Correlation Functions in (2,2) and (0,2) Hybrid Models

Mauricio Romo (IAS)

Kavli IPMU, 2017

M. Bertolini & M.R.

Arxiv: 17....

OUTLINE

- (2,2) HYBRID MODELS
- (0,2) HYBRID MODELS
- APPLICATION: (0,2) LG MODELS
- FUTURE DIRECTIONS

(2,2) Hybrids

<u>Data:</u> $(Y, W, \mathbb{C}^\times_V)$

Y - Kähler manifold

$W \in H^0(\mathcal{O}_Y)$

$\mathbb{C}^*_V \circlearrowright Y \quad \mathcal{L}_V W = W$ (quasi-homogeneity)

<u>Conditions:</u>

$Y = \mathrm{Tot}(\bigoplus_i X_i \longrightarrow B)$ $\quad$ B - compact smooth Kähler,

$V(B) = 0 \qquad V(X_i) = q_i X_i$

$c_1(Y) = 0$ (CY)

$dW^{-1}(0) = B$

Bertolini, Melnikov, Plesser '15

(2,2) Hybrids

Examples:

1) Y - compact, smooth, $W \equiv 0$ (σ-model)

2) $Y = \mathbb{C}^N$, $B = pt.$ $W \in \mathbb{C}[\phi_1, \dots, \phi_N]$ (LG)

3) $Y = \mathbb{C}^N / \Gamma$ $B = pt$ $W \in \mathbb{C}[\phi_1, \dots, \phi_N]^\Gamma$ (LG orb.)

4) $Y = \left(\mathcal{O}(-2) \oplus \mathcal{O}^{\oplus 3} \longrightarrow \mathbb{P}^1\right) / \mathbb{Z}_4$

$$W = \frac{1}{8}(x_1^8 + x_2^8)\phi_1^4 + \frac{1}{4}(\phi_2^4 + \phi_3^4 + \phi_4^4) - \psi x_1 x_2 \phi_1 \phi_2 \phi_3 \phi_4$$

$[x_1, x_2] \in \mathbb{P}^1$ $\phi_1 \in \mathcal{O}(-2)$ $\phi_{2,3,4} \in \mathcal{O}$

(Phase of $\mathbb{P}^4(11222)[8]$)

(2,2) Hybrids

Chiral Ring

$\mathcal{N} = (2,2) \Rightarrow Q_\pm, \bar{Q}_\pm$ 4 supercharges

$$\bar{Q}_{(c,c)} := \bar{Q}_+ + \bar{Q}_- \quad \text{(B-Twist)}$$

$$\mathcal{H}_{\bar{Q}} := \bar{Q}_{(c,c)}\text{-cohomology} \quad \text{(physical observables)}$$

$$\mathcal{O}_i \in \mathcal{H}_{\bar{Q}}$$

$$\mathcal{O}_i \cdot \mathcal{O}_j = c_{ij}{}^k \mathcal{O}_k$$

$$c_{ijk} = \langle \mathcal{O}_i \mathcal{O}_j \mathcal{O}_k \rangle_{\Sigma = S^2} = \int \mathcal{D}\phi \mathcal{D}\chi \, e^{-\int_\Sigma L(\phi : \Sigma \to Y, W)\, d^2 z} \, \mathcal{O}_1 \cdot \mathcal{O}_2 \cdot \mathcal{O}_3$$

(B-Twisted)

(2,2) Hybrids

Geometric Interpretation:

$$\mathcal{H}_{\bar{Q}} = \bigoplus_{q=0}^{\hat{c}} \mathcal{H}^{q}_{\bar{Q}} \cong (PV(Y), \delta)$$

$$PV(Y) := \Omega^{(0,*)}(Y, \Lambda^* T_Y) \qquad \delta := \bar{\partial} - \iota_J \,, \; J = dW$$

$$\mathcal{O}(\omega) \longrightarrow \omega^L_{\bar{I}} \, d\bar{z}^{\bar{I}} \otimes \partial_L$$

Example: LG (orbifold) $Y = \mathbb{C}^N$ $\quad W(e^{iq_i\lambda}\phi_i) = e^{i\lambda} W(\phi)$

$$\mathcal{H}^{\text{untwisted}}_{\bar{Q}} = \left[\frac{\mathbb{C}[\phi_1, \ldots, \phi_N]}{(\partial_1 W, \ldots, \partial_N W)} \right]^{\Gamma} = \left[\mathrm{Jac}(W) \right]^{\Gamma}$$

(2,2) Hybrids

B-twist + Localization ($\Sigma = S^2$):

$$\boxed{\langle \mathcal{O}_1 \mathcal{O}_2 \mathcal{O}_3 \rangle_{S^2} = \int_Y \Omega_Y \wedge \left(\Omega_Y \lrcorner\, e^{\hat{L}} \circ \omega_1 \wedge \omega_2 \wedge \omega_3 \right)} \quad \left(\overset{Y\text{- compact}}{=} \int_Y \Omega_Y \wedge \left(\Omega_Y \lrcorner\, \omega \right) \right)$$

$$\Omega_Y \lrcorner : \Omega^{(0,s)}(Y, \Lambda^r T_Y) \longrightarrow \Omega^{(d-r,s)}(Y)$$

$$\hat{L} := -\frac{v}{4} \|J\|^2 + \frac{v}{4} \bar{\partial} \bar{J}^\alpha \partial_\alpha , \qquad v = \mathrm{Vol}(\Sigma)$$

$$J_\alpha = \partial_\alpha W \qquad \bar{J}^\alpha = g^{\alpha \bar{\beta}} \bar{J}_{\bar{\beta}}$$

$$\|J\|^2 = g^{\alpha \bar{\beta}} J_\alpha \bar{J}_{\bar{\beta}}$$

(2,2) Hybrids

<u>In General</u> $\omega \in (PV, \delta)$

$$\langle \omega \rangle = \int_{\partial(Y \setminus B)} \Xi(\omega) \quad , \quad \Xi(\omega) \in H^{2d-1}_{DR}(Y \setminus B)$$

<u>Example</u>: LG (orbifold) $\omega \in [\mathrm{Jac}(W)]^{\Gamma}$

$$\langle \omega \rangle = \int_{S^{N-1}} A_{N-1}\, \omega = \int_{\{|\partial_i W| = \varepsilon_i\}_{i=1}^{N}} \frac{\omega\, d\phi_1 \wedge \cdots \wedge d\phi_N}{\prod_i \partial_i W} = \operatorname*{Res}_{(\phi=0)}(\omega)$$

$$A_{N-1} \sim \wedge\, \bar{\partial} \rho_j \in H^{2N-1}_{DR}((\mathbb{C}^N)^*) \, , \quad \rho_j = \frac{|\partial_i W|^2}{\|dW\|^2}$$

(2,2) Hybrids

Properties of Correlators:

i) $\partial(Y \setminus B) = M$ $\qquad S^{2d-2b-1} \longrightarrow M \xrightarrow{\pi} B$

ii) Holomorphic (in the parameters of W) $\quad \bar{J} \to \bar{J} + \delta \bar{J}$

iii) Independent of infinitesimal variations

$$v \longrightarrow v + \delta v \qquad g_{\alpha\bar{\beta}} \longrightarrow g_{\alpha\bar{\beta}} + \delta g_{\alpha\bar{\beta}}$$

iv) Satisfy a "transformation law"

(0,2) Hybrids

(Bertolini & Plesser)

<u>Data:</u> $(\mathcal{E}, Z, \mathbb{C}^\times_V, J \in \Gamma(\mathcal{E}^\vee))$

Z - Kähler $\qquad \mathcal{L}_V J \sim J$

$\mathcal{E} \to Z$ - Holomorphic

<u>Conditions</u>

$$\Lambda^R \mathcal{E} \cong K_Z^\vee \qquad R = \operatorname{rank}(\mathcal{E}) \qquad \left(\Rightarrow c_1(\mathcal{E}) + c_1(TZ) = 0\right)$$

$$ch_2(\mathcal{E}) = ch_2(T_Z)$$

$Z = X \xrightarrow{\pi} B \qquad J^{-1}(0) = B$

[<u>Remark:</u>
$\mathcal{E} = TZ \Rightarrow (2,2)$ Hybrid]

(0,2) Hybrids

$\mathbb{Z}_2$-twisted correlators (ignoring instantons):

$$\boxed{\langle \omega \rangle = \int_Z \Omega_{\mathcal{E}} \lrcorner \left(e^{\hat{L}} \cdot \omega \right)}$$

$$\omega = \omega_1 \wedge \omega_2 \wedge \omega_3$$

$$\Omega_{\mathcal{E}} \in \Gamma(\Lambda^R \mathcal{E} \otimes K_Z) = \Gamma(\mathcal{O}_Z)$$

$$\omega \in \left(\Omega^{(0,*)}(Z, \Lambda^* \mathcal{E}), \delta = \bar{\partial} - \iota_J \right) \cong \mathcal{H}^{\text{hol}}_{\bar{Q}_+}$$

Application : (0,2) LG models

$Z = \mathbb{C}^2$ $\quad$ RANK$(\mathcal{E}) = 3$

$$J = \begin{pmatrix} \phi_1^2 \phi_2 \\ \phi_2(\phi_1^2 - \psi\phi_2) \\ \phi_1(\phi_1^2 - \psi\phi_2) \end{pmatrix}, \quad J \neq \partial W, \quad J^{-1}(0) = 0$$ (NOT A COMPLETE INTERSECTION)

$$\mathcal{H} = \frac{\mathbb{C}[\phi_1, \phi_2]}{(J)} \oplus \left\{ \phi_1^k \phi_2^\ell \cdot \partial_A \right\}$$

$\exists\, \mathcal{B} \in \mathrm{Mat}_{3\times 3}(\mathbb{C}[\phi_1, \phi_2])$ (TRANSFORMATION LAW)

s.t. $$\mathcal{B} \cdot J = \begin{pmatrix} 1 & 0 & 0 \\ \psi & 0 & \phi_1 \\ \psi^{-1} & -\psi^{-1} & 0 \end{pmatrix} \begin{pmatrix} J_1 \\ J_2 \\ J_3 \end{pmatrix} = \begin{pmatrix} \phi_1^2 \phi_2 \\ \phi_1^4 \\ \phi_2^2 \end{pmatrix} = T$$

[GENERAL CASE: $\mathcal{B} \in \mathrm{Hom}(\mathcal{E}^\vee, \mathcal{E}^\vee)$]

Application : (0,2) LG models

$\alpha \in \mathcal{H}$ $\quad M := B^{B}_{A} \bar{\partial} \bar{T}^{A} \partial_{B} \in \Omega^{(0,1)}(\mathbb{C}^2, \mathcal{E})$

$\hookrightarrow$ ONLY DEPENDS ON ψ

$$\langle \alpha \rangle_{S^2} = \int_{\mathbb{C}^2} d^2\phi_1 \, d^2\phi_2 \, \epsilon_{ABC} \, \epsilon^{\bar{\alpha}\bar{\beta}} M^{A}_{\bar{\alpha}} M^{B}_{\bar{\beta}} \alpha^{A}_{4} \, e^{-\frac{V}{4}\|T\|^2} = \sum_{a} f_a(\psi) \, c_a$$

So :

$$\langle \phi_1^{k} \phi_2^{l} \, \partial_1 \rangle = c \cdot \psi^{-1} \cdot \delta_{k,2} \, \delta_{l,1}$$ $\leftarrow$ SING. AT $\psi = 0$

WHERE $$c = \int_{\mathbb{R}^2_{\geq 0}} dr_1 \, dr_2 \, e^{-\frac{V}{4}\|T\|^2} r_1^{a} r_2^{b} > 0$$

Application : (0,2) LG models

At special values of ψ, families of these models are conjecture to flow to coset models

e.g.

$$J_1 = \phi_1^m$$

$$J_2 = \phi_2^m \qquad \Longrightarrow \qquad \left(\frac{SU(2)_{mm-1}}{U(1)_{2(mm-1)}} \times \left(U(1) \times U(1)\right)_a \right) \otimes \overline{\left(\frac{SU(2)_{mm-1} \times SO(2)_1}{U(1)_{2(mm+1)}} \right)}$$

$$J_3 = \phi_1 \phi_2$$

Gadde & Putrov '16

FUTURE DIRECTIONS

- ADDING BOUNDARIES

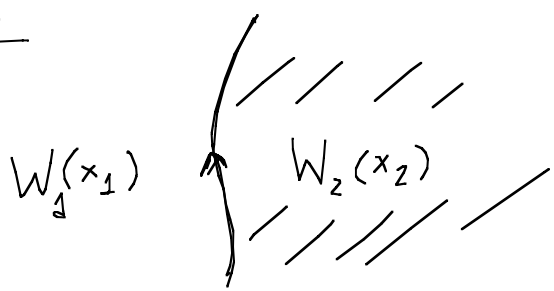
B-BRANES : $(\mathcal{D}, \mathcal{E})$

$E = \mathcal{E}_0 \oplus \mathcal{E}_1 \longrightarrow Y$

$\mathcal{D} \in \Gamma(Y, \mathrm{End}(E))$

$\mathcal{D}^2 = W \cdot \mathrm{id}$

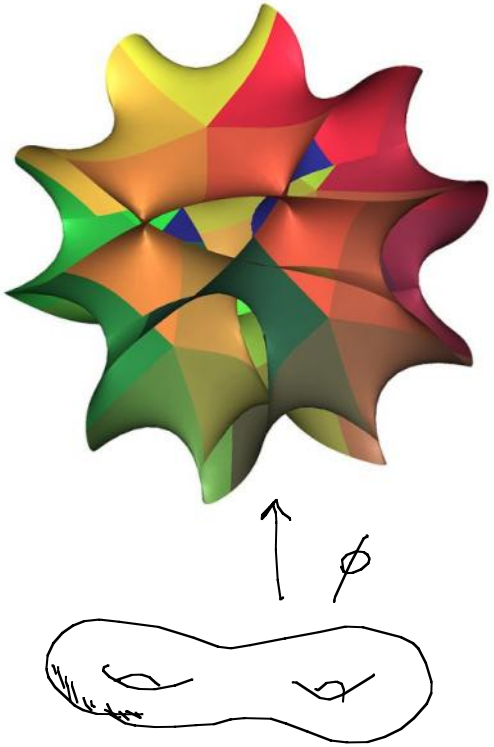
- DEFECTS



2-CATEGORIES

(DEFCT TQFT)

- ORBIFOLDS

Happy 10th Anniversary!

$\mathbb{R}^3 \times$