Status and Progress on Resummed Calculations in Soft-Collinear Effective Theory

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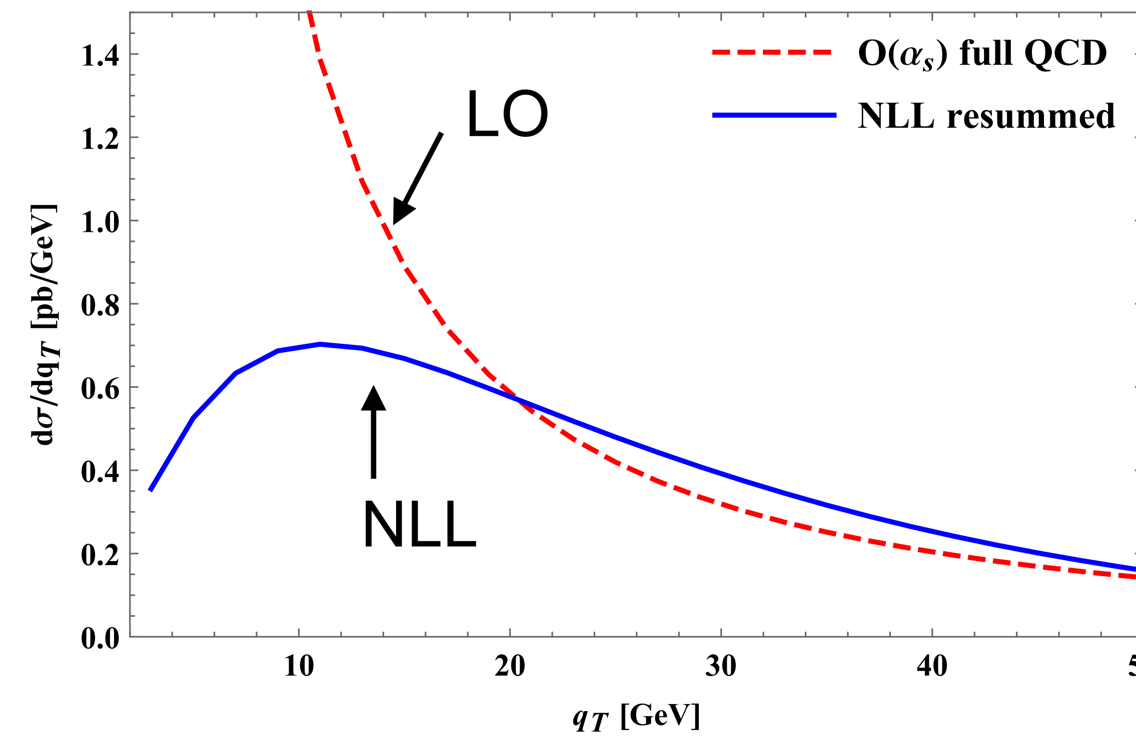
Introduction

- **Perturbative calculation at high energy collider often involve large ratio of scales:**
    - pT of W/Z/H O(GeV - 10 GeV) v.s. $M_W/M_Z/M_H$ O(100 GeV)
    - Jet veto scales O(10 GeV) v.s. pT of jet O(0.1-1 TeV)
    - jet mass O(10 GeV) v.s. pT of jet O(0.1 - 1 TeV)
- **The large ratios can potentially lead to large logarithms, which invalidate, or at least reduce the applicability of naive perturbation theory**
- **Physically, the large logarithms come from incomplete cancellation of IR divergence. They can be systematically resummed to all orders (QCD resummation, or RG improved perturbation theory)**
- **A successful program of LHC at precision frontier calls for development in resummation methods**



Higgs pT spectrum in naive perturbation theory and RG improved perturbation theory

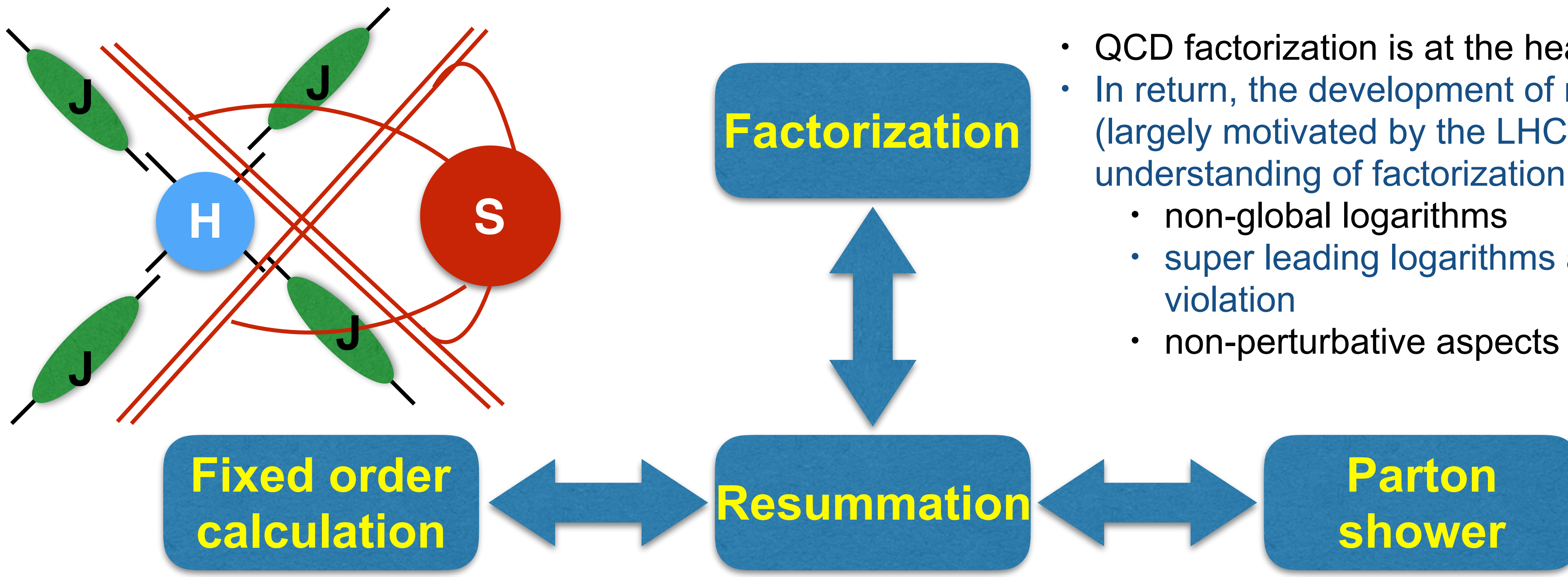
$$
\begin{aligned}
\log\sigma = &+\alpha_s(\log^2\tau + \log\tau + 1 + \dots) \\
&+\alpha_s^2(\log^3\tau + \log^2\tau + \log\tau + 1 + \dots) \\
&+\alpha_s^3(\log^4\tau + \log^3\tau + \log^2\tau + \log^1\tau + \dots) \\
&+\dots
\end{aligned}
$$

LL NLL NNLL N³LL

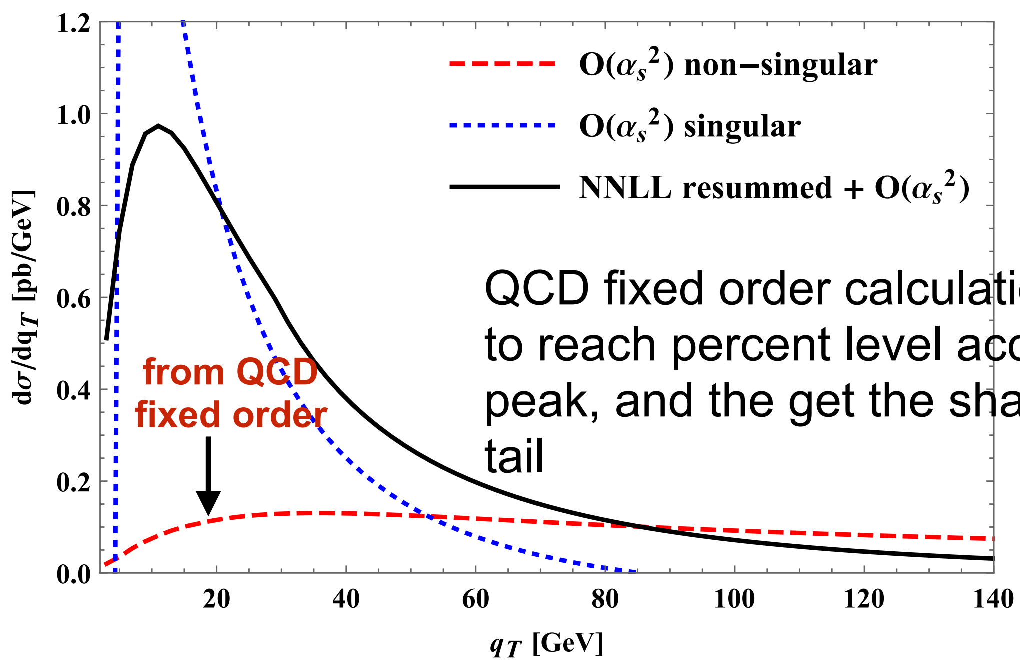
τ: large ratio of scales $\qquad \log\tau \gg 1$

Interplay of resummation and other pQCD subfields

- **QCD resummation has close connection with other subfields of pQCD**

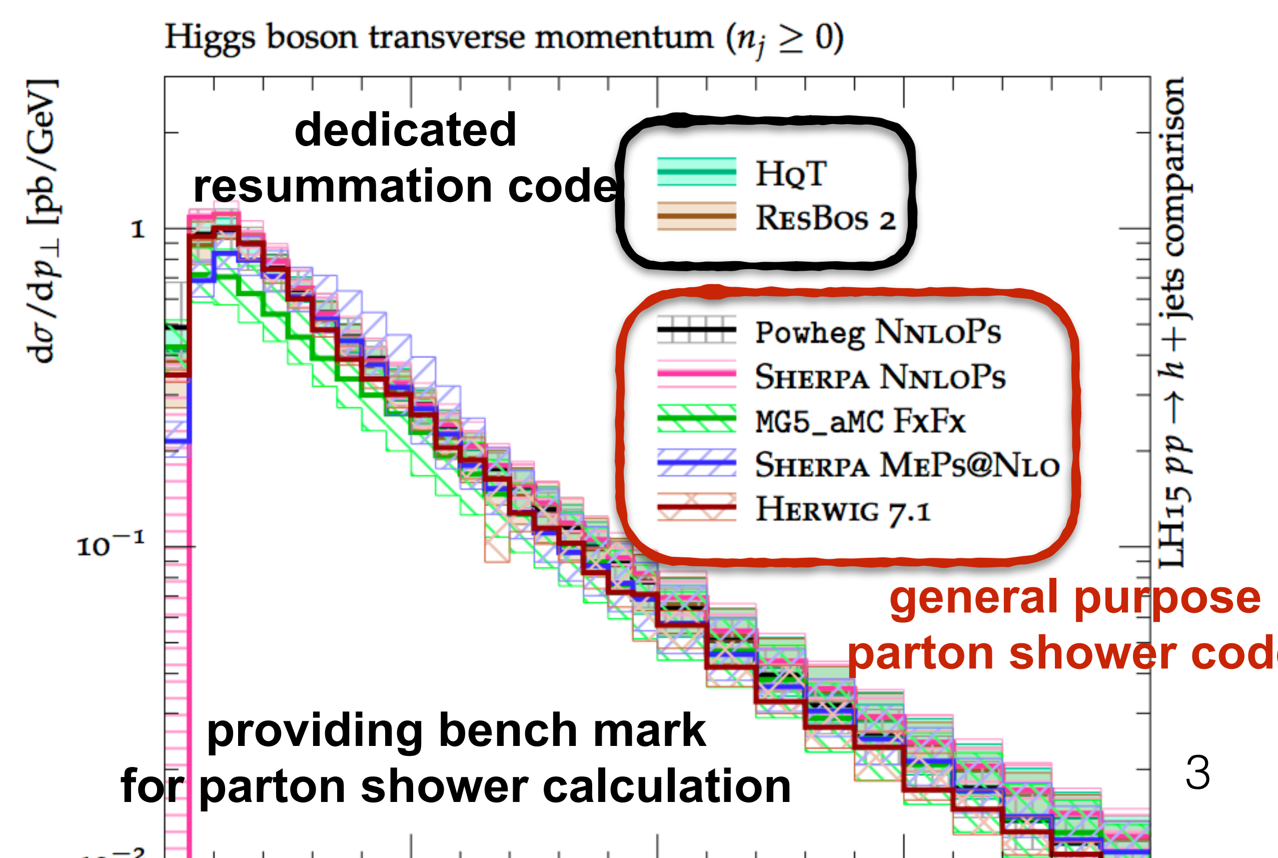


- QCD factorization is at the heart of resummation.
- In return, the development of resummation (largely motivated by the LHC program) push the understanding of factorization to a new level
    - non-global logarithms
    - super leading logarithms and factorization violation
    - non-perturbative aspects of factorization



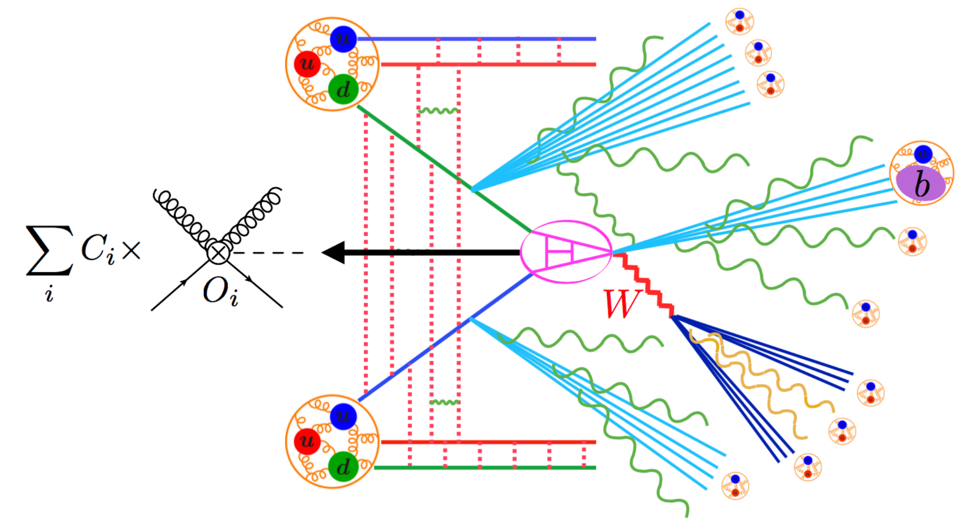
QCD fixed order calculation is needed to reach percent level accuracy at the peak, and the get the shape right at the tail

- **Resummation inspired infrared regularization method for fixed order calculation: qT subtraction, N-jettiness subtraction, …**



Continuous progress in resummation methods

- **Soft-Collinear Effective Theory(SCET) has evolved into a powerful framework for QCD resummation**

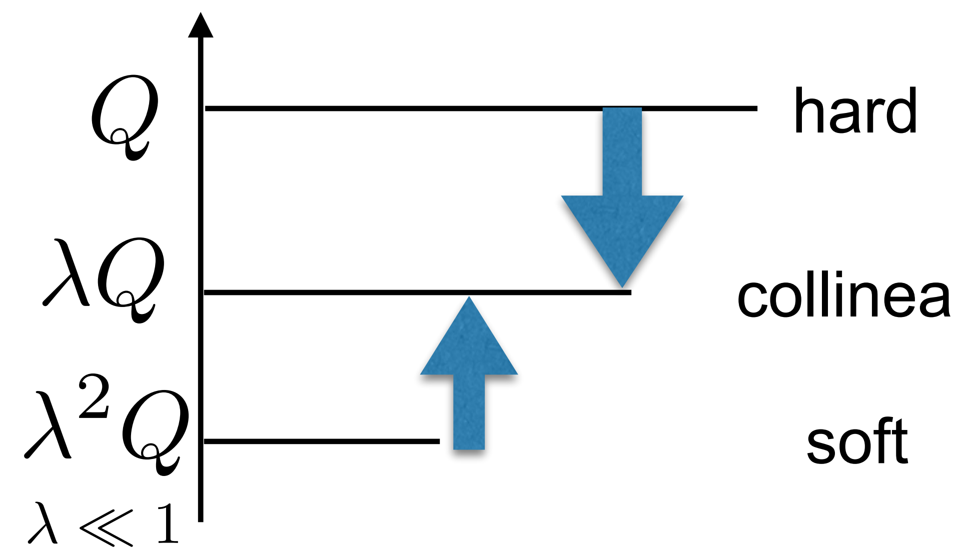


courtesy of I. Moult

Factorized Lagrangian

$$\mathcal{L}_{\mathrm{SCET}} = \underset{\substack{\uparrow \\ \text{soft part}}}{\mathcal{L}_S(\psi_S, A_S)} + \sum_{n_i} \underset{\substack{\uparrow \\ \text{collinear part}}}{\mathcal{L}_{n_i}(\xi_{n_i}, A_{n_i})}$$

$$\frac{d\sigma}{d\tau} = H B_1 \otimes B_2 \otimes J_1 \otimes \ldots \otimes J_n \otimes S$$



$$\mu \frac{dH}{d\mu} = \gamma_H H \qquad \mu \frac{dB_i}{d\mu} = \gamma_{B_i} B_i$$

$$\mu \frac{dJ_i}{d\mu} = \gamma_{J_i} J_i \qquad \mu \frac{dS}{d\mu} = \gamma_S S$$

double logarithmic evolution, e.g.,

$$\gamma_H = \Gamma_{\mathrm{cusp}} \ln \frac{Q}{\mu} + \gamma_h$$

$$\gamma_J + \sum_i \gamma_{B_i} + \sum_i \gamma_{J_i} + \gamma_S = 0$$

consistency condition

- **Resummation achieved by solving (linear) RG equation**

Resummation of Next-to-Leading Power Logarithms

$$\frac{d\sigma}{d\tau} = \boxed{\sum_{n=0}^{\infty} \left(\frac{\alpha_s}{\pi}\right)^n \left[c_n^{(0)}\, \delta(\tau) + \sum_{m=0}^{2n-1} c_{nm}^{(0)} \left(\frac{\log^m \tau}{\tau}\right)_+ \right] \quad \text{Leading Power (LP)}}$$

$$\boxed{+ \sum_{n=1}^{\infty} \left(\frac{\alpha_s}{\pi}\right)^n \sum_{m=0}^{2n-1} c_{nm}^{(2)} \log^m \tau \quad \text{Next to Leading Power (NLP)} \\ + \sum_{n=1}^{\infty} \left(\frac{\alpha_s}{\pi}\right)^n \sum_{m=0}^{2n-1} c_{nm}^{(4)}\, \tau \log^m \tau}$$

$$+ \cdots$$

- High precision phenomenology
- Improve fixed order Calculations (phase space slicing)
- Processes where NLP is crucial (e.g. B physics)

$$\mathcal{L}_{\rm SCET} = \sum_{i\geq 0} \mathcal{L}_{\rm hard}^{(i)} + \sum_{i \geq 0} \mathcal{L}_{\rm dyn}^{(i)}$$

Can be studied by systematic expansion of SCET Lagrangian in terms of power counting parameter λ

- Full operator basis for Drell-Yan/ Higgs worked out to O(λ²)
  - 1 at LP; 13 at NLP
  - Number of operator reduced drastically for Leading Log

- Universal subleading power Lagrangian
- Systematic corrections to soft and collinear radiation

Moult, Stewart, Tackmann, Waalewijn
Feige, Kolodrubetz, Moult, Stewart
Moult, Vita, Stewart

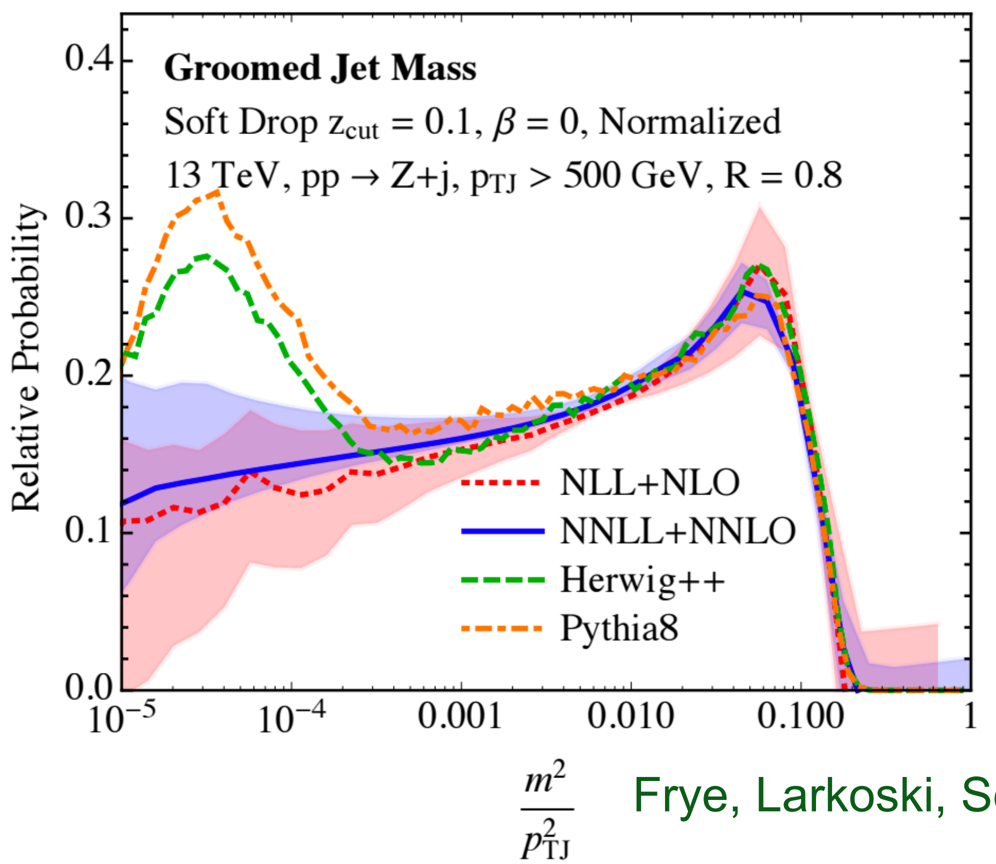
- **Factorization and resummation of NLP Leading Logarithms possible; extra convolution in hard function** [Moult, Stewart, Vita, HXZ]

$$\frac{d\sigma^{(n)}}{d\tau} = \sum_j H_j^{(n_{Hj})} \otimes J_j^{(n_{Jj})} \otimes S_j^{(n_{Sj})}$$

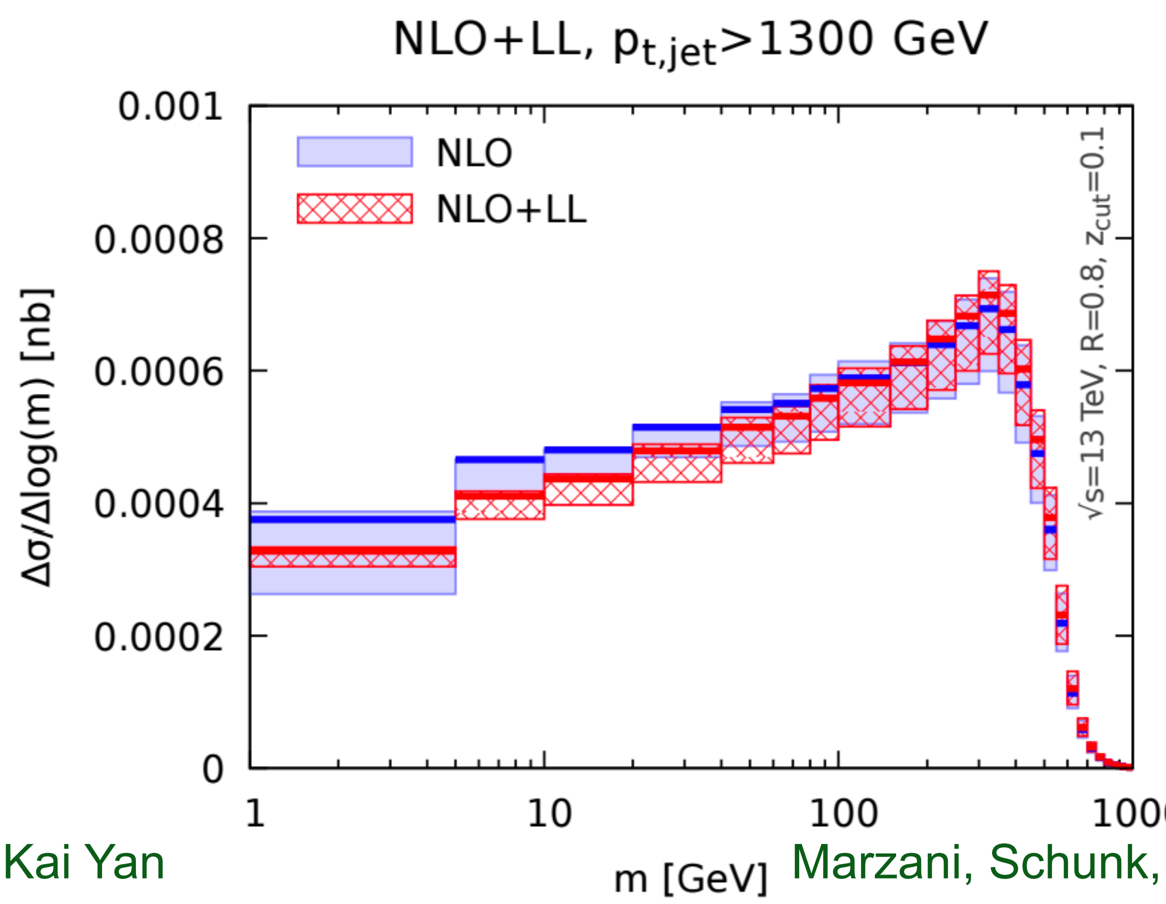
- **Confirmed/checked by explicit N3LO calculation of the NLP Leading Log**

Resumming jet structure

- Substructure of jet as a tool to discriminate jet from QCD and new physics
- e.g.: Groomed jet mass, remove soft particle contribution to jet mass by soft drop procedure
- Need precision resummed calculation to understand the systematics and assess their robustness



Frye, Larkoski, Schwartz, Kai Yan



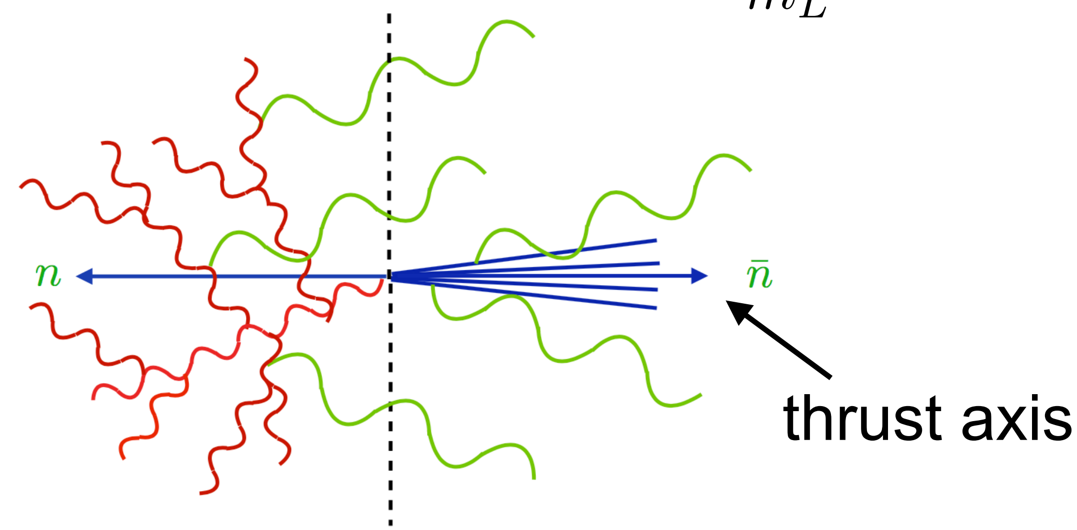
Marzani, Schunk, Soyez

- The leading log is a a single log, $\alpha_s^n \log^n m$
- Peak at larger value compared with inclusive jet mass distribution
- Relatively insensitive to hadronization/pile up effects

Non-global logarithms

- **NGLs are observable measured on jets or other restricted phase space region** [Dasgupta, Salam]
- **Example: heavy/light hemisphere jet mass distribution**
- **NGLs in hemisphere jet mass distribution obey a non-linear integro-differential equation, the BMS equation**

$$m_H \gg m_L \qquad \ln \frac{m_H}{m_L} \gg 1$$

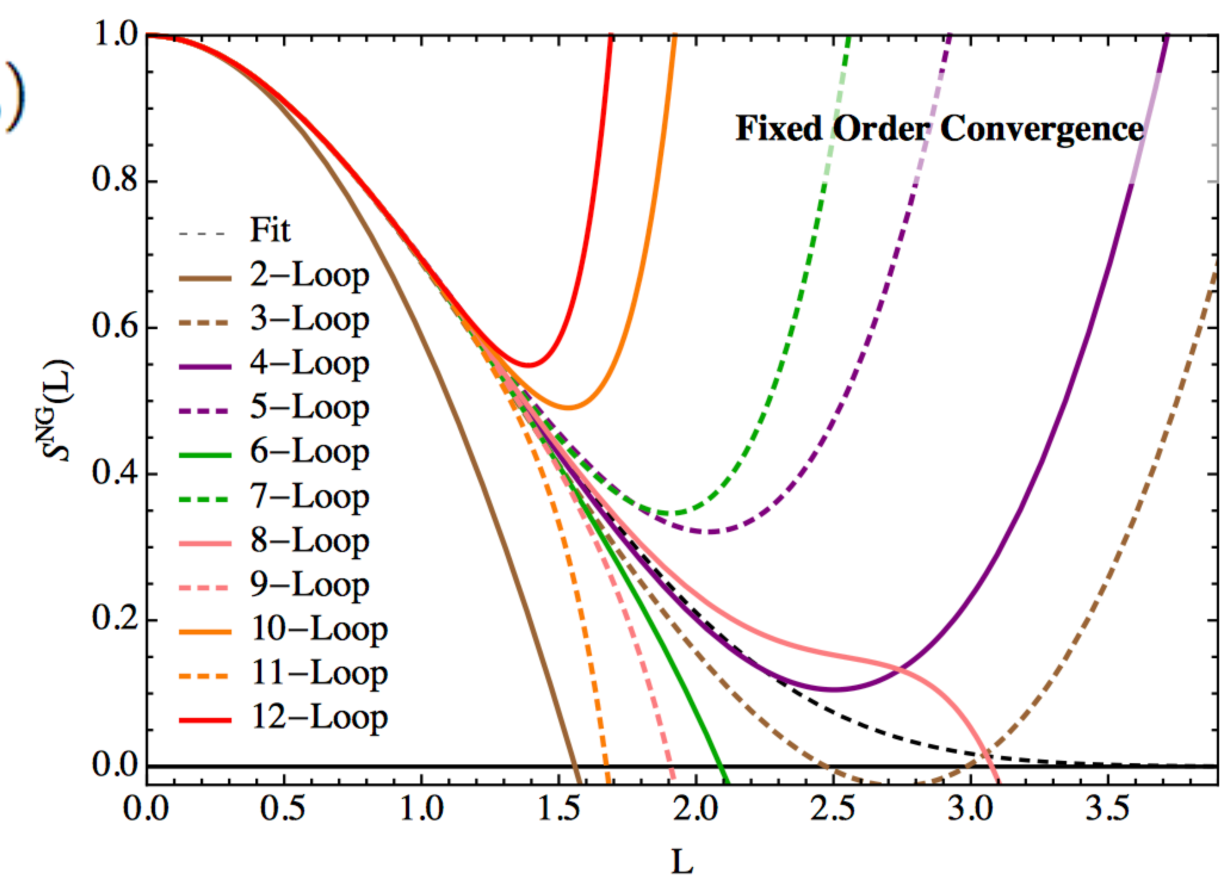


$$\partial_L g_{ab} = \int_{\text{heavy}} \frac{d\Omega_j}{4\pi} W_{ab}(j)\left(U_{abj}(L) g_{aj} g_{jb} - g_{ab}\right)$$

- **No analytic solution exist for BMS eq.**
- **One can attempt an iterative solution in the expansion of α_s** [Schwartz, HXZ; Caron-Huot]

$$g = 1 + \alpha_s g^{(1)} + \alpha_s^2 g^{(2)} + \alpha_s^3 g^{(3)} + \ldots$$

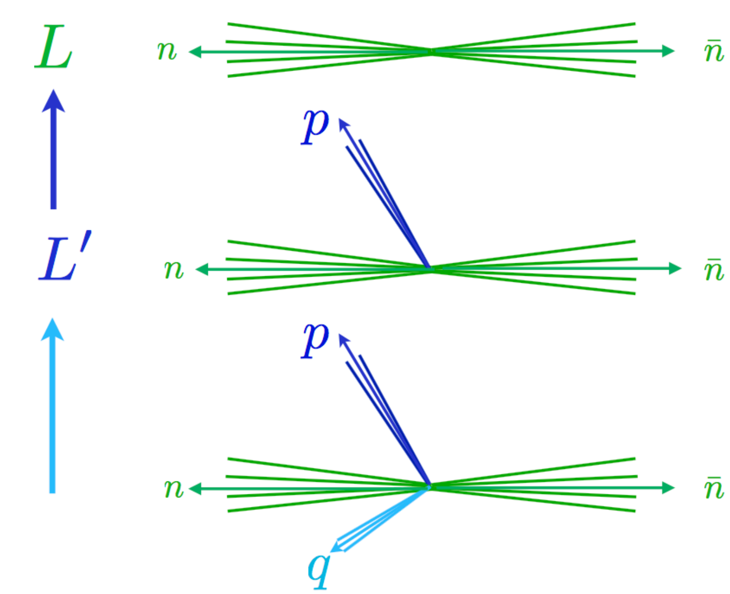
- **Gives perturbative series with finite convergence radius in $L=\alpha_s \mathrm{Log}\, m_H/m_L$**
- **Not enough in phenomenological application**



Dress gluon expansion

$$\frac{d\sigma}{dm_H dm_L} = d\sigma_{0-sj} + \int_1 d\sigma_{1-sj} + \int_2 d\sigma_{2-sj} + \ldots$$

- **Perturbative expansion in terms of sub jets (dress gluon)** [Larkoski, Moult, Neill]
- **Mathematically equivalent to successive approximation method for differential equation**
- **The first sub jet expansion already reveals the singularity of the solution**

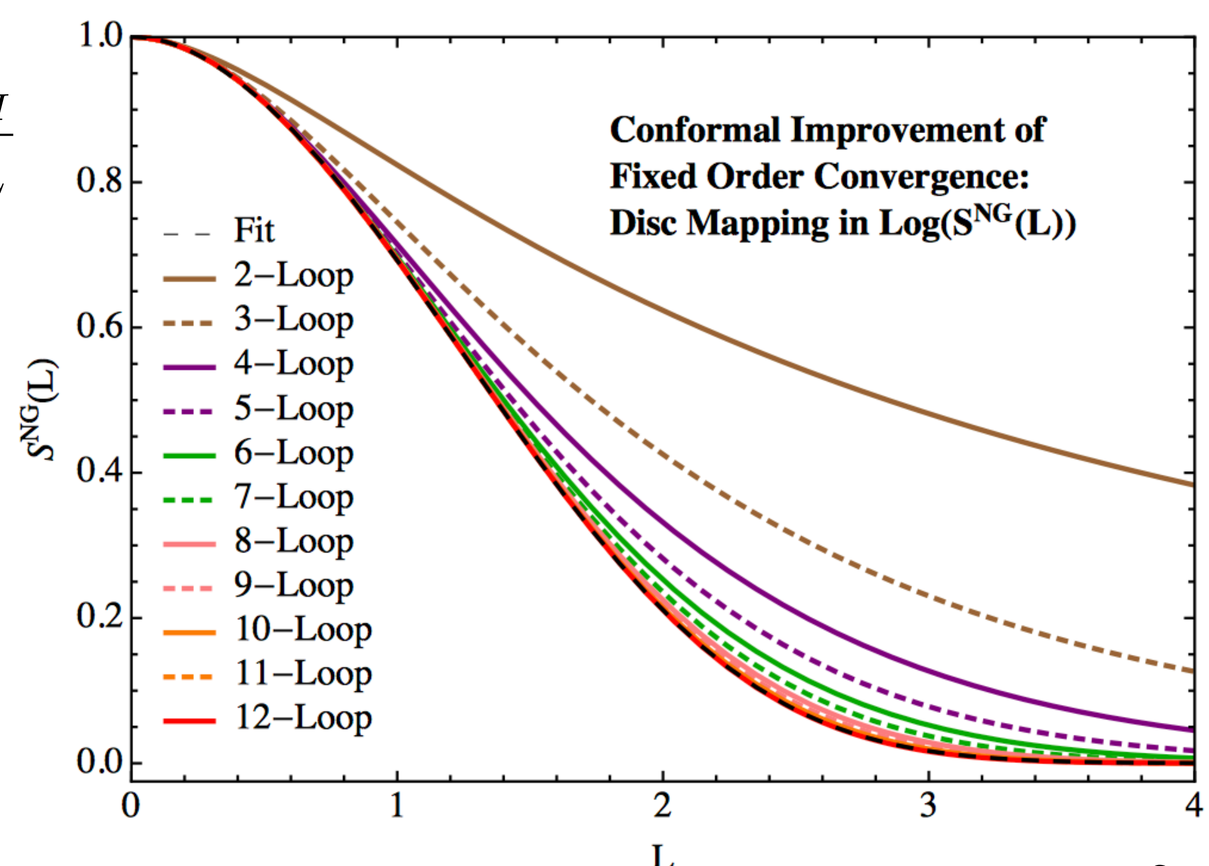


$$\Phi^{(1)} = -\frac{\gamma_E}{2} L - \frac{1}{2} \ln \Gamma(1+L) \qquad L = \frac{\alpha_s N_c}{\pi} \ln \frac{m_H}{m_L}$$

- **Inspire a change of variable to the original naive perturbation series**

$$L \to u(L) \qquad u(L) = \begin{cases} \frac{\sqrt{1+L}-1}{\sqrt{1+L}+1}, \\ \log(1+L)\,. \end{cases}$$

- **Significantly accelerate the convergent of the perturbation series**



[Larkoski, Moult, Neill]

SCET and Glauber gluon

- Glauber modes are off-shell modes with small time and longitudinal component
- Responsible many interesting phenomena: BFKL, factorization violation, super leading logarithms, etc
- Can be included into SCET as potential operator

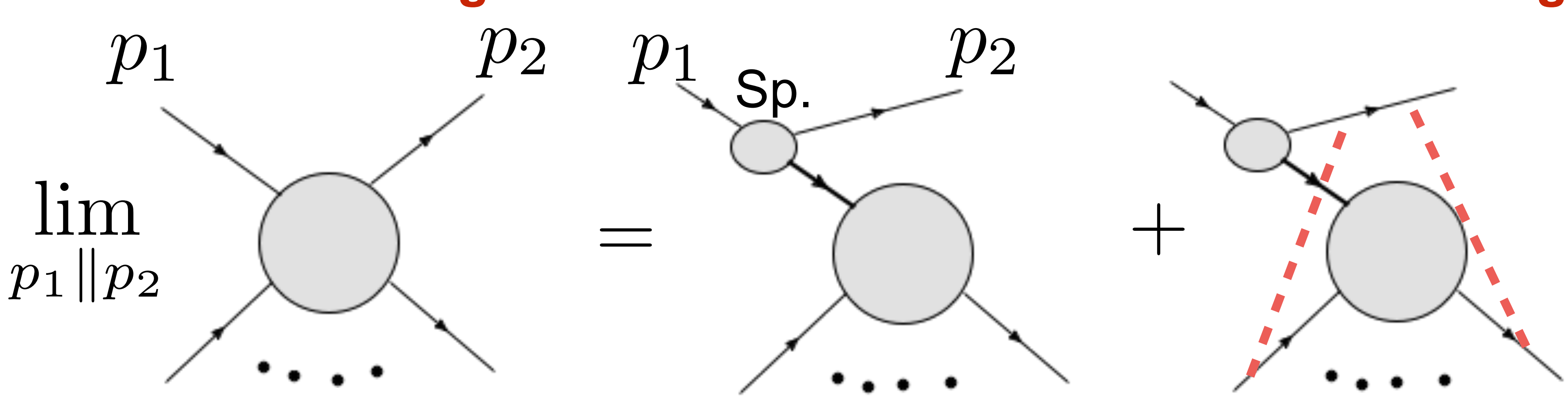
$$P_{\text{Glauber}} \sim Q(\lambda^2, \lambda^2, \lambda)$$

$$\mathcal{L}_{\text{SCET}_{\text{II}}} = \mathcal{L}_S(\psi_S, A_S) + \sum_{n_i} \mathcal{L}_{n_i}(\xi_{n_i}, A_{n_i}) + \mathcal{L}_{\text{Glauber}}$$

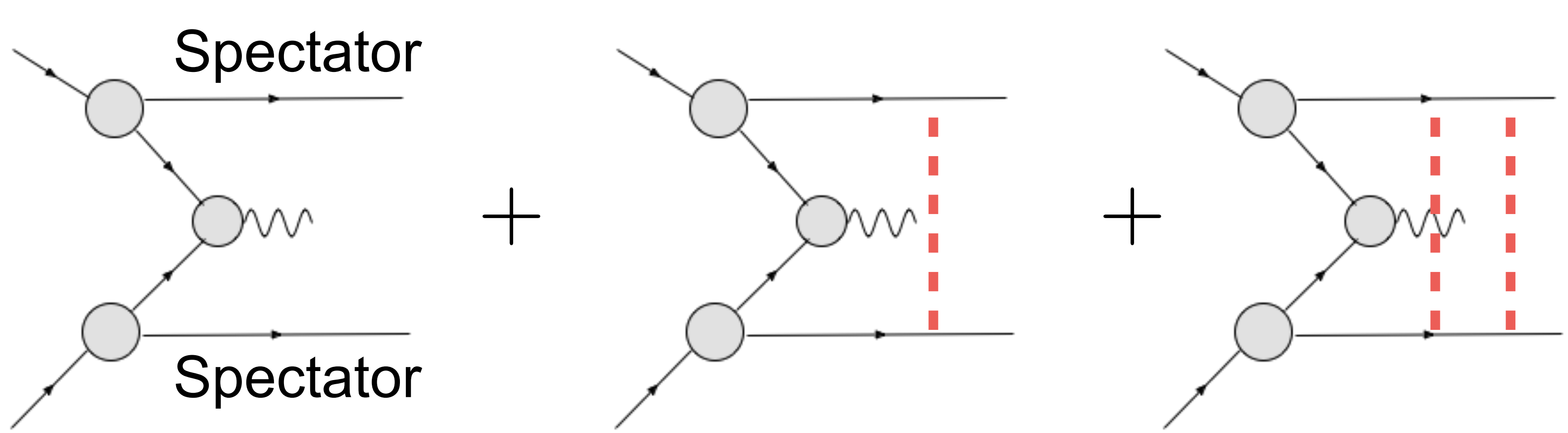
Rothstein, Stewart

$$\mathcal{L}_{\text{Glauber}} = \sum_{n,\bar{n}} \sum_{j=q,g} \mathcal{O}_n^{iB} \frac{1}{\mathcal{P}_\perp^2} \mathcal{O}_s^{BC} \frac{1}{\mathcal{P}_\perp^2} \mathcal{O}_{\bar{n}}^{jC} + \sum_{n} \sum_{i,j=q,g} \mathcal{O}_n^{iB} \frac{1}{\mathcal{P}_\perp^2} \mathcal{O}_s^{j_n B}$$

- Enable investigation of factorization violation effects using EFT



- Violation of collinear factorization at amplitude level
- First derived from structure of IR poles for two-loop amplitude [Catani, de Florian, Rodrigo]
- Direct calculated using SCET+Glauber operator, including finite terms [Schwartz, Y. Kai, HXZ]



- Re-derive CSS factorization of Drell-Yan pT distribution by direct amplitude calculation [Schwartz, Y. Kai, HXZ]
- Straightforward to extend to other observable: E_T, beam thrust, transverse thrust

Gauge boson production at small transverse momentum

- Gauge boson production at small qT enjoys a TMD factorization

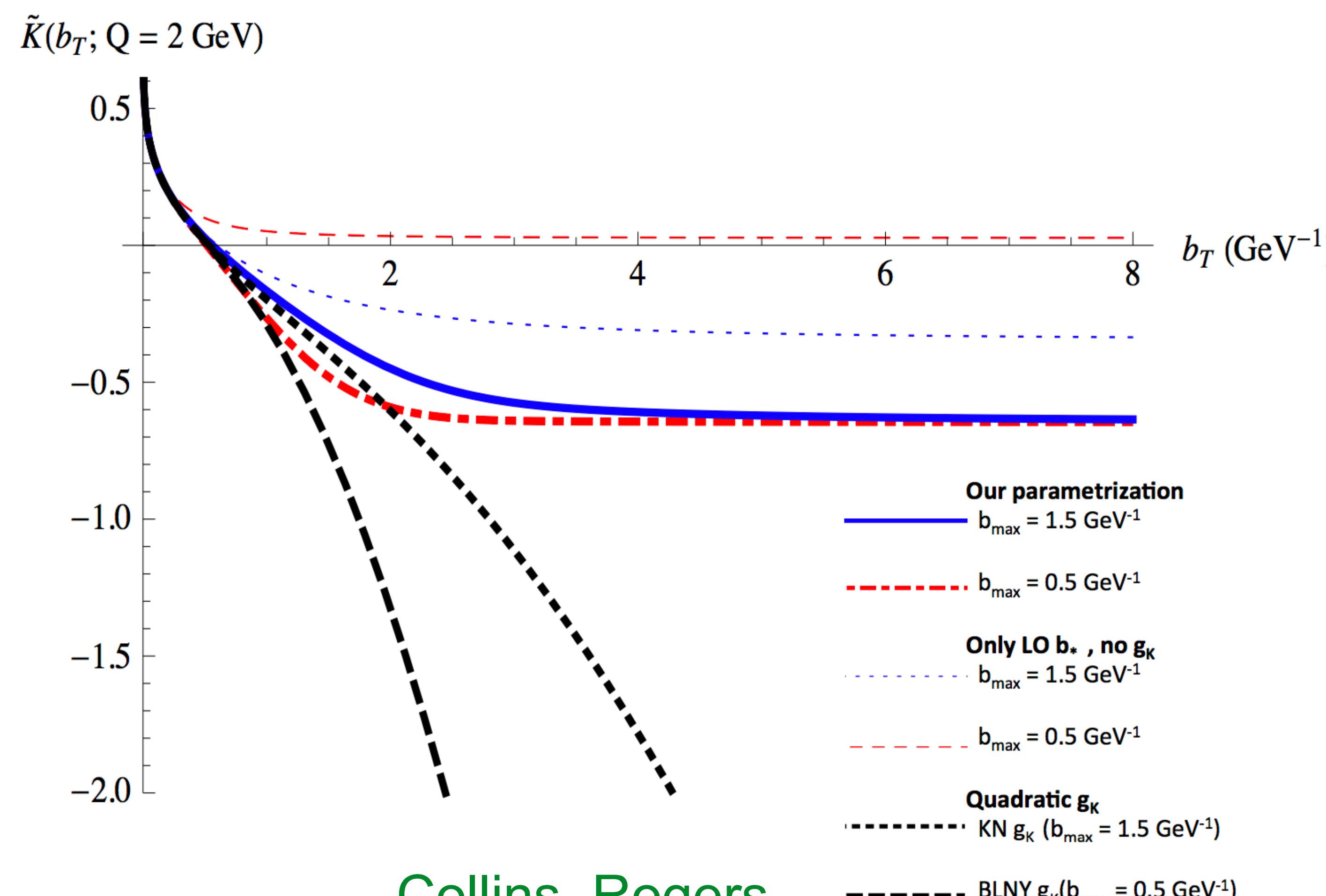
$$\frac{d\sigma}{d^4q\,d\Omega} = \frac{2}{s}\sum_j \frac{d\hat{\sigma}_{j\bar{j}}(Q,\mu_Q,\alpha_s(\mu_Q))}{d\Omega}\int d^2\boldsymbol{b}_T\, e^{i\boldsymbol{q}_T\cdot\boldsymbol{b}_T}\,\tilde{f}_{j/A}(x_A,\boldsymbol{b}_T;Q_0^2,\mu_0)\,\tilde{f}_{\bar{j}/B}(x_B,\boldsymbol{b}_T;Q_0^2,\mu_0)$$

$$\times \boxed{\left(\frac{Q^2}{Q_0^2}\right)^{\tilde{K}(b_T;\mu_0)}} \exp\left\{\int_{\mu_0}^{\mu_Q}\frac{d\mu'}{\mu'}\left[2\gamma_j(\alpha_s(\mu');1) - \ln\frac{Q^2}{(\mu')^2}\gamma_K(\alpha_s(\mu'))\right]\right\}$$

$$+\ \text{polarized terms} + \text{large } q_T \text{ correction, } Y + \text{p.s.c.}$$

Functional form fixed by soft-collinear factorization

- Lots of data from low energy Semi-Inclusive DIS process
- Important to understand the Q^2 evolution of TMD PDF: $\tilde{K}$
- From SCET viewpoint, $\tilde{K}$ originates from rapidity divergence: a result of boost invariant
- Regulated by finite Q in full theory
- Fit of $\tilde{K}$ from data depends strongly on its non-perturbative modeling

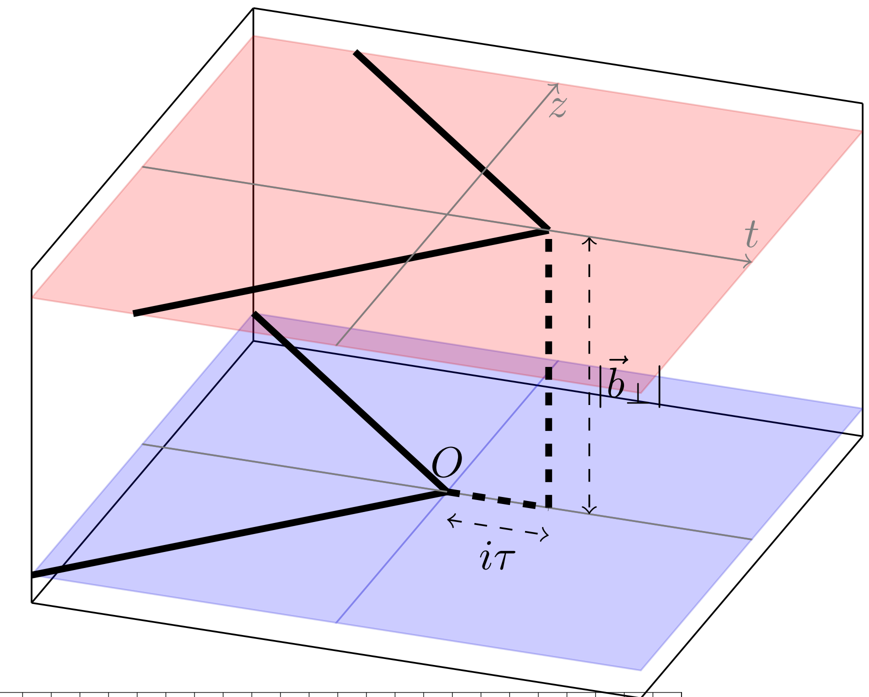


Collins, Rogers

Rapidity anomalous dimension

- **A new operator definition for rapidity anomalous dimension (related to $\tilde{K}$)** [Y. Li, Neill, HXZ]
- **Logarithmic divergence of cusp Wilson loop**

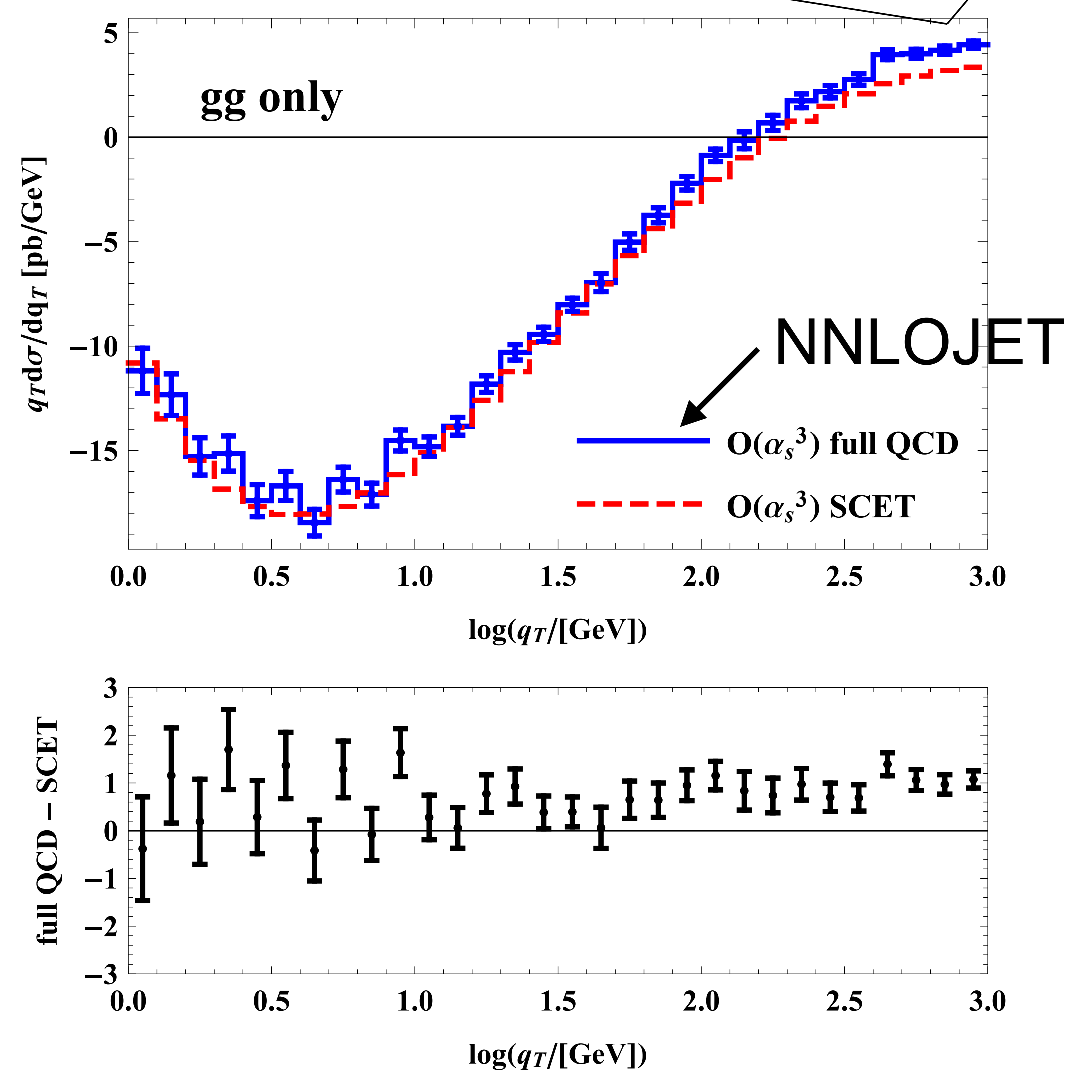
$$\lim_{\tau \to 0}$$



$$S_{\mathrm{F.D.}} = \frac{\mathrm{Tr}}{C} \langle 0 | T\{S^\dagger_{\bar{n}} S_n(0,0,0)\} \overline{T}\{S^\dagger_n S_{\bar{n}}(i\tau, i\tau, \vec{b}_\perp)|0\rangle$$

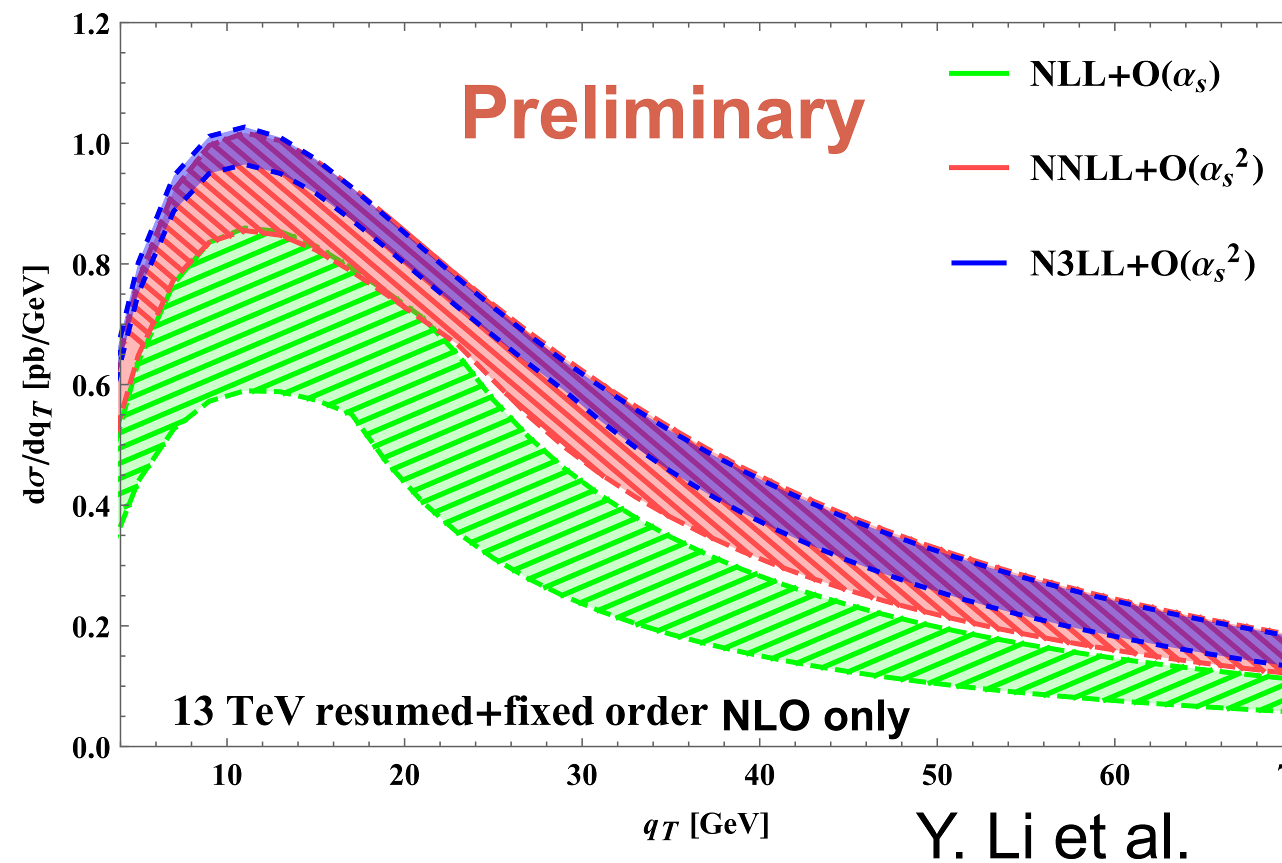
$$S_n(x) = \mathrm{P}\exp\left(i g_s \int_{-\infty}^{0} ds\, n \cdot A(x + ns)\right)$$

- **The convenient operator definition allows the perturbative calculation of rapidity anomalous dimension through to three loops** [Y. Li, HXZ]
- **Lattice calculation?**
- **Resummation formula predicts the singular q_T distribution at NNLO**
- **Highly non-trivial agreement with H+jet production at NNLO** [NNLOJET: X. Chen et al.]

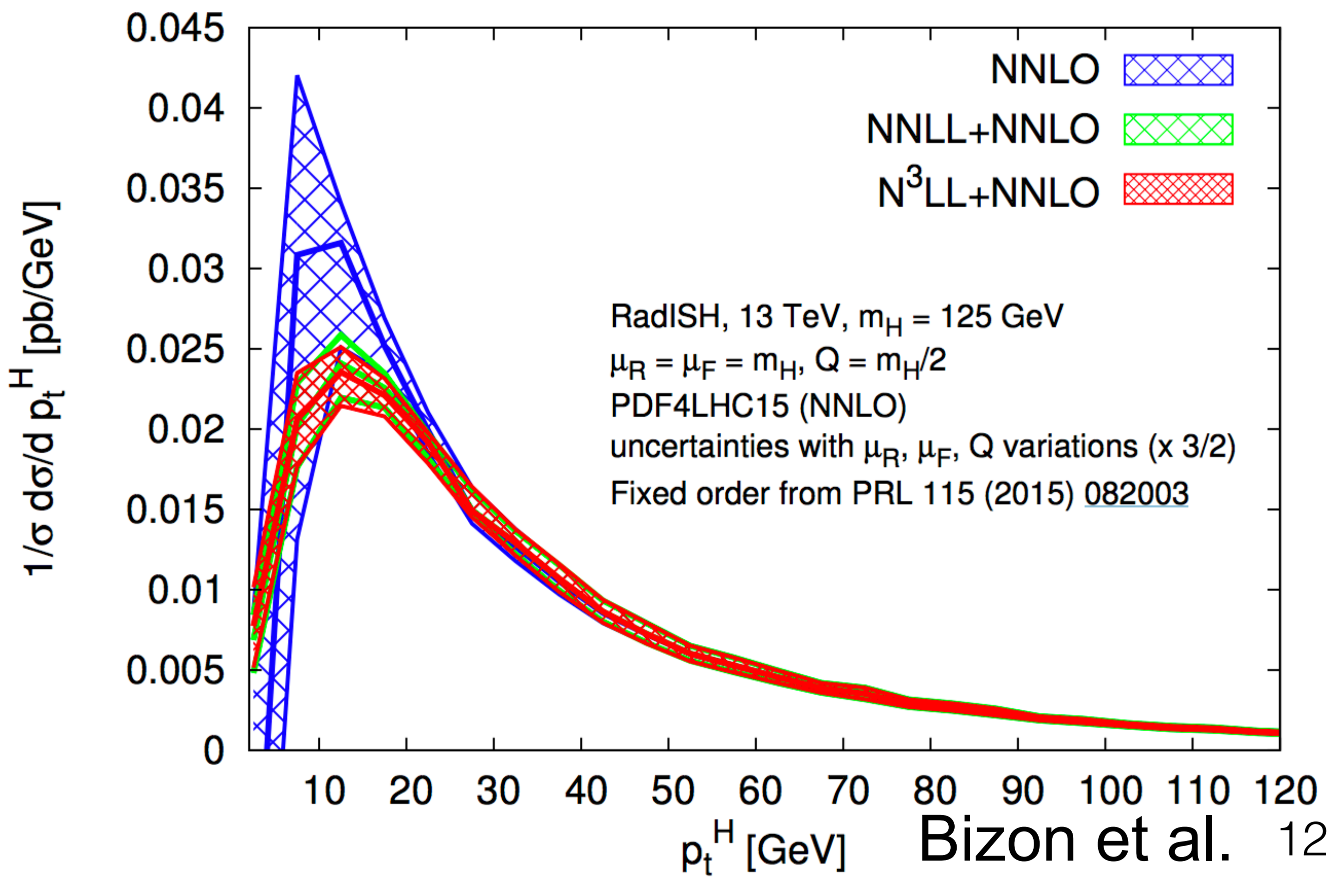


pT resummation for Higgs at N3LL

- First example of N^{3}LL resummation for differential distribution at hadron collider (with the exception of 4-loop cusp anomalous dim.)
  - N^{3}LL resummation for thrust in e+e- in about 10 years ago [Becher, Schwartz]
  - Complications at hadron collider: PDF convolution, definition of observable, anomalous dim.
- N^{3}LL resummaiton in the rapidity renormalization group formalism [Y. Li et al.]
  - Good convergence of RG improved perturbation theory
  - Smooth switch between resummed and fixed order beyond NLL
- Momentum space N^{3}LL resummation [Bizon et al.]



Y. Li et al.



Bizon et al.

Summary

- Precision measurement at LHC requires precision theoretical predictions
- Resummation is necessary to enlarge the range of validity of perturbation theory
- SCET as a powerful framework for resumming large logarithms at colliders
  - Progress towards resummation at Next-to-Leading power
  - Jet structure observable
  - Observable with non-linear evolution equation
  - Galuber operator in SCET
  - Resummation at N^{3}LL accuracy at hadron collider
  - ……
- Expect continuous development in the coming years