Fixed-order predictions for precision observables

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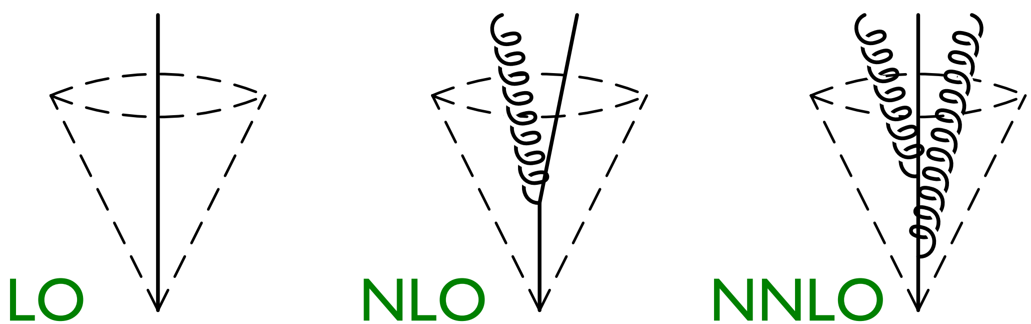
Higher order corrections

- Fixed order: expand in powers of coupling constants
  - One more order = one extra particle or one more loop
- Leading order
  - Capture basic features of scattering process
  - Usually input to parton shower (multiple radiation)
- Higher orders
  - Increasingly accurate in shape and normalization
  - Describe more and more details of final state



NLO multi-particle production

- Why NLO?
  - reduce scale uncertainty of LO theory prediction
  - reliable normalization and shape
  - accounts for effects of extra radiation
  - jet algorithm dependence

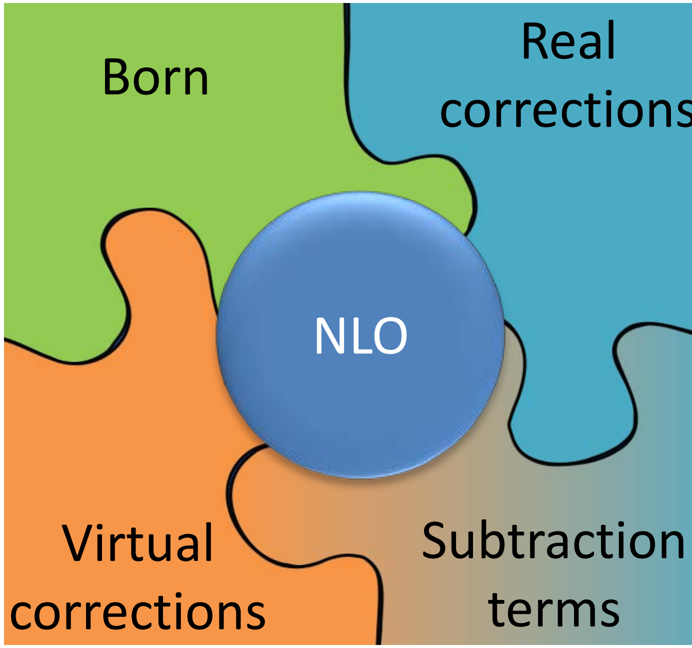
- Typical observations
  - sizable NLO corrections
  - corrections not constant, but kinematics-dependent
  - remaining uncertainty at NLO typically 10-20%

NLO multi-particle production

- Leading-order multi-purpose tools available for 25+ years
- Enormous progress in getting NLO predictions for $2 \to (4,5,6,\ldots)$ processes over the last years
- Made possible by
  - Improved techniques for loop amplitudes
  - Crucial: a high level of automation
- Well-defined interfaces (Binoth Les Houches accord)
  - combine different ingredients from different codes

NLO automation

- One-loop amplitudes
  - BlackHat (Z. Bern, L. Dixon, F. Febres Cordero, S. Höche, H. Ita, D. Kosower, D. Maitre)
  - GoSam (G.Cullen, N.Greiner, G.Heinrich, G Luisoni, P. Mastrolia, G.Ossola, F.Tramontano)
  - RECOLA (S. Actis, A. Denner, L. Hofer, J.N. Lang, A. Scharf, S. Uccirati)
  - OpenLoops (F. Cascioli, P. Maierhöfer, S. Pozzorini)
  - NJet (S. Badger, B. Biedermann, P. Uwer, V. Yundin)
  - MadLoop/aMC@NLO (R. Frederix et al.)
  - CutTools (G. Ossola, C. Papadopoulos, R. Pittau)
- Real radiation, subtraction terms and phase space (infrastructure)
  - From event generator programs

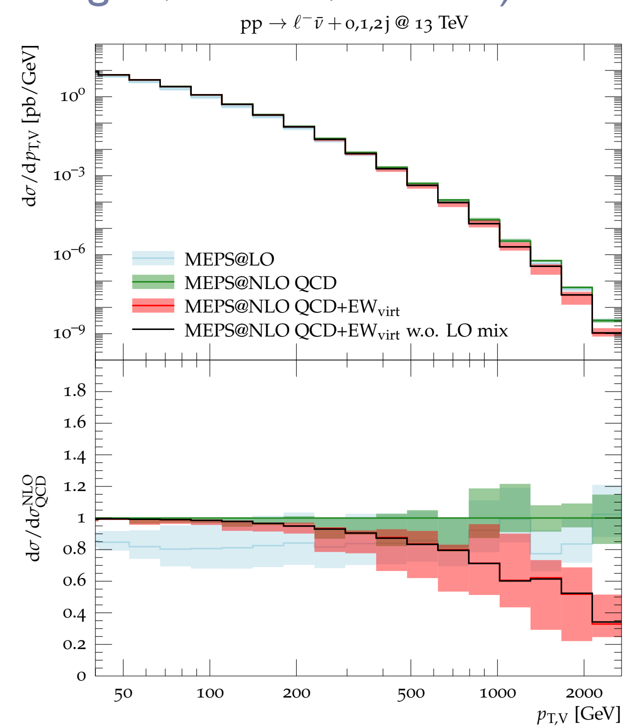


Tools for NLO calculations

- **MCFM, VBFNLO** (J. Campbell, K. Ellis, C. Williams; D. Zeppenfeld et al.)
  - Extensive libraries of NLO QCD processes
- **MG5_aMC@NLO** (F. Maltoni, S. Frixione et al.)
  - Full event generation with automation of one-loop amplitudes
  - Matching to parton shower (MC@NLO method)
- **SHERPA** (F. Kraus et al.)
  - Interfaces to one-loop codes (OpenLoops, BlackHat, Njet, GoSam)
  - Matching to parton shower (MC@NLO, POWHEG methods)
  - Matching of NLO multiplicities (MENLOPS)
- **HERWIG** (S. Gieseke, S. Plätzer, P. Richardson et al.)
  - Full event generation with one-loop from GoSam or VBFNLO
  - Matching to parton shower (MC@NLO method)

NLO QCD+EW

- Extension of automated NLO codes to include electroweak corrections
  - OpenLoops (S. Kallweit, J. Lindert, P. Maierhöfer, S. Pozzorini, M. Schönherr)
  - MG5_aMC@NLO (S. Frixione, V. Hirschi, D. Pagani, H. Shao, M. Zaro)
  - RECOLA (S. Actis, A. Denner, L. Hofer, J.N. Lang, A. Scharf, S. Uccirati)
  - Issue of ordering of corrections: QCDxEW or QCD+EW
- Applications
  - Top pair production (MG5_aMC@NLO)
  - Vector boson plus multijet (OpenLoops+Sherpa)

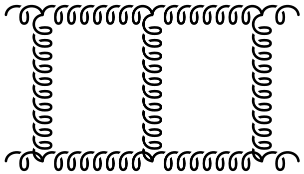


NNLO accuracy for at hadron colliders

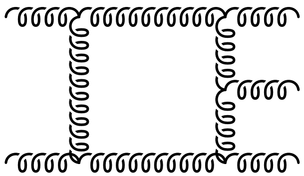
- NNLO predictions
  - expected to have a per-cent level accuracy
  - yielding first reliable estimate of theoretical uncertainty
- For processes measured to few per cent accuracy
    - jet production
    - vector boson (+jet) production
    - vector boson pair prodcution
    - top quark pair production
- For processes with potentially large perturbative corrections
  - New channels and/or phase space regions open up
    - Higgs or vector boson pair production

NNLO calculations

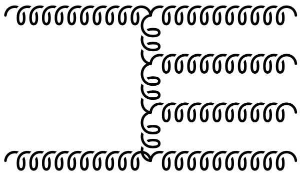
- Require three principal ingredients
  - two-loop matrix elements
    - explicit infrared poles from loop integral
      - known for all massless $2 \to 2$ processes



  - one-loop matrix elements
    - explicit infrared poles from loop integral
    - and implicit poles from single real emission
      - usually known from NLO calculations



  - tree-level matrix elements
    - implicit poles from double real emission
      - known from LO calculations



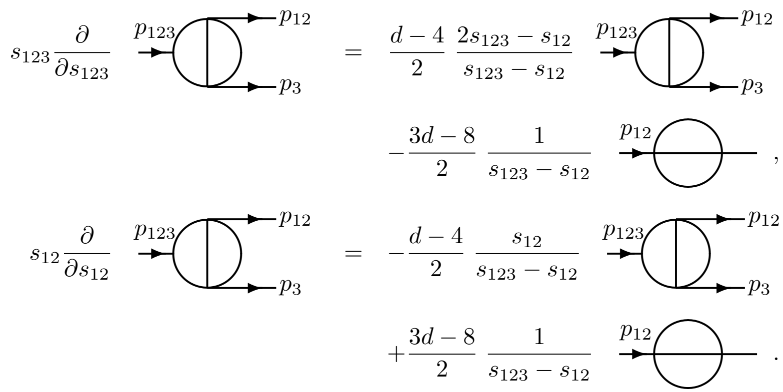
- Infrared poles cancel in the sum
- Challenge: combine contributions into parton-level generator

Multi-loop amplitudes

- Key ingredient to higher order QCD corrections
- Two challenges
  - Expression of amplitude in terms of master integrals
  - Calculation of master integrals
- Integral reduction techniques
  - Integration-by-parts (K. Chetyrkin, F. Tkachev; S. Laporta)
  - Unitarity-based methods
  - Integrand reduction
- Calculation of integrals
  - Direct evaluation (numerical: sector decomposition)
  - Differential equations (A. Kotikov; E. Remiddi, TG; J. Henn)

Two-loop amplitudes

- Master integrals from differential equations
  - Differentiate integrand with respect to masses and momenta
  - Apply integration-by-parts identities



  - Integrate differential equations and match boundary
- Systematic approach
  - dlog-form, alphabet of process (J. Henn)
  - Integration: polylogarithms, symbol, coproduct (A. Goncharov; C.Duhr)

Two-loop amplitudes

- Analytic $2 \rightarrow 2$ results for processes with jets
  - Di-jet production (C. Anastasiou, E.W.N. Glover, C. Oleari, M. Tejeida-Yeomans; Z. Bern, L. Dixon, A. De Freitas)
  - Vector-boson-plus-jet production (L. Garland, E.W.N. Glover, A. Koukoutsakis, E. Remiddi, L. Tancredi, E. Weihs, TG)
  - Higgs-boson-plus-jet production (E.W.N. Glover, M. Jaquier, A. Koukoutsakis, TG)
- Top quark pair production
  - Numerical representation (P. Bärnreuther, M. Czakon, P. Fiedler)
- Vector boson pairs
  - Vector-boson-plus-photon (L. Tancredi, TG)
  - Pair of massive vector bosons: W^+W^-, Z^0Z^0, $W^\pm Z^0$ (F. Caola, J. Henn, K. Melnikov, A&V. Smirnov; A. von Manteuffel, L. Tancredi, TG)

Real radiation at NNLO: factorization

- Single unresolved radiation at one loop
  - One-loop correction to collinear splitting factors (Z. Bern, V. Del Duca, W. Kilgore, C. Schmidt)
  - One-loop correction to soft eikonal factor (S. Catani, M. Grazzini)
- Double unresolved radiation factors at tree level (J. Campbell, E.W.N. Glover; S. Catani, M. Grazzini)
  - Double soft
  - Soft/Collinear
  - Triple collinear
  - Double single collinear
- Require method to extract singular contributions

Real radiation at NNLO: methods

- Sector decompostion
  (T. Binoth, G. Heinrich; C. Anastasiou, K. Melnikov, F. Petriello)
  - Partition of phase space into sectors of non-overlapping singularities
  - Laurent expansion of integrand

$$(1-z)^{-1-\epsilon} = -\frac{1}{\epsilon}\delta(1-z) + \sum_n \frac{(-\epsilon)^n}{n!}\left(\frac{\ln^n(1-z)}{1-z}\right)_+$$

  - Numerical sector integrals
- Sector-improved subtraction schemes
  (M. Czakon; R. Boughezal, K. Melinkov, F. Petriello)
  - Residue subtraction: universal singular factors
  - Integration of subtraction terms using sector decomposition

Real radiation at NNLO: methods

- **q_T-subtraction** (S. Catani, M. Grazzini)
  - Production of colourless final states at hadron colliders
  - Universal behaviour for small transverse momentum from resummation
  - Cut off real radiation phase space at small transverse momentum

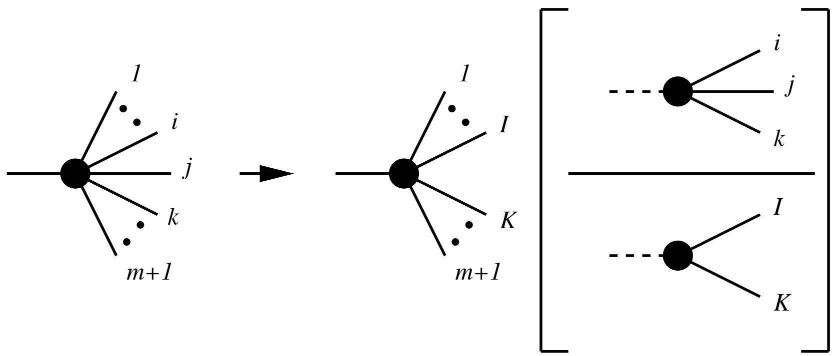
$$d\sigma^F_{NNLO} = \mathcal{H}^F_{NNLO} \otimes d\sigma^F_{LO} + \left[d\sigma^{F+\text{jet}}_{NLO} - d\sigma^{CT}_{NLO} \right]$$

- **N-jettiness subtraction**
  (R. Boughezal, X. Liu, F. Petriello; J. Gaunt, M. Stahlhofen, F. Tackmann, J. Walsh)
  - Use N-jettiness variable as cut-off for N-jet final state

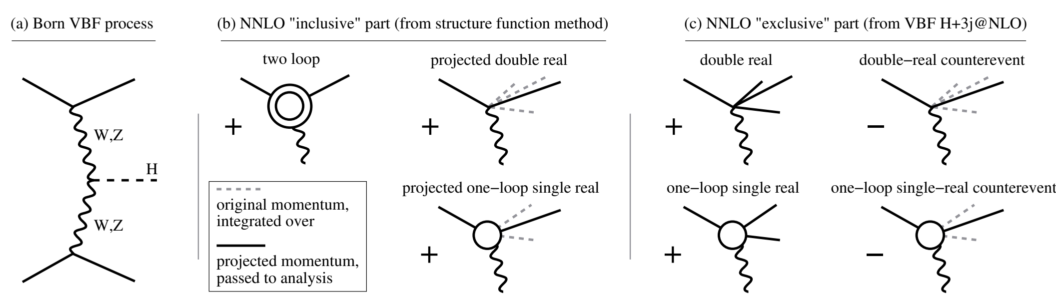
$$\tau_N = \frac{2}{Q^2} \sum_k \min \{ q_a \cdot p_k, q_b \cdot p_k, q_1 \cdot p_k, \ldots q_N \cdot p_k \}$$

Real radiation at NNLO: methods

- **Antenna subtraction** (A. Gehrmann-De Ridder, E.W.N. Glover, TG)
  - Subtraction terms constructed from antenna functions
  - Antenna function contains all emission between two partons



- **Projection-to-Born** (M. Cacciari, F. Dreyer, A. Karlberg, G. Salam, G. Zanderighi)



NNLO calculations: results

- Sector decomposition
  - pp → H, pp → V, including decays
    (C. Anastasiou, K. Melnikov, F. Petriello; S. Bühler, F. Herzog, A. Lazopoulos, R. Müller)
- Sector-improved residues
  - pp → tt (M. Czakon, P. Fiedler, A. Mitov)
  - pp → H+j (R. Boughezal, F. Caola, K. Melnikov, F. Petriello, M. Schulze)
  - pp → t+j (M. Brucherseifer, F. Caola, K. Melnikov)
- q_T subtraction
  - pp → H, pp → V, pp → $\gamma\gamma$, pp → VH
    (S. Catani, L. Cieri, D. de Florian, G. Ferrera M. Grazzini, F. Tramontano)
  - pp → VV (M. Grazzini et al.)
- N-Jettiness subtraction
  - pp → H+j (R. Boughezal, C. Focke, W. Giele, X. Liu, F. Petriello)
  - pp → W+j (R. Boughezal, C. Focke, X. Liu, F. Petriello)
  - pp → Z+j (R. Boughezal, J. Campbell, K. Ellis, C. Focke, W. Giele, X. Liu, F. Petriello)
  - pp → ɣ+j (J. Campbell, K. Ellis, C. Williams)

Real radiation at NNLO: results

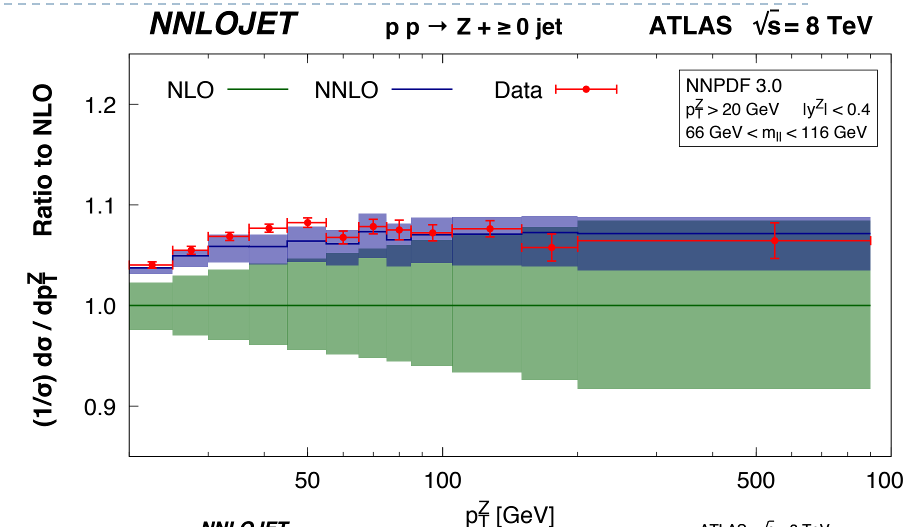
- Antenna subtraction (NNLOJET)
  - pp $\to$ H+j (J. Cruz-Martinez, X. Chen, E.W.N. Glover, M. Jaquier, TG)
  - pp $\to$ Z+j (A. Gehrmann-De Ridder, E.W.N. Glover, A. Huss, T. Morgan, TG)
  - pp $\to$ 2j (J. Currie, A. Gehrmann-De Ridder, E.W.N. Glover, A. Huss, J. Pires, TG)
  - ep $\to$ 2j (J. Currie, E.W.N. Glover, A. Huss, J. Niehues, TG)
- Projection-to-Born
  - pp $\to$ H+2j (VBF) (M. Cacciari, F. Dreyer, A. Karlberg, G. Salam, G. Zanderighi)

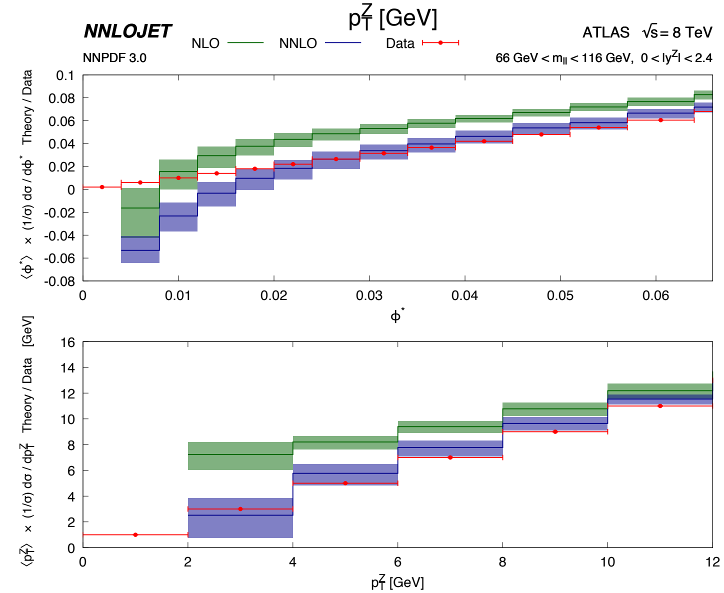
Z p_T-distribution at NNLO

- **Classical QCD observable**
  - Measured: p_T= 1...1000 GeV
  - Constrains gluon distribution
    (R.Boughezal, A. Guffanti, F. Petriello, M. Ubiali)
  - NNLO improves description of data in shape and normalization
- **Related observable**

$$\phi^* = \tan\left(\frac{\pi - \Delta\phi}{2}\right)\sin(\theta^*_\eta) \approx \frac{p_T^Z}{2m_{ll}}$$

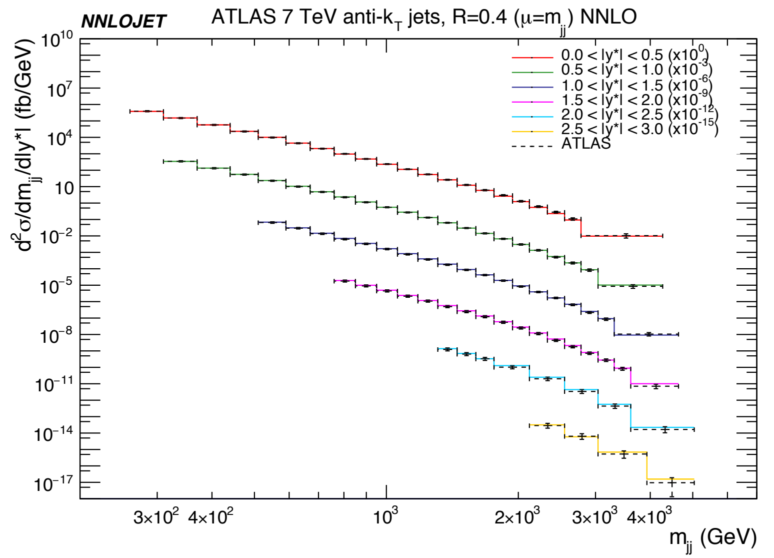
  - purely from lepton directions
  - Higher resolution at low pT
  - NNLO reliable to p_T=5 GeV
  - Challenge for numerics

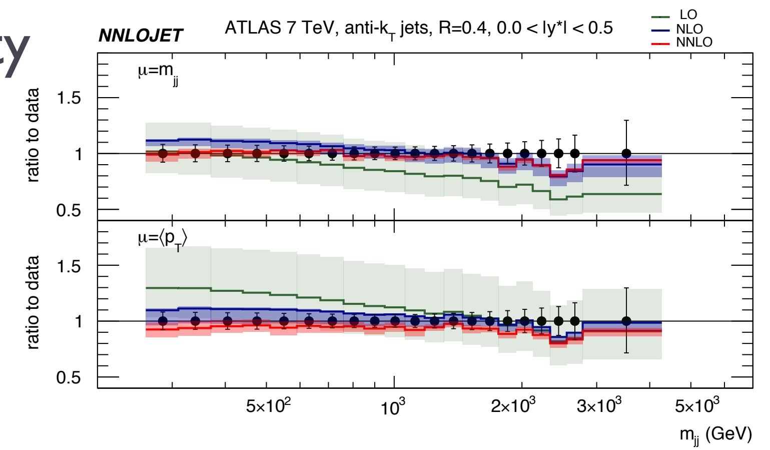




Di-jet production at NNLO

- Inclusive jet and di-jet production
  - Constraints on parton distributions
  - Determination of α_s
- Hadron collider: LHC/Tevatron
  (J.Currie, et al.)
  - Per-cent level accuracy at NNLO
  - Reduced spread among scale choices
  - Improved description of dependence on transverse momentum and rapidity
- Deep inelastic scattering: HERA
  (J. Currie, A. Huss, J. Niehues, TG)
- Prepare interfaces to PDF
  - Revisit gluon distribution





Towards NNLO automation

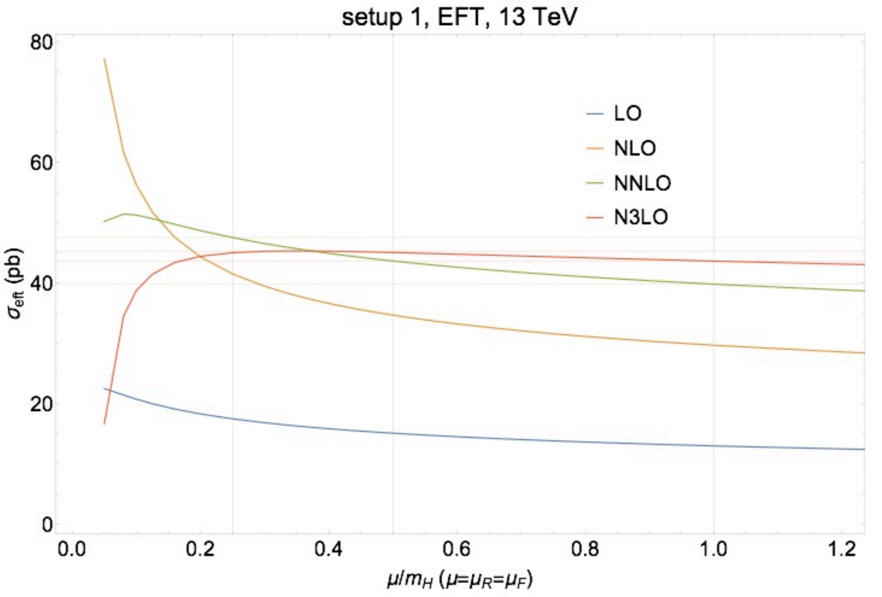
- Methods for real radiation at NNLO becoming mature
  - q_T subtraction
  - N-Jettiness subtraction
  - Sector-improved schemes
  - Antenna subtraction
- Issues
  - Automation of code generation
  - Numerical efficiency and stability
  - Dissemination of results
    - K-factors, event N-tuples, coefficient grids, … ?
  - Availability of two-loop amplitudes

Multi-loop amplitudes

- Limit in complexity at two loops: 2 → 2
  - Up to two external masses, one internal mass
    - pp → V_1V_2, pp → tt available
    - 2 → 3 ongoing: gg → ggg (S. Badger, G. Mogull, A. Ochirov, D. O'Connell; J. Henn. A. Lo Presti, TG; C. Papadopoulos, D. Tomassini, C. Wever)
- Limit in complexity at three loops: 2 → 1 (or equivalent)
  - DIS three loop splitting and coefficient functions (S. Moch, A. Vogt, J. Vermaseren)
  - Heavy quark DIS coefficient functions at NNLO (J. Ablinger, A. Behring, J. Blümlein, A. Hasselhuhn, A. von Manteuffel, C.G. Raab, M. Round, C. Schneider, F. Wissbrock)

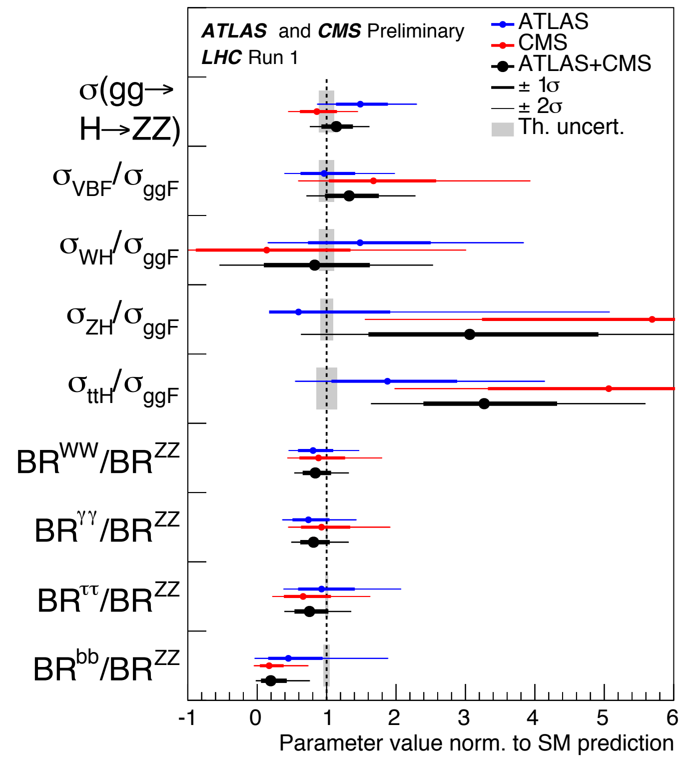
Higgs production in gluon fusion at N3LO

- **N3LO contributions to Higgs production** (P. Baikov, K. Chetyrkin, V. Smirnov, A. Smirnov, M. Steinhauser; N. Glover, T. Huber, N. Ikizlerli, C. Studerus, M. Jaquier, A. Koukoutsakis, C. Anastasiou, C. Duhr, F. Dulat, F. Herzog, B. Mistlberger, E. Furlan, Y. Li, A. von Manteuffel, R. Schabinger, H.X. Zhu, C. Anzai, A. Hasselhuhn, M. Hoschele, J. Hoff, W. Kilgore, M. Steinhauer, T. Ueda, TG)
  - Three-loop form factor
  - Two-loop single real
  - One-loop double real
  - Tree-level triple real
- **Coefficient function** (C. Anastasiou, C. Duhr, F. Dulat, F. Herzog, B. Mistlberger)
  - From high-order expansion around threshold



Higgs production in gluon fusion

- Precise prediction mandatory for coupling extraction
- N3LO coefficient function plus
  - Quark mass corrections at NLO
  - Electroweak corrections
- Sources of uncertainty
  - PDF and strong coupling
  - Resummation effects
  - Truncation error
- Prediction for 13 TeV



(C. Anastasiou, C. Duhr, F. Dulat, E. Furlan, F. Herzog, A. Lazopoulos, B. Mistlberger, TG)

$$\boxed{\sigma = 48.58\,\mathrm{pb}^{+2.22\,\mathrm{pb}\,(+4.56\%)}_{-3.27\,\mathrm{pb}\,(-6.72\%)}\ (\mathrm{theory}) \pm 1.56\,\mathrm{pb}\,(3.20\%)\ \ (\mathrm{PDF}+\alpha_s)}$$

Where do we stand?

- Witnessed an NLO revolution
  - Previously unthinkable NLO QCD+EW multi-particle calculations now feasible due to technological breakthroughs
  - High-level of automation
  - Standarization of interfaces: combine different codes (providers)
  - Interface to experiment (codes, ntuples, histograms,..)?
- Substantial progress on NNLO calculations
  - Several different methods available
  - Close interplay with resummation
  - Calculations on process-by-process basis
  - Codes typically require HPC infrastructure

Future Directions

- NNLO automation
  - Uncover analytical structures to organize calculation
  - Develop standard interfaces
  - Interface to experiment ?
- Beyond NNLO
  - N^3LO precision for benchmark processes