THE SYK MODEL

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10th Anniversary

KAVLI IPMU INSTITUTE FOR THE PHYSICS AND MATHEMATICS OF THE UNIVERSE

SACHDEV-YE KITAEV Model

NEW CLASS OF LARGE N CFTS

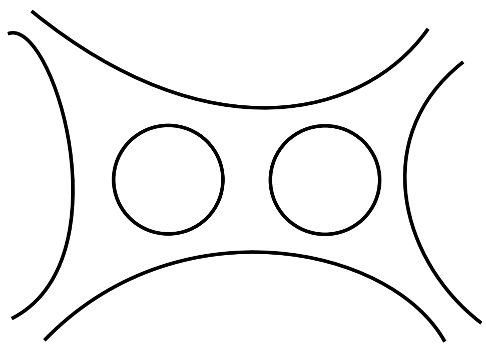
Quantum Mechanics of N Majorana
fermions with q-body
quenched random interactions

BROKEN 1D DIFFEOMORPHISM IN IR -

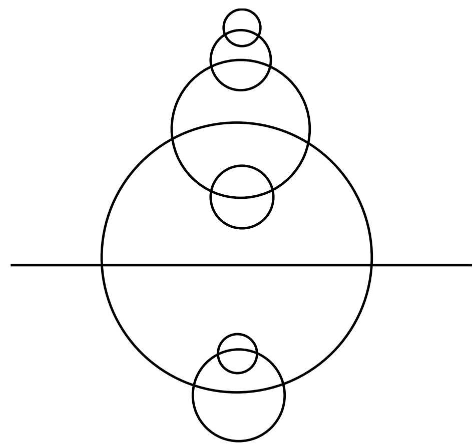
SOLUBLE CFT - Maximally chaotic -
HOLOGRAPHIC TO
GRAVITY (STRING) IN AdS_2 BULK

SYK: an new class of Large N Theories

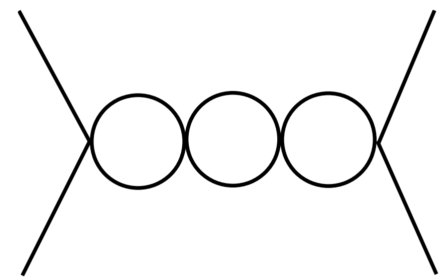
Hard — Easy



Matrix model
planar diagrams



SYK
melon diagrams



vector model
bubble diagrams

Boundary:	SUSY-YM	SYK	O(N) Wilson Fisher
Bulk:	Large gap Critical string Theory in AdS	Tower of massive particles ?	Tower of massless particles Vasielev high spin theories

$$H = \frac{1}{q!} \sum_{i_1,\ldots,i_q=1}^{N} J_{i_1 \ldots i_q} \chi_{i_1} \ldots \chi_{i_q}$$

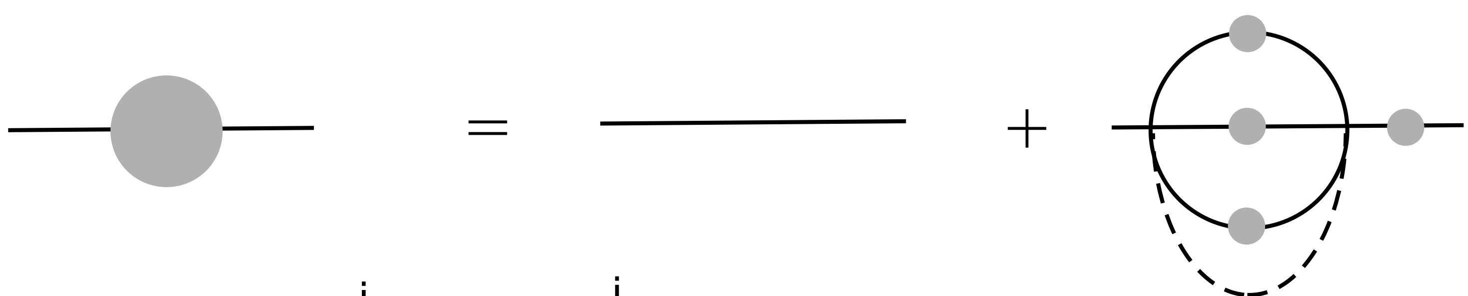
- Majorana fermions $\{\chi_i, \chi_j\} = \delta_{ij}$
- $J_{i_1 \ldots i_q}$ are Gaussian random, O(N) symmetric
- Soluble for Large N
- (Near) Conformal invariance for large J t (IR)
- Maximally Chaotic (Kitaev)

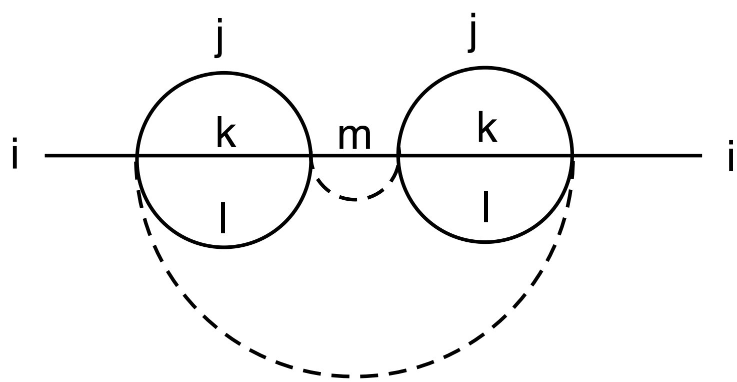
2-pt function

Sachdev Ye '93; Georges, Parcollet, Sachdev '01; Kitaev '15

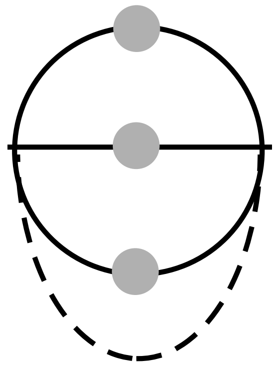
$$L = \sum_i \frac{1}{2}\chi_i \frac{d}{d\tau}\chi_i - \sum_{i,j,k,l} J_{ijkl}\, \chi_i \chi_j \chi_k \chi_l$$

- SYK solvable as a result of having a small & well-organized set of Feynman diagrams: nested MELONS.





$$\sim \frac{J^4}{N^2}$$



$$\Sigma(\tau) = J^2 G(\tau)^3$$

$$G(\omega)^{-1} = -i\omega - \Sigma(\omega)$$

In IR, drop $i\omega$

$$\int d\tau\, G(\tau_1, \tau)\Sigma(\tau, \tau_2) = -\delta(\tau_1 - \tau_2)$$

Conformal (time reparameterization) invariance $\tau \;\text{->}\; f(\tau)$

$$G(\tau_1, \tau_2) \to [f'(\tau_1) f'(\tau_2)]^{\Delta}\, G(f(\tau_1), f(\tau_2))$$

$$G(\tau) \equiv \langle T\chi_i(\tau)\chi_i(0)\rangle = \begin{cases} \frac{1}{2}\mathrm{sgn}(\tau)\ , & |J\tau| \ll 1 \\ \frac{1}{\sqrt{4\pi}} \frac{\mathrm{sgn}(\tau)}{|J\tau|^{1/2}}\ , & |J\tau| \gg 1 \end{cases}$$

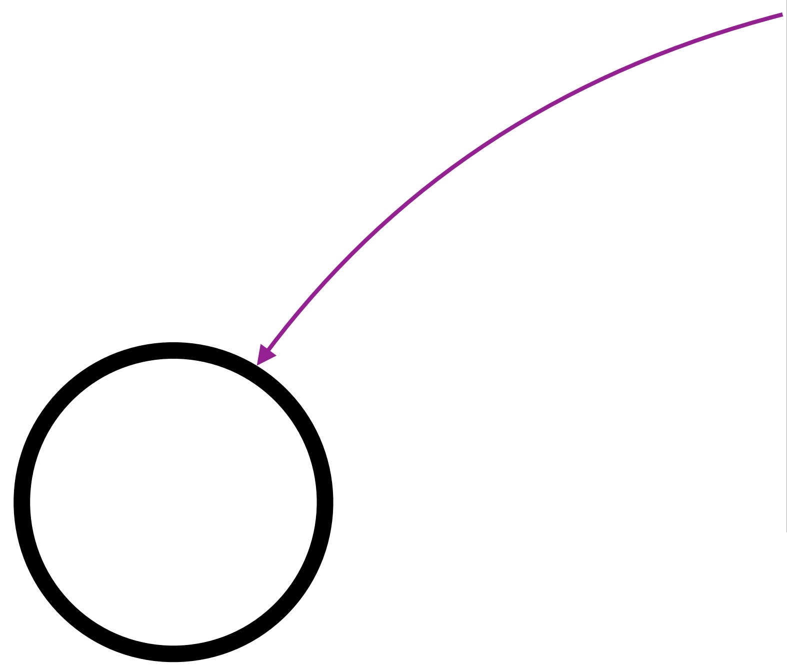
$\Delta = 0$

$\Delta = \frac{1}{4}\quad , \frac{1}{q}$

$$G(\tau, \tau') \propto \frac{1}{(\tau - \tau')^{2\Delta}}$$

$$f(\tau) = \frac{\beta}{\pi} \tan \frac{\pi \tau}{\beta}$$

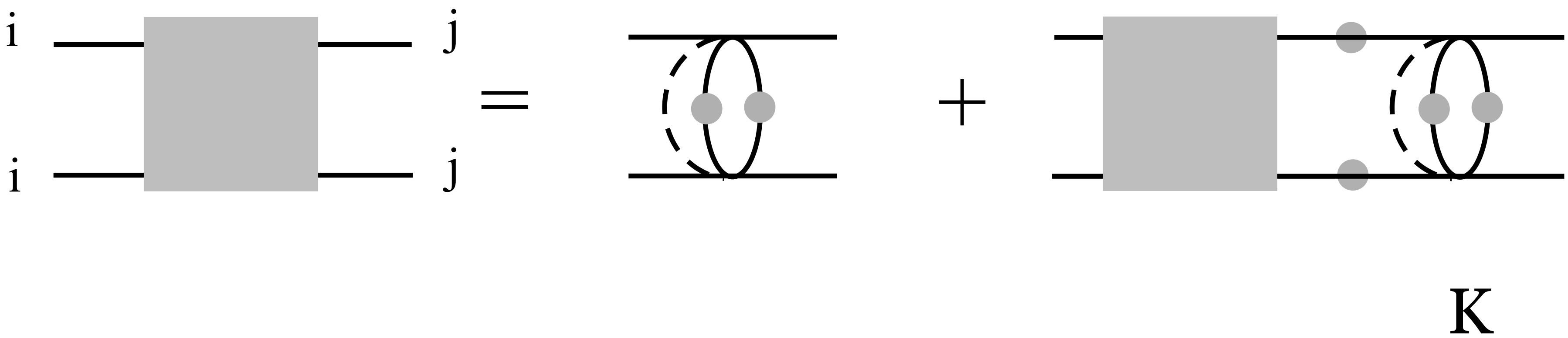
$$G_f = \left[\frac{\pi}{\beta \sin \frac{\pi \tau}{\beta}} \right]^{2\Delta}$$



4-pt function

Kitaev ’15; Polchinski, Rosenhaus ‘16; Maldacena, Stanford ‘16

- Only ladder diagrams



SYK spectrum

- After disorder average, SYK has $O(N)$ symmetry

- Singlet operators are $O_n \sim \sum_{i=1}^{N} \chi_i \, \partial_\tau^{1+2n} \chi_i$

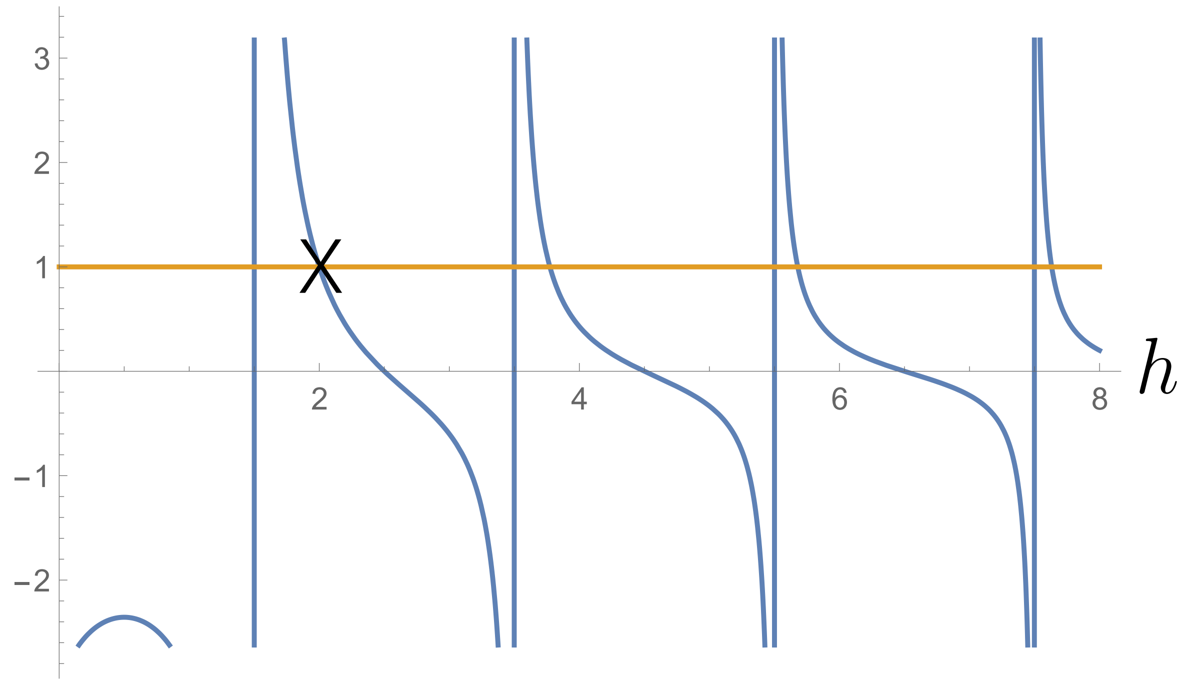
- Dimensions are when eigenvalues of kernel of four-point function equal 1. At strong coupling, we have:

SYK spectrum

Kitaev '15; Polchinski, V.Rosenhaus '16; Maldacena, Stanford '16

Dimensions h: h for which g(h) = 1

$$g(h) = -\frac{3}{2h-1}\tan\frac{\pi}{2}(h-\frac{1}{2})$$



$$h_n \approx 2\Delta + 2n + 1, \qquad n \gg 1$$

Conformal symmetry breaking

- Divergence due to $h = 2$

- Result of IR limit ($|J\tau| \gg 1$). Eliminate by including $\frac{1}{|J\tau|}$ corrections to IR two-point function appearing in kernel.

- Analogous to breaking that occurs in AdS_2 as studied by Almheiri, Polchinski ’14

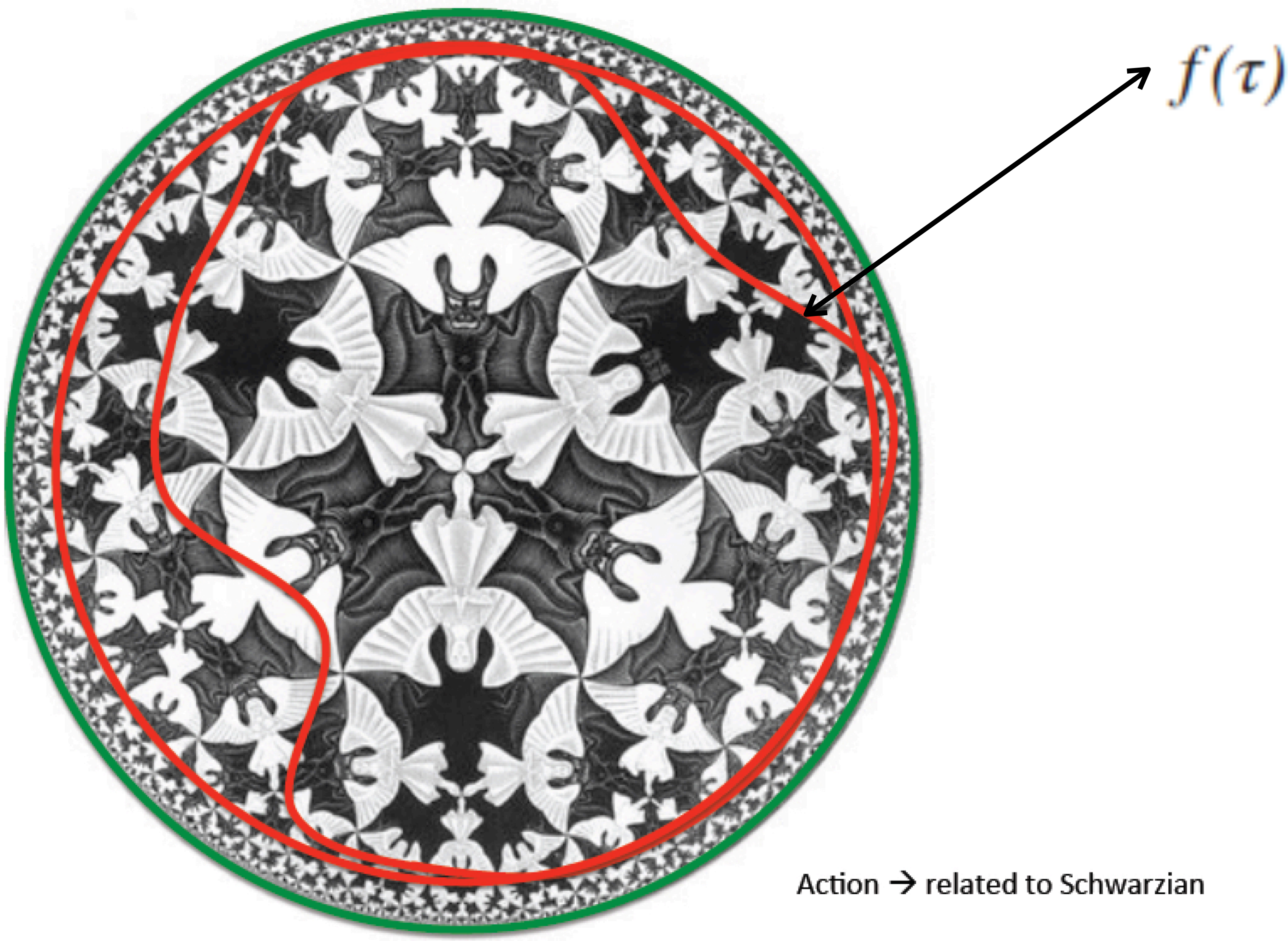
- Detailed story

Kitaev ’15; Jensen, ’16
Maldacena, Stanford, Yang ’16;
Engelsoy, Mertens, Verlinde ’16;
Almheiri, Kang ’16; Jevicki, Suzuki, Yoon, ’16;

- Simplest model:

Teitelboim Jackiw
Almheiri Polchinski

$$\int d^2x\sqrt{g}\phi(R+2)+\phi_0\int d^2x\sqrt{g}R$$



Action → related to Schwarzian

The Schwarzian

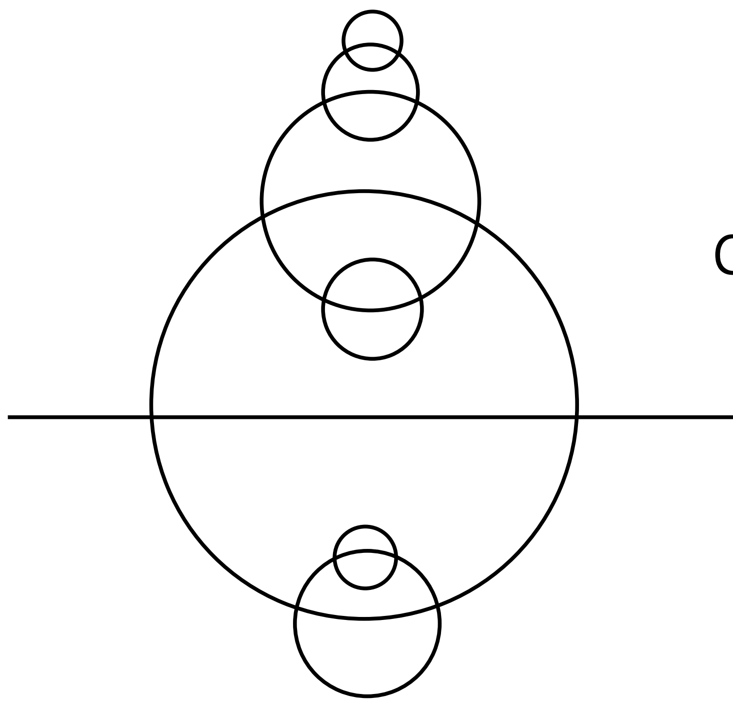
- Keep first correction to IR action

$$S \sim \frac{N}{J} \int d\tau \{f(\tau), \tau\} \qquad \{f(\tau), \tau)\} = \frac{f'''}{f'} - \frac{3}{2}\left(\frac{f''}{f'}\right)^2$$

- This is low energy effective action of SYK. In bulk: action of dilaton (boundary term)
- Maximial chaos comes from this Schwarzian action
- One loop exact. Stanford & Witten

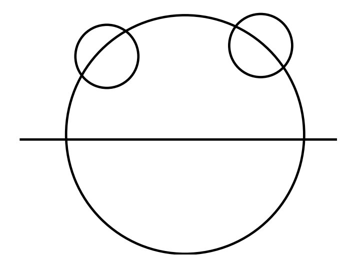
Large q simplification

Maldacena Stanford '16
DG, Rosenhaus. '16



With a single cut,
diagram splits into a tree

suppressed



SUSY SYK

Giatto, Maldacena, Sachdev, Fu '16

$$H = \sum J_{ijk}\phi_i\chi_j\chi_k$$

ϕ_i is auxillary scalar

$$\Delta_\chi = \frac{1}{6} \qquad \Delta_\phi = \Delta_\chi + \frac{1}{2}$$

$$Q = \sum J_{ijk}\chi_i\chi_j\chi_k \qquad H=Q^2$$

2 d SUSY

Stanford, Witten, 2017

Higher Dimensional SYK

Gu, Xi, Stanford, '16

Balents, et, '17

$$H = \sum_a \left(\sum_{ijkl} J^a_{ijkl} \chi^a_i \chi^a_j \chi^a_k \chi^a_l + \sum_{ijkl} \tilde{J}^a_{ijkl} \chi^a_i \chi^a_j \chi^{a+1}_k \chi^{a+1}_l \right)$$

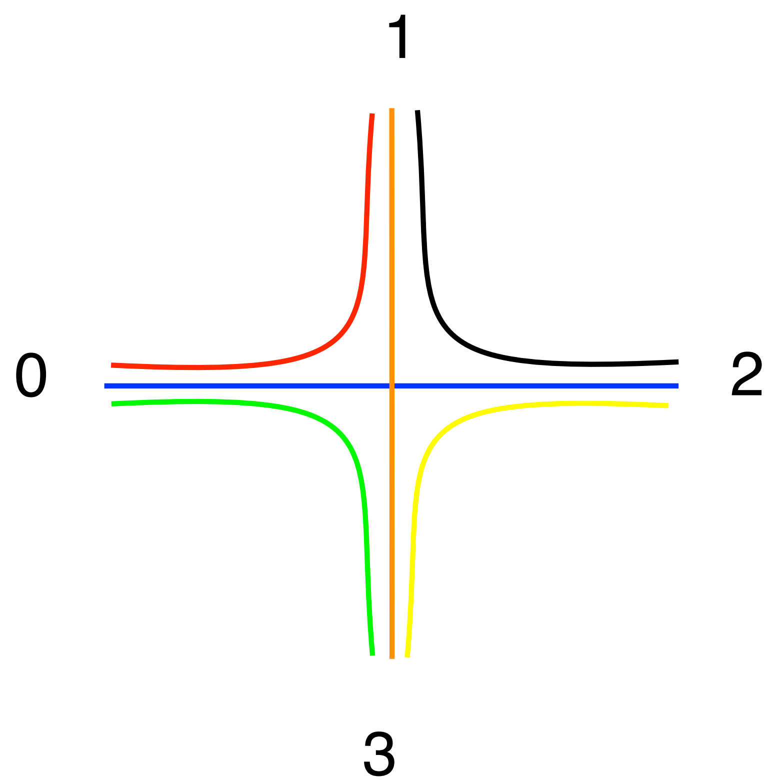
- Interpret flavor as site, on a 1d lattice

Tensor SYK

Witten '16

$$H = \sum \chi^0_{ijk}\chi^1_{klm}\chi^2_{mjp}\chi^3_{pli}$$

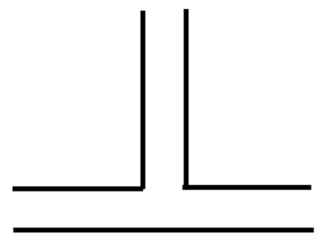
- $4n^3$ variables
- $O(n)^6$ symmetry

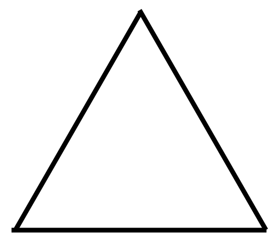


Colored Tensor Models

Gurau, Rivasseau, …

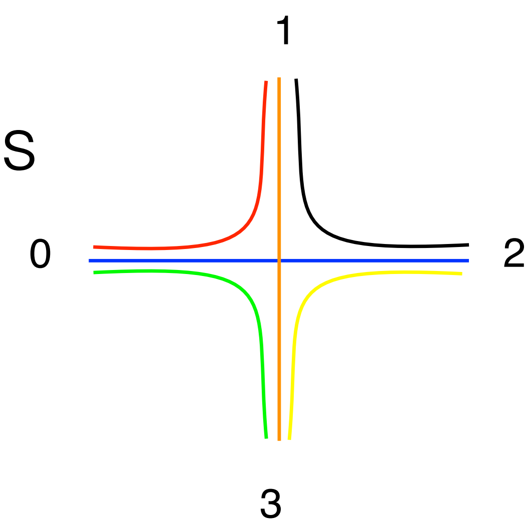
- Matrix models

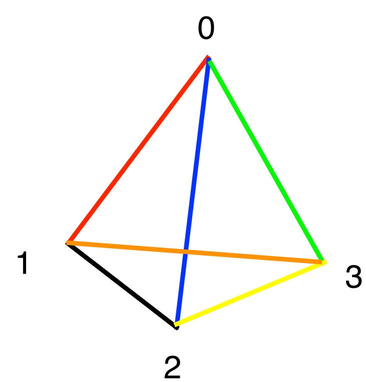




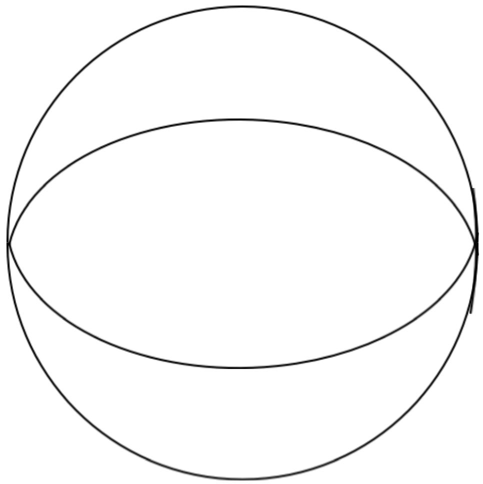
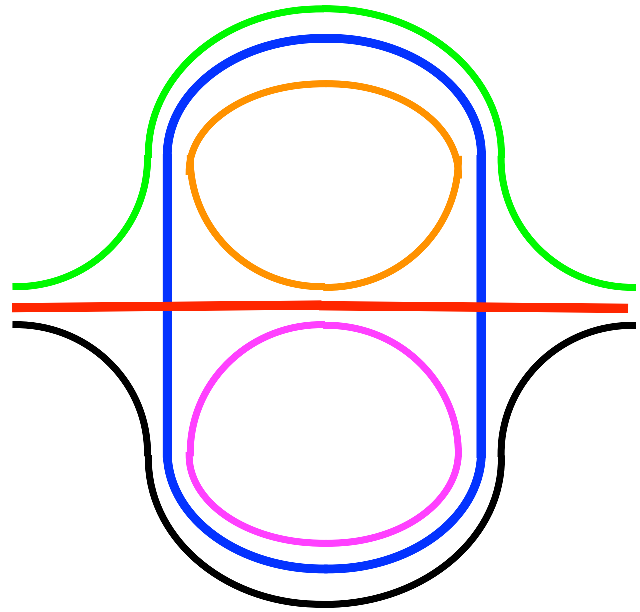
2d geometry

- Tensor models





3d geometry?

The 1/n expansion is in powers of $1/n^{p}$.

p= The sum of the genus of all planar graphs obtained by delating two colors .

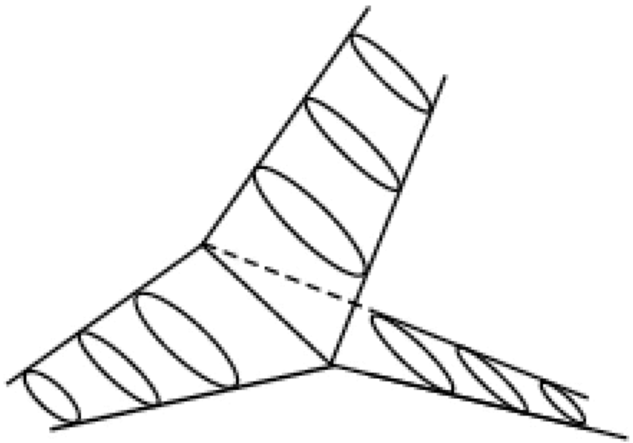
Differs from the 1/n expansion of SYK!

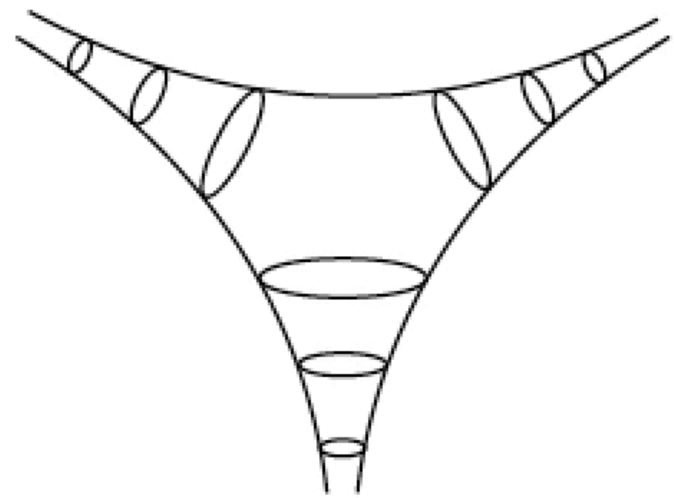
WHAT IS THE BULK GRAVITY/STRING THEORY?

Direct Approach :

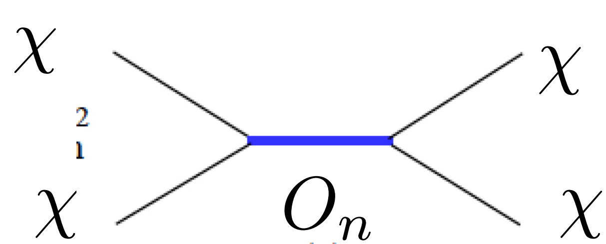
D.G, V.Rosenhaus
hep-th 1702.08016

- Need to know interactions of bulk fields. 3-pt





$$S_{bulk} = \int d^2x\sqrt{g}\left[\frac{1}{2}(\nabla\phi_n)^2 + \frac{1}{2}m_n^2\phi_n^2 + \frac{1}{\sqrt{N}}\lambda_{nmk}\,\phi_n\phi_m\phi_k\right]$$

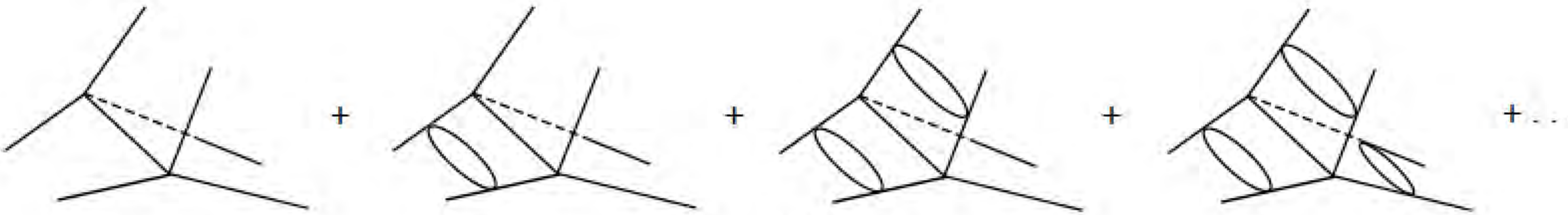


Conformal invariance implies that: $< O_n(\tau_1) O_m(\tau_2) O_k(\tau_3) >=$

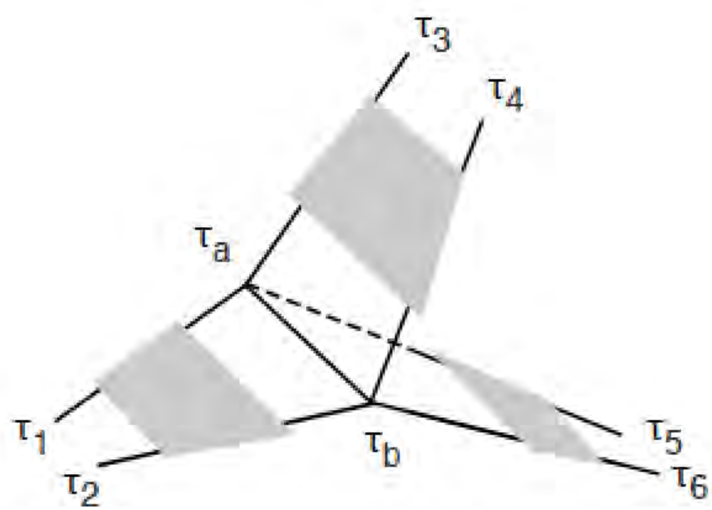
$$I(\tau_1, \tau_2, \tau_3) = \frac{c_{nml}}{h_n h_m h_l \, c_n c_m c_l} \frac{1}{|\tau_{23}|^{h_m+h_l-h_n} |\tau_{13}|^{h_n+h_l-h_m} |\tau_{12}|^{h_n+h_m-h_l}}$$

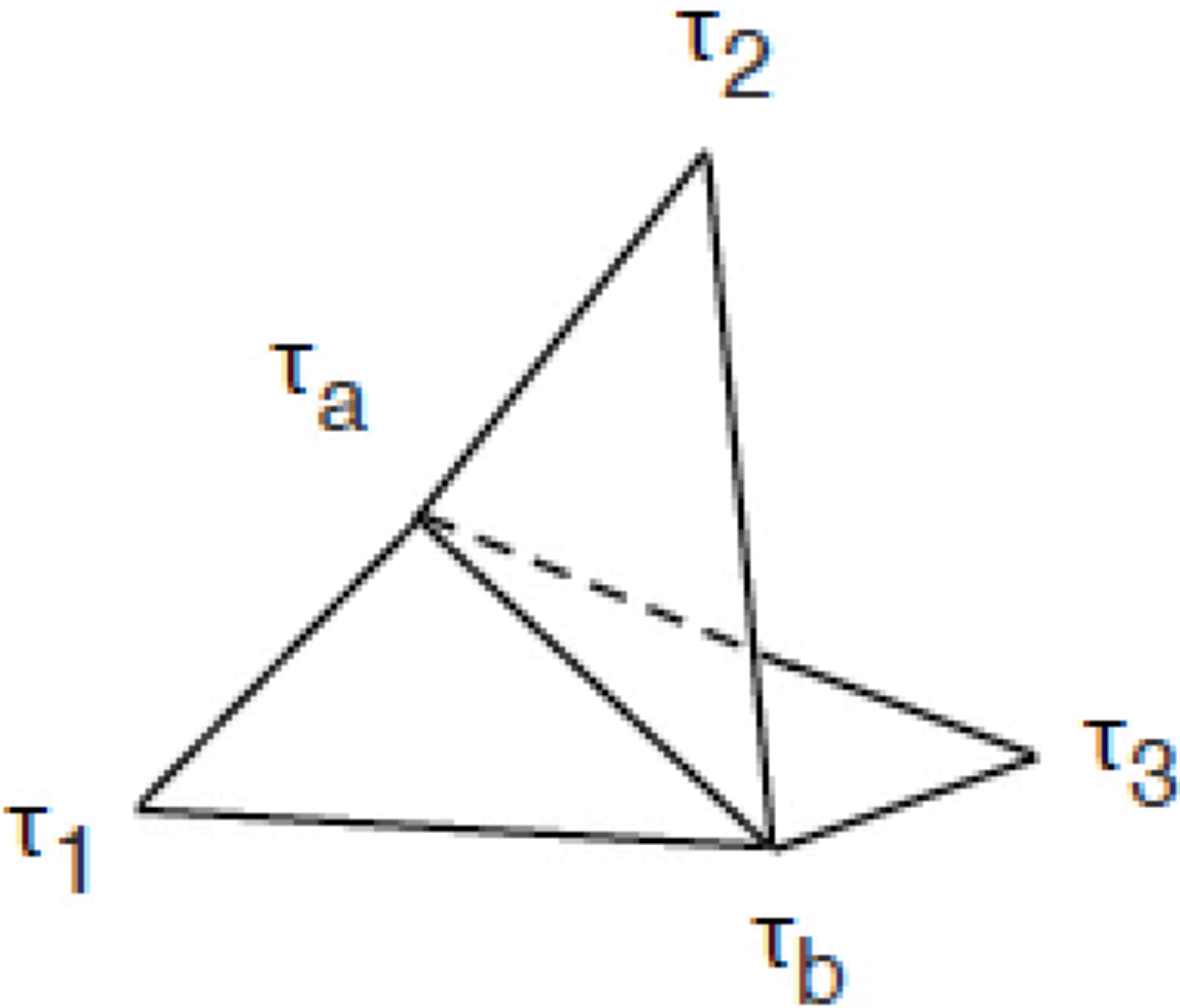
The coefficients $C_{n,m,l}$

determine the 3-pt function of the massive fields in the bulk:



(a)





τ2
τa
τ1
τ3
τb

$$\mathcal{I}^{(1)}_{nmk} = \frac{\sqrt{\pi}\, 2^{h_n+h_m+h_k-1}\, \Gamma(1-h_n)\Gamma(1-h_m)\Gamma(1-h_k)}{\Gamma\left(\frac{3-h_n-h_m-h_k}{2}\right)} [\rho(h_n,h_m,h_k)+\rho(h_m,h_k,h_n)+\rho(h_k,h_n,h_m)] \tag{3.27}$$

where,

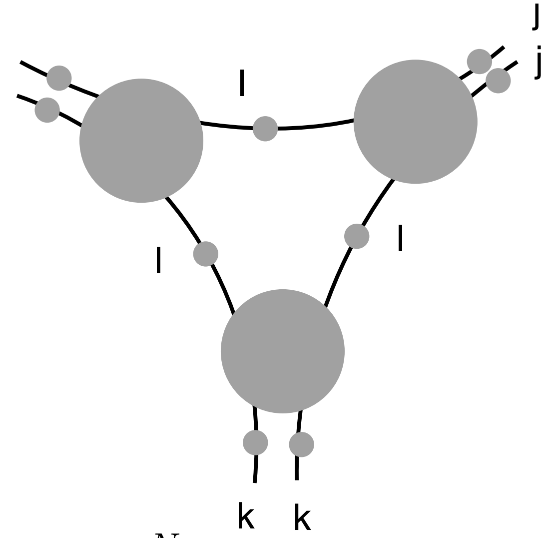
$$\rho(h_n,h_m,h_k) = \frac{\Gamma(\frac{h_m+h_k-h_n}{2})}{\Gamma(\frac{2-h_n-h_m+h_k}{2})\Gamma(\frac{2-h_n-h_k+h_m}{2})} \left(1 + \frac{\sin(\pi h_m)}{\sin(\pi h_k) - \sin(\pi h_n + \pi h_m)}\right) . \tag{3.28}$$

For Large q

$$\lambda^{(1)}_{nmk} = -(-1)^{n+m+k} \frac{16}{\sqrt{\pi}} q \left(\epsilon_n + \epsilon_m + \epsilon_k\right) \alpha_n \alpha_m \alpha_k \ ,$$

$$\alpha_n = \sqrt{\frac{n(1+4n)(1+2n)}{\left(n(1+2n)+1\right)\left(n(1+2n)-1\right)}} \ . \qquad \epsilon_n = \frac{1}{q} \frac{n(2n+1)+1}{n(2n+1)-1} \ ,$$

PLANAR

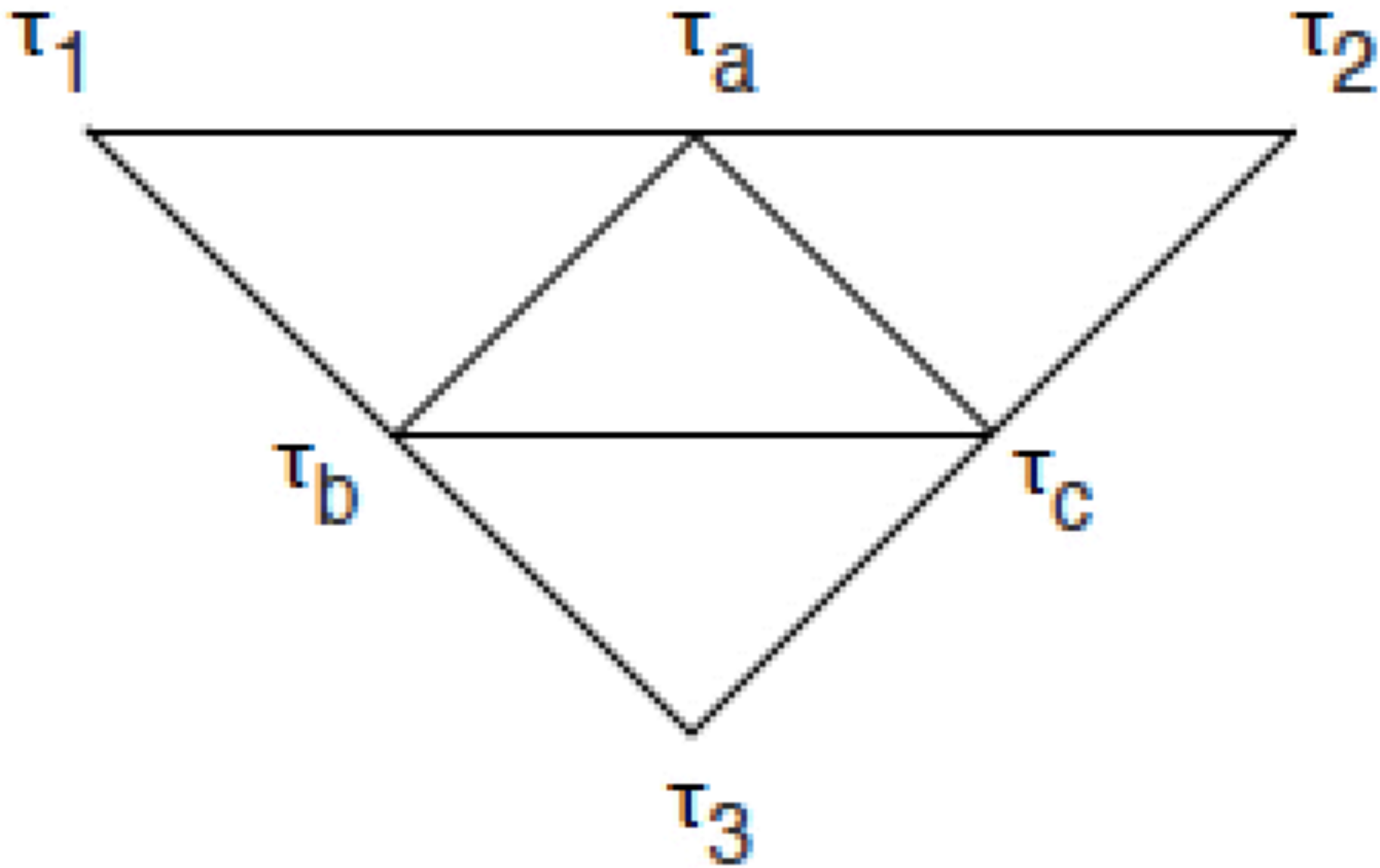


In the large q limit, the operators $\Phi_n = \sum_{i=1}^{N} \chi_i \, \partial_\tau^{1+2n} \chi_i$ greatly simplify.

$\text{Dim}[\Phi_n] = \text{h}_\text{n}$; $h_n \sim 2n + 1 + \epsilon_n$ $\epsilon_n = \frac{2}{q} \frac{(2n^2 + n + 1)}{(2n^2 + n - 1)}$

and the 3-point function : $< \Phi_n(\tau_1) \Phi_m(\tau_2) \Phi_l(\tau_3 >)$ is given by

$$I(\tau_1, \tau_2, \tau_3) = \int d\tau_a d\tau_b d\tau_c \, \text{sgn}(\tau_{1a}) \, \text{sgn}(\tau_{1b}) \text{sgn}(\tau_{2a}) \text{sgn}(\tau_{2c}) \text{sgn}(\tau_{3b}) \text{sgn}(\tau_{3c})$$
$$\frac{|\tau_{ab}|^{h_n - 1} |\tau_{ca}|^{h_m - 1} |\tau_{bc}|^{h_l - 1}}{|\tau_{1a}|^{h_n - 1} |\tau_{1b}|^{h_n + 1} |\tau_{2c}|^{h_m - 1} |\tau_{2a}|^{h_m + 1} |\tau_{3b}|^{h_l - 1} |\tau_{3c}|^{h_l + 1}}$$



τ_1
τ_a
τ_2
τ_b
τ_c
τ_3

$$\langle \mathcal{O}_1(\tau_1)\mathcal{O}_2(\tau_2)\mathcal{O}_3(\tau_3)\rangle_2 = c_1c_2c_3\,\xi(h_1)\xi(h_2)\xi(h_3)\, I^{(2)}_{123}(\tau_1,\tau_2,\tau_3)\ , \tag{3.23}$$

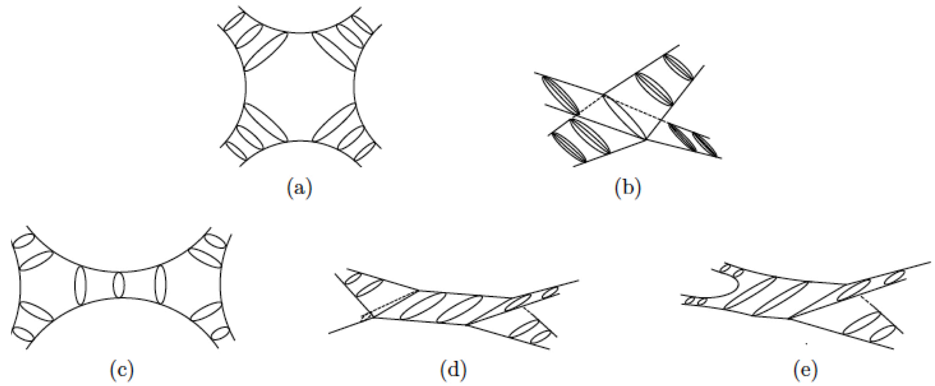
where [6],

$$I^{(2)}_{123}(\tau_1,\tau_2,\tau_3) = \int d\tau_a d\tau_b d\tau_c \frac{-\mathrm{sgn}(\tau_{1a}\tau_{1b}\tau_{2a}\tau_{2c}\tau_{3b}\tau_{3c})|\tau_{ab}|^{h_1-1}|\tau_{ca}|^{h_2-1}|\tau_{bc}|^{h_3-1}}{|\tau_{1a}|^{h_1-1+2\Delta}|\tau_{1b}|^{h_1+1-2\Delta}|\tau_{2c}|^{h_2-1+2\Delta}|\tau_{2a}|^{h_2+1-2\Delta}|\tau_{3b}|^{h_3-1+2\Delta}|\tau_{3c}|^{h_3+1-2\Delta}}. \tag{3.24}$$

$$\begin{aligned}
\xi(h) &= \frac{1}{\sqrt{\pi}}\frac{\Gamma(\frac{2\Delta+1}{2})}{\Gamma(1-\Delta)}\frac{\Gamma(\frac{1-h}{2})}{\Gamma(\frac{h}{2})}\frac{\Gamma(\frac{2-2\Delta+h}{2})}{\Gamma(\frac{1+2\Delta-h}{2})}\ , \\
&= \overline{\alpha}_1\ {}_4F_3\left[\begin{matrix}1-h_1 & h_1 & 2\Delta-h_3 & 1-h_3\\ 1+h_2-h_3 & 2\Delta & 2-h_2-h_3\end{matrix};1\right] \\
&+ \overline{\alpha}_2\, z^{h_3-h_2}\ {}_4F_3\left[\begin{matrix}1-h_1-h_2+h_3 & h_1-h_2+h_3 & 2\Delta-h_2 & 1-h_2\\ 2-2h_2 & 1-h_2+h_3 & 2\Delta-h_2+h_3\end{matrix};1\right] \\
&+ \overline{\alpha}_3\, z^{1-2\Delta}\ {}_4F_3\left[\begin{matrix}2-h_1-2\Delta & 1+h_1-2\Delta & 1-h_3 & 2-h_3-2\Delta\\ 2+h_2-h_3-2\Delta & 3-h_2-h_3-2\Delta & 2-2\Delta\end{matrix};1\right] \\
&+ \overline{\alpha}_4\, z^{h_2+h_3-1}\ {}_4F_3\left[\begin{matrix}h_2+h_3-h_1 & h_1+h_2+h_3-1 & h_2-1+2\Delta & h_2\\ 2h_2 & h_2+h_3-1+2\Delta & h_2+h_3\end{matrix};1\right] .
\end{aligned}$$

$$
\begin{aligned}
\alpha_1 &= -\frac{\Gamma(\frac{2\Delta+1}{2})^2}{\Gamma(1-\Delta)^2}\prod_{i=1}^{3}\frac{\Gamma(\frac{1-h_i}{2})}{\Gamma(\frac{h_i}{2})}\;\frac{\Gamma(\frac{3-h_2-2\Delta}{2})\Gamma(\frac{2+h_2-2\Delta}{2})}{\Gamma(\frac{h_2+2\Delta}{2})\Gamma(\frac{1-h_2+2\Delta}{2})}\,\frac{\Gamma(\frac{h_3-h_2}{2})\Gamma(\frac{h_2+h_3-1}{2})}{\Gamma(\frac{2-h_2-h_3}{2})\Gamma(\frac{1+h_2-h_3}{2})}\,\frac{\Gamma(\frac{h_1+h_2-h_3}{2})}{\Gamma(\frac{1-h_1-h_2+h_3}{2})}\,, \\
\alpha_2 &= -\frac{\Gamma(\frac{2\Delta+1}{2})^3}{\Gamma(1-\Delta)^3}\,\frac{\Gamma(\frac{1-h_1}{2})}{\Gamma(\frac{h_1}{2})}\,\frac{\Gamma(\frac{1-h_2}{2})^2\,\Gamma(\frac{2h_2-1}{2})}{\Gamma(\frac{h_2}{2})^2\,\Gamma(\frac{2-2h_2}{2})}\,\frac{\Gamma(\frac{3-h_2-2\Delta}{2})}{\Gamma(\frac{h_2+2\Delta}{2})}\,\frac{\Gamma(\frac{2+h_3-2\Delta}{2})}{\Gamma(\frac{1-h_3+2\Delta}{2})} \\
&\quad\cdot\frac{\Gamma(\frac{h_2-h_3}{2})\Gamma(\frac{h_2-h_3+2-2\Delta}{2})}{\Gamma(\frac{1-h_2+h_3}{2})\Gamma(\frac{h_3-h_2+1+2\Delta}{2})}\,\frac{\Gamma(\frac{h_1-h_2+h_3}{2})}{\Gamma(\frac{1-h_1+h_2-h_3}{2})}\,, \\
\alpha_3 &= -\frac{\Gamma(\frac{2\Delta+1}{2})^3\,\Gamma(\Delta)}{\Gamma(1-\Delta)^3\,\Gamma(\frac{3-2\Delta}{2})}\prod_{i=1}^{3}\frac{\Gamma(\frac{1-h_i}{2})\Gamma(\frac{2+h_i-2\Delta}{2})\Gamma(\frac{3-h_i-2\Delta}{2})}{\Gamma(\frac{h_i}{2})\Gamma(\frac{1-h_i+2\Delta}{2})\Gamma(\frac{h_i+2\Delta}{2})} \\
&\quad\cdot\frac{\Gamma(\frac{h_3-h_2+2\Delta}{2})\Gamma(\frac{h_2+h_3-1+2\Delta}{2})}{\Gamma(\frac{3+h_2-h_3-2\Delta}{2})\Gamma(\frac{4-h_2-h_3-2\Delta}{2})}\,\frac{\Gamma(\frac{h_1+h_2-h_3}{2})}{\Gamma(\frac{1-h_1-h_2+h_3}{2})}\,, \\
\alpha_4 &= -\frac{\Gamma(\frac{2\Delta+1}{2})^3}{\Gamma(1-\Delta)^3}\,\frac{\Gamma(\frac{1-h_1}{2})}{\Gamma(\frac{h_1}{2})}\,\frac{\Gamma(\frac{1-2h_2}{2})}{\Gamma(h_2)}\,\frac{\Gamma(\frac{2+h_2-2\Delta}{2})}{\Gamma(\frac{1-h_2+2\Delta}{2})}\,\frac{\Gamma(\frac{2+h_3-2\Delta}{2})}{\Gamma(\frac{1-h_3+2\Delta}{2})} \\
&\quad\cdot\frac{\Gamma(\frac{1-h_2-h_3}{2})}{\Gamma(\frac{h_2+h_3}{2})}\,\frac{\Gamma(\frac{3-h_2-h_3-2\Delta}{2})}{\Gamma(\frac{h_2+h_3+2\Delta}{2})}\,\frac{\Gamma(\frac{h_1+h_2-h_3}{2})\Gamma(\frac{-h_1+h_2+h_3}{2})\Gamma(\frac{h_1+h_2+h_3-1}{2})}{\Gamma(\frac{1-h_1-h_2+h_3}{2})\Gamma(\frac{1+h_1-h_2-h_3}{2})\Gamma(\frac{2-h_1-h_2-h_3}{2})}\,. \qquad (3.32)
\end{aligned}
$$

FUTURE DIRECTIONS



- Can to use same methods to evaluate 4-pt, 5-pt,… couplings in the bulk.
- Derivative couplings? Is the bulk theory “local”?
- Are there hidden symmetries for large q?
- Can SYK be embedded in critical string theory? Or in a new kind of “string theory”?

?

THE END