Anomaly constraints on QCD phase transition

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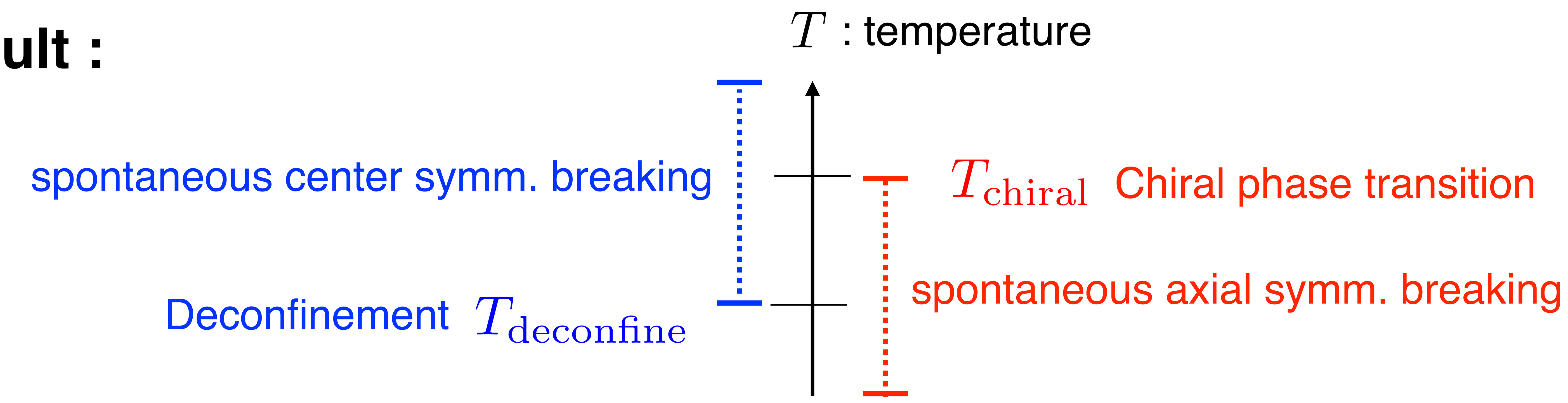
Introduction :

Four dimensional gauge theories such as massless QCD have thermal phase transitions associated to

- Deconfinement
- Chiral phase transition

Remark: the definition of deconfinement is subtle in QCD. See below.

Result :



For strongly coupled gauge theories specified below,

$$T_{\mathrm{deconfine}} \leq T_{\mathrm{chiral}}$$

The equality is possible only for 1st order transition

up to more exotic possibilities (see our paper and [Gaiotto-Kapstin-Komargodski-Seiberg,2017]).

A little more detail on critical temperatures:

T_{chiral} : defined as the breaking/restoration of the discrete axial symmetry $Z_M^{\mathrm{axial}} \subset U(1)_{\mathrm{axial}}$
Z_M^{axial} is the axial symmetry which is not broken by instantons.

$T_{\mathrm{deconfine}}$: defined as the breaking/restoration of the (subgroup of) center symmetry

$$W = \mathrm{tr} P \exp(i \oint_{S^1} A_\mu dx^\mu)$$: Polyakov loop (holonomy around the circle)

$$Z_{N_c}^{\mathrm{center}} : \quad W \to e^{2\pi i/N_c} W$$

1. For gauge theories with adjoint fermions (such as Super-Yang-Mills), $Z_{N_c}^{\mathrm{center}}$ exists.
2. For QCD-like theories with fundamental fermions with imaginary baryon chemical potential $\mu_B = \pi$, a subgroup $Z_2 \subset Z_{N_c}^{\mathrm{center}} \rtimes (\text{parity on } S^1)$ exists.
3. For more generic imaginary chemical potential including zero, see our paper.

Very brief sketch of derivation:

$SU(N_c)$ gauge theory with (1)adjoint fermions or (2)fundamental fermions with $\mathrm{gcd}(N_c, N_f) \neq 1$

New mixed 't Hooft anomaly of center and axial symmetry:

$SU(N_c)$ bundle → (1) $[SU(N_c)]/Z_{N_c}$ bundle for adjoint
(2) $[SU(N_c) \times SU(N_f)]/Z_{\mathrm{gcd}(N_c,N_f)}$ bundle for fundamental

Fractional instanton number is possible under nontrivial bundles:
Z_M^{axial} has 't Hooft anomaly which forbids a trivial gapped state by anomaly matching.