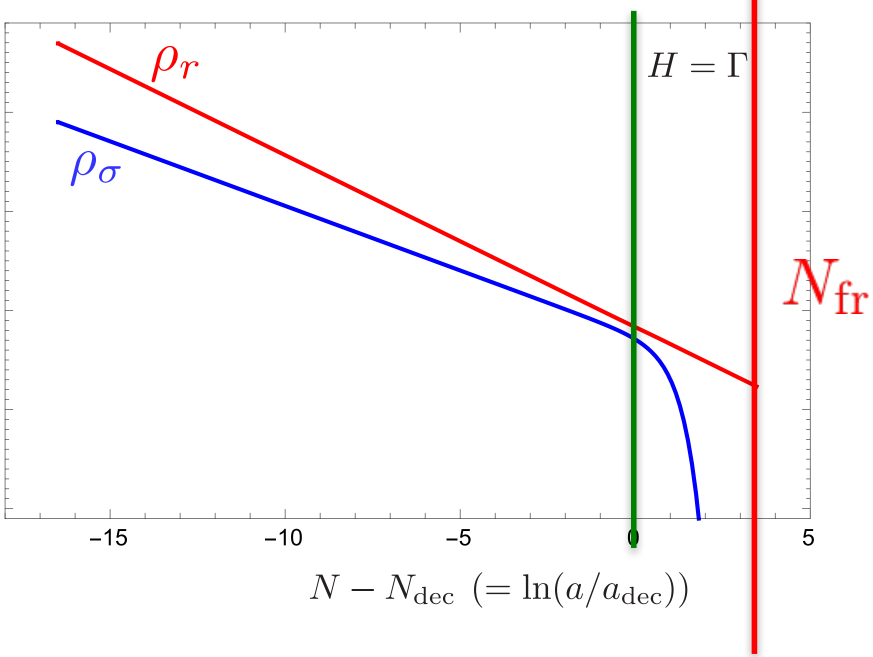


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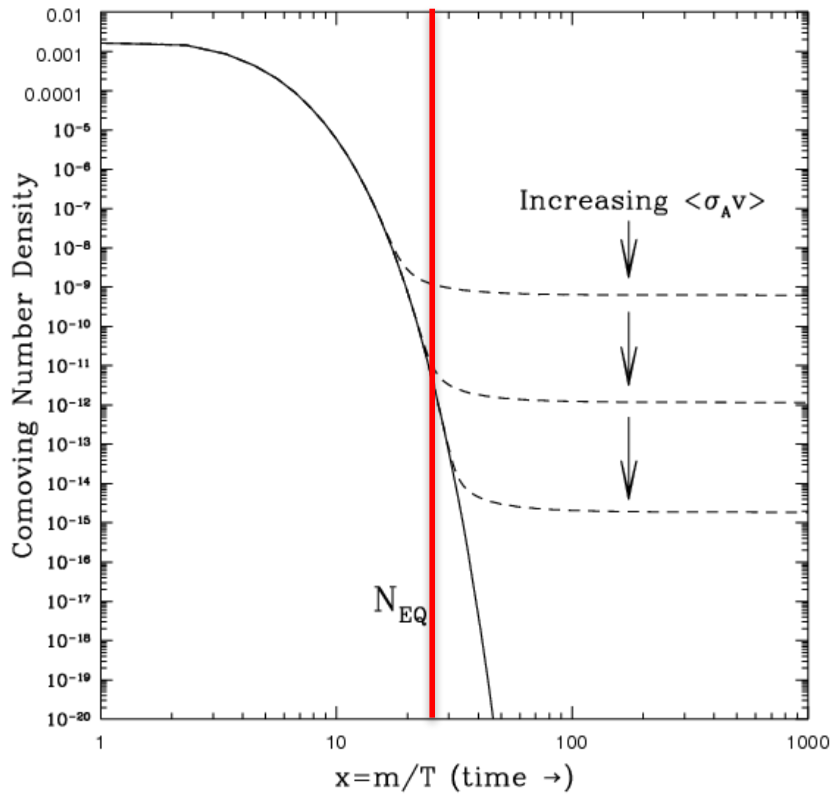
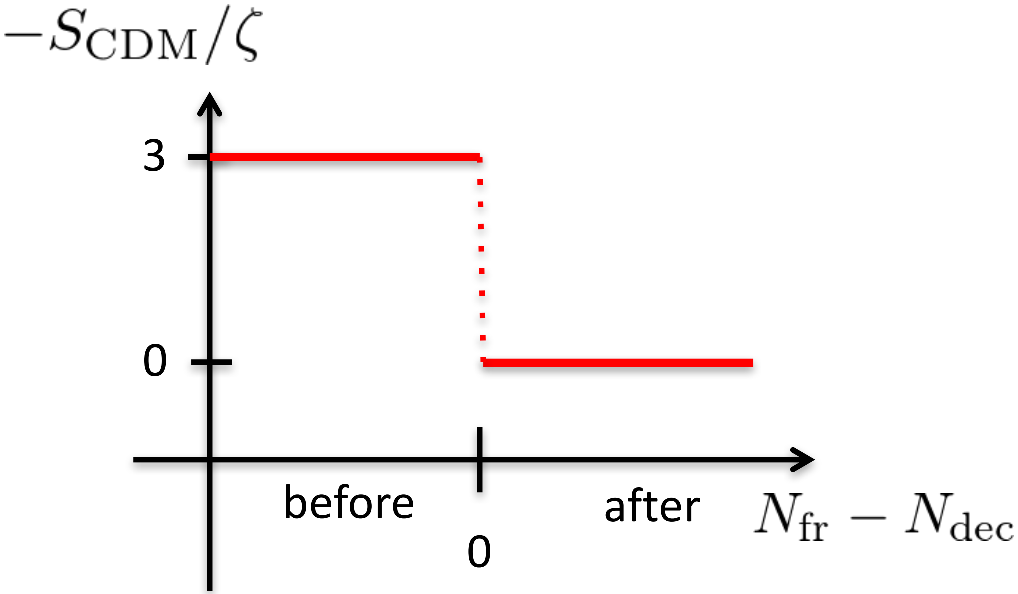



after the curvaton decay, the Universe evolves adiabatically and

$$\zeta_{\rm CDM} = \zeta \ \rightarrow \ S_{\rm CDM} = 0$$

# Constraints

- Known result




Current observational limit;

$$\beta_{\rm iso} < 0.04 \quad \text{;correlated}$$

Planck 2015

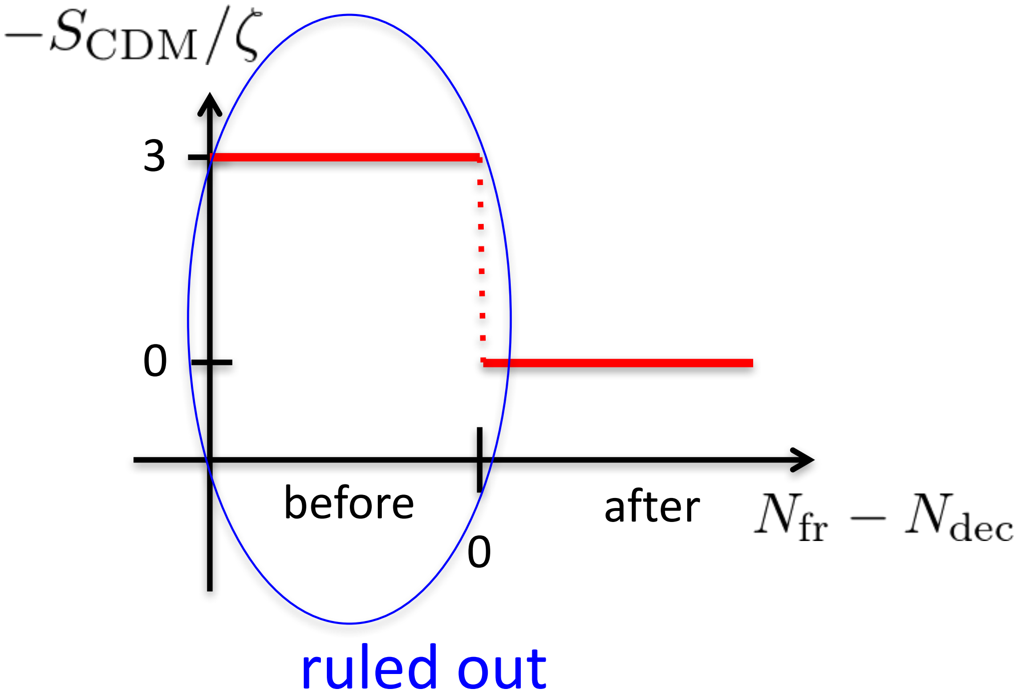
Basically,

$$\boxed{S_{\rm CDM} \ll \zeta}$$

$$\beta_{\rm iso}(k) = \frac{\mathcal{P}_{SS}(k)}{\mathcal{P}_{\zeta\zeta}(k) + \mathcal{P}_{II}(k)}$$

# Constraints

- Known result

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More detailed study!!

# Revisit 1

$$r_s := \left.\frac{3\rho_\sigma}{4\rho_r + 3\rho_\sigma}\right|_{N=N_{\rm dec}}$$

- In case with $r_s \sim 1$

Planck 2015;

$r_s > 0.19$

(T+E,95%CL)

From a constraint
on the non-Gaussianity

Effect of fluctuations of ``freeze-out" hypersurface

$$H = \lambda\, n_{\rm CDM}$$

$n_{\rm CDM}^{\rm (eq)} = g_* \left(\frac{mT}{2\pi}\right)^{3/2} e^{-m/T}$, ; depends only on the temperature (radiation energy dens.)

$H^2 = \frac{(\rho_r + \rho_\sigma)}{3M_{\rm pl}^2}$ ; depends both on the radiation and the curvaton energy dens.

$\zeta_{\rm CDM}$ would be affected by curvaton fluctuations even for N_dec > N_fr

# Revisit 1

- Taking into account of this effect

``sudden freeze-out” approximation

Lyth and Wands (2003)

$$\boxed{S_{\rm CDM}/\zeta = 3\left[\frac{\Omega_{\sigma,{\rm fr}}}{r_s}\left(\frac{m}{T_{\rm fr}} - \frac{3}{2}\right)\frac{1}{2(\alpha_{\Lambda,{\rm fr}} - 2) + \Omega_{\sigma,{\rm fr}}} - 1\right]}$$

m; CDM mass

$$\alpha_{\Lambda,{\rm fr}} \approx \frac{m}{T_{\rm fr}}$$

$$\Omega_{a,{\rm fr}} = \rho_a/(3M_{\rm pl}^2 H^2)|_{\rm fr}\ (a = r, \sigma)$$

- for $m/T_{\rm fr} \sim 20$

$$S_{\rm CDM}/\zeta \approx 3\left(\frac{\Omega_{\sigma,{\rm fr}}}{2r_s} - 1\right)$$

$\rightarrow -1.5$ for $\Omega_{\sigma,{\rm fr}} \rightarrow r_s \sim 1$ still large …

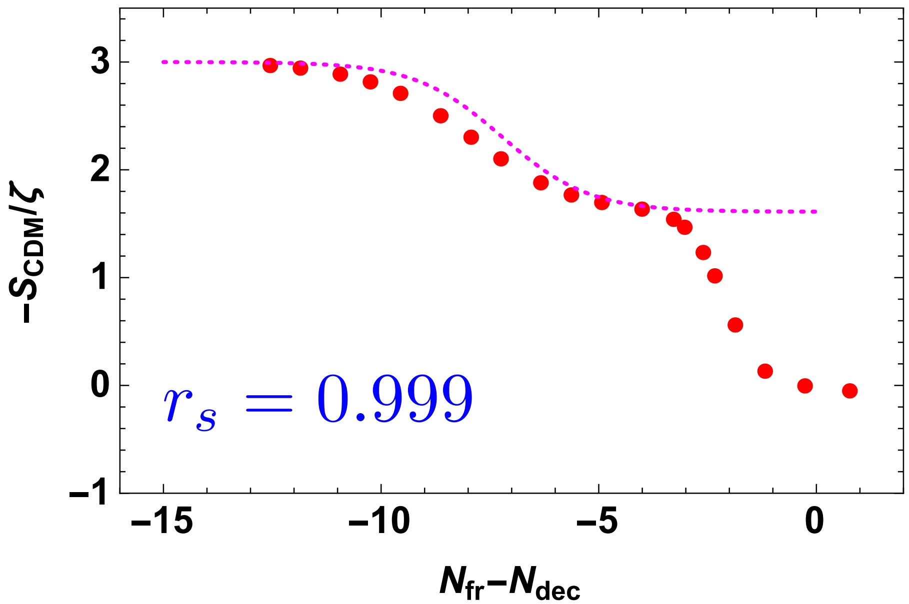
# Our work

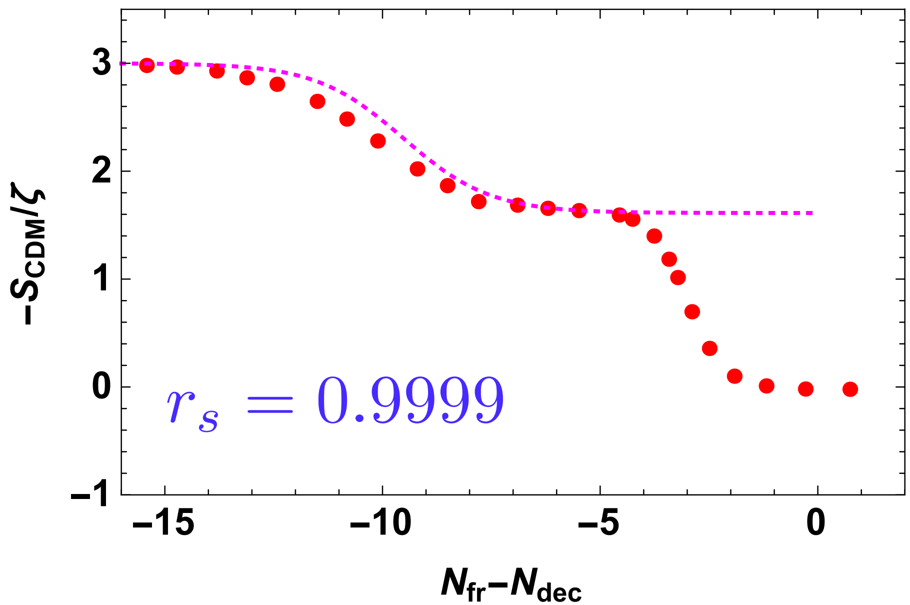
Kitajima, Langlois, Takahashi, SY in prep.

Kitajima, Langlois, Takahashi, SY in prep.

# Numerical study

For $r_s \sim 1$




red points; numerical result

magenta dotted line; sudden f.o. formula

$$\Omega_{\sigma,\mathrm{fr}} \simeq r_s\, e^{-(N_{\mathrm{fr}}-N_{\mathrm{dec}})}$$

Numerical calculation based on delta N formalism

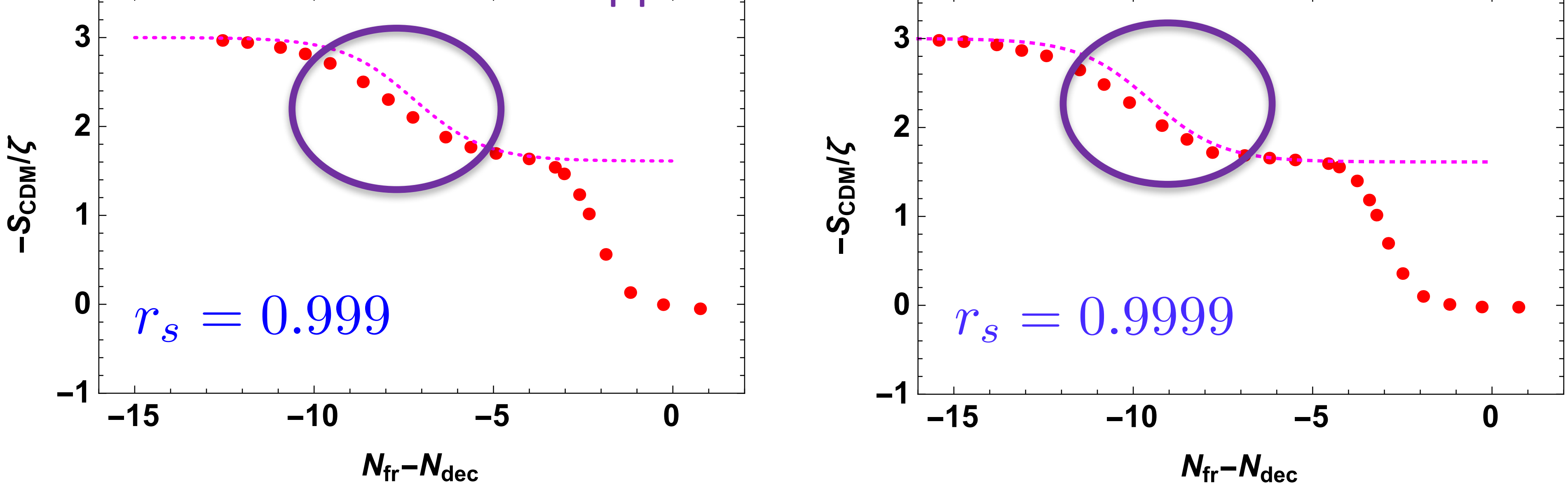
Starobinsky (1985), Sasaki and Stewart (1995), Sasaki and Tanaka (1996), Kodama and Hamazaki (1998), …

Kitajima, Langlois, Takahashi, SY in prep.

# Numerical study

For $r_s \sim 1$

Small suppression can be realized ...

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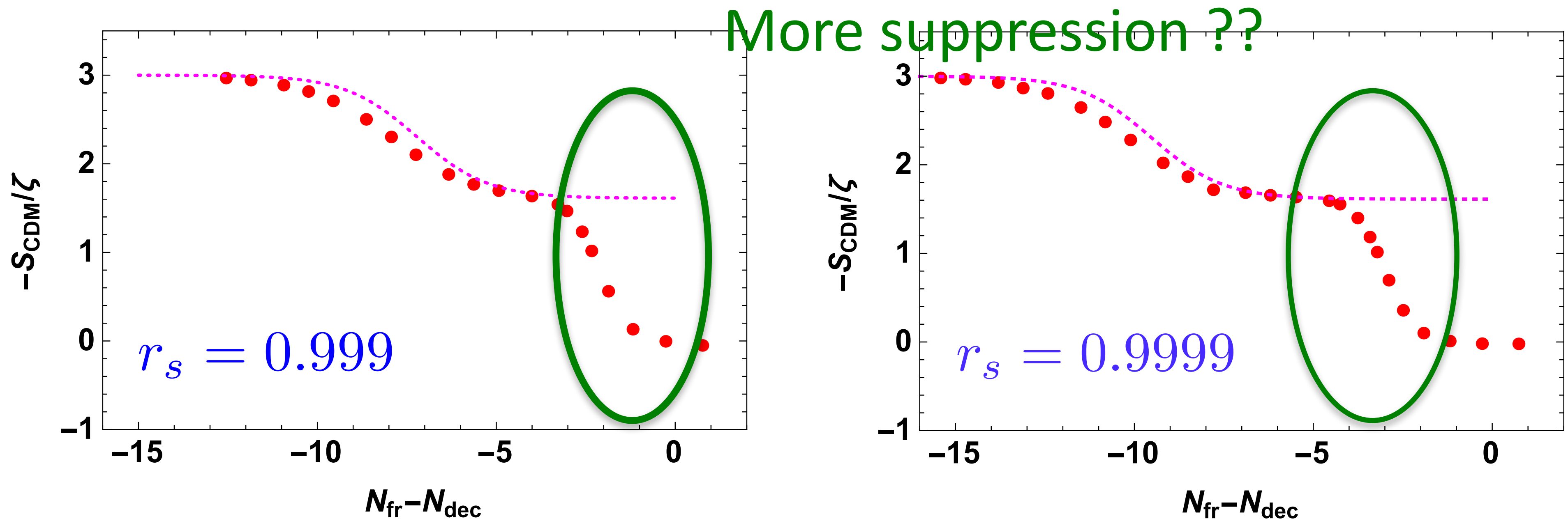


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# Point

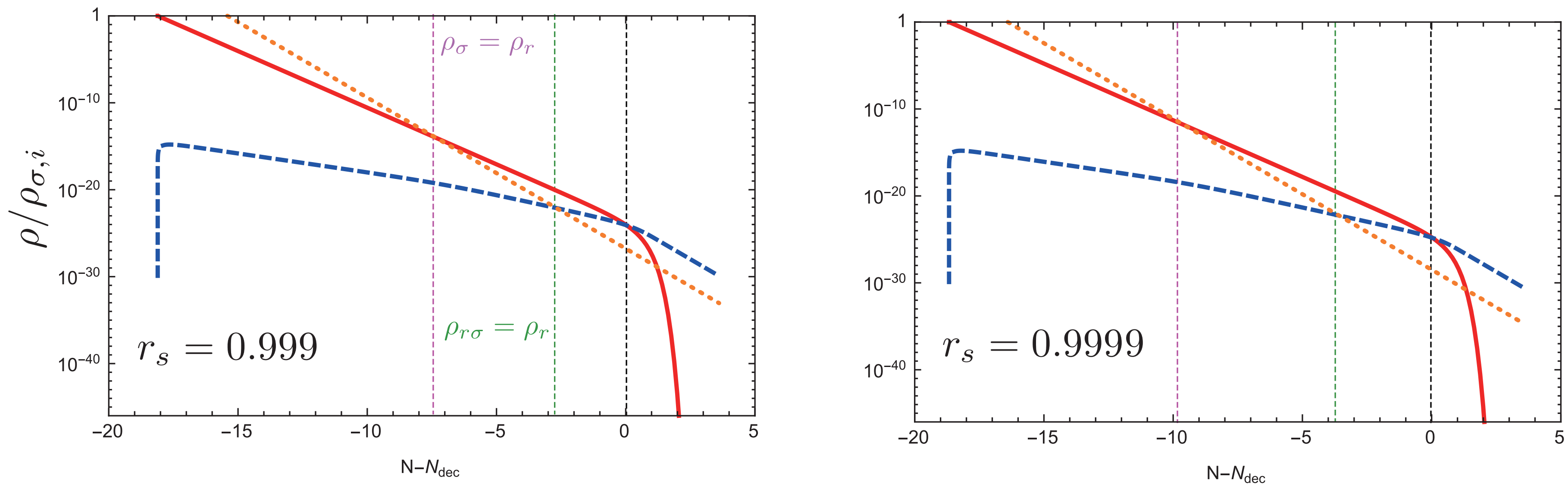
Even for the case where the freeze-out occurs before the curvaton decay,

isocurvature perturbations seem to be much suppressed compared with sudden-f.o. formula.

What's the cause? ⇒ "Hint" is in the background evolution.

Kitajima, Langlois, Takahashi, SY in prep.

# ``curvaton-radiation” effect (revisit 2)




✓ formally decompose the radiation into inflaton-origin comp. and curvaton-origin comp. (curvaton-rad.)

✓ curvaton (matter-like comp.)

$$\rho_r + \rho_{r\sigma} + \rho_\sigma = 3M_{\rm pl}^2 H^2$$

Kitajima, Langlois, Takahashi, SY in prep.

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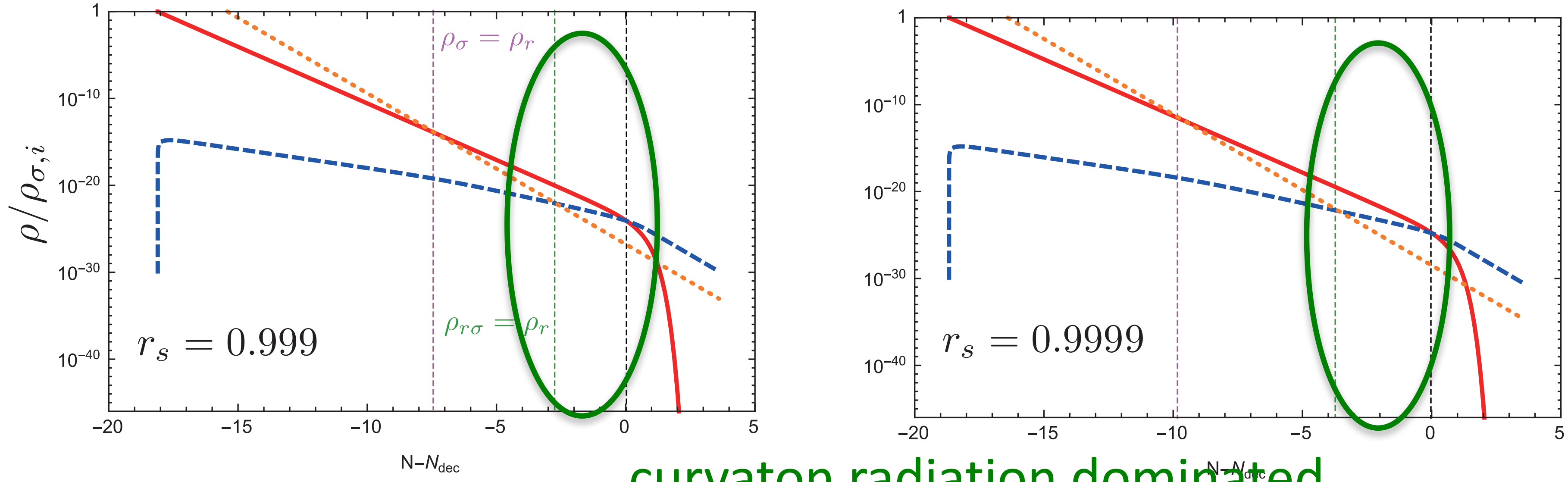


curvaton radiation dominated

- ✓ formally decompose the radiation into inflaton-origin comp. and curvaton-origin comp. (curvaton-rad.)
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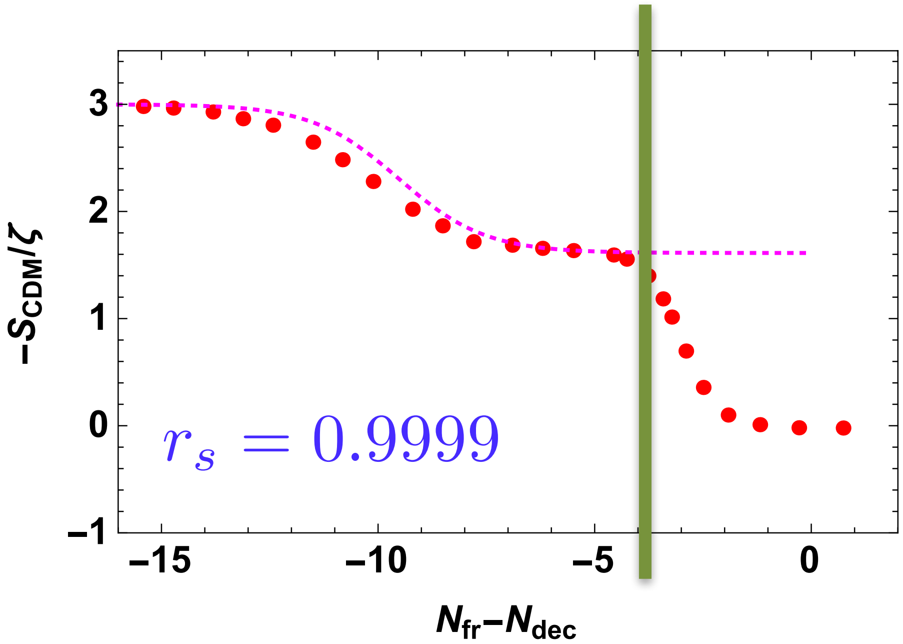
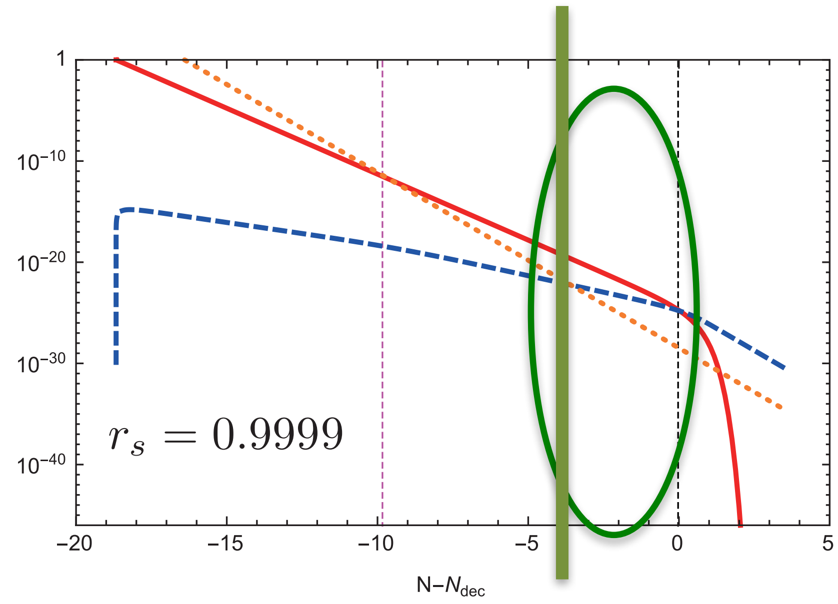



Iso-curvature perturbations




background

Key; a time when the curvaton-radiation dominates over the radiation component; $N_{\rm dom}$

$$\rho_r = \rho_{r\sigma}$$

Kitajima, Langlois, Takahashi, SY in prep.

# ``curvaton-radiation” effect (revisit 2)

Sudden ``freeze-out” approximation with curvaton-radiation

$$
\begin{aligned}
S_{\mathrm{CDM}}/\zeta &= 3\left[\frac{1}{r_s}\frac{\partial \ln Z_f}{\partial \ln Y_i} - 1\right] \\
&= 3\left\{\frac{1}{r_s}\left[3\frac{\partial N_{\mathrm{fr}}}{\partial \ln Y_i} + \left(\frac{3}{2} + \frac{m}{T_{\mathrm{fr}}}\right)\frac{\partial \ln T_{\mathrm{fr}}}{\partial \ln Y_i}\right] - 1\right\} \\
&= \frac{3}{r_s}\left[\left(\frac{3}{2} - \frac{m}{T_{\mathrm{fr}}}\right) + \left(\frac{3}{2} + \frac{m}{T_{\mathrm{fr}}}\right)\left(1 - \frac{p}{4}\right) f_{r\sigma}\right]\frac{\partial N_{\mathrm{fr}}}{\partial \ln Y_i} \\
&\qquad + \frac{3}{4}\frac{f_{r\sigma}}{r_s}\left(\frac{3}{2} + \frac{m}{T_{\mathrm{fr}}}\right)\left(1 + \frac{(p-3)}{3}r_s\right) - 3,
\end{aligned}
$$

with

$$
\frac{\partial N_{\mathrm{fr}}}{\partial \ln Y_i} = \frac{\Omega_{r\sigma,\mathrm{fr}} + \Omega_{\sigma,\mathrm{fr}} - \frac{\alpha_{\Lambda,\mathrm{fr}}}{2} f_{r\sigma} + \frac{(p-3)r_s}{3}\left(\Omega_{r\sigma,\mathrm{fr}} - \frac{\alpha_{\Lambda,\mathrm{fr}}}{2} f_{r\sigma}\right)}{4 + (p-4)\Omega_{r\sigma,\mathrm{fr}} - \Omega_{\sigma,\mathrm{fr}} - 2\alpha_{\Lambda,\mathrm{fr}}\left[1 - \left(1 - \frac{p}{4}\right) f_{r\sigma}\right]}.
$$

$$\rho_{r\sigma} \propto a^{-p}$$

( p = 1.5)

Kitajima, Langlois, Takahashi, SY in prep.

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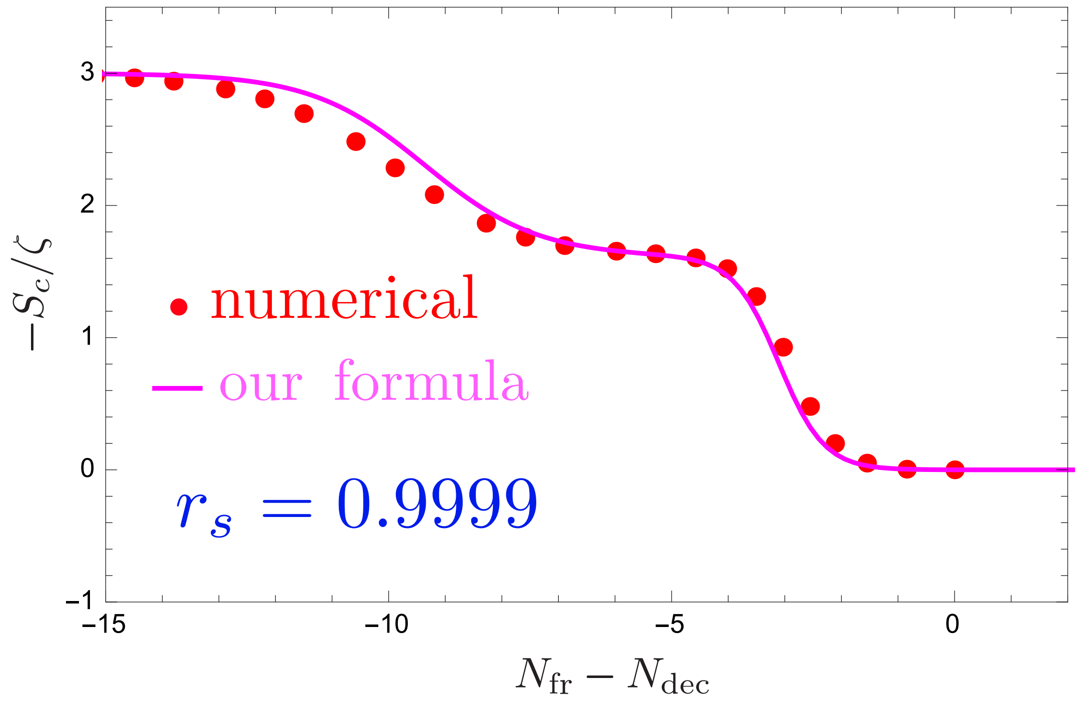
# Refined sudden f.o. formula




much improvement!

red points; numerical result

magenta line; refined sudden f.o. formula

Kitajima, Langlois, Takahashi, SY in prep.

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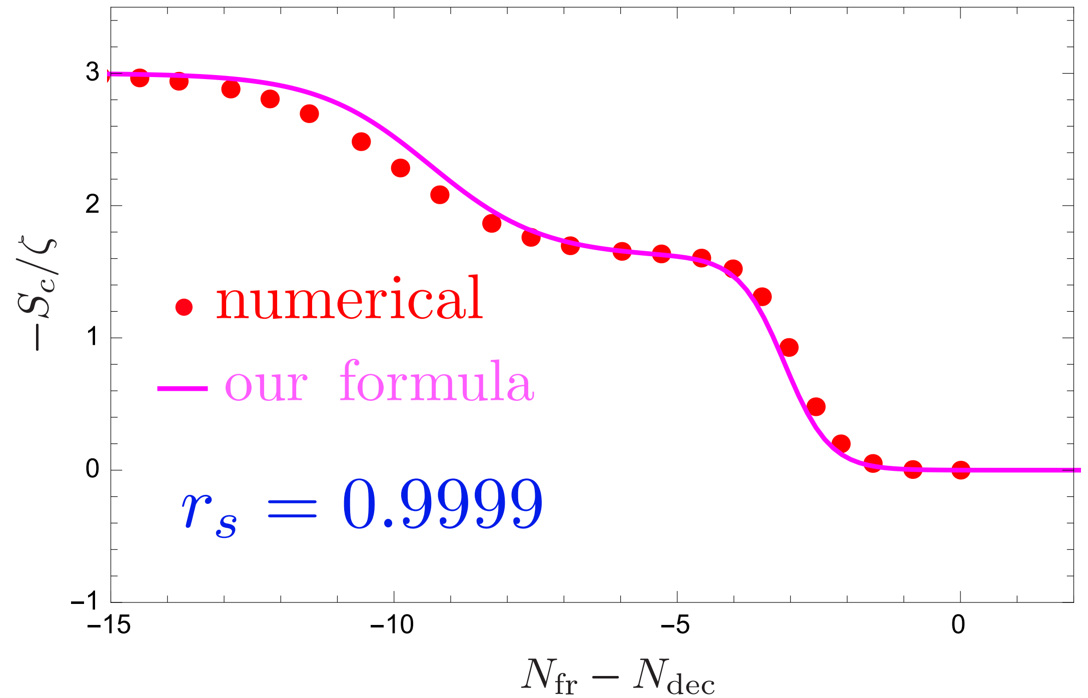
# Refined sudden f.o. formula




much improvement!

In the context of building a model, important point is

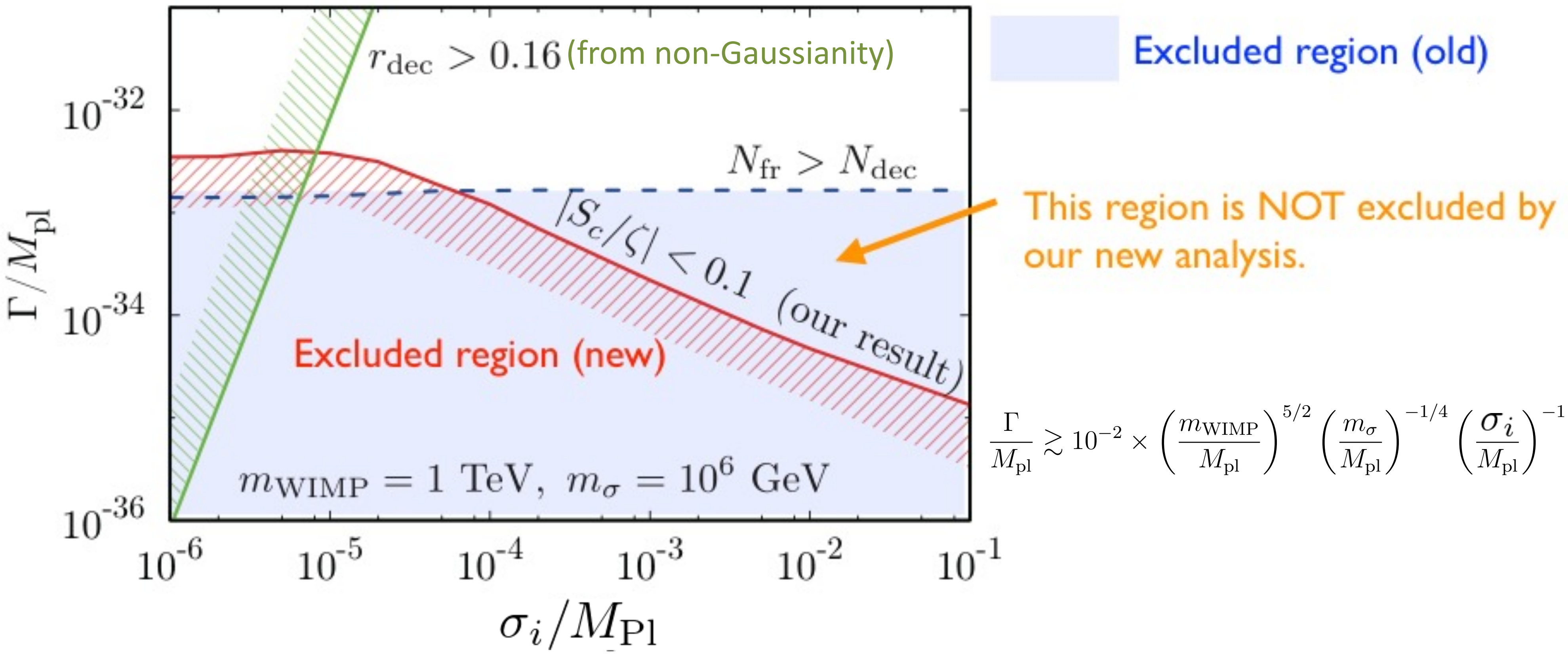
$$-S_{\rm CDM}/\zeta \sim 0 \quad \text{for} \quad N_{\rm dom} < N_{\rm fr}$$

for $r_s \sim 1$

Kitajima, Langlois, Takahashi, SY in prep.

# Implication (parameter constraint)

- Parameter space for the curvaton




$$\frac{\Gamma}{M_{\rm pl}} \gtrsim 10^{-2} \times \left(\frac{m_{\rm WIMP}}{M_{\rm pl}}\right)^{5/2} \left(\frac{m_\sigma}{M_{\rm pl}}\right)^{-1/4} \left(\frac{\sigma_i}{M_{\rm pl}}\right)^{-1}$$

Isocurvature constraints for the curvaton scenario is not severe than previously considered for larger $\sigma_i$ !

# summary

- We revisit the CDM isocurvature perturbations in curvaton scenario, especially for the case where the curvaton dominates the Universe before decay.

  (favored from the current constraints for the non-Gaussianity)

- We find that for such a case iscurvature perturbations are completely suppressed for $N_{\rm dom} < N_{\rm fr}$

  (related to the evolution of the curvature perturbation)

- We find isocurvature constraints for the curvaton scenario is not severe than previously considered!

# Need to include ``curvaton-radiation”

$$-S_{\rm CDM}/\zeta \sim 0$$ for $$N_{\rm dom} < N_{\rm fr}$$

Actually, this result seems to be trivial.

For N > N_dom, the Universe is completely dominated by curvaton,
and this means that the Universe evolves adiabatically.
(cf. reheating by the inflaton decay)

Weinberg 2004

# Entropy production

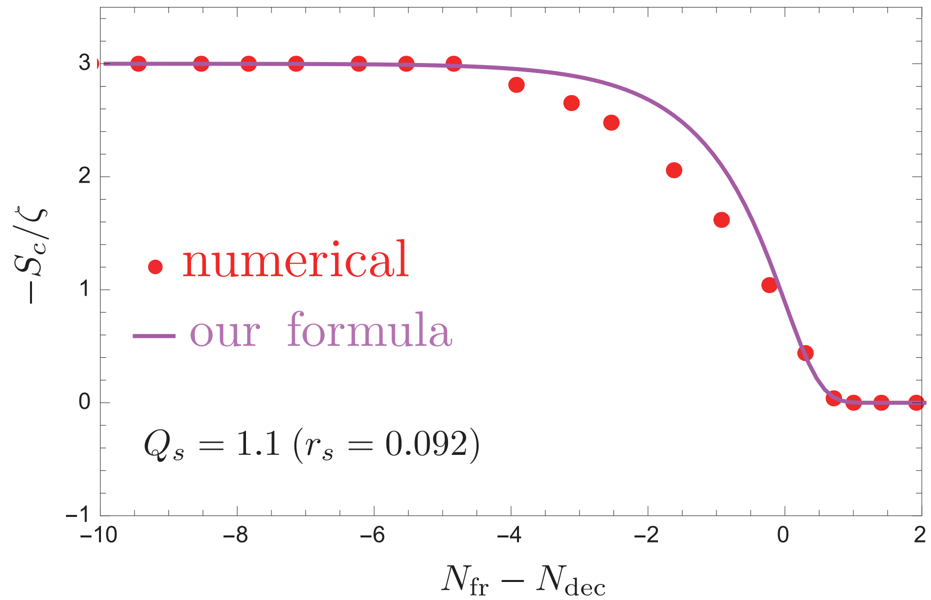
$r_s = 0.9999 \Leftrightarrow S_f / S_i \sim 1000$

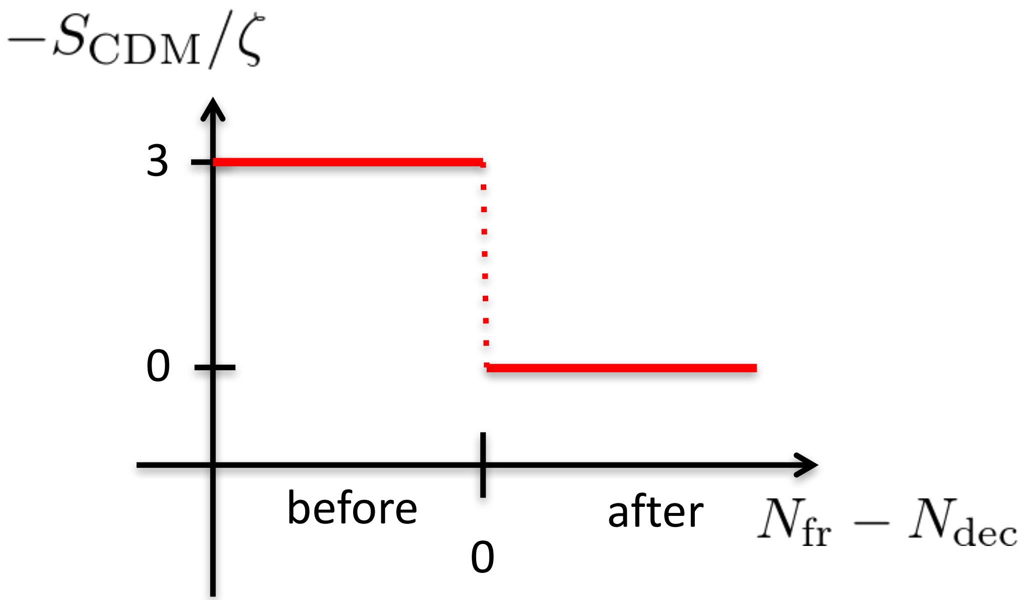
; entropy production rate by the curvaton decay

S; entropy density in comoving volume

# backup

- Relatively small rs




Conventional understanding seems to be good.

# CDM iso curvature in terms of zeta

$$\zeta_{\rm CDM} = \zeta_{\rm fr} + \frac{2\alpha_{\Lambda,\rm fr} - \left(\frac{3}{2} + \frac{m}{T_{\rm fr}}\right)}{4 - \Omega_\sigma - 2\alpha_{\Lambda,\rm fr}} \frac{3\Omega_{\sigma,\rm fr}}{4 - \Omega_{\sigma,\rm fr}} S_{\sigma,\rm fr}$$

- CDM isocurvature perturbations
  trace the evolution of the curvature perturbations !

- ``curvaton-rad.” is required to express the evolution of the curvature perturbations.

In usual, we only need to know the final curvature perturbations
in the curvaton scenario, not important to trace the evolution.

# Axion DM isocurvature in curvaton