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Operator bases: understanding the maze of effective field theory

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Experiments at a given energy scale explore a little portion of the 'maze' of our universe

The little girl in the picture is 'playing with pions' at energies of a few GeV

Her father is exploring the quarks, gluons, etc. of the Standard Model at LHC energies of a few TeV

Each sees their own part of the maze, near to where they are standing, and can describe it using an effective field theory with a local Lagrangian

$$\mathcal{L} = \sum_i c_i \mathcal{O}_i \qquad ✺$$

What can we understand about the structure of this maze in general?

S-matrix kinematics

We can define an operator basis as the set of all operators in equation ✺ that give rise to independent physical effects

$$\mathcal{K} = \{\mathcal{O}_i\}$$

			Equivalences between operators
S-matrix elements →	**On-shell**	$\sim$	**Equation of motion**
S-matrix elements →	**Momentum conservation**	$\sim$	**Integration by parts**

A major result of our work is to treat these equivalences systematically

Example: EFT of a real scalar field using conformal representation theory

Consider the following conformal irrepr. as a 'building block' of the Lagrangian:

$$R_{[(d-2)/2,0]} = \begin{pmatrix} \phi \\ \partial_\mu \phi \\ \partial_{\{\mu_1}\partial_{\mu_2\}}\phi \\ \vdots \end{pmatrix}$$

Scaling dimension Δ — Spin l

Equation of motion is 'built in' via the traceless condition (= { . . . })

Build all possible operators with n phi fields:

Tensor decomposition back into conformal irreps.

$$\mathrm{Sym}^n R_{[(d-2)/2,0]} = \sum_{\Delta,l} c_{\Delta,l} R_{[\Delta,l]}$$

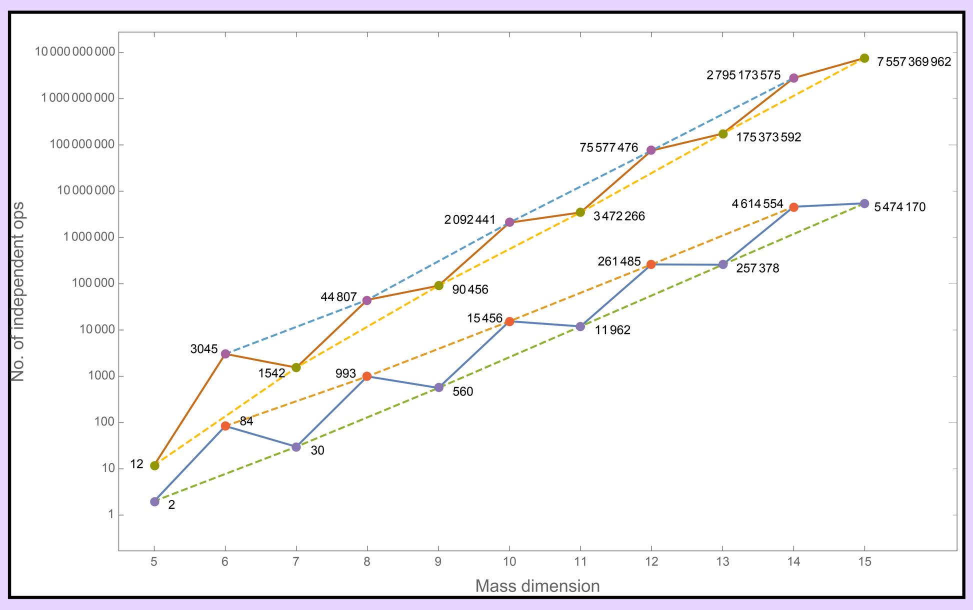
Lowering operator on irrep. is a derivative, meaning descendent (non-highest weight) operators are total derivatives:

Integration by parts is treated by keeping only primary (highest weight) operators

Counting operators in the Standard Model EFT

Hilbert series $\quad H = \mathrm{Tr}_{\mathcal{K}}\, \omega = \sum_{\mathcal{O}\in\mathcal{K}} \omega(\mathcal{O})$

A 'partition function for the S-matrix'—sum with some weighting function ω over all independent operators in the basis



For example, in the SM EFT, weighting by particle content, and conclusively solving a long-running problem of determining higher dimension operators in this theory

Can we develop an understanding of the all-order structure of the operator basis 'maze'?

Non-linearly realized symmetry group

In the maze above, the little girl and her father describe QCD using different degrees of freedom: he uses quarks and gluons, whereas she uses pions and the QCD chiral Lagrangian built from the matrix (and its derivatives)

$$\xi(x) = e^{i\pi^i(x)X^i/f_\pi}$$

pions

We can also treat non-linearly realized symmetry groups, working with the Maurer-Cartan forms

$$w_\mu \equiv \xi^{-1}\partial_\mu \xi = u^i_\mu X^i + v^a_\mu T^a = u_\mu + v_\mu,$$

X = Broken, T = Unbroken generators

Building block:

$$R_u = \begin{pmatrix} u_\mu \\ D_{\{\mu_1}u_{\mu_2\}} \\ D_{\{\mu_1}D_{\mu_2}u_{\mu_3\}} \\ \vdots \end{pmatrix}$$

Defining $D_\mu = \partial_\mu + v_\mu$

EOM is built in again

But, **not a conformal irrep.** However, can still treat IBP systematically, and similarly (appeal to Hodge theory)

Can we understand how to relate the girl's and her father's operator bases?

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