B/2 correlators in (0,2) hybrid models

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A quest for (0,2) mirror symmetry

Just what is (0,2) mirror symmetry?

- (2,2) mirror symmetry: Let M be a CY manifold.

$$(M, T_M) \longleftrightarrow (\widetilde{M}, T_{\widetilde{M}})$$

A model (Kähler moduli) $\longleftrightarrow$ B model, cx.str. moduli

- (0,2) mirror symmetry: Let M Calabi-Yau + holomorphic stable bundle $\mathcal{E}$.

$$(M, \mathcal{E}) \overset{?}{\longleftrightarrow} (\widetilde{M}, \widetilde{\mathcal{E}})$$

A/2 model $\overset{?}{\longleftrightarrow}$ B/2 model

- (Some) open questions:
  - Do B/2 model correlators receive quantum corrections?
  - If yes, is there a subset of theories where they do not?
  - If yes, is there a split in the moduli space, which is exchanged by a mirror map? What about the bundle moduli?

(0,2) hybrid conformal field theories

Let $\boldsymbol{Y}$ be a Kähler manifold such that

$$\boldsymbol{Y} = \text{tot}\left(X \xrightarrow{\pi} B\right) .$$

A hybrid model can be thought as a fibration of a (0,2) Landau-Ginzburg (LG) model varying adiabatically over a compact and smooth manifold B. Let $\mathcal{E} \to \boldsymbol{Y}$ be a holomorphic vector bundle which couples to the left-moving fermions of the theory.

Definition 1. The quadruple $(\boldsymbol{Y}, \mathcal{E}, V, J)$ defines a nonsingular good hybrid model when it admits

1. *Chiral symmetries*.
2. *Potential condition*: the (0,2) superpotential $J \in \Gamma(\mathcal{E}^*)$ is such that $J^{-1}(0) = B$.

This UV theory will RG flow to a family of IR SCFTs parametrized by

1. Kähler class of the Kähler metric of $\boldsymbol{Y}$;
2. Complex structure of $\boldsymbol{Y}$;
3. Complex structure of $\mathcal{E}$ as a holomorphic bundle;
4. Superpotential J.

B/2 heterotic topological ring

(0,2) SCFTs possess a (infinite) chiral ring defined by the cohomology of the supercharge $\overline{\boldsymbol{Q}}$, $\mathcal{H}_{\overline{\boldsymbol{Q}}}$. Two projections are possible within $\overline{\boldsymbol{Q}}$-cohomology

- $\mathcal{H}_{A/2}$ ring;
- $\mathcal{H}_{B/2}$ ring.

Unitarity bounds imply that the B/2 genus zero correlators

$$c_{\alpha\beta\gamma} = \langle \mathcal{O}_\alpha \mathcal{O}_\beta \mathcal{O}_\gamma \rangle ,$$

are topological in the sense that they are independent from the worldsheet metric and from the insertion points of the operators.

Result 1. *In (0,2) hybrid models, the $\mathcal{H}_{\overline{\boldsymbol{Q}}}$ chiral ring consists of elements*

$$[\mathcal{O}] \in H^{\bullet}_{\overline{\boldsymbol{Q}}}(\boldsymbol{Y}, \Lambda^\bullet \mathcal{E}) ,$$

where $\mathcal{O} \in \Omega^{0,\bullet}(\Lambda^\bullet \mathcal{E})$ is a anti-holomorphic horizontal form (in the sense of the fiber/base geometry of $\boldsymbol{Y}$) valued in the sheaf $\Lambda^\bullet \mathcal{E}$.

Localization and S^2 correlators

S^2 localization of the B/2 twisted theory yields the following:

Result 2. *The S^2 correlators are given by the following formula*

$$\langle \mathcal{O}_\alpha \mathcal{O}_\beta \mathcal{O}_\gamma \rangle = \int_{\boldsymbol{Y}} \Omega_{\boldsymbol{Y}} \wedge \left(\omega_{\mathcal{E}} \lrcorner \left(\exp\left(-\frac{\mathbf{v}}{4} \|J\|^2 + \frac{\mathbf{v}}{4} \overline{\partial J}^A \partial_A \right) \mathcal{O}_\alpha \mathcal{O}_\beta \mathcal{O}_\gamma \right) \right) ,$$

where, $\Omega_{\boldsymbol{Y}}$ is the holomorphic top form on $\boldsymbol{Y}$, ∂_A is a basis for $\mathcal{E}$, $\omega_{\mathcal{E}}$ is a section of $\bigwedge^R \mathcal{E}^\vee$, $R = \text{rank}\, \mathcal{E}$, and $\mathbf{v}$ is the volume of the worldsheet $\Sigma = \mathbb{P}^1$. The operator $\omega_{\mathcal{E}} \lrcorner$ acts by contraction with the section $\omega_{\mathcal{E}}$.

The formula above possesses some formal properties, which are expected for a B/2 correlator.

Result 3. *Let $\alpha = \mathcal{O}_1 \mathcal{O}_2 \mathcal{O}_3$, it is possible to show the following*

1. *Independence from the choice of the representative of cohomology classes*

$$\langle \overline{\boldsymbol{Q}} \alpha \rangle = 0 .$$

2. *Independence from the worldsheet volume* $\mathbf{v}$

$$\langle \alpha \rangle \big|_{\mathbf{v}+\delta\mathbf{v}} = \langle \alpha \rangle \big|_{\mathbf{v}} .$$

3. *Independence from Kähler class of the Kähler metric g of $\boldsymbol{Y}$*

$$\langle \alpha \rangle \big|_{g+\delta g} = \langle \alpha \rangle \big|_{g} .$$

4. *Holomorphicity*

$$\langle \alpha \rangle \big|_{\overline{J}+\delta\overline{J}} = \langle \alpha \rangle \big|_{\overline{J}} . \tag{1}$$

Transformation law

Conjecture 1. *There exist $\mathcal{B} \in \text{Hom}(\mathcal{E}^*, \mathcal{E}^*)$ such that $T = \mathcal{B}J \in \Gamma(\mathcal{E}^*)$ does not depend on the complex structure parameters and $T^{-1}(0) = B$. Then*

$$\langle \mathcal{O}_\alpha \mathcal{O}_\beta \mathcal{O}_\gamma \rangle = \sum_n \left(\frac{\mathbf{v}}{4}\right)^n \int_{\boldsymbol{Y}} \Omega_{\boldsymbol{Y}} \wedge \left(\mathcal{M}(\mathcal{B})_n \lrcorner \left(e^{-\frac{\mathbf{v}}{4}\|T\|^2} \left(\overline{\partial T}^A \partial_A \right)^n \mathcal{O}_\alpha \mathcal{O}_\beta \mathcal{O}_\gamma \right) \right) ,$$

where

$$\mathcal{M}(\mathcal{B})_n := \omega_{\mathcal{E}} \lrcorner \underbrace{\mathcal{B}^t \wedge \cdots \wedge \mathcal{B}^t}_{n\text{-times}} .$$

This form allows to compute the dependence of the correlators from the parameters of J explicitly.

Conclusions

- We find perfect agreement with other GLSM phases.
- We obtain a non-trivial prediction for correlators involving twisted fields (from the LG point of view).
- Instanton corrections vanish in all the examples we have checked, suggesting that at least in a large class of theories $B/2$ correlators are classical. Can we find examples where this does not hold?