Complex dynamics on derived categories of K3 surfaces

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Complex dynamics on algebraic varieties

X: algebraic variety, $f : X \to X$: endomorphism
$\rightsquigarrow$ We can regard a pair (X, f) as a complex dynamical system on X.
Study behavior of iteration of f.
e.g. periodic points, topological entropy $h_{\text{top}}(f)$ of complex dynamical system etc
From the point of view of complex dynamics, we can measure complexity of symmetry of algebraic varieties.

Complex dynamics on derived categories

X: algebraic variety
$D^b(\text{Coh}X)$: derived category of coherent sheaves on X
$\Phi : D^b(\text{Coh}X) \to D^b(\text{Coh}X)$: endofunctor
$\rightsquigarrow$ $(D^b(\text{Coh}X), \Phi)$: categorical dynamical system ("complex dynamical system on $D^b(\text{Coh}X)$")
e.g. periodic orbit?, categorical entropy $h_{\text{cat}}(\Phi)$ of categorical dynamical system
(Dimitrov-Haiden-Kazarkov-Kontsevich 2014) etc

Symmetry of algebraic varieties

The automorphism group $\text{Aut}(X)$ indicates symmetry of X.

$$\text{Aut}(X) \subset \underset{\substack{\upuparrows \\ \text{more symmetry of } X}}{\text{Aut}(D^b(\text{Coh}X))}$$

Question

Are there geometric meaning of elements in $\text{Aut}(D^b(\text{Coh}(X))$?

Complex dynamics on K3 surfaces

McMullen and Oguiso et al constructed examples of K3 surfaces with automorphisms of positive topological entropy.
However,
X: K3 surface of Picard number one
$\Rightarrow \text{Aut}(X) = 1$ or $\mathbb{Z}_2$
$\rightsquigarrow$ X has only automorphisms of null topological entropy.

$\exists \Phi \in \text{Aut}(D^b(\text{Coh}(X)))$ such that $h_{\text{cat}}(\Phi) > 0$
$\rightsquigarrow$ The K3 surface X has a "wild" symmetry at the level of derived category.
$\rightsquigarrow$ Study the meaning of such autoequvalences.

Complex dynamics on moduli spaces

Theorem

We can give a categorical dynamical system $(D^b(\text{Coh}(X)), \Phi)$ on a K3 surface X such that the following properties hold.

- $\exists \sigma$: Bridgeland stability condition on $D^b(\text{Coh}(X))$, $\exists v \in H^*(X, \mathbb{Z})$ s.t. Φ induces an automorphism $\Phi_{\sigma,v} : M_\sigma(v) \to M_\sigma(v)$.
  $M_\sigma(v)$: moduli space of σ-stable objects with Mukai vector v.
- $\dim M_\sigma(v) \cdot h_{\text{cat}}(\Phi) \geq 2h_{\text{top}}(\Phi_{\sigma,v}) > 0$

The moduli space $M_\sigma(v)$ is an example of a projective hyperKähler manifold. This theorem gives new examples of projective hyperKähler manifolds with automorphisms of positive topological entropy. Moreover, symmetry of derived categories sometimes induces symmetry of moduli spaces.