Formation of primordial black holes from cosmological fluctuations

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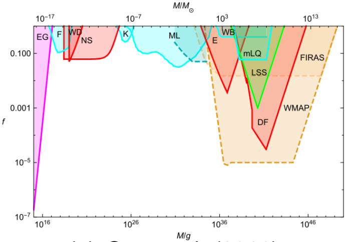
This talk focuses on our recent work.

- Harada, Yoo (Nagoya) & Kohri (KEK), arXiv: 1309.4201
- Carr (QMUL) & Harada, 1405.3624
- Harada, Yoo, Nakama (JHU) & Koga (Rikkyo), 1503.03934
- Harada, Yoo, Kohri, Nakao (OCU) & Jhingan (YGU), 1609.01588
- Harada, Yoo, Kohri, & Nakao, 1707.03595

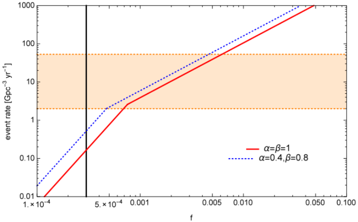
Outline

Primordial black hole (PBH)

- PBH = Black hole formed in the early Universe
  - Probe into the early Universe, high-energy physics, and quantum gravity through Hawking radiation, dark matter, and gravitational waves (Carr et al. (2010), Carr et al. (2016))
  - LIGO BBH events may be sourced by PBHs. (Sasaki et al. (2016), Bird et al. (2016), Clesse & Garcia-Bellido (2017)) Some information about BH spins of GW170104 obtained (Abbott et al. (2017))
  - BH spins have been measured. (McClintock 2011) Spinning PBHs may affect the CMB. (Pani & Loeb (2013))



(a) Carr et al. (2016)



(b) Sasaki et al. (2016)

PBH formation and evaporation

- The mass M and the cosmological time t at the formation are directly related by

$$M \simeq M_H(t) \simeq \frac{c^3}{G} t.$$

- Hawking radiation: nearly black-body radiation

$$T_H = \frac{\hbar c^3}{8\pi G M k_B}, \quad \frac{dE}{dt} = -\frac{dM}{dt} c^2 = g_{\rm eff} 4\pi R_g^2 \sigma T_H^4, \quad R_g = \frac{2GM}{c^2},$$

$$t_{ev} \simeq \frac{G^2 M^3}{g_{\rm eff} \hbar c^4} \simeq 10 \text{ Gyr} \left(\frac{M}{10^{15}\text{g}}\right)^3.$$

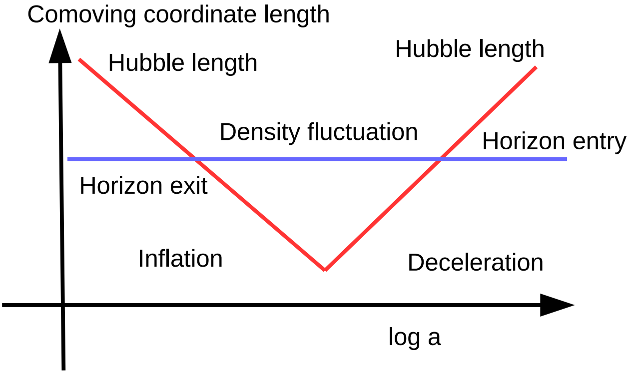
- $f(M)$ ($M > 10^{15}$g) in terms of the production probability $\beta_0(M)$

$$f(M) := \frac{\Omega_{PBH}(M)}{\Omega_{CDM}} \simeq 4.8 \Omega_{PBH}(M) \simeq 4 \times 10^8 \beta_0(M) \left(\frac{M}{M_\odot}\right)^{-1/2},$$

where PBHs are assumed to form in the radiation-dominated era. (Carr et al. (2010), Carr et al. (2016))

PBH formation by primordial fluctuations

- Primordial fluctuations with long wavelength are generated in the accelerated-expansion phase. They re-enter the horizon in the decelerated-expasion phase and can grow to form black holes.



- The decelerated-expansion may be radiation-dominated or matter-dominated.

Outline

PBH formation in the radiation-dominated (RD) era

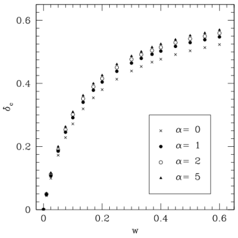
- Pioneered by Carr (1975)
- δ_H: density perturbation at horizon entry
- The threshold $\delta_{\rm th}$ of PBH formation together with the probability distribution of δ_H determines the formation probability β_0.
- If δ_H obeys a Gaussian distribution with standard deviation σ_H, β_0 is given by

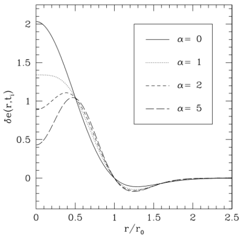
$$\beta_0 \simeq \sqrt{\frac{2}{\pi}}\frac{\sigma_H}{\delta_{\rm th}}e^{-\delta_{\rm th}^2/(2\sigma_H^2)}.$$

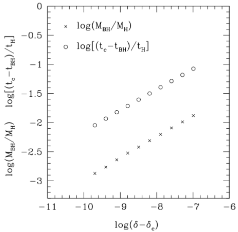
- β_0 is sensitive to the threshold $\delta_{\rm th}$ as $\delta_{th} = O(1)$ and $\sigma_H \ll 1$.
- The threshold $\delta_{\rm th}$ is basically determined by the Jeans criterion. Carr's formula: $\delta_{\rm th} \simeq w$ for $p = w\rho$.
- Caution: δ_H depends on the slicing condition. The comoving slice is usually used.

Numerical relativity simulation in spherical symmetry

- Pioneered by Nadezhin, Novikov, & Polnarev (1978)
- Long wavelength solutions $\zeta(r)$ (Shibata & Sasaki (1999)) or asymptotically quasi-homogeneous solutions $K(r)$ (Polnarev & Musco (2007)) as initial conditions
- All recent results for radiation converge to the range $\delta_{\rm th} \simeq 0.42 - 0.66$ (Musco & Miller (2013), Harada, Yoo, Nakama & Koga, 1503.03934, ...).
- EOS ($p = w\rho$) dependence, profile dependence, critical behaviour (Musco & Miller (2013))

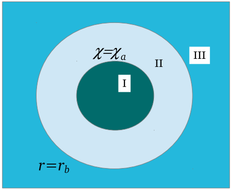






Conventional analytic formula of the threshold

- 3-zone model: I: $K = 1$ FLRW, II: compensating layer, III: K=0 FLRW



- Jeans' criterion: If and only if the radius of region I at maximum expansion is larger then the Jeans length $R_J = c_s\sqrt{3/(8\pi G\rho)}$ with $c_s = \sqrt{w}$ for $p = w\rho$, the pertubation grows to a black hole.

$$\delta_{CMC,th} = w, \quad \text{and} \quad \delta_{th} = \frac{3(1+w)}{5+3w}w,$$

in the CMC and comoving slices, respectively. (Cf. Carr (1975))

- The choice of R_J in GR is not so straightforward...

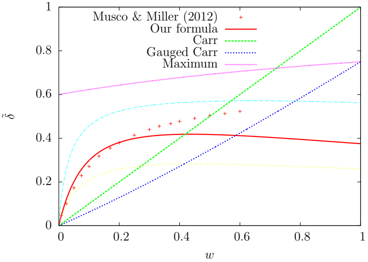
New analytic formula (Harada, Yoo & Kohri, 1309.4201)

- Reformulated Jeans' criterion: If and only if the free-fall time is shorter than the sound-crossing time, the perturbation grows to a black hole.

$$\delta_{\rm th} = \frac{3(1+w)}{5+3w}\sin^2\left(\frac{\pi\sqrt{w}}{1+3w}\right), \quad \delta_{\rm max} = \frac{3(1+w)}{5+3w}.$$

(The maximum corresponds not to the separate universe but to the hemisphere. (Kopp et al. (2011), Carr & Harada, 1405.3624))

- The new formula agrees well with the result of numerical relativity for $0.01 \le w \le 0.6$. It gives $\delta_{\rm th} \simeq 0.4135$ for $w = 1/3$.



Threshold for curvature perturbation

- 3+1 decomposition

$$ds^2 = -\alpha^2 dt^2 + e^{-2\zeta} a^2(t)\tilde{\gamma}_{ij}(dx^i + \beta^i dt)(dx^j + \beta^j dt),$$

where $\det(\tilde{\gamma}_{ij})$ is the determinant of the flat 3-metric. This ζ in the uniform-density slice is often called as curvature perturbation.

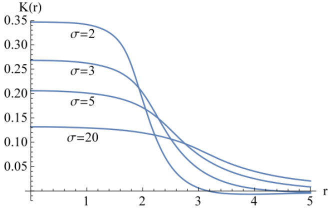
- In the long wavelength and linear-order approximation, we find

$$\delta_H \simeq -\frac{4}{9}\,\hat{\zeta}_k\big|_{k=k_{BH}}, \quad \sigma_H^2 \simeq \left(\frac{4}{9}\right)^2 P_\zeta(k_{BH}),$$

where $P_\zeta(k)$ is the power spectrum of ζ. For $w = 1/3, \delta_{\rm th} \simeq 0.42$ implies $|\hat{\zeta}_k|_{\rm th} \simeq 0.95$, although the linear-order approximation cannot be fully justified. (See Harada, Yoo, Kohri & Nakao, 1707.03595)

Profile dependence (Harada, Yoo, Nakama & Koga 1503.03934)

- $\mathbf{0.42 \lesssim \delta_{th} \lesssim 0.56}$, where δ_{th} is smaller for the gentler transition. $|\zeta|_{\rm peak,th}$ shows the opposite behaviour.



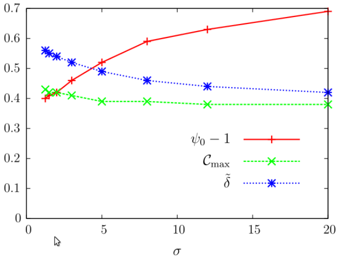


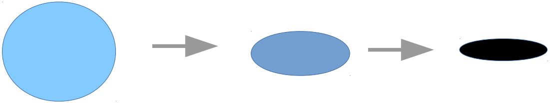
Figure: $\psi = e^{-2\zeta}$ and ψ_0 is its peak value. The larger σ means the gentler transition to the surrounding homogeneous region.

- In fact, $\zeta_{\rm peak}$ is strongly affected by a density perturbation of much larger scale, while δ_H or $\hat{\zeta}_k$ is not affected.(Cf. Young & Byrnes (2015))

Outline

PBH formation in the matter-dominated (MD) era

- Pioneered by Khlopov & Polnarev (1980). Recently motivated by early MD phase scenarios such as inflaton oscillations, phase transitions, and superheavy metastable particles.
- If pressure is negligible, nonspherical effects play crucial roles.
  - The triaxial collapse of dust leads to a "pancake" singularity. (Lin, Mestel & Shu 1965, Zeldovich 1969)



  - A rotational mode is decaying in the cosmological linear perturbation theory. However, we will look at it later.
  - We here rely on the Newtonian approximation to deal with nonspherical dynamics analytically.

Outline

Zeldovich approximation

- Zeldovich approximation (ZA) (1969)
  Extrapolate the Lagrangian perturbation theory in the linear order in Newtonian gravity to the nonlinear regime.

$$r_i = a(t)q_i + b(t)p_i(q_j),$$

  where $b(t) \propto a^2(t)$ denotes a linear growing mode.
- We can take the coordinates in which

$$\frac{\partial p_i}{\partial q_j} = \mathrm{diag}(-\alpha, -\beta, -\gamma),$$

  where we can assume $\infty > \alpha \geq \beta \geq \gamma > -\infty$.
- We assume that α, β and γ are constant over the smoothing scale.
- The linearised density perturbation is given by

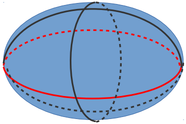
$$\delta = (\alpha + \beta + \gamma)\frac{b}{a} \propto a, \quad \delta_H := \delta(t_i) = \alpha + \beta + \gamma,$$

  if we normalise b so that $(b/a)(t_i) = 1$ at horizon entry $t = t_i$.

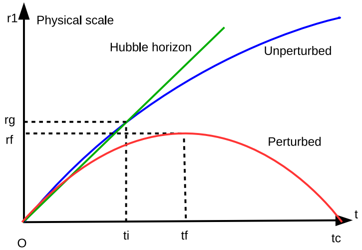
Anisotropic collapse in the ZA

- The triaxial ellipsoid of a Lagrangian ball (assumption)

$$\begin{cases} r_1 = (a - \alpha b)q \\ r_2 = (a - \beta b)q \\ r_3 = (a - \gamma b)q \end{cases}$$



- Evolution of the collapsing region:
  - Horizon entry ($t = t_i$): $a(t_i)q = cH^{-1}(t_i) = r_g := 2Gm/c^2$.
  - Maximum expansion ($t = t_f$): $\dot{r}_1(t_f) = 0$ giving $r_f := r_1(t_f) = r_g/(4\alpha)$.
  - Pancake singularity ($t = t_c$): $r_1(t_c) = 0$ giving $a(t_c)q = 4r_f$.

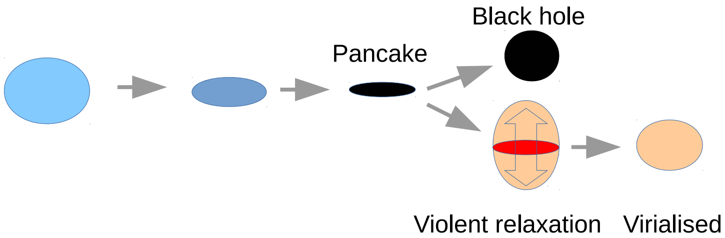


Application of the hoop conjecture to the pancake collapse

- Hoop conjecture (Thorne 1972): The collapse results in a BH if and only if $C \lesssim 4\pi GM/c^2$, where C is the circumference of the pancake singularity.
- Then, we obtain a BH criterion:

$$h(\alpha,\beta,\gamma) := \frac{C}{4\pi Gm/c^2} = \frac{2}{\pi}\frac{\alpha-\gamma}{\alpha^2}E\left(\sqrt{1-\left(\frac{\alpha-\beta}{\alpha-\gamma}\right)^2}\right) \lesssim 1,$$

  where $E(e)$ is the complete elliptic integral of the second kind.
- If $h \gtrsim 1$? : It does not immediately collapse to a BH.



Doroshkevich's probability distribution

- α, β, and γ obey $w(\alpha,\beta,\gamma)$ from the assumption of the Gaussian distribution of the deformation tensor components (Doroshkevich 1970)

$$
\begin{aligned}
& w(\alpha,\beta,\gamma)d\alpha d\beta d\gamma \\
= \quad & -\frac{27}{8\sqrt{5}\pi\sigma_3^6}\exp\left[-\frac{3}{5\sigma_3^2}\left\{(\alpha^2+\beta^2+\gamma^2)-\frac{1}{2}(\alpha\beta+\beta\gamma+\gamma\alpha)\right\}\right] \\
& \cdot(\alpha-\beta)(\beta-\gamma)(\gamma-\alpha)d\alpha d\beta d\gamma
\end{aligned}
$$

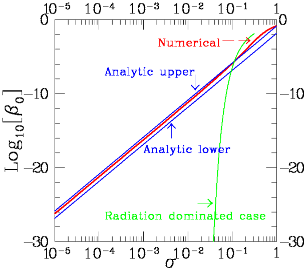
- Linearised density perturbation at horizon entry:

$$
\delta_H(\alpha,\beta,\gamma)=\alpha+\beta+\gamma,\ \ \sigma_H = \sqrt{\langle\delta_H^2\rangle} = \sqrt{5}\sigma_3.
$$

PBH production rate

- Triple integral for β_0 ($\theta(x)$ is a step function.)

$$\beta_0 \simeq \int_0^\infty d\alpha \int_{-\infty}^{\alpha} d\beta \int_{-\infty}^{\beta} d\gamma \theta[1 - h(\alpha,\beta,\gamma)] w(\alpha,\beta,\gamma).$$



- We derive a semi-analytic formula. (Cf. Khlopov & Polnarev (1980))

$$\beta_0 \simeq 0.05556\sigma_H^5.$$

Outline

Perturbation in Newtonian theory

- Standard cosmological perturbation in the Eulerian coordinates $\mathbf{x} := \mathbf{r}/a$, $\mathbf{u} := aD\mathbf{x}/Dt$, $\delta := (\rho - \rho_0)/\rho_0$, and $\psi := \Psi - \Psi_0$. The background is chosen to the flat FLRW with $a = a_0 t^{2/3}$.
- Linear perturbation in the momentum space

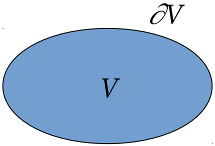
$$\mathbf{u}_1 = \sum_{\mathbf{k}} \hat{\mathbf{u}}_{1,\mathbf{k}}(t) e^{i\mathbf{k}\cdot\mathbf{x}}, \ \delta_1 = \sum_{\mathbf{k}} \hat{\delta}_{1,\mathbf{k}}(t) e^{i\mathbf{k}\cdot\mathbf{x}}, \ \psi_1 = \sum_{\mathbf{k}} \hat{\psi}_{1,\mathbf{k}}(t) e^{i\mathbf{k}\cdot\mathbf{x}},$$

- Growing modes

$$\hat{\delta}_{1,\mathbf{k}} = A_{\mathbf{k}} t^{2/3}, \hat{\psi}_{1,\mathbf{k}} = -\frac{2}{3}\frac{a_0^2}{k^2} A_{\mathbf{k}}, \ \ \hat{\mathbf{u}}_{1,\mathbf{k}} = i a_0 \frac{\mathbf{k}}{k^2}\frac{2}{3} A_{\mathbf{k}} t^{1/3}.$$

Angular momentum

- Region V: to collapse in the future



- Angular momentum within V

$$
\begin{aligned}
\mathbf{L}_c &= \int_{a^3V} \rho \mathbf{r} \times \mathbf{v} d^3\mathbf{r} = \rho_0 a^4 \left(\int_V \mathbf{x} \times \mathbf{u} d^3\mathbf{x} + \int_V \mathbf{x}\delta \times \mathbf{u} d^3\mathbf{x} \right), \\
\mathbf{L} &= \mathbf{L}_c - \mathbf{R} \times \mathbf{P} \\
&= \rho_0 a^4 \left(\int_V \mathbf{x} \times \mathbf{u} d^3\mathbf{x} + \int_V \mathbf{x}\delta \times \mathbf{u} d^3\mathbf{x} - \frac{1}{V} \int_V \mathbf{x}\delta d^3\mathbf{x} \times \int_V \mathbf{u} d^3\mathbf{x} \right)
\end{aligned}
$$

where $\mathbf{L}$ is the angular momentum with respect to the COM, $\mathbf{R}$ is the shift of the COM and $\mathbf{P}$ is the linear momentum.

1st-order effect

$$\mathbf{L} = \rho_0 a^4 \left(\int_V \mathbf{x} \times \mathbf{u} d^3\mathbf{x} + \int_V \mathbf{x}\delta \times \mathbf{u} d^3\mathbf{x} - \frac{1}{V} \int_V \mathbf{x}\delta d^3\mathbf{x} \times \int_V \mathbf{u} d^3\mathbf{x} \right)$$

- If ∂V is not a sphere, the 1st term contribution grows as $\propto a \cdot \mathbf{u} \propto t$.
- If we assume V is a triaxial ellipsoid with axes (A_1, A_2, A_3), we find

$$\langle \mathbf{L}_{(1)}^2 \rangle^{1/2} \simeq \frac{2}{5\sqrt{15}} q \frac{MR^2}{t} \langle \delta^2 \rangle^{1/2},$$

where $r_0 := (A_1 A_2 A_3)^{1/3}$, $R := a(t) r_0$ and $q := \sqrt{\frac{Q_{ij}Q_{ij}}{3\left(\frac{1}{5}Mr_0^2\right)^2}}$ is a nondimensional reduced quadrupole moment of V. (Cf. Catelan & Theuns 1996)

2nd-order effect

$$\mathbf{L} = \rho_0 a^4 \left(\int_V \mathbf{x} \times \mathbf{u} d^3\mathbf{x} + \int_V \mathbf{x}\delta \times \mathbf{u} d^3\mathbf{x} - \frac{1}{V} \int_V \mathbf{x}\delta d^3\mathbf{x} \times \int_V \mathbf{u} d^3\mathbf{x} \right)$$

- Even if ∂V is a sphere, the remaining contribution grows as 1st order × 1st order $\propto a \cdot \delta \cdot \mathbf{u} \propto t^{5/3}$.

$$\langle \mathbf{L}_{(2)}^2 \rangle^{1/2} = \frac{2}{15} \mathcal{I} \frac{MR^2}{t} \langle \delta^2 \rangle,$$

where δ hereafter is the density perturbation averaged over V.
$R := a(t) r_0$. We assume $\mathcal{I} = O(1)$. (Cf. Peebles 1969)

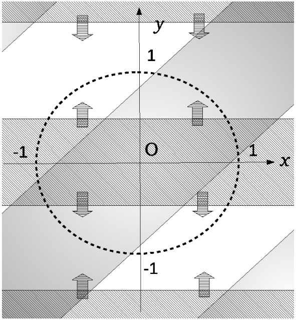


Figure: Coupling of two independent growing modes

The application of the Kerr bound to the PBH formation

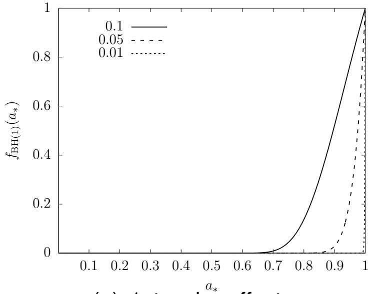
- Time evolution of V and angular momentum
  - Horizon entry ($t = t_H$): $ar_0 = cH^{-1}$, $\delta_H := \delta(t_H)$, $\sigma_H := \langle \delta_H^2 \rangle^{1/2}$
  - Maximum expansion ($t = t_m$): $\delta(t_m) = 1$, typically $t_m = t_H \sigma_H^{-3/2}$
  - $a_* := L/(GM^2/c)$ at $t = t_m$

$$\langle a_{*(1)}^2 \rangle^{1/2} = \frac{2}{5}\sqrt{\frac{3}{5}} q \sigma_H^{-1/2}, \langle a_{*(2)}^2 \rangle^{1/2} = \frac{2}{5} \mathcal{I} \sigma_H^{-1/2}, a_* \simeq \max\left(\langle a_{*(1)}^2 \rangle, \langle a_{*(2)}^2 \rangle\right)$$

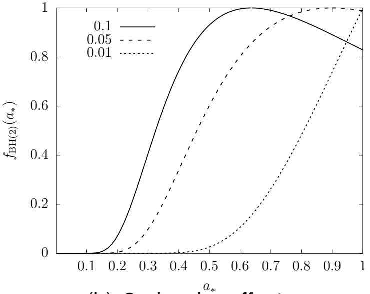
  - For $t > t_m$, the evolution of V decouples from the cosmological expansion and hence a_* is kept almost constant.
- Consequences
  - Supercritical angular momentum: typically $\langle a_*^2 \rangle^{1/2} \gtrsim 1$ if $\sigma_H \lesssim 0.1$
  - Suppression: The Kerr bound implies that a_* is typically too large for direct collapse to a BH.
  - Most of the PBHs have $a_* \simeq 1$. This contrasts with small spins ($a_* \lesssim 0.4$) of PBHs formed in the RD era. (Chiba & Yokoyama (2017))

Spin distribution

- Spin distribution of PBHs formed in the MD era



(a) 1st-order effect



(b) 2nd-order effect

Figure: The distribution function normalised by the peak value. We assume a Gaussian distribution for the density perturbation.

Numerical calculation of PBH production rate

- A threshold $\delta_{\rm th}$ appears for PBH formation against the Kerr bound.
- Triple integral for β_0 ($\theta(x)$ is a step function.)

$$\beta_0 \simeq \int_0^\infty d\alpha \int_{-\infty}^{\alpha} d\beta \int_{-\infty}^{\beta} d\gamma \theta[\delta_H(\alpha,\beta,\gamma) - \delta_{\rm th}]\theta[1 - h(\alpha,\beta,\gamma)] w(\alpha,\beta,\gamma).$$

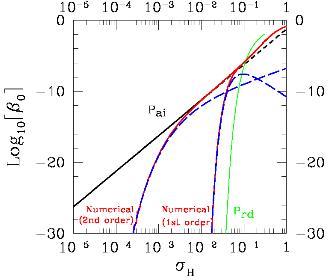


Figure: The red lines are due to both angular momentum and anisotropy. The 1st-order effect depends on q. The black solid line is solely due to anisotropy.

Discussion of PBH production

- Semianalytic estimate (black dashed line and blue dashed line)

$$\beta_0 \simeq \begin{cases} 2\times 10^{-6} f_q(q_c)\mathcal{I}^6\sigma_H^2 \exp\left[-0.15\dfrac{\mathcal{I}^{4/3}}{\sigma_H^{2/3}}\right] & \text{(2nd-order effect)} \\ 3\times 10^{-14}\dfrac{q^{18}}{\sigma_H^4}\exp\left[-0.0046\dfrac{q^4}{\sigma_H^2}\right] & \text{(1st-order effect)} \\ 0.05556\sigma_H^5 & \text{(anisotropic effect)} \end{cases},$$

  where $f_q(q_c)$: the ratio of regions with $q < q_c = O(\sigma_H^{1/3})$.

- σ_H in terms of P_ζ:

$$\sigma_H^2 \simeq \left(\frac{2}{5}\right)^2 P_\zeta(k_{BH}).$$

Outline

Summary

- PBHs may form in the RD era as well as in the (early) MD era by primordial cosmological fluctuations.
- The Jeans criterion applies to the formation of PBHs in the RD era. The threshold has been studied in detail.
- Nonspherical effects, such as anisotropy and angular momentum, play crucial roles in the formation of PBHs in the MD era.
- PBHs formed in the MD era mostly have large spins at least when they are formed.

Application of the Kerr bound to the rotating collapse

- Technical assumption

$$|\mathbf{L}_{(1)}| \simeq \frac{2}{5\sqrt{15}} q \frac{MR^2}{t}\delta, \ |\mathbf{L}_{(2)}| \simeq \frac{2}{15}\mathcal{I}\frac{MR^2}{t}\langle\delta^2\rangle^{1/2}\delta.$$

- The above assumption implies

$$a_{*(1)} = \frac{2}{5}\sqrt{\frac{3}{5}} q\delta_H^{-1/2}, \ \ a_{*(2)} = \frac{2}{5}\mathcal{I}\sigma_H\delta_H^{-3/2}, \ \ a_* = \max(a_{*(1)}, a_{*(2)}).$$

- The Kerr bound $a_* \le 1$ gives a threshold $\delta_{\rm th}$ for δ_H, where

$$\delta_{\rm th} = \max(\delta_{\rm th(1)}, \delta_{\rm th(2)}), \ \ \delta_{\rm th(1)} := \frac{3\cdot 2^2}{5^3}q^2, \ \ \delta_{\rm th(2)} := \left(\frac{2}{5}\mathcal{I}\sigma_H\right)^{2/3}.$$