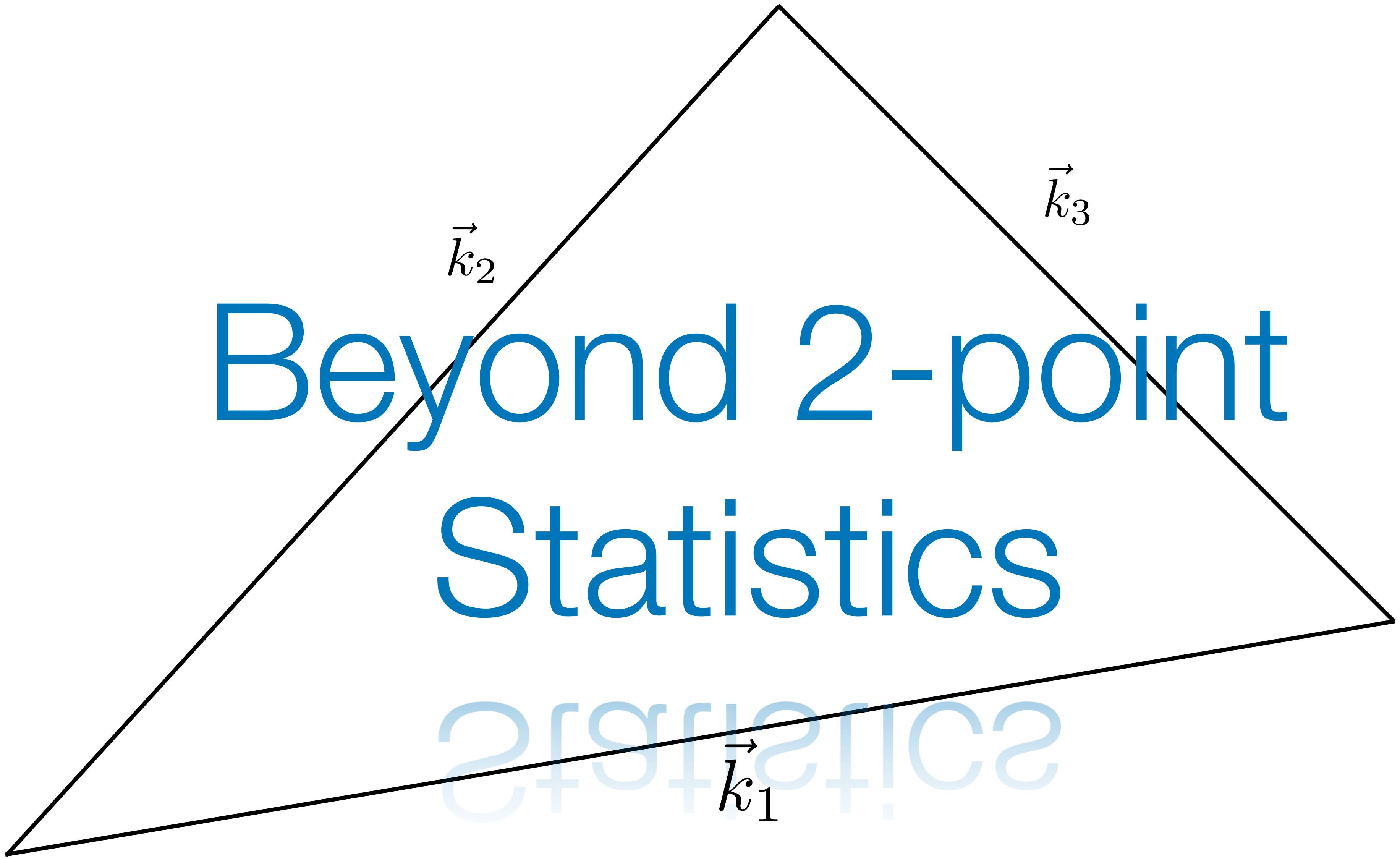
Beyond 2-point Statistics



Daan Meerburg

B-mode from space workshop

Beyond 2-point statistics: motivation

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-If we want to learn anything new probably have to seek beyond 2-point statistics

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Provide powerful tests of the inflationary paradigm (Maldacena 2002, Creminelli and Zaldarriaga 2004 ++)

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Put in (optimistic!) another way: Smoking gun for new physics (Lee, Baumann & Pimentel 2016, Baumann et al in prep)

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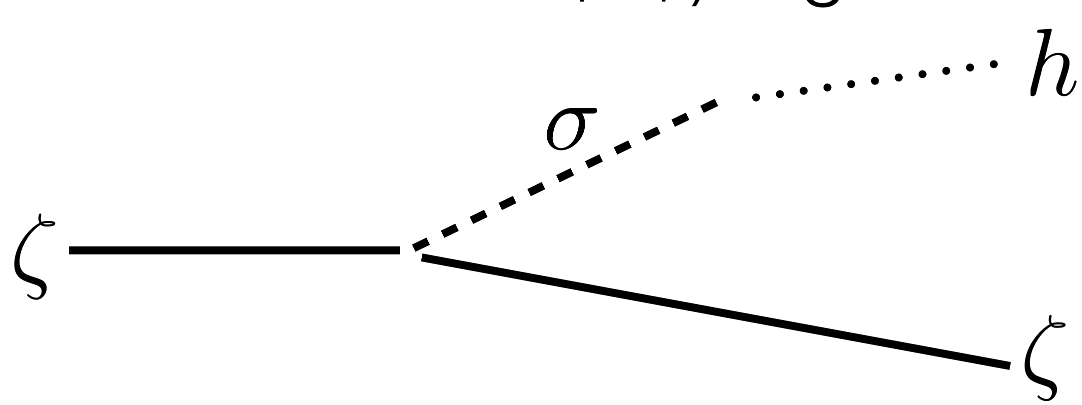
Massive higher order spin fields (with 'free' coupling, Lee, Baumann and Pimentel 2016, Baumann et al in prep) e.g.



SEE ALSO ARKANI-HAMED & MALDACENA (2015)

Beyond 2-point statistics: simple ansatz

Example, correlating two scalar fluctuations with one tensor (model independent):

$$\langle \zeta\zeta h \rangle \propto \sqrt{r} f_{\rm NL}^{\zeta\zeta h} \delta\left(\sum \vec{k}_i\right) \mathcal{I}(k_1, k_2, k_3)\epsilon_{ij}(k_3)\hat{k}_1^i \hat{k}_2^j$$

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Note that we assume $\sqrt{\langle hh \rangle} \propto \mathcal{O}(\sqrt{r})$

$$f_{\rm NL}^{\zeta\zeta h} \equiv \langle \zeta\zeta h \rangle / (P_{\zeta)}^{3/2} P_h^{1/2})$$

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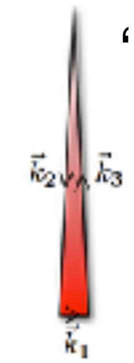

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'equilateral'

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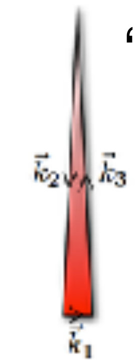

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Future constraints on scalar NGs (TTT,TTE,TEE,EEE)

Type	*Planck* actual (forecast)	CMB-S4	CMB-S4 + low-ℓ *Planck*	Rel. improvement
Local	$\sigma(f_{\rm NL}) = 5\,(4.5)$	$\sigma(f_{\rm NL}) = 2.6$	$\sigma(f_{\rm NL}) = 1.8$	2.5
Equilateral	$\sigma(f_{\rm NL}) = 43\,(45.2)$	$\sigma(f_{\rm NL}) = 21.2$	$\sigma(f_{\rm NL}) = 21.2$	2.1
Orthogonal	$\sigma(f_{\rm NL}) = 21\,(21.9)$	$\sigma(f_{\rm NL}) = 9.2$	$\sigma(f_{\rm NL}) = 9.1$	2.4

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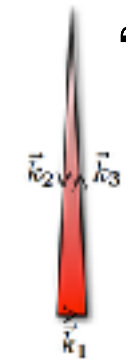

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Future constraints on tensor NGs (BTT only)

Type	Planck	CMB-S4	rel. improvement
local	$\sigma(\sqrt{r} f_{\rm NL}) = 15.2$	$\sigma(\sqrt{r} f_{\rm NL}) = 0.3$	50.7
equilateral	$\sigma(\sqrt{r} f_{\rm NL}) = 200.5$	$\sigma(\sqrt{r} f_{\rm NL}) = 7.4$	27.1
local $(r = 0.01)$	$\sigma(\sqrt{r} f_{\rm NL}) = 15.2$	$\sigma(\sqrt{r} f_{\rm NL}) = 0.7$	25.3
equilateral $(r = 0.01)$	$\sigma(\sqrt{r} f_{\rm NL}) = 200.8$	$\sigma(\sqrt{r} f_{\rm NL}) = 14.7$	13.7

CMBS4 science book, 2016

Beyond 2-point statistics: from space

Full-Sky	$\sum_n \ell_n$ = even	$\sum_n \ell_n$ = odd
Flat-Sky	Left-Handed = Right-Handed	Left-Handed = (−) Right-Handed
Non-Vanishing In Parity-Conserving Universe	$\langle TTT\rangle$, $\langle TEE\rangle$, $\langle TTE\rangle$, $\langle EEE\rangle$, $\langle BBE\rangle$, $\langle BBT\rangle$	$\langle BTT\rangle$, $\langle BEE\rangle$, $\langle BET\rangle$, $\langle BBB\rangle$

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Delta's, cosines and sines become Wigner j's

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Include all possible measures and all possible sources:

$$\langle\zeta\zeta\zeta\rangle \rightarrow \langle TTT\rangle,\ \langle TTE\rangle,\ \langle TEE\rangle,\ \langle EEE\rangle$$

$$\langle h\zeta\zeta\rangle \rightarrow \langle TTT\rangle,\ \langle TTE\rangle,\ \langle TEE\rangle,\ \langle EEE\rangle$$
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Beyond 2-point statistics: challenges

Fast and optimal estimator: Komatsu, Spergel and Wandelt (KSW) estimator (Assuming isotropic noise and diagonal covariance)

$$\hat{f}_{\rm NL}^{X_1 X_2 X_3} = \frac{1}{N} \sum_{pqrijk} \sum_{\ell m} \frac{B^{\ell_1 \ell_2 \ell_3, pqr}_{m_1 m_2 m_3 X_1 X_2 X_3}}{C^{pi}_{\ell_1} C^{qj}_{\ell_2} C^{rk}_{\ell_3}} a^i_{\ell_1 m_1} a^j_{\ell_2 m_2} a^k_{\ell_3 m_3}$$

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Speed depends on loops; naively ℓ^5

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$$\hat{f}_{\mathrm{NL}}^{\zeta\zeta\zeta} \propto \int y^2 dy \int d\Omega A^{\zeta}(\hat{n}, y) B^{\zeta}(\hat{n}, y) C^{\zeta}(\hat{n}, y)$$

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With *A, B* and *C* filtered maps. Computations per map $\mathcal{O}(\ell^3)$

Determines overall scaling algorithm, i.e. $\ell^5 \rightarrow \ell^3$

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Maps exist. Are complex maps, and estimator becomes

$$\hat{f}_{\rm NL}^{\zeta\zeta h} \propto \sum_{Mm_am_b} \begin{pmatrix} 1 & 1 & 2 \\ m_a & m_b & M \end{pmatrix} \int_{\mathbb{R}+} r^2 dr \int_{\mathbb{S}^2} d\Omega A^{\zeta}_{m_a}(\hat{n},r) B^{\zeta}_{m_b}(\hat{n},r) C^{h}_{M}(\hat{n},r)$$

Duivenvoorden et al in prep

Beyond 2-point statistics: outlook & conclusions

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-Similar to scalars, make sense to extend the search beyond 2-point statistics

-Contrary to scalars, not as easy to produce large violations, but at same time huge improvements on constraints are expected in the near future

-Could provide smoking gun for new physics

-Some very interesting models, such as heavy particles as relics of string theory, naturally produce effects at 3-level

-Actively building tools to apply to explore data (SO, CMBS4, litebird)

-Promising results, resolved the scaling problem of the KSW estimator