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Compensating strong coupling with large charge

Susanne Reffert
University of Bern

based on work arXiv:1505.01537, 1610.04495, 1707.00710 with:
L. Alvarez-Gaume (CERN/SCGP), S. Hellerman (IPMU), O. Loukas (U. Bern),
D. Orlando (U. Bern/INFN Torino), M. Watanabe (IPMU)

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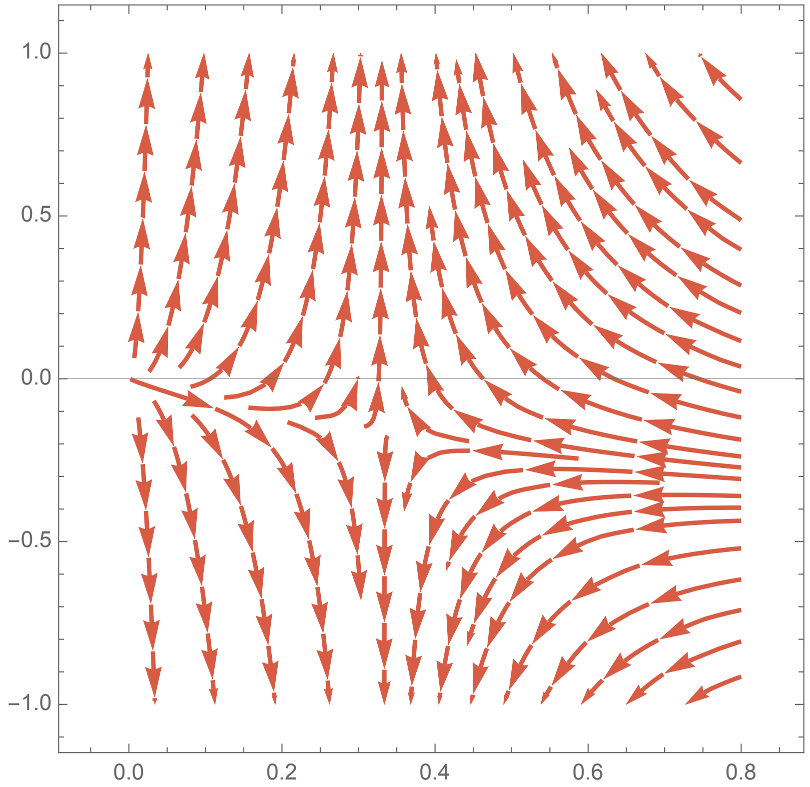
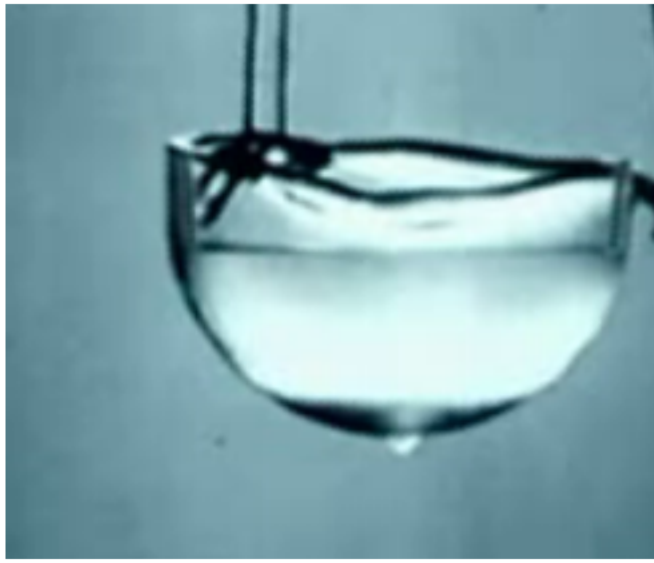
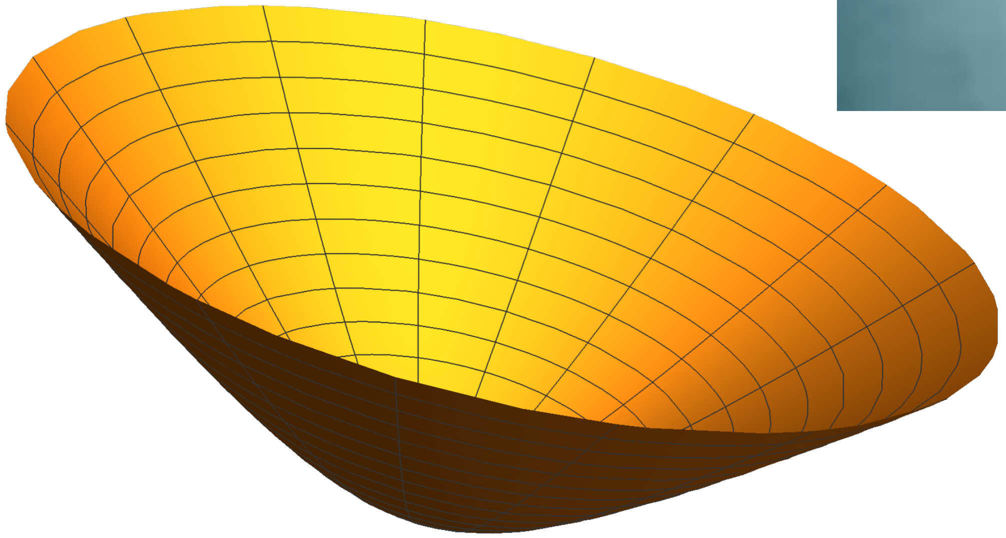
Introduction

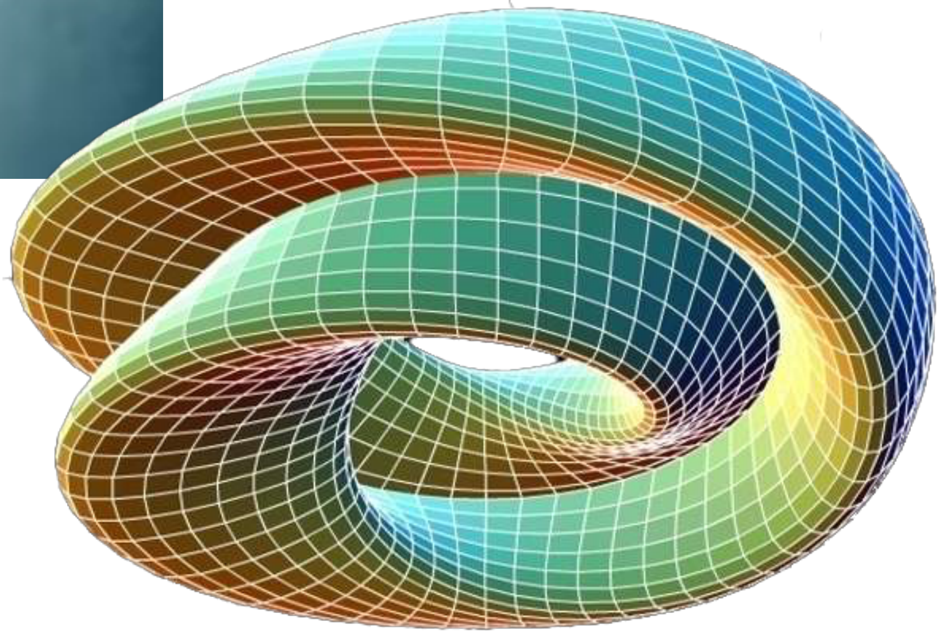
Conformal field theories (CFTs) play an important role in theoretical physics:

- fixed points in RG flows
- critical phenomena
- quantum gravity
- string theory









Introduction

BUT: most CFTs do not have small parameters in which to do a perturbative expansion: couplings are O(1).

Difficult to access.
Possibilities: analytic (2d), conformal bootstrap (d>=3), lattice calculations, non-perturbative methods…

Make use of symmetries, look at special subsectors where things simplify.

Here: study theories with a global symmetry group. Study subsectors with large charge Q.

Large charge Q becomes controlling parameter in a perturbative expansion!

NOT bootstrap!

Introduction

Basic idea: consider model at the (infrared) Wilson-Fisher fixed point.

Write down Wilsonian effective action. In general: infinitely many terms - not so useful.

Subsector of large charge Q: find classical ground state (condensate). Large charge breaks Lorentz invariance. Look for homogeneous ground state (homogeneous in space, but time dependent).

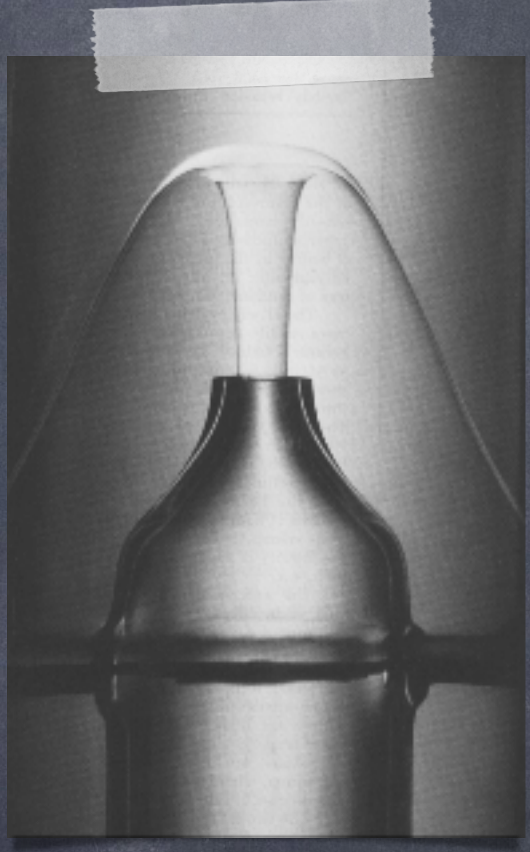
Then: describe (quantum) fluctuations around the condensate. Encoded by Goldstone bosons.

vacuum + Goldstones + 1/Q-suppressed corrections

Wilsonian action has only a handful of terms that are not suppressed by the large charge. Useful!

Overview

- Introduction
- The O(2) model
    - semi-classical treatment
    - quantum treatment
    - results and lattice confirmation
- The O(2N) vector model
- Beyond the vector models
- Summary/Outlook



The O(2) model

The O(2) model

Consider simple model: O(2) model in (2+1)d.

$$\mathcal{L}_{\text{UV}} = \partial_\mu \phi^* \, \partial^\mu \phi - g^2 (\phi^* \phi)^2$$

Flows to Wilson-Fisher fixed point in IR.

Global U(1) symmetry: $\varphi_{IR} = a\, e^{ib\chi} \qquad \chi \to \chi + \text{const.}$

Look at scales: put system in box of scale R

Second scale given by U(1) charge Q: $\rho^{1/2} \sim Q^{1/2}/R$

Study the CFT at the fixed point in a sector with

$$\frac{1}{R} \ll \Lambda \ll \frac{Q^{1/2}}{R} \ll g^2$$

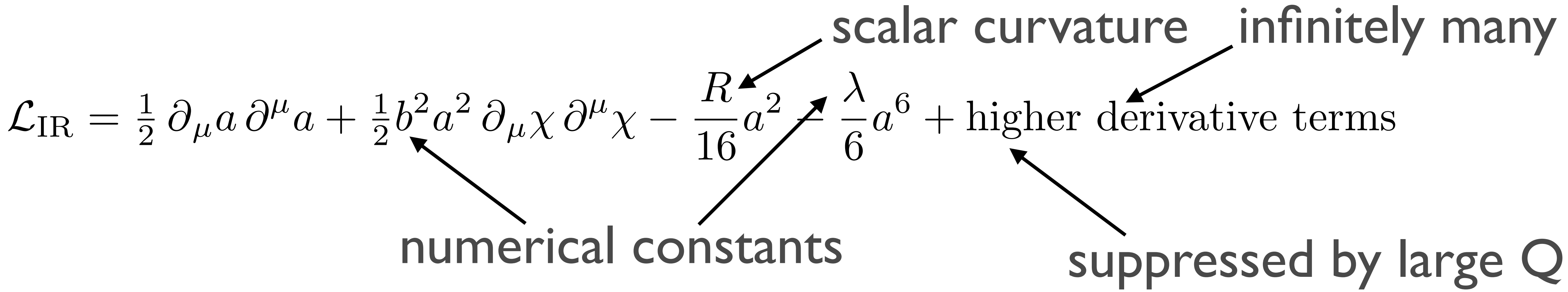
UV scale

cut-off of effective theory

Write Wilsonian action.

The O(2) model

Assume large vev for a: $\Lambda \ll a^2 \ll g^2$



scalar curvature

infinitely many

$$\mathcal{L}_{\mathrm{IR}} = \tfrac{1}{2}\,\partial_\mu a\,\partial^\mu a + \tfrac{1}{2} b^2 a^2\,\partial_\mu \chi\,\partial^\mu \chi - \frac{R}{16} a^2 - \frac{\lambda}{6} a^6 + \text{higher derivative terms}$$

numerical constants

suppressed by large Q

Lagrangian is approximately scale-invariant.

Do semi-classical analysis: solve classical e.o.m. at fixed Noether charge.

$$\rho = \frac{\delta \mathcal{L}_{\mathrm{IR}}}{\delta \dot\chi} = b^2 a^2 \dot\chi \qquad Q \sim 4\pi R^2 b \sqrt{\lambda} a^4$$

Classical solution at lowest energy and fixed global charge becomes the vacuum of the quantum theory.

The O(2) model

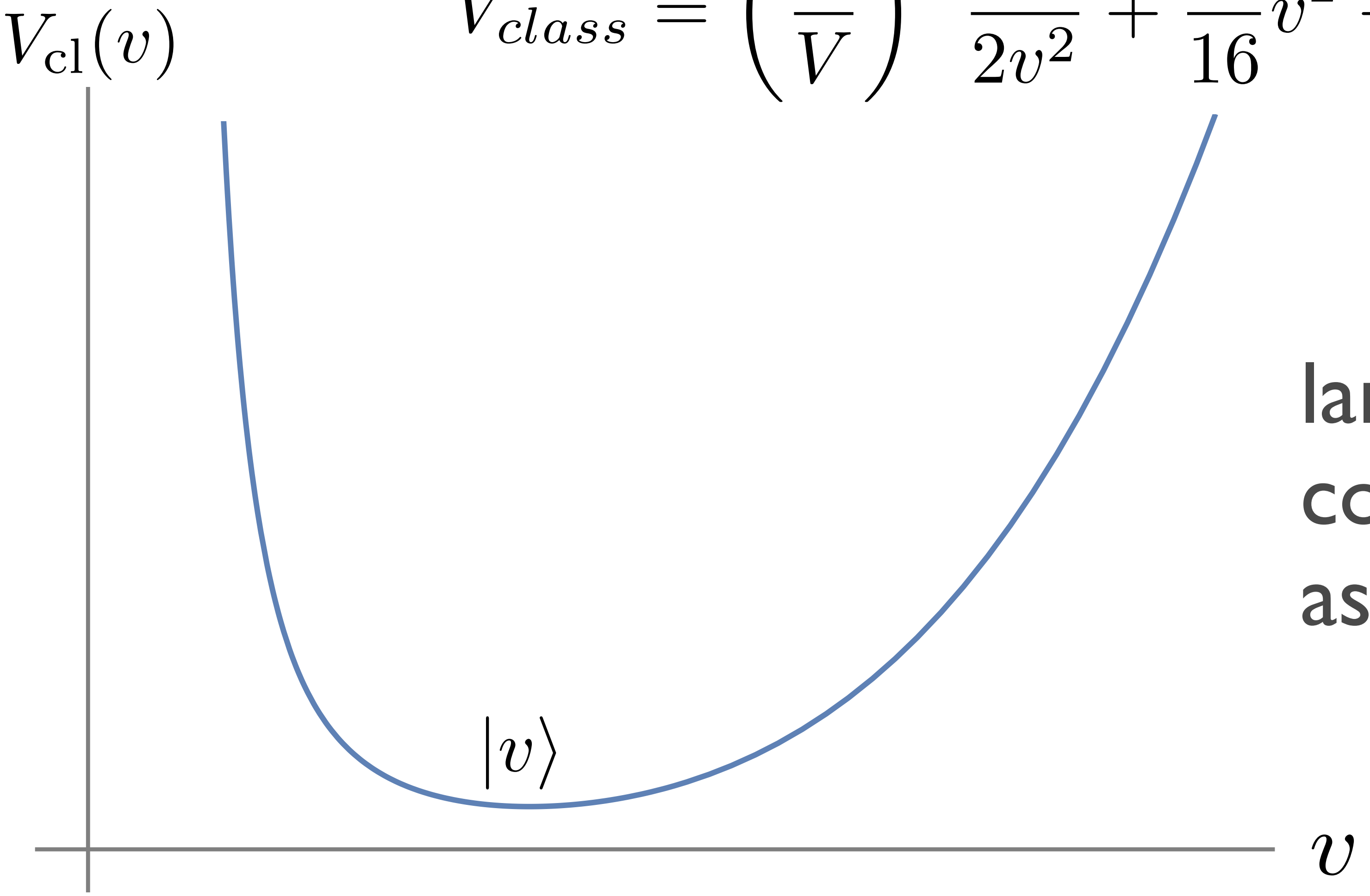
⇒ non-trivial condensate

non-const. vev

$$\langle a \rangle = v, \qquad \langle \dot{\chi} \rangle = \mu = \frac{Q}{V \cdot v^2}, \qquad \langle \chi \rangle = \mu\, t$$

Fixed charge ground state is homogeneous in space. Determine radial vev by minimizing the classical potential:

$$V_{class} = \left(\frac{Q}{V}\right)^2 \frac{1}{2v^2} + \frac{R}{16} v^2 + \frac{\lambda}{6} v^6$$



$$v \sim Q^{1/4}$$

large condensate is compatible with our assumption $a \gg 1$

The O(2) model

Quantum story: study the low-energy spectrum

Parametrize fluctuations on top of the classical vacuum

$$a = v + \hat{a} \qquad \chi = \mu t + \frac{\hat{\chi}}{v}$$

massive mode not relevant for low-energy spectrum

$m \sim \mathcal{O}(\sqrt{Q})$

spontaneous breaking of time-translation invariance

⇒ relativistic Goldstone

$$\omega_{\vec{p}} = \frac{|\vec{p}|}{\sqrt{2}}$$

⇒ superfluid phase of O(2) model

Are fluctuations controlled?

Diagonalize quantum Hamiltonian ⇒ all higher orders are suppressed by inverse powers of Q

vacuum + Goldstone + 1/Q-suppressed corrections

The O(2) model

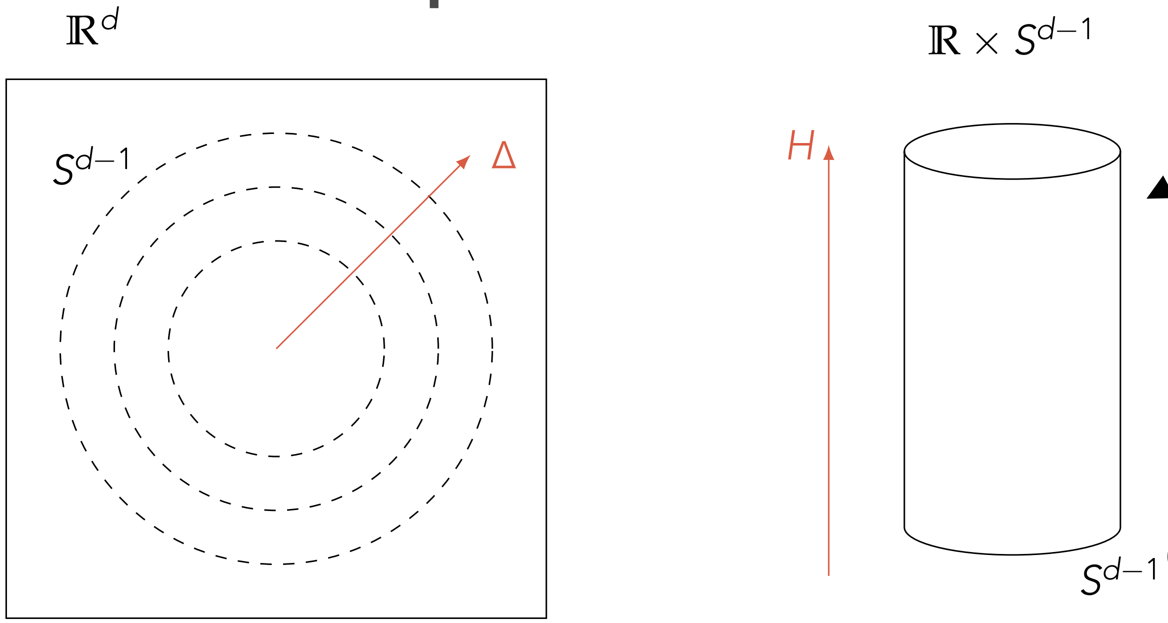
Energy of classical ground state at fixed charge:

2 universal parameters (b, λ)

$$E_\Sigma(Q) = \frac{c_{3/2}}{\sqrt{V}} Q^{3/2} + \frac{c_{1/2}}{2} R\sqrt{V} Q^{1/2} + \mathcal{O}(Q^{-1/2})$$

dependence on manifold

Use state-operator correspondence of CFT:



anomalous dimension

energy

one-loop vacuum energy of Goldstone

Anomalous dimension of lowest operator of charge Q:

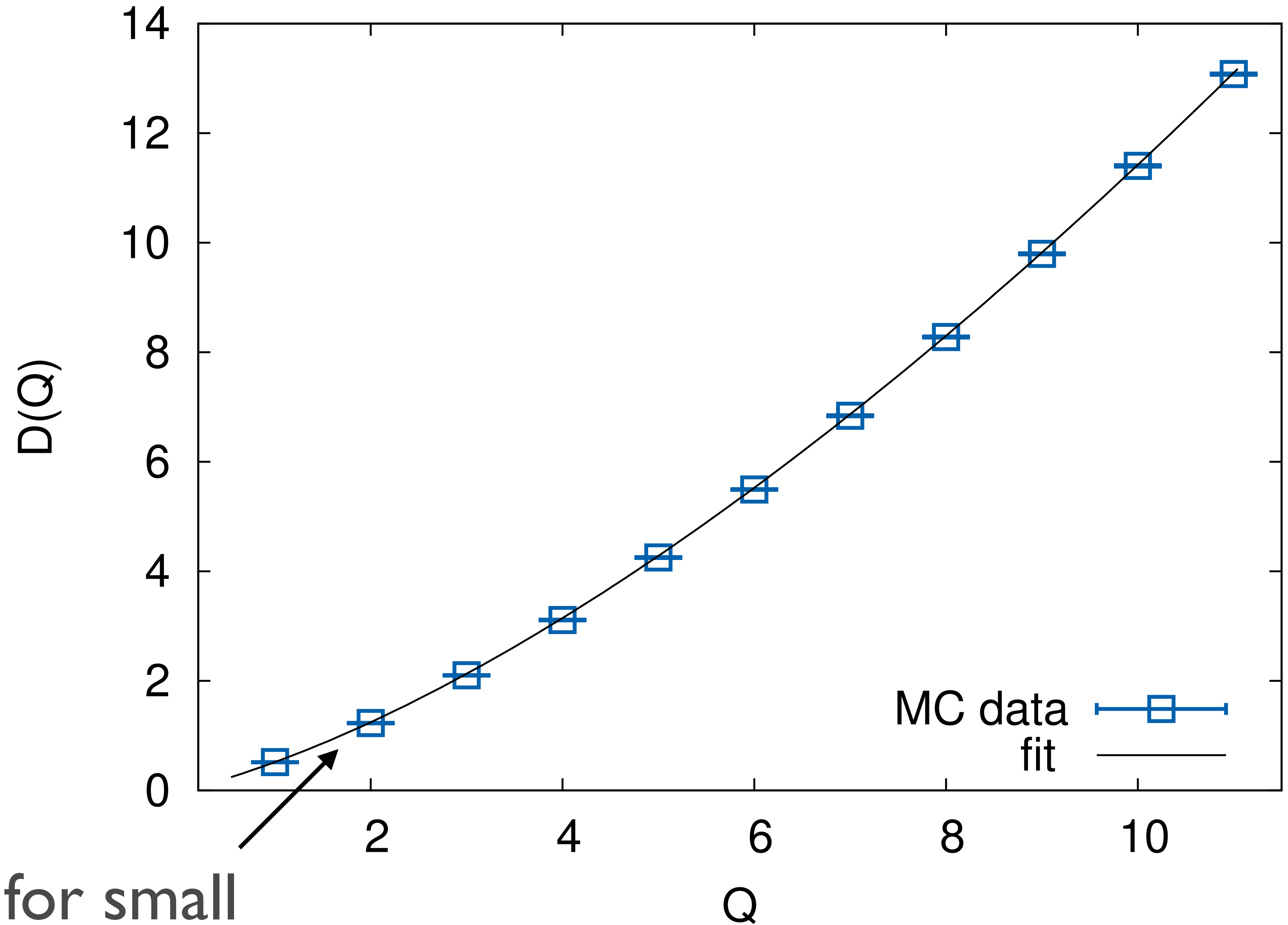
$$D(Q) = \frac{c_{3/2}}{2\sqrt{\pi}} Q^{3/2} + 2\sqrt{\pi}\, c_{1/2} Q^{1/2} - 0.094 + \mathcal{O}(Q^{-1/2})$$

S. Hellerman, D. Orlando, S. R., M. Watanabe, arXiv:1505.01537 [hep-th]

The O(2) model

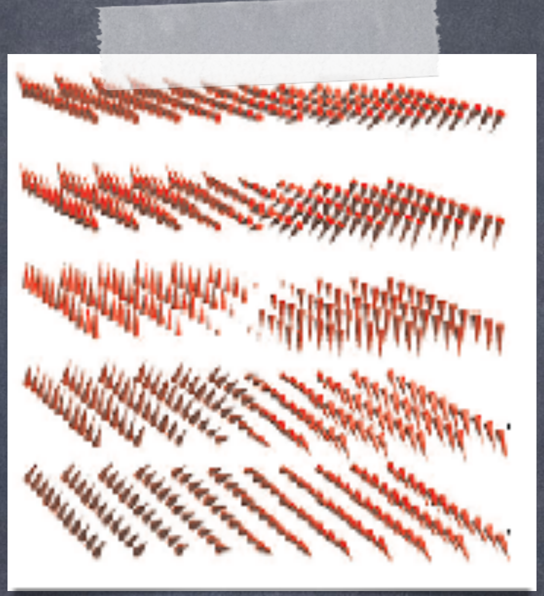
$$D(Q) = \frac{c_{3/2}}{2\sqrt{\pi}} Q^{3/2} + 2\sqrt{\pi}\, c_{1/2} Q^{1/2} - 0.094 + \mathcal{O}(Q^{-1/2})$$

Confirmation from the lattice:



works for small charge. Why??

D. Banerjee, S.h Chandrasekharan, D. Orlando, arXiv:1707.00711 [hep-lat]



The O(2N) vector model

The O(2N) vector model

Treatment generalizes to O(2N):

Fix k<=N U(1) charges.

Symmetry breaking O(2N) to U(k) to U(k-1).

Homogeneous ground state: can fix only sum of charges

There are now

- 1 relativistic Goldstone $\omega \propto p$
- k-1 non-relativistic Goldstones (count double) $\omega \propto p^2$

Nielsen and Chadha; Murayama and Watanabe

1 + 2(k-1) = dim[U(k)/U(k-1)]

Non-relativistic Goldstones are suppressed by large Q
⇒ low-energy physics governed by relativistic Goldstone

The O(2N) vector model

Same formula for anomalous dimensions:

N-dependent

universal for O(2N)

$$D(Q) = \frac{c_{3/2}}{2\sqrt{\pi}} Q^{3/2} + 2\sqrt{\pi}\, c_{1/2} Q^{1/2} - 0.094 + \mathcal{O}(Q^{-1/2})$$

L. Alvarez-Gaume, O. Loukas, D. Orlando and S. R., [arXiv:1610.04495 [hep-th]]

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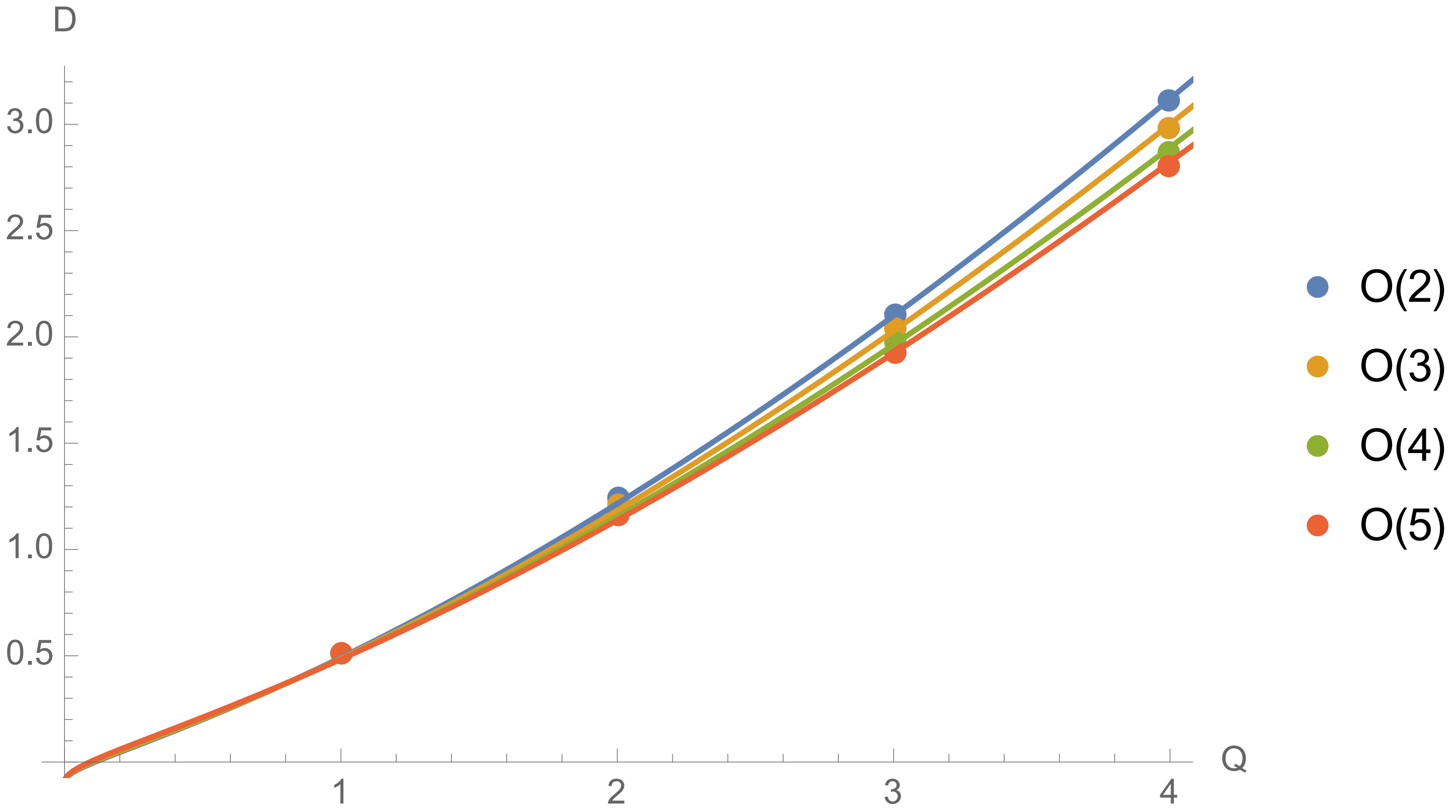
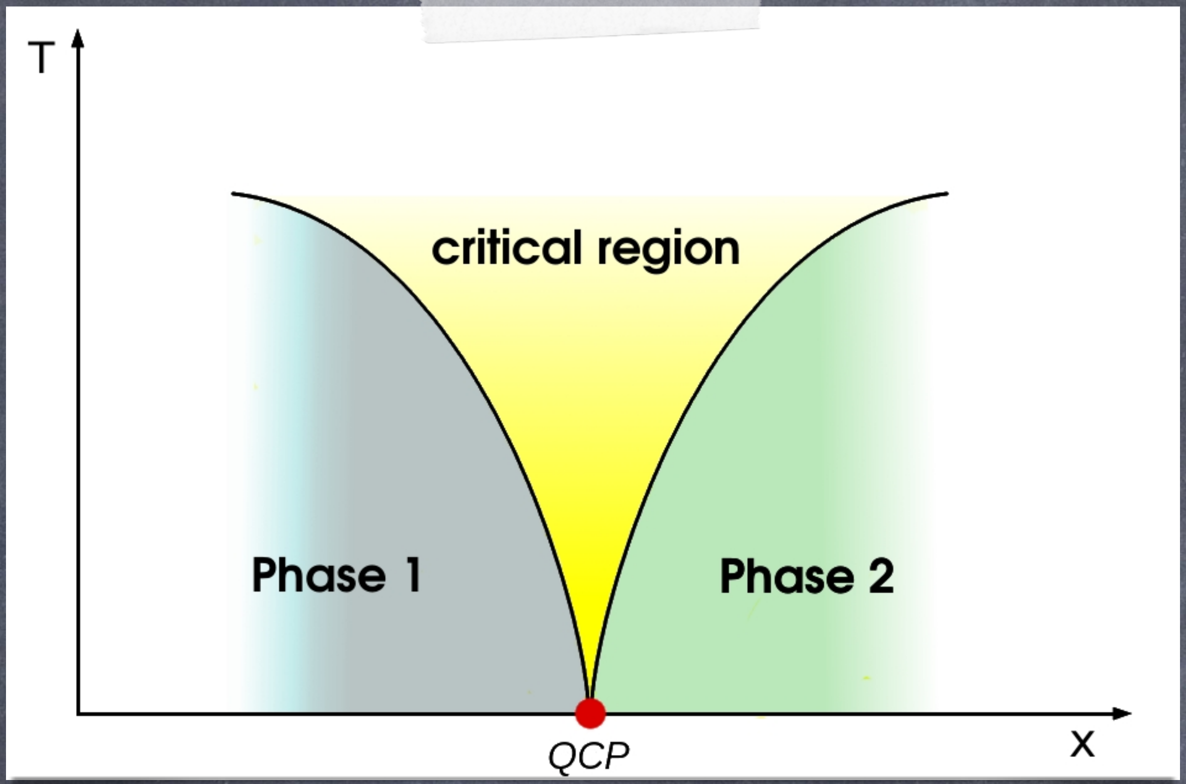


Hasenbusch, Vicari



Beyond the vector models

Matrix models

Want to go beyond vector models.
Study models with matrix-valued order parameter, global SU(N) symmetry.

SU(3) matrix model in 3d: can fix only one U(1)-charge if you want a homogeneous ground state.

Low-energy physics is again governed by a single relativistic Goldstone boson.

Anomalous dimension has again the same form as for the vector model.

Calculated the 3-point functions as well.

O. Loukas, D. Orlando and S. R., [arXiv:1707.00710 [hep-th]]

SU(4) matrix model: new effects appear. Can fix more than one U(1) charge. Can distinguish more than one IR fixed point at large charge.

O. Loukas, to appear

Summary

Summary

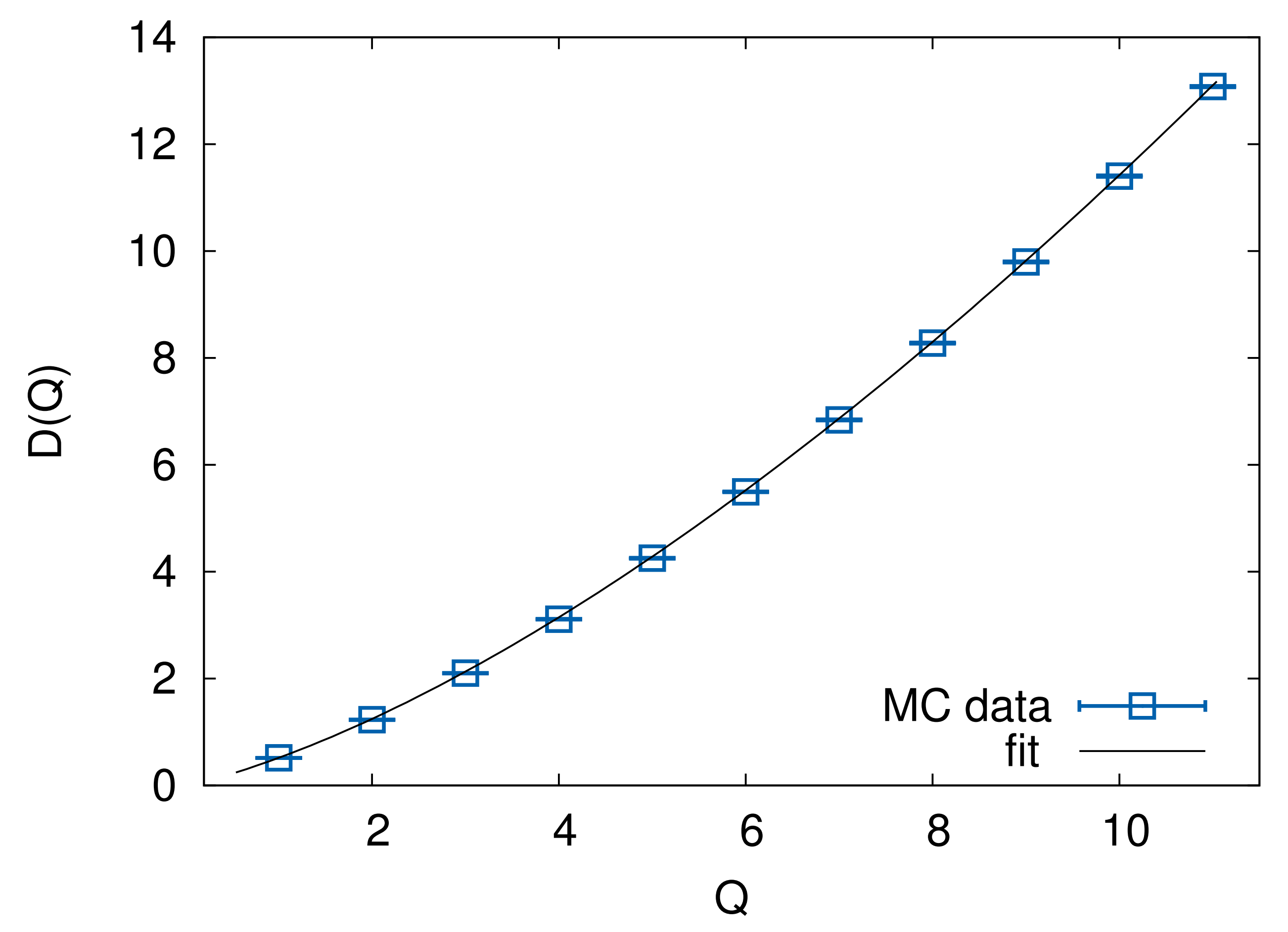
- Concrete examples where a strongly-coupled CFT simplifies in a special sector.
- O(2N) model in 3d: in the limit of large U(1) charge Q, we computed the conformal dimensions in a controlled perturbative expansion:

$$D(Q) = \frac{c_{3/2}}{2\sqrt{\pi}} Q^{3/2} + 2\sqrt{\pi}\, c_{1/2} Q^{1/2} - 0.094 + \mathcal{O}(Q^{-1/2})$$

- Can be applied beyond vector model: SU(N) matrix models

Summary

- Further study of non-homogeneous ground states
  Hellerman et al.
- Study fermionic models at large charge
- Study supersymmetric models at large R-charge
  Hellerman et al.
- Connection to holography (gravity duals)
- Connection to large spin?
- Understand dualities semi-classically at large charge
- Use/check large charge results in conformal bootstrap
  Jafferis and Zhiboedov
- Can large charge approach be used for QCD (e.g. large baryon number)?



Thank you for your attention!