Operator dimensions from moduli

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Introduction

- Conformal field theories are characterized by the scaling dimensions and the three-point functions of their operators. Generically, it is hard to compute them analytically since the theories are strongly coupled.
- Nonetheless, we can write down a conformally invariant effective field theory [Hellerman et al. (2015)] which is weakly coupled in the limit where the charge density is larger than the infrared scale.

Conformally invariant effective field theory at large charge

- In radial quantization when the total charge is large, the lowest state with a given charge is always automatically in the regime where the charge density is large compared to the infrared scale.
- We then choose the Wilsonian cutoff Λ to be much larger than the infrared energy scale but much smaller than the inverse mean distance between charges $\rho^{-1/2}$.
- In this limit both higher derivative operators in the effective action, and quantum loop corrections to observables, are both suppressed by powers of the density in the denominator.
- Thus, operator dimensions and other properties of low-lying states of large charge J, are calculable perturbatively in J^{-1}.

Large charge with a vacuum manifold

- Superconformal field theories with an infinite chiral ring have flat directions and their J-scaling is entirely different from theories that do not have flat directions.
- For 3d $\mathcal{N}=2$ theories with a non-nilpotent chiral ring there is at least one state satisfying the BPS condition $\Delta_J = J_{\mathrm{R}}$ for arbitrarily $\mathrm{U}(1)_{\mathrm{R}}$-charge J_{R}.
- The dimension Δ_J of the BPS operators receives no quantum corrections, so the scaling of the lowest state is kind of boring.
- However we can still ask about things like the dimensions of near-BPS states such as the second- and third-lowest operators of a given large $\mathrm{U}(1)_{\mathrm{R}}$-charge J_{R}.

The superconformal XYZ model

- We compute the dimensions of near-BPS states with large J_{R} in the 3d $\mathcal{N}=2$ superconformal fixed point obtained by starting with three free chiral superfields X, Y, and Z, and giving them a superpotential $W = XYZ$. [Hellerman et al. (2017)].
- This model has a vacuum manifold consisting of three branches connected at the origin, where each one has a vacuum expectation value for one of the three chiral superfields.
- There is an obvious S_3 symmetry permuting the branches. So, without losing any generality we will study the X-branch, whose coordinate ring consists of all the operators of the form X^J.

Superconformal effective field theory

- Integrating out the heavy states, we get a simple description in terms of an effective action on the X-branch of moduli space.
- The X-branch is described at leading order in derivatives by the unique conformally invariant Kähler potential $K = c_K \left|\bar{X}X\right|^{3/4}$, where c_K is a coefficient that we do not know how to calculate without knowing the full form of the Wilsonian action at the fixed point.
- K is trivial with respect to a new variable $\Phi := c_K^{1/2} X^{3/4}$ and its conjugate. That is, at leading order this effective description predicts that the physics of the X-branch is described by a free field theory.
- There are no possible conformal corrections to the form of the Kähler potential. So, any corrections to the free spectrum must come from higher-derivative D-terms, which has to be super-Weyl-invariant.
- The leading higher-derivative D-term for a chiral superfield with a nonzero scaling dimension is the Fradkin-Tseytlin-Paneitz-Riegert (FTPR) four-derivative term. On $\mathbb{S}^2 \times \mathbb{R}$ with radius r, it is of the form

$$\mathcal{L}_{\mathrm{FTPR}} = \kappa \frac{1}{\bar{\phi}}\left[\left(\nabla^2\right)^2 - \frac{3}{2r^2}\nabla^2 + \frac{4}{r^2}\partial_t^2 - \frac{9}{16r^4}\right]\frac{1}{\phi} + (\text{fermions}).$$

Energy shift in large-J perturbation theory

- There is an super-algebraic explanation for the fact that the lowest and the second-lowest states with charge J do not receive quantum corrections. So let us look for the correction to the third-lowest state with large $\mathrm{U}(1)_{\mathrm{R}}$ charge J.
- we evaluate the matrix element of the FTPR term in the state

$$|2; J+2\rangle := (2(J+2)!)^{-1/2}\left(a^\dagger_{\bar{\phi}}\right)^2\left(a^\dagger_{\phi}\right)^{J+2}|0\rangle\,,$$

  which we approximate by $2^{-1/2}\left(a^\dagger_{\bar{\phi}}\right)^2 |[J+2]\rangle$ where $|[J+2]\rangle$ is a coherent state whose average occupation number is $J+2$, and evaluate the latter matrix element by perturbation theory in a nontrivial vacuum expectation value of order J.
- By an explicit calculation, one can show that the third-lowest scalar primary operator with large charge J has its dimension

$$\Delta = J + 2 - \frac{192\pi^2\kappa}{J^3} + O(J^{-4}).$$

Sign constraint

- The coefficient κ is a "non-universal" coefficient in the effective action, that we do not know how to compute.
- However we can bound the FTPR coefficient below at zero by a non-bootstrap arguement due to [Adams et al. (2006)].
- If we examine the purely bosonic component of the FTPR term in flat space, we find that it is equal to $\mathcal{L}^{\text{flat, bosonic}}_{\mathrm{FTPR}} = 4\kappa|\partial\phi|^4/|\phi|^6$.
- [Adams et al. (2006)] have pointed out that κ must always be positive in a consistent effective field theory. A negative κ would lead to violations of unitarity in low-energy moduli scattering, as well as superluminal propagation in backgrounds where a scalar gradient gets an expectation value.
- So the energy shift of the dimension of the third-lowest scalar primary operator is negative definite by some consistency condition that is entirely obscure from the point of view of the underlying CFT.

Three-point functions

- Physics at large charge for superconformal field theories with moduli space is governed by a free field theory with corrections suppressed by charge density.
- The chiral-chiral-antichiral three-point function $\left\langle \phi^{J-j}(\infty)\phi^j(1)\bar{\phi}^J(0)\right\rangle$ depends on flat space only on the two-point function $\left\langle \phi^J(1)\bar{\phi}^J(0)\right\rangle$, which, in the case of theories with eight or more supercharges in four dimensions, is exactly calculable by supersymmetric localization [Gerchkovitz et al. (2017)].
- In free field theory, we have $\left\langle \phi^J(1)\bar{\phi}^J(0)\right\rangle = (J+1)!$.
- However, for 4d $\mathcal{N}=4$ super-Yang-Mills with gauge group SU(2), exact localization computation gives $(J+1)!$, which is bigger than the free theory value by a factor of J.
- This mismatch is compensated by a contribution from the Wess-Zumino action S_{WZ} for the conformal anomaly [Komargodski and Schwimmer (2011)]. At large charge, ϕ has a vacuum expectation value (vev) of order J, so the conformal dilaton $\tau := \log\phi + \text{const.}$ has the vev $\langle\tau\rangle = \log J + O(1)$. Among others S_{WZ} has a term $S_{\mathrm{WZ}} \supset (a_{\mathrm{UV}} - a_{\mathrm{IR}})\int(\mathrm{Euler}_4)\tau$, which, in the case of 4d $\mathcal{N}=4$ SU(2) SYM, $S_{\mathrm{WZ}} = -\log J + O(1)$. The Euclidean path integral gets a multiplicative contribution of $\exp(-S_{\mathrm{WZ}}) = J$, which gives the factor we need to make up the difference above.
- We believe this kind of calculation works for any 4d superconformal field theories with one-dimensional Coulomb branch.

References

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