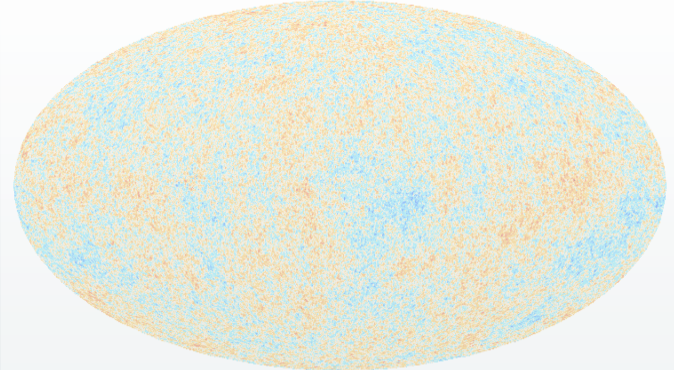


Ω_0

n_s H_0

Ω_b

τ

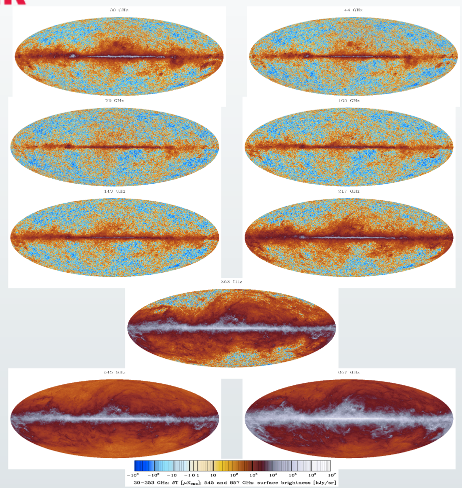
σ_8

Maps2Science

Graça Rocha
JPL/Caltech

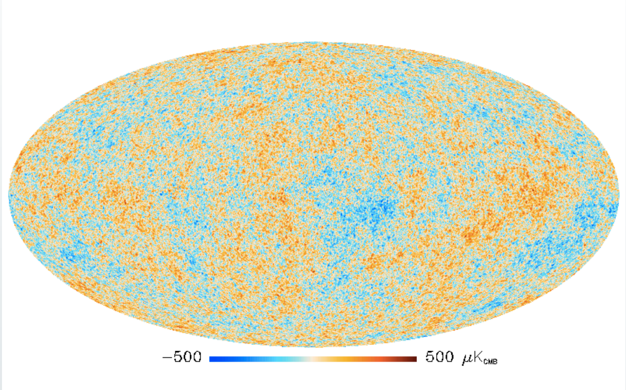
B-modes from space – Berkeley, 6th of December 2017

Parameter Extraction from the CMB Sky Maps (in a Nutshell)



Frequency maps

Component Separation



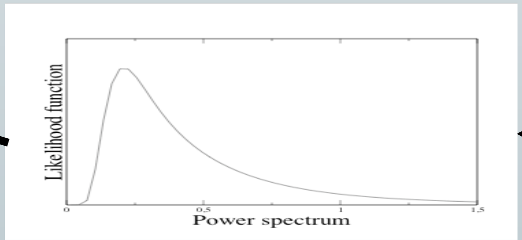
Cleaned CMB map

directly from sky maps to the likelihood

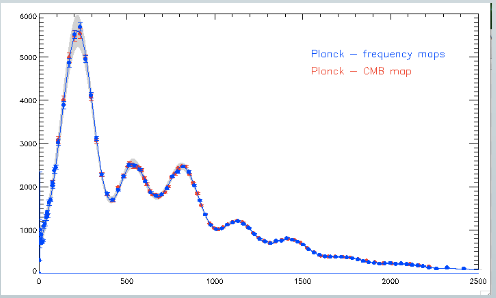
n_s Ω_b σ_8 Ω_0 H_0 τ

Cosmological parameters

MCMC



Likelihood



Angular Power spectrum

Hybrid Likelihoods

Low - ℓ vs High - ℓ

- For Gaussian fluctuations the Likelihood is a Multivariate Gaussian of the observed data:

$$L(\mathbf{d}|\mathbf{p}) = \frac{1}{2\pi^{N/2}|\mathbf{C}|^{1/2}} \exp\left(-\frac{1}{2}\mathbf{d}\,\mathbf{C}^{-1}\mathbf{d}^t\right)$$

- Direct extraction of science from the high-resolution pixelized maps is computationally expensive and in fact unfeasible + different sensitivities to systematic errors at different angular scales + increasingly Gaussian likelihood at smaller angular scales
- For low - ℓ : feasible for low-resolution maps, for l up to 30 or so
- For high - ℓ :
  - Resort to data compression – estimate angular power spectrum
  - Resort to Likelihood (computationally tractable) approximations
  - The exact Likelihood in harmonic space takes the form of a inverse Wishardt distribution (for a perfect CMB sky):

$$-2\ln P(\hat{C}_\ell|C_\ell) = (2\ell+1)\left(\ln|C_\ell| + \mathrm{Tr}\left(\hat{C}_\ell C_\ell^{-1}\right)\right),$$

  - In real world need to account for noise, mask, beams and residuals foregrounds
  - For sufficient large number of modes – CLT - Gaussian (correlated) likelihood

Types of Power Spectrum Estimators
In the beginning…

MLE **Maximum Likelihood Estimator**	**MADspec** **Cambridge ML** **BolPol**	**Computes the Power Spectrum C_l that maximizes the Likelihood:** $L(d \mid p) = \frac{1}{2\pi^{N/2} \mid C \mid^{1/2}} \exp\left(-\frac{1}{2} dC^{-1} d^t\right)$
	Teasing	**Importance Sampling combined with a Copula based approximation to the Likelihood**
	Gibbs sampling **Commander** **MAGIC**	**MCMC posterior estimation code that samples from the full CMB posterior by a Gibbs sampling scheme.** **It iteratively samples from the conditional densities:** **P(signal I power spectrum, data), P(power spectrum I signal, data)**
Pseudo - C_l **Master-based** **Monte Carlo**	**(Pol)Spice** **ROMAster** **Xpol** **CrossSpec**	**Estimates C_l : or $C(\theta)$:** $C_l = \frac{1}{2l+1}\sum_{m=-l}^{l} \mid a_{lm} \mid^2$; $C(\theta) = \langle T(q_1)T(q_2)\rangle = \sum_l \frac{l+1/2}{l(l+1)} C_l P_l(\cos\theta)$ **Uses either fast spherical transforms or fast evaluation of the 2-point Correlation function (Spice); Covariance estimated from MCs or from analytic appproximations**
Hybrid	**XFaster**	**Computes the C_l that maximizes the Likelihood:** $C_l = \frac{1}{2}\sum_{l'} F_{ll'}^{-1} Tr\left[C^{-1}\frac{\partial S}{\partial C_{l'}} C^{-1}\left(C^{obs} - N\right)\right]$ $C^{obs}_{lm,l'm'} = a^{obs}_{lm} a^{obs*}_{l'm'}$ **Covariance estimated via Fisher**
	GL- Hybrid	**Combines a Quadratic MLE at low-l and Pseudo - C_l at high-l** **With a smooth transition**

How do they compare ?

MADspec **Cambridge-ML** **BolPol** **Teasing**	**Exact**	Computationally expensive –> can estimate PS on low-resolution full sky maps (can compute high-resolution small patches of the sky)	**suitable for PSE @ low-l**
Gibbs sampling **Commander**	**Exact** **Bayesian**	Slow MCMC convergence for high-l due to low S/N, (this has been solved now) Unique code that provides the posterior distribution of the C_l Unique statistical framework for full propagation of errors to parameters	
Pseudo - C_l	**Approx**	Have to assume an approximation to the Likelihood –> Possibly ok for high-l, There is no good approximation at low-l Auto-Spectra- requires MC's of noise; signal, signal+noise –> computationally expensive; Cross-Spectra	**Suitable for PSE @ high-l**
Xfaster	**Approx**	Same as above but: Auto-Spectra - requires MC's for noise and signal separately - no need for MC's for signal+noise; Cross-Spectra	

Likelihood approx. for high l

- **Approximations :**

  Bond, Jaffe, Knox
  Verde et al.
  Contaldi
  ...

  - **Gaussian**

    $$\mathcal{P}_{Gauss}(\hat{\mathbf{C}}|\mathbf{C}) \propto exp\left\{-\frac{1}{2}(\hat{\mathbf{C}}-\mathbf{C})^T\mathbf{S}^{-1}(\hat{\mathbf{C}}-\mathbf{C})\right\}$$

  - **Lognormal, Offset Lognormal**

    $$\mathcal{P}_{LN}(\hat{\mathbf{C}}|\mathbf{C}) \propto exp\left\{-\frac{1}{2}(\hat{\mathbf{z}}-\mathbf{z})^T\mathbf{M}(\hat{\mathbf{z}}-\mathbf{z})\right\}$$

    $$z_\ell = \ln(C_\ell + x_\ell)$$
    $$M_{\ell\ell'} = (C_\ell + x_\ell)S^{-1}_{\ell\ell'}(C_{\ell'} + x_{\ell'})$$

  - **Equal Variance**

    $$\ln \mathcal{L} = -\frac{1}{2}G\left[e^{-(z-\hat{z})} - (1-(z-\hat{z}))\right]$$

    $$z = \ln(q_b + q_b^N)$$
    $$\sigma_z = \sqrt{F^{-1}_{bb'}}/(q_b + q_b^N)$$
    $$G = \left[e^{-\sigma_z} - (1-\sigma_z)\right]^{-1}$$

  - **WMAPLike**

    $$\ln \mathcal{P}_{WMAP}(\hat{\mathbf{C}}|\mathbf{C}) = \frac{1}{3}\ln \mathcal{P}_{Gauss} + \frac{2}{3}\ln \mathcal{P}_{LN}$$

  - **Offset Lognormal bandpower (Gaussian for TE and Offset Lognormal for TT, EE, BB)**
  - **SCR Likelihood (1/3 Like)** — **Gaussian on** $\hat{x} = \hat{C}_l^{1/3}$ — Smith, Challinor, Rocha

- **Attempts to properly account for the Temperature and Polarization**
  - **Xfaster Likelihood** — Contaldi, Rocha

    $$\ln L = -\frac{1}{2}\sum_\ell g_\ell(2\ell+1)\left[\frac{C_\ell^{obs}}{(\tilde{C}_\ell + \langle N_\ell\rangle)} + \ln\left(\tilde{C}_\ell + \langle N_\ell\rangle\right)\right]$$

  - **HL Likelihood** — Hamimeche, Lewis

    $$\ln L(C_l\,|\,\hat{C}_l) = -\frac{1}{2}\frac{2l+1}{2}\sum_{i}[g(D_{l,ii})]^2 = \frac{2l+1}{2}Tr[g(D_l^2)] \leftrightarrow with \leftrightarrow g(x) = sign(x-1)\sqrt{2(x-\ln(x)-1)}$$

Sometime later..
Planck Hybrid Likelihood

- Low-l
  - **Commander** – Gibbs sampling – Temperature
  - **BFLike** – Pixel-based - Polarization
  - **Lollipop**
  - **SimBal, SimLow**
- High-l
  - Spectra-based:
    - **PLIK – baseline**
    - **CamSpec**
    - **Hilipop, Mspec, ….**
  - Map-based:
    - **XFcmb - XFaster**

High - ℓ
In the end …

A. Spectra based - analysis of Multifrequency maps: 70, 100, 143, 217 GHz

- Employ a pseudo-Cl approach :

$$C_l = \frac{1}{2l+1}\sum_{m=-l}^{l} |a_{lm}|^2$$

Start with a numerical spherical harmonic transform (anafast) of the full-sky map -> debias and deconvolve to account for the noise, mask and beam – estimate Cross-spectra of frequency maps

Estimate somehow the bandpowers covariance matrix (MCs, analytical, Fisher,…) – needs to account for the noise + foreground residuals in the maps

Build the Gaussian correlated Likelihood for the full set of cross-spectra

Account for diffuse and extra-galactic foregrounds at spectra level and separate while estimating parameters

B. Map based - analysis of CMB maps (cleaned from diffuse foregrounds):

- Employ an approximation to the iterative, Maximum likelihood, quadratic band power estimator based on a diagonal approximation to the quadratic Fisher matrix estimator
- Estimate auto-spectra (can do cross-spectra) of the CMB maps generated with the 4 CompSep methods

The CMB map, with diffuse foreground removed, still has extragalactic foregrounds residuals –these are treated at Likelihood level and marginalised over

Some common complications

- Noise
- Accurate characterizing the noise of the instrument is difficult - noise is not white: 1/f noise, correlated noise,.. - psf measured, fit, use simulations, resort to splits of data eg Half-rings, end2end simulations

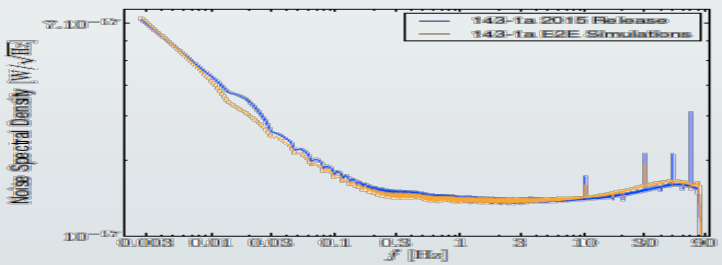


Figure 27. The PSD of the true noise for bolometer 143–1a as compared to that generated using the end-to-end simulation.

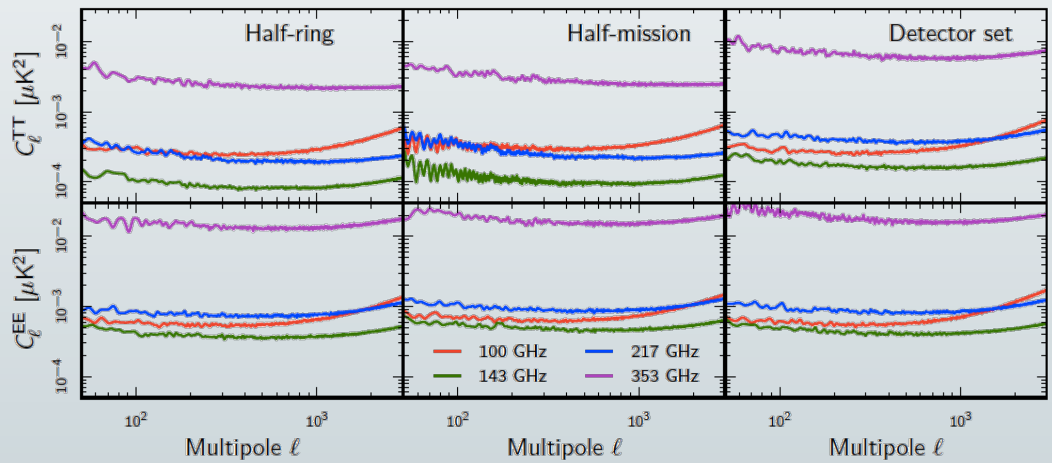
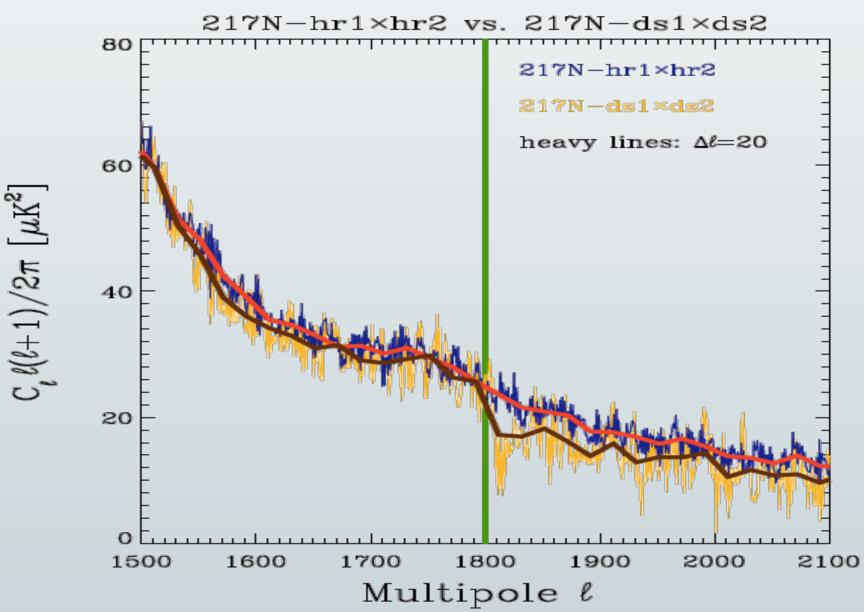


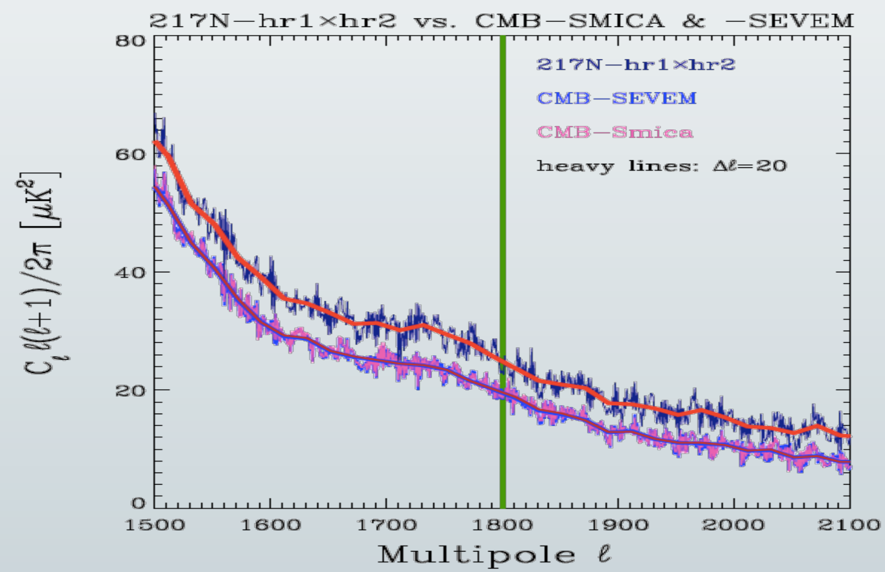
Fig. 9. *TT* and *EE* power spectra reconstructed from the half-difference between data subset maps for the dipole-calibrated channels.

- Beams
  - Estimate the beam transfer function use different approaches (FEBeCoP, QuickBeam) , it might vary spatially (it does for Planck), account for cross-polar component or show it is negligible, etc..
  - Formalism to estimate the full Tensorial beam description in place(for beam and window or transfer function)

Some common complications

- An example - features in the power spectrum
  - For example: l=1800 feature due to he 4K line removal





  - rhs HR data input to CompSep analysis - full data coaddition washes the feature away – hence this feature is mitigated by the compsep step
  - On the other hand - poor understanding of the residuals in the CMB maps – resort to simulations to characterize them

Sometime later..
Planck Hybrid Likelihood

- Low-l
  - **Commander** – Gibbs sampling – Temperature
  - **BFLike** – Pixel-based - Polarization
  - **Lollipop**
  - **SimBal, SimLow**

- High-l
  - Spectra-based:
    - **PLIK – baseline**
    - **CamSpec**
    - **Hilipop**
    - **.....**
  - Map-based:
    - **XFcmb - XFaster**

Low- ℓ
In the end…

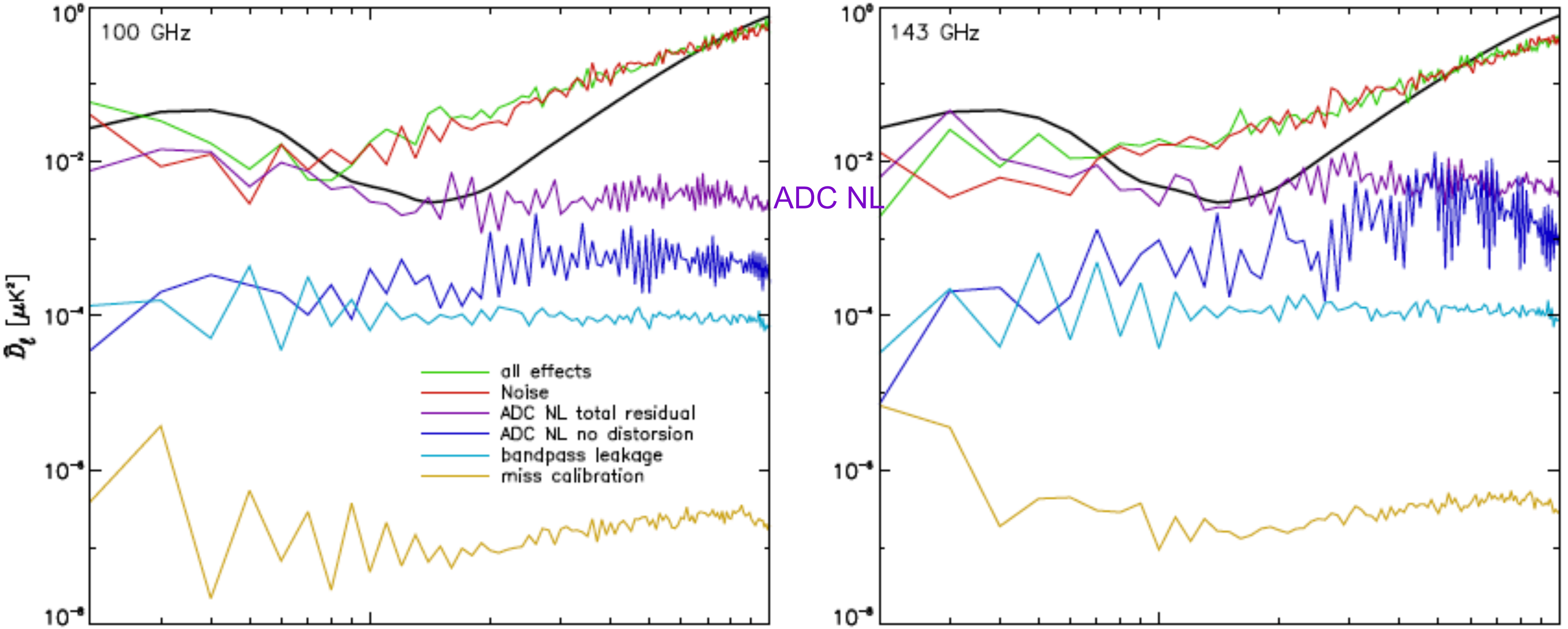
A

- Commander – Gibb Samples the Power Spectra and constructs Blackwell-Rao likelihood
- BFLike - Pixel-based Likelihood – needs an accurate description of the pixel noise covariance matrix (non-diagonal)
- SimLow

B

- Estimates spectra cross - spectra: PCL or QML – still needs a description of the noise covariance matrix to construct the bandpowers covariance matrix
- Lollipop is an approximation to the Hamimeche & Lewis Likelihood for cross-spectra at low multipoles
- SimBal is a simulation based Likelihood targeted at τ

Simulated systematics propagated to EE-spectra for HFI



Systematics at low ell in HFI channels dominated by ADC nonlinearity

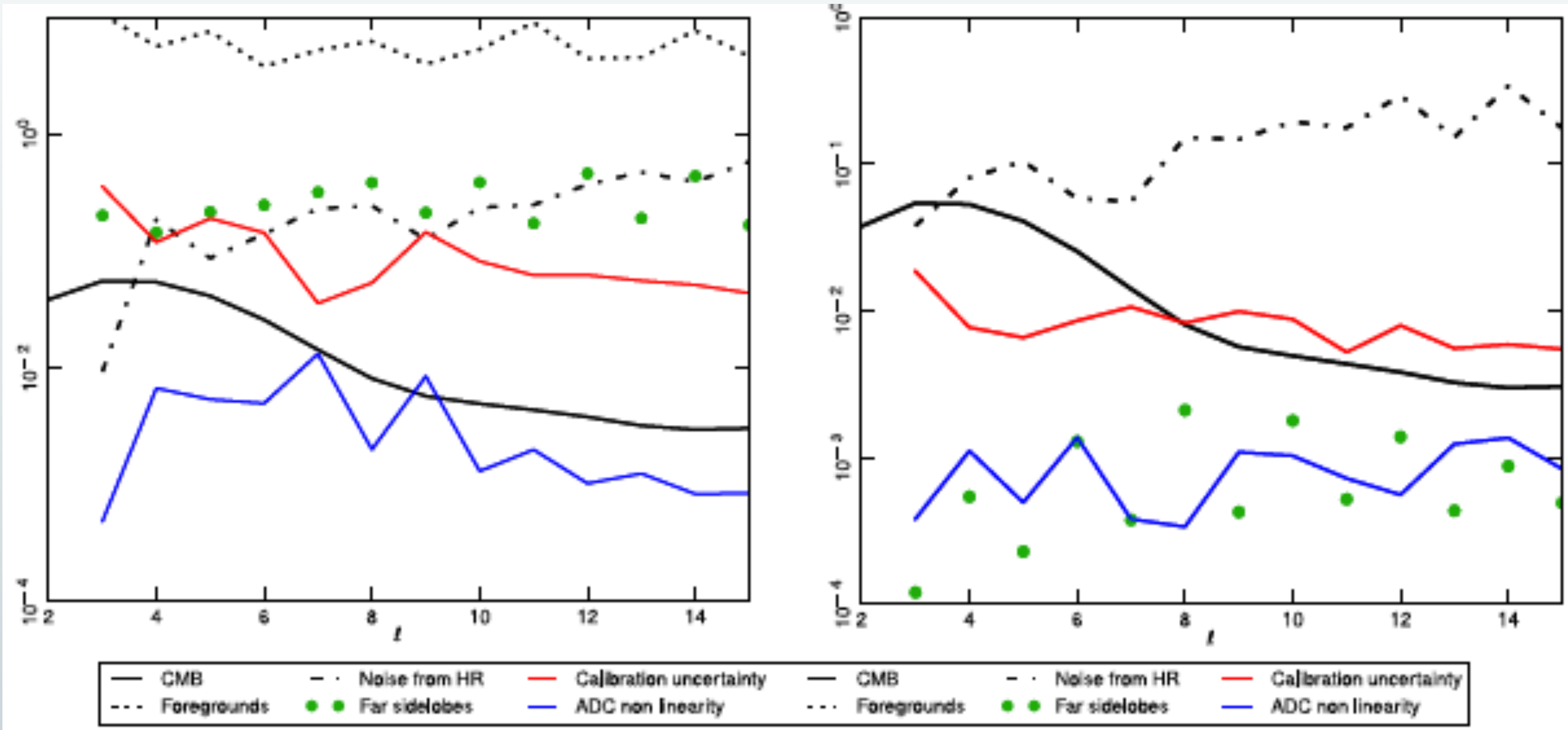
Purely instrumental -> Analog-to-Digital Converter (ADC) nonlinearity; Time response residuals; Relative gain between detectors; Possible time-variable gain

Scan-strategy related -> Far sidelobe pickup; Zodiacal light emission; Bandpass mismatch T → P leakage

ADC nonlinearity: **Can be (mostly) corrected by applying a time-variable linear gain correction.**

Simulated LFI systematics in EE spectra

30 GHz 70 GHz



Systematics at low ell are mostly dominated by

- **calibration uncertainty**
- **far sidelobe pickup at 30 GHz**

XFaster

Band powers estimated with XFaster for each of the CMB maps generated by

COMMANDER, NILC, SEVEM, SMICA

XFaster: approximation to the iterative, Maximum likelihood, quadratic band power estimator - based on a diagonal approximation to the quadratic Fisher matrix estimator

$$\tilde{C}_\ell = \sum_b q_b \tilde{C}^S_{b\ell} = \sum_b \left(\frac{1}{2} \sum_{b'} \mathcal{F}^{-1}_{bb'} \sum_\ell (2\ell+1) g_\ell \frac{\tilde{C}^S_{b'\ell}}{(\tilde{C}_\ell + \langle \tilde{N}_\ell \rangle)^2})(\tilde{C}^{obs}_\ell - \langle \tilde{N}_\ell \rangle) \right) \tilde{C}^S_{b\ell}$$

$$\mathcal{F}_{bb'} = \frac{1}{2} \sum_\ell (2\ell+1) g_\ell \frac{\tilde{C}^S_{b\ell} \tilde{C}^S_{\ell b'}}{(\tilde{C}_\ell + \langle \tilde{N}_\ell \rangle)^2}$$

The iterative scheme starts from a flat or some initial guess spectrum model - the result is a band power spectrum and the associated Fisher matrix (eg. the uncertainty of the band powers)

- Half-Difference of data splits used to estimate the noise bias in the power spectra extracted from the Half-Sum maps

XFaster - XFcmb

- Use a Gaussian Correlated likelihood and a MCMC sampler (CosmoMC) for 50 <l<2000
  - 6 cosmological parameters
    - impose a Gaussian prior on tau: eg $\tau = 0.07 \pm 0.006$
    - also considered the low- l Likelihood (TEB)
  - 5 (6) foreground parameters: $\left(A_{ps}^{TT}, A_{ps}^{EE}, A_{cib}, (n_{cib}), A_{tsz}, A_{ksz}\right)$
    - A_{ps} for TT and EE - the amplitude of a Poisson component, $C_l = A_{ps}$=constant
    - A_{cl} ie $_{ACIB}$ - the amplitude of a clustered component with shape $D_\ell = \ell(\ell+1)C_\ell / 2\pi \propto \ell^{0.8}$, D_l at l = 3000 in units of μK^2
    - A_{tsz}, A_{ksz} Amplitude of thermal and kinetic SZ template with amplitude set at l=1000
  - Also considered code tailored FG templates based on FFP8 simulations

Component Separation

- Commander – parametric model fitting in pixel space
- NILC – needlet internal linear combination in harmonic space
- SEVEM – template fitting in pixel space
- SMICA – parameter fitting in harmonic space

Masks

UT78

UP78

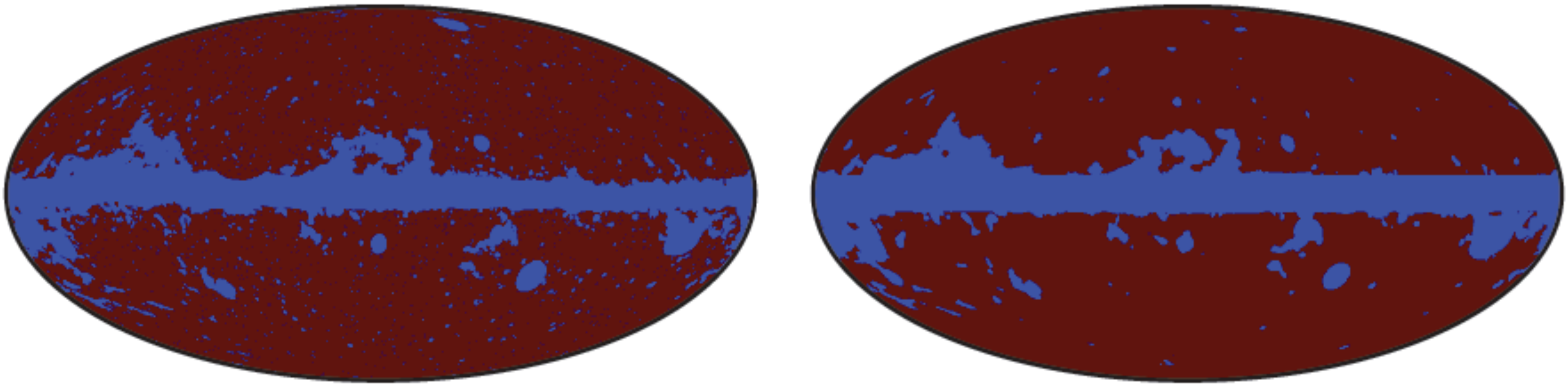


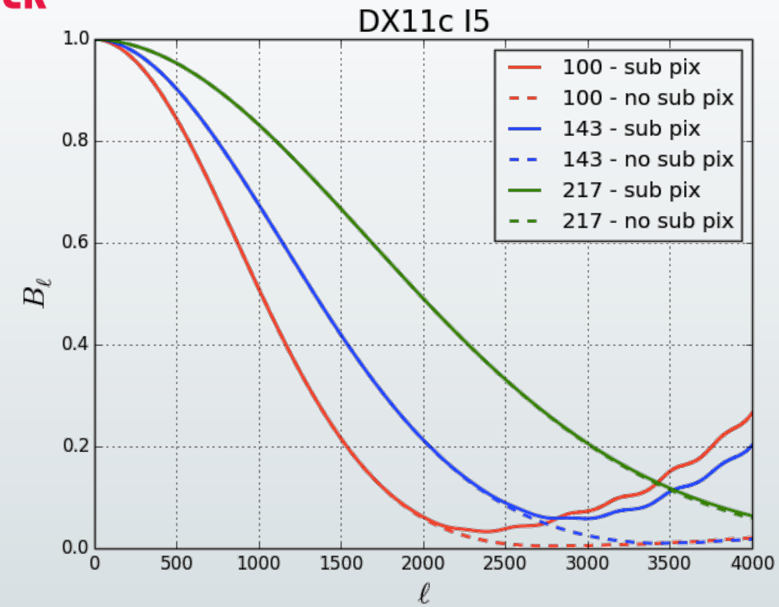
Fig. 8. Total intensity (left) and polarization (right) union masks.

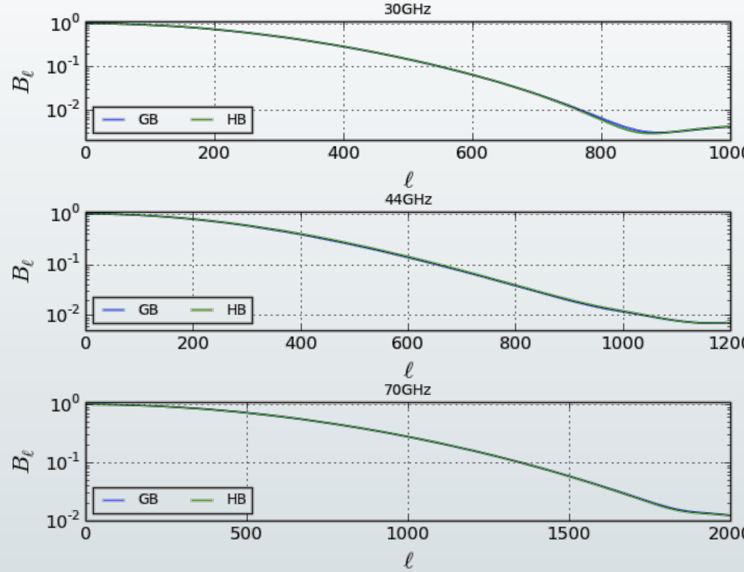
Beams

FEBeCoP

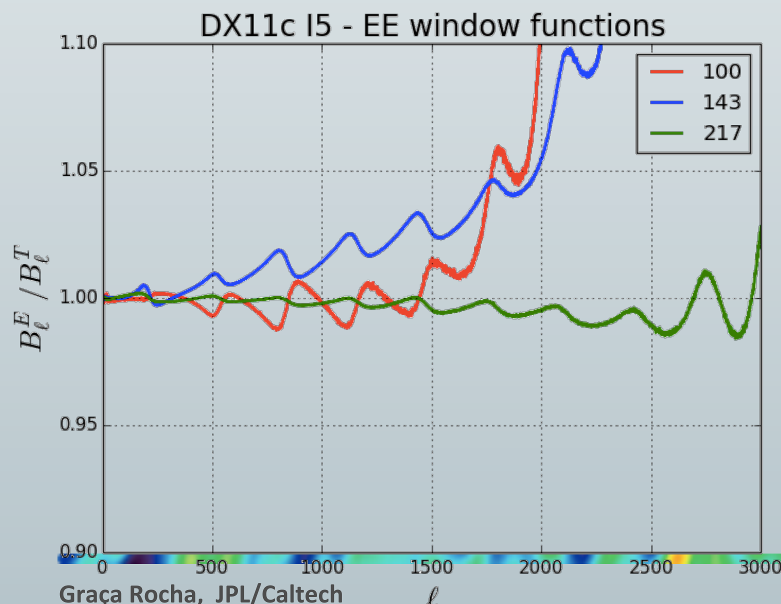
HFI

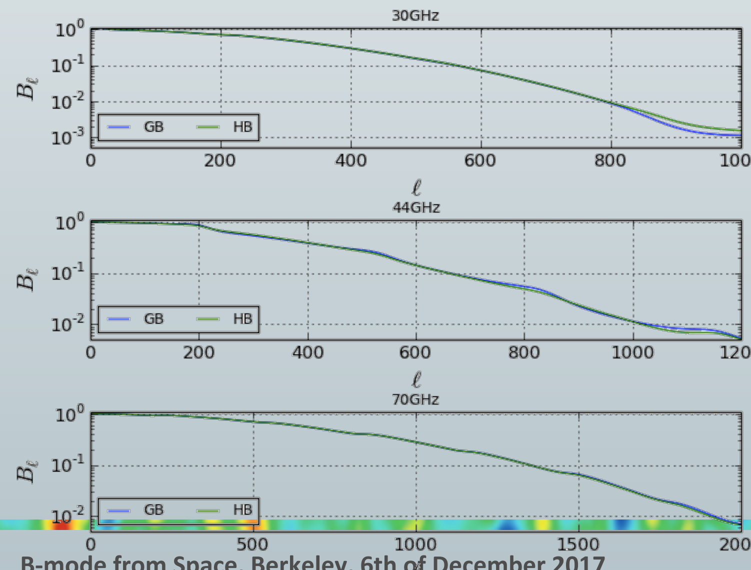
LFI





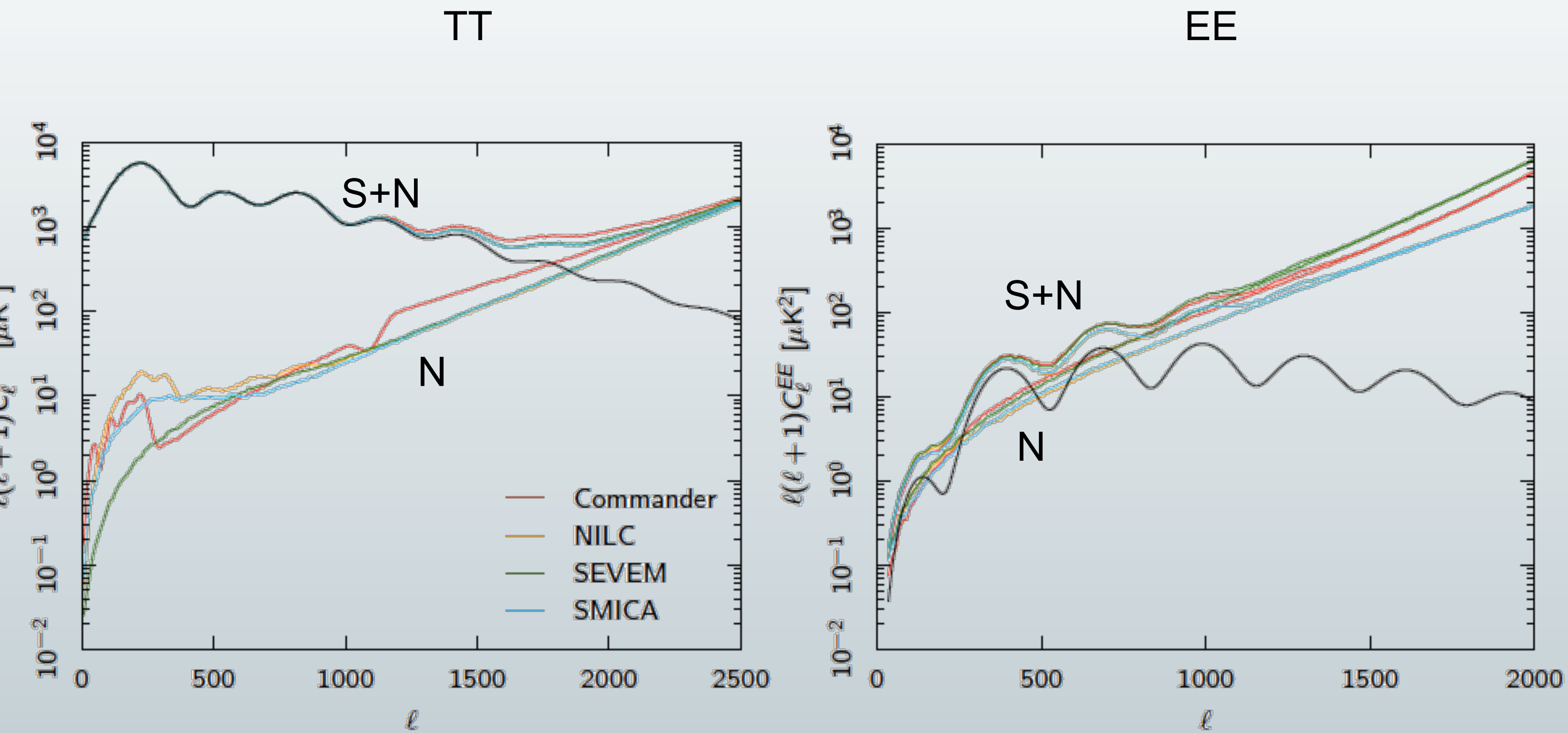
TT





EE

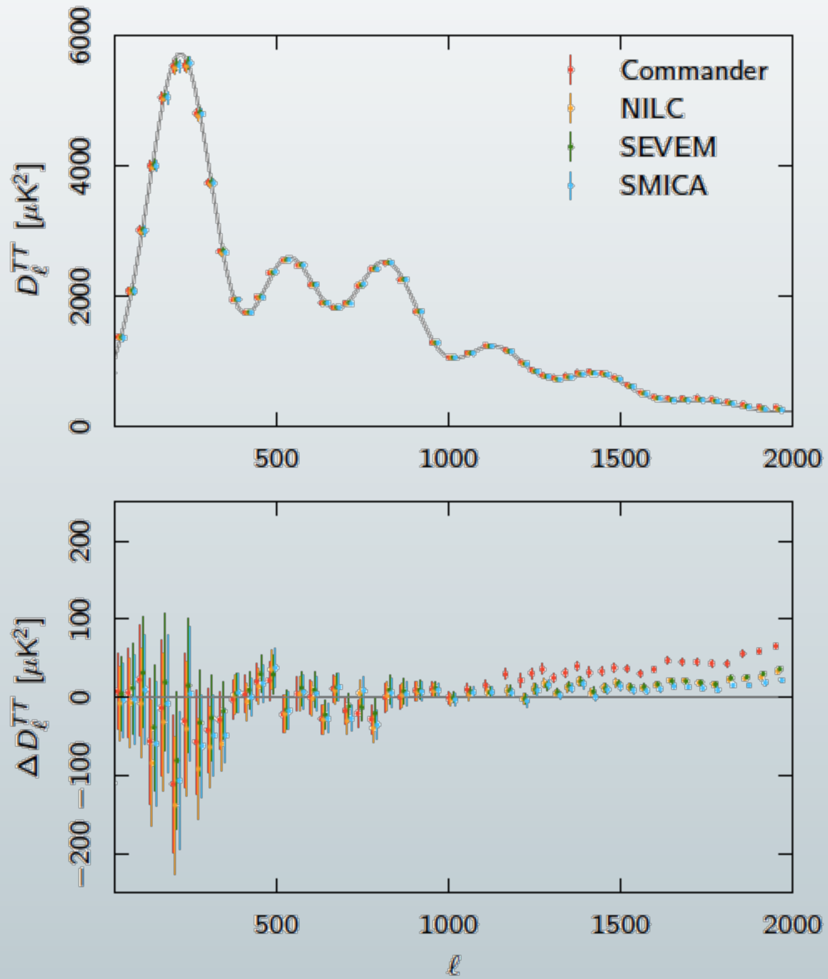
Power Spectra of CMB maps



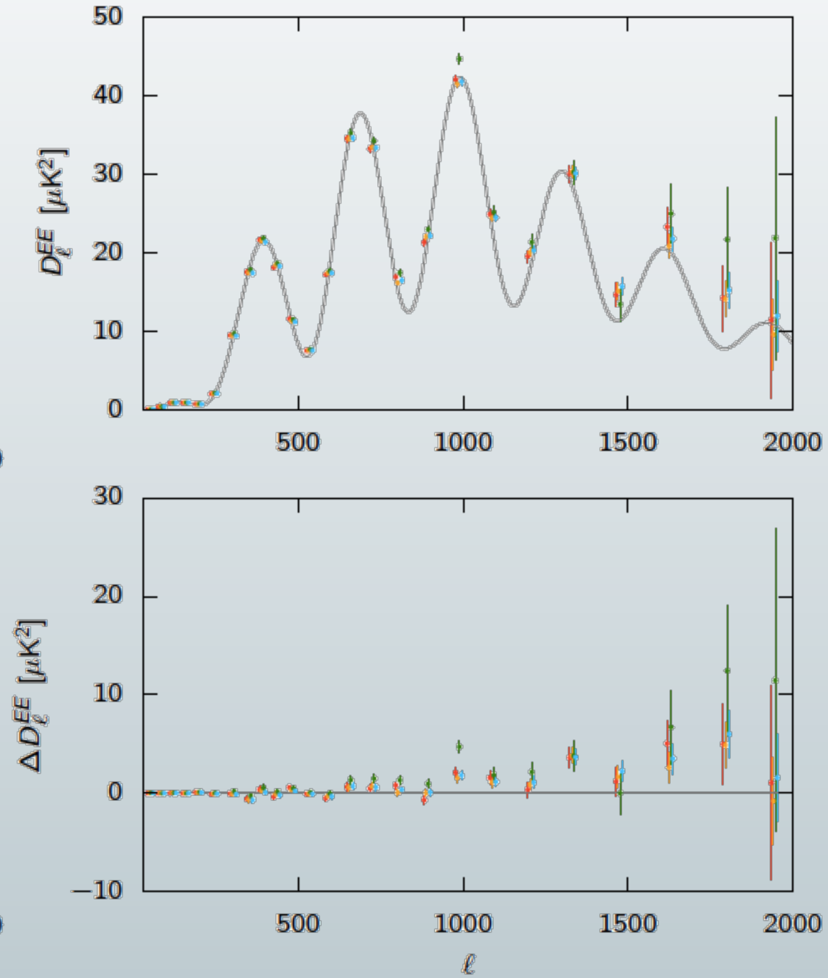
Signal+Noise -> half-mission half-sum (HMHS)
Noise -> half-mission half-difference (HMHD)

XFaster BandPowers
(before EGF subtraction)

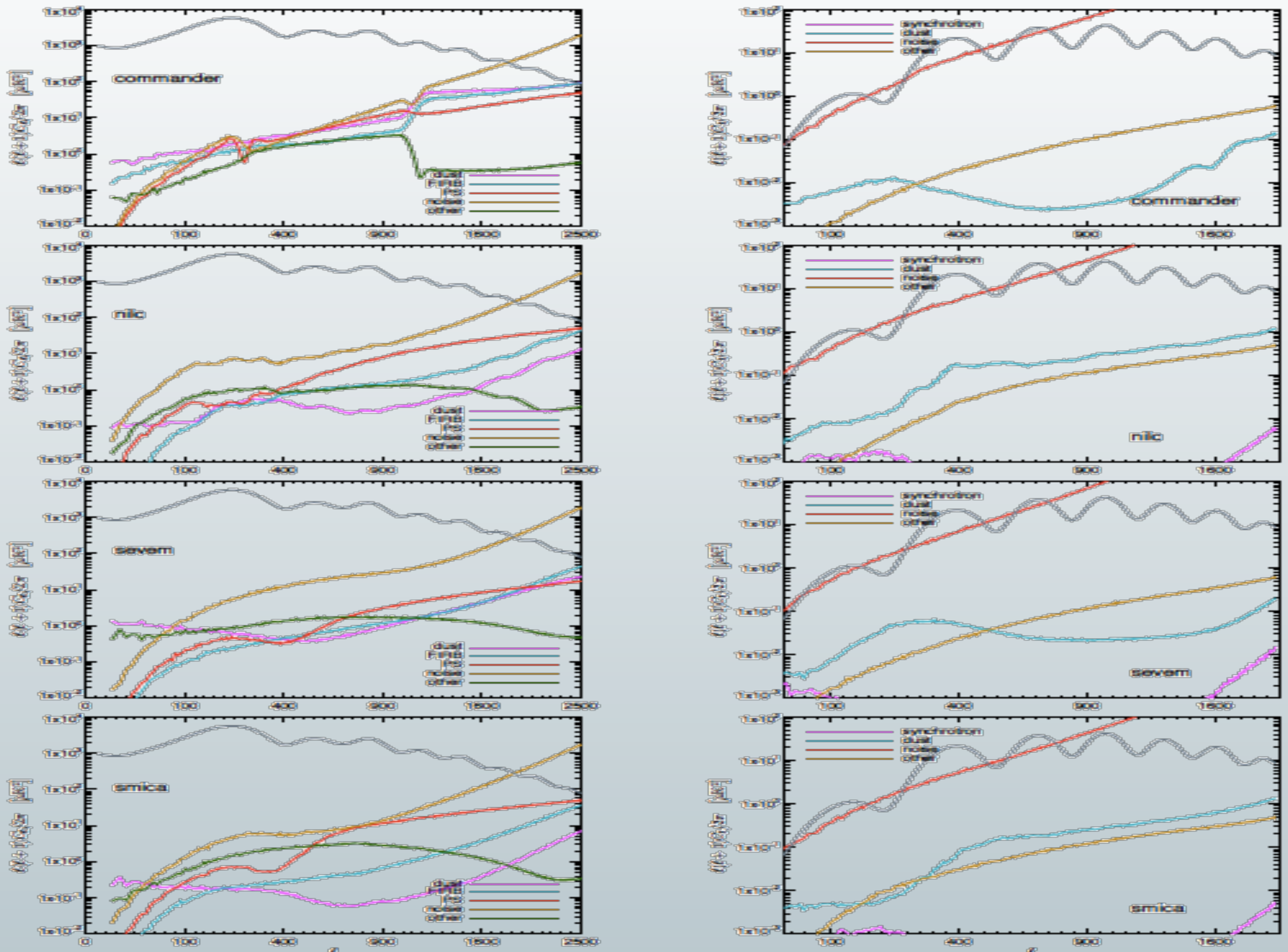
TT



EE

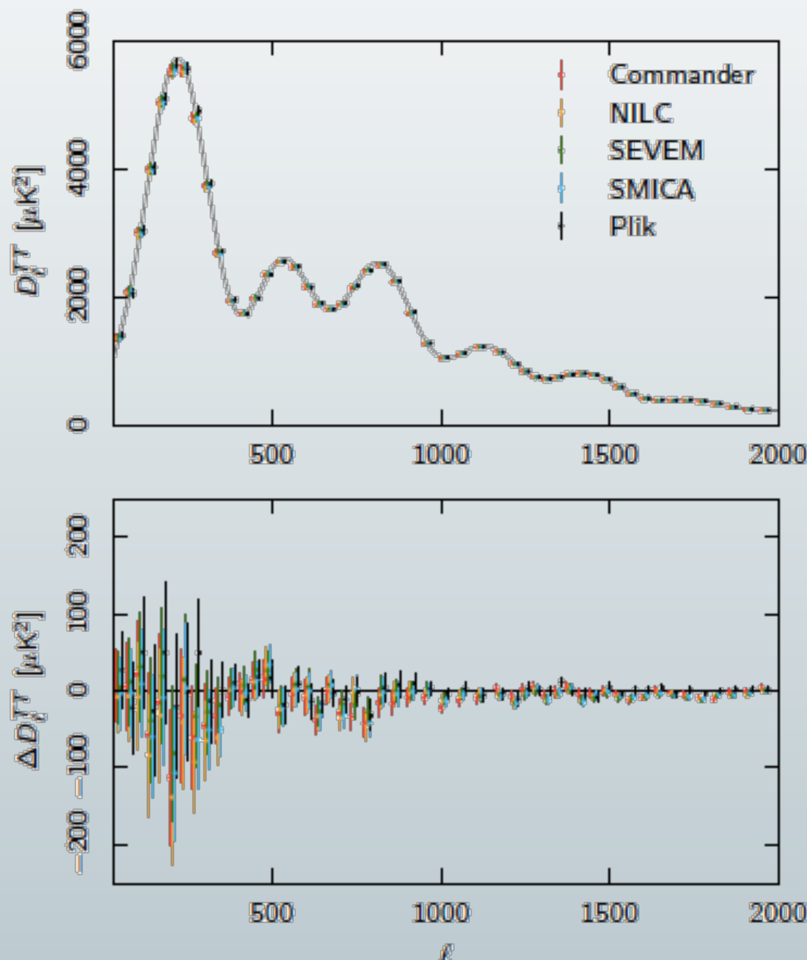


Foreground model
Method-tailored full-sky from (FFP8) simulations.

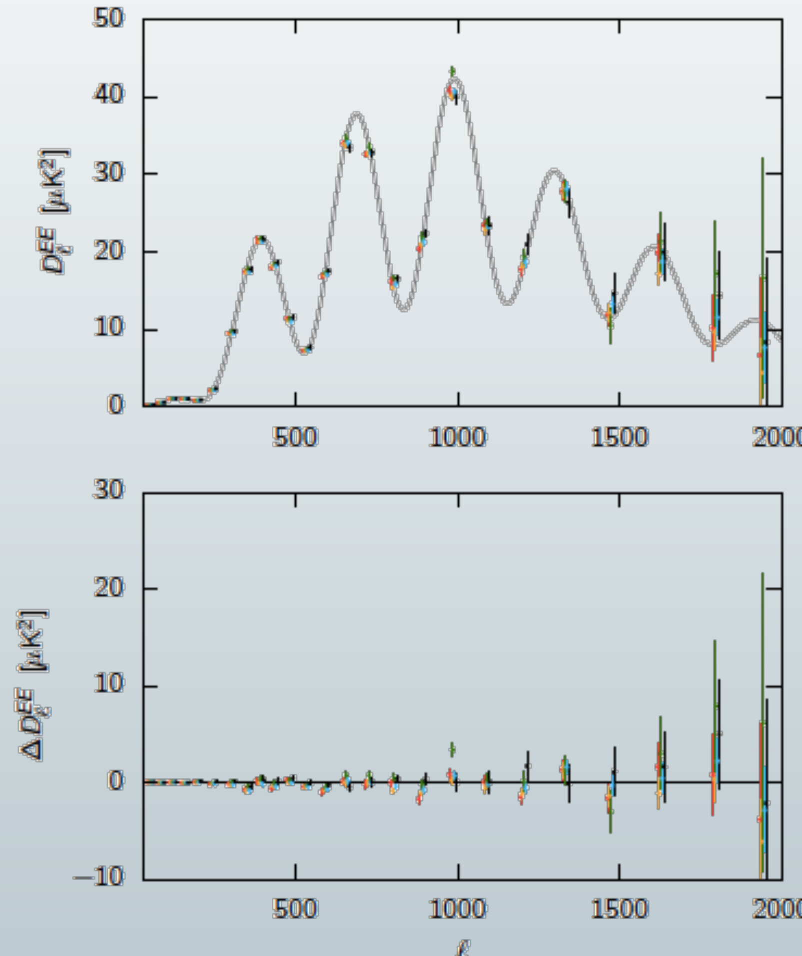


XFaster BandPowers
(after EGF subtraction)

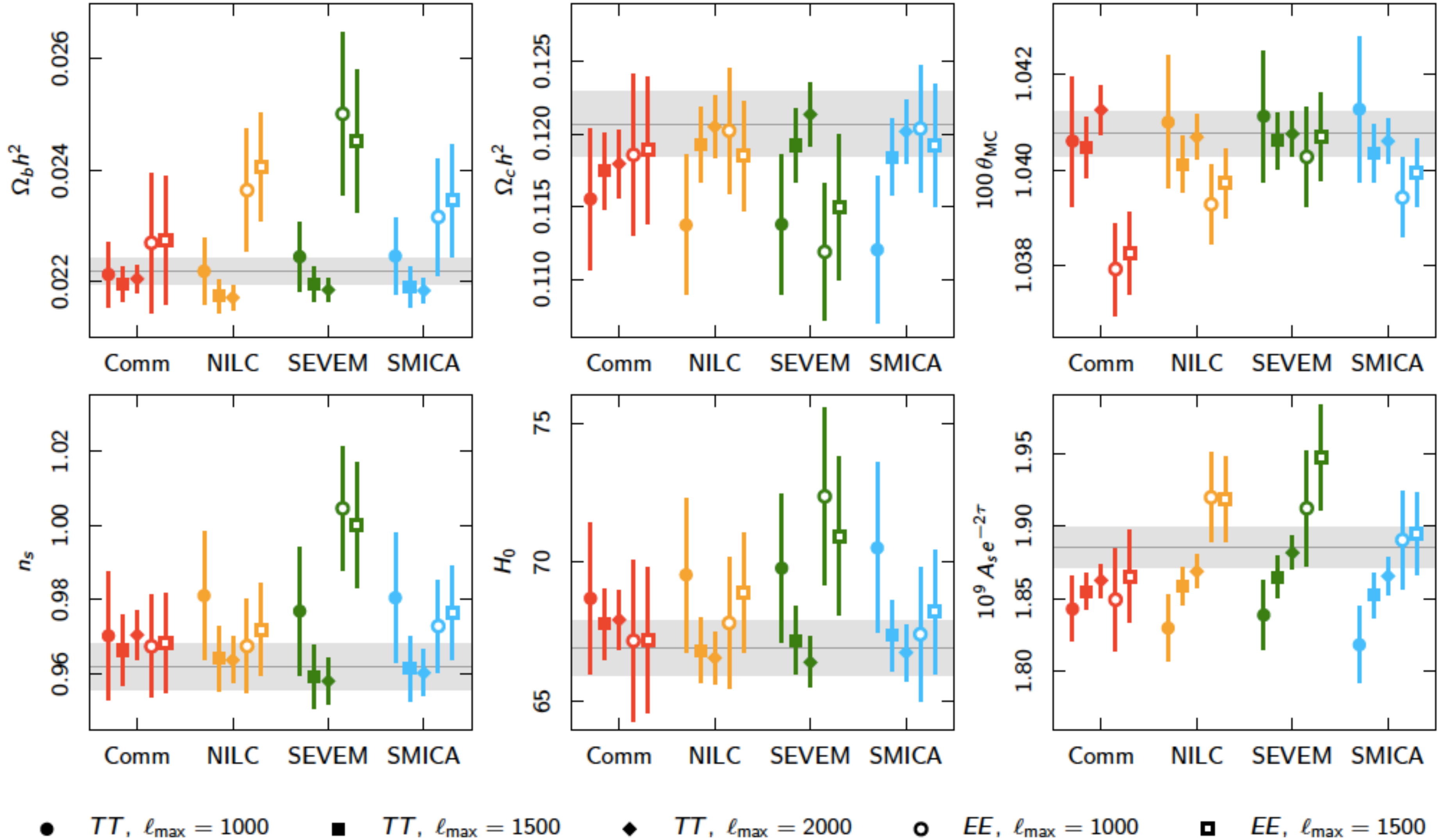
TT



EE



Cosmological Parameters



Higher order statistics

- Non – Gaussianity

  - After subtraction of the lensing-ISW correlation contribution,
    For SMICA - within 1 sigma from other methods and FG cleaned maps

$$f_{NL}^{local} = 0.6 \pm 5.0$$

  - Temperature alone and same SMICA map

$$f_{NL}^{local} = 1.3 \pm 5.7$$

  - Polarization alone

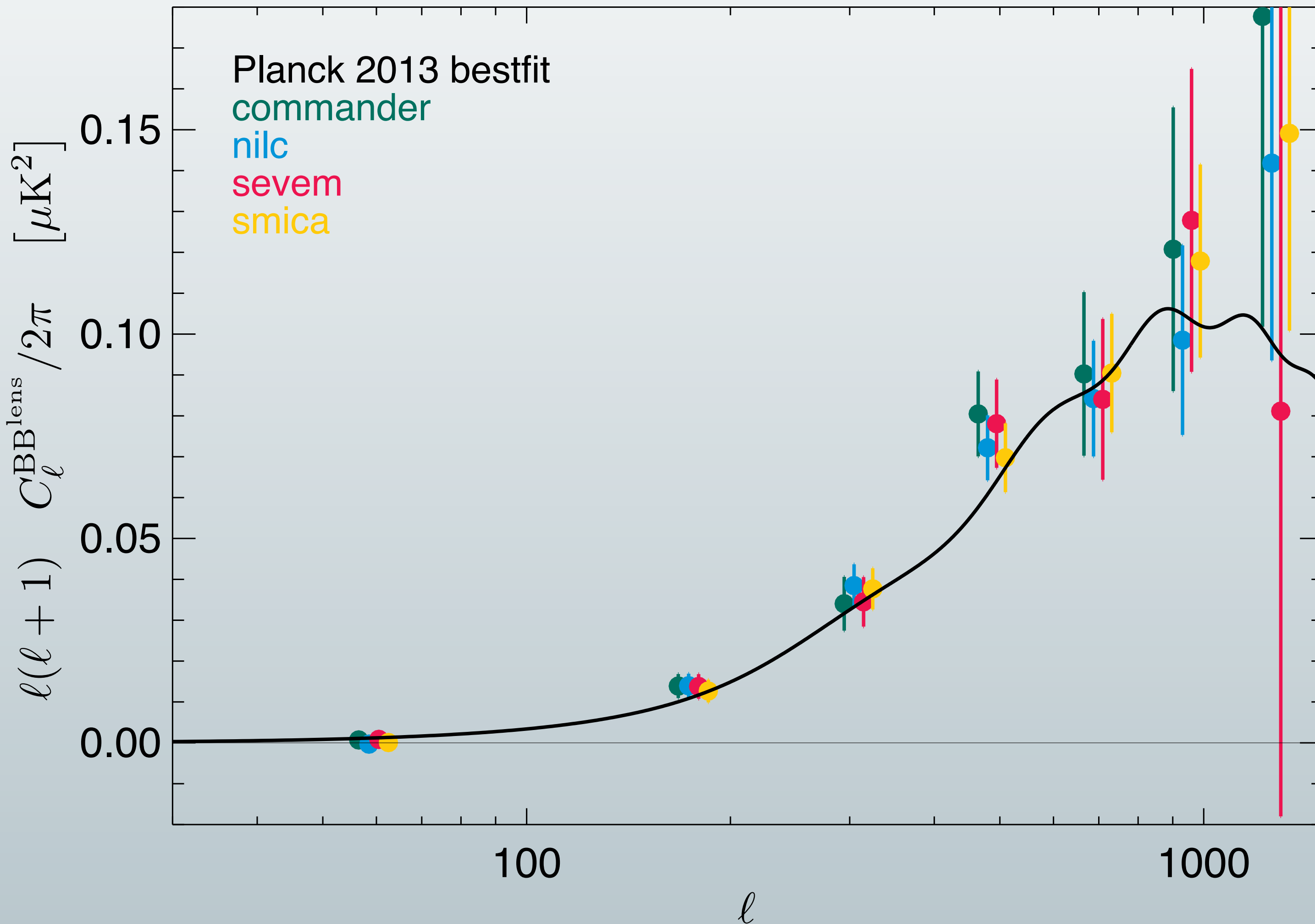
$$f_{NL}^{local} = 28.4 \pm 31.0$$

Consistency with Gaussianity

Lensing B-modes

- Gravitational lensing by large scale structure

Predicted lensing B modes of CMB polarization



Alens~1

Planck Low-l Likelihood
Commander

For l ≤ 50 - we adopt Gibbs sampling approach as implemented in **Commander**

Data model - > multi-frequency obs + set of foreground signal:

$$\mathbf{d}_\nu = \mathbf{s} + \sum_i \mathbf{f}^i_\nu + \mathbf{n}_\nu.$$

CMB field - Gaussian random field with power spectrum $\mathbf{C}_l$,

Noise - Gaussian with covariance $\mathbf{N}_\nu$

- Model: single low-frequency foreground comp (sum of synchrotron, anomalous microwave emission, and free-free emission), a carbon monoxide (CO)comp, and thermal dust component, in addition to unknown monopole and dipole comp at each frequency.
- Map out the full posterior distribution, **P** (**s**; **f**^{i} ;**C**$_l$|**d**), using a Gibbs sampling (MC sampling) . Directly drawing samples from **P** (**s**; **f**^{i} ;**C**$_l$|**d**) is computationally prohibitive, but this algorithm achieves the same by iteratively sampling from each corresponding **conditional** distribution:

$$\mathbf{s} \leftarrow P(\mathbf{s}|\mathbf{f}, C_\ell, \mathbf{d})$$

Multivariate Gaussian distribution

$$\mathbf{f} \leftarrow P(\mathbf{f}|\mathbf{s}, C_\ell, \mathbf{d})$$

does not have a closed analytic form, but can be mapped out numerically

$$C_\ell \leftarrow P(C_\ell|\mathbf{s}, \mathbf{f}^i, \mathbf{d}).$$

Inverse Gamma distribution

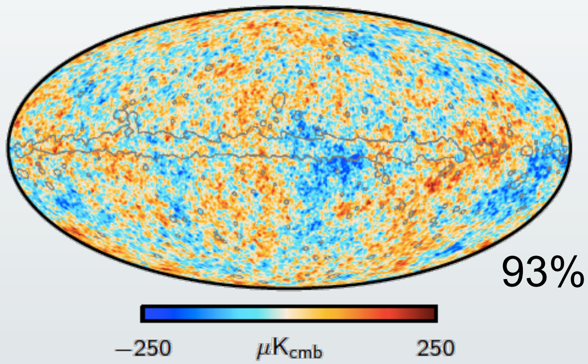
- For CMB Likelihood Ensemble of CMB sky samples , s^k

$$\mathcal{L}^k(C_\ell) \propto \frac{\sigma_{\ell,k}^{\frac{2\ell-1}{2}}}{C_\ell^{\frac{2\ell+1}{2}}} e^{-\frac{2\ell+1}{2}\frac{\sigma_{\ell,k}}{C_\ell}}. \quad \text{->} \quad \mathcal{L}(C_\ell) \propto \sum_{k=1}^{N_{\rm samp}} \mathcal{L}^k(C_\ell).$$

BR

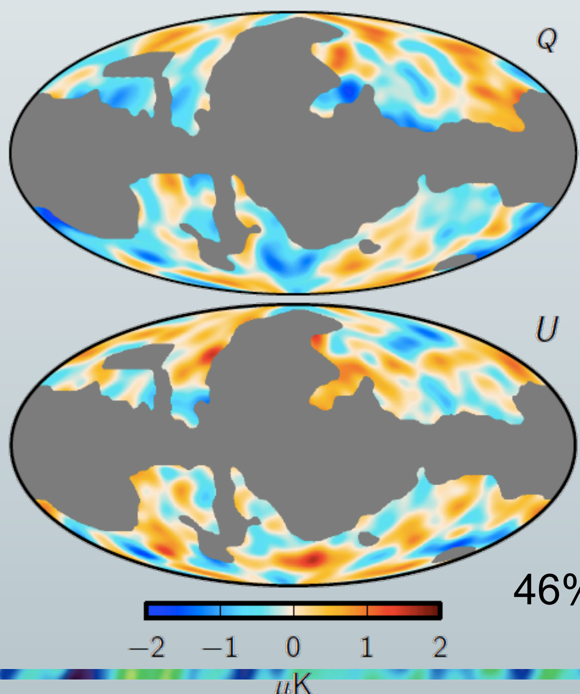
Low- ℓ

Commander T map



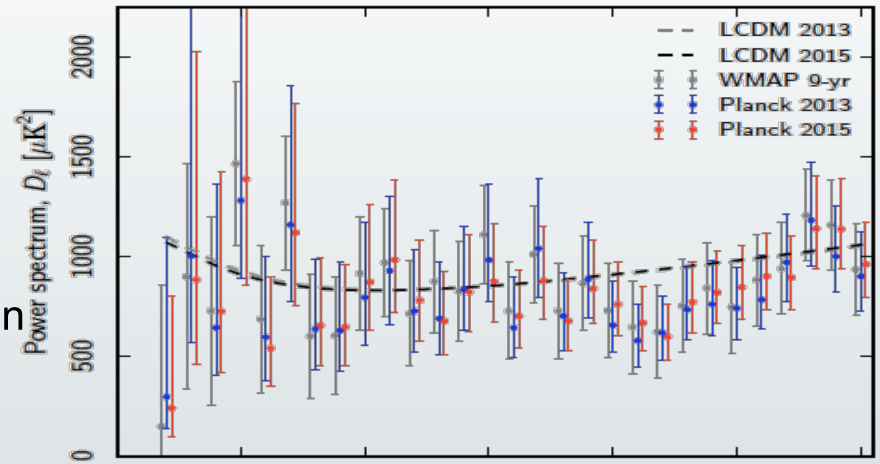
93% sky fraction

70GHz pol. Foreground cleaned with 30GHz and 353GHz

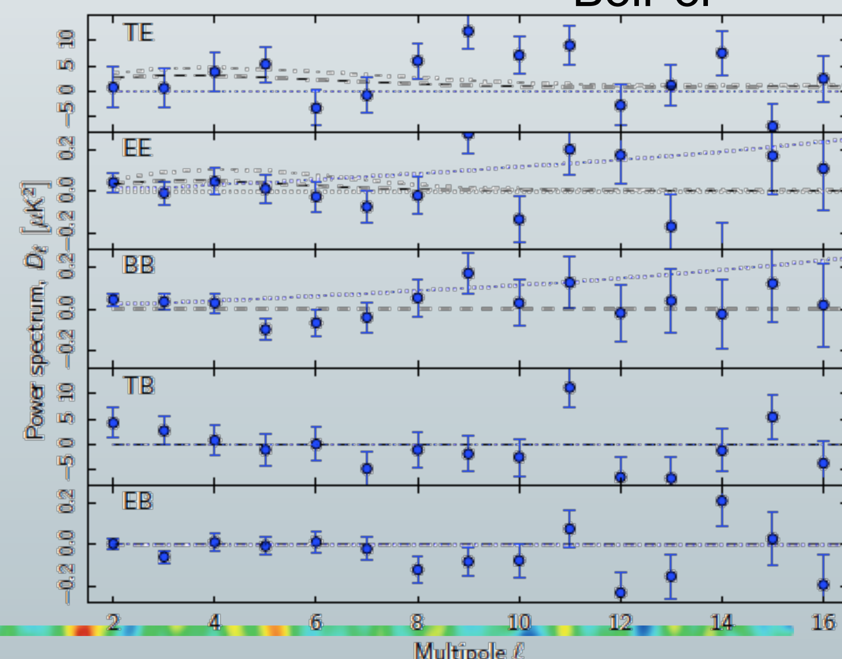


46% sky fraction

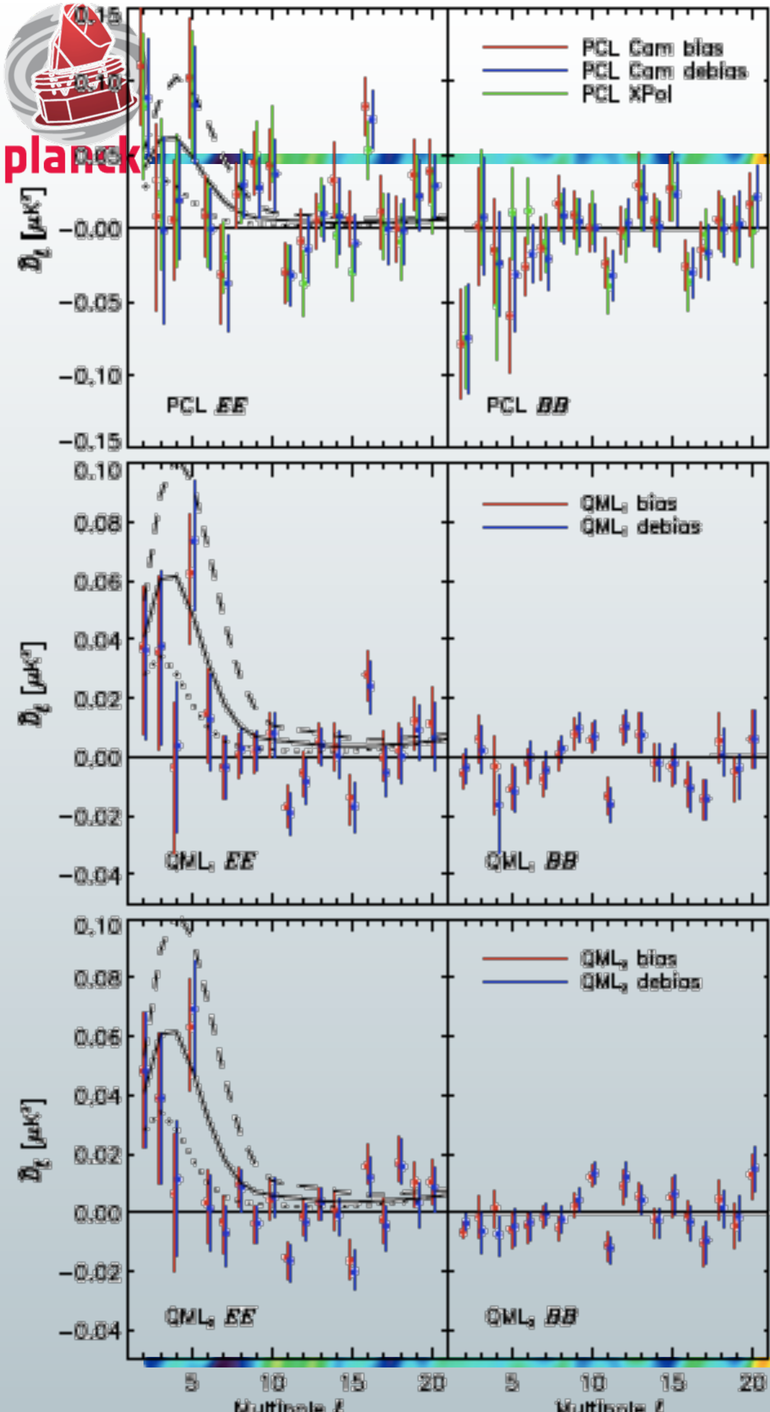
BR



BolPol



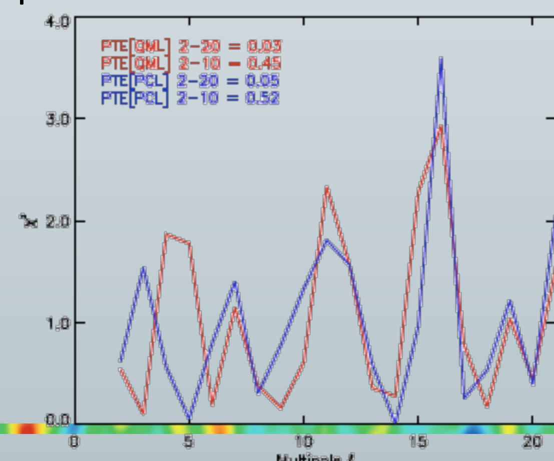
Low- ℓ



- Main results: Use 100 x 143 (foreground cleaned)
- Cross checks from 100 x 70 and 143 x 70, 100x100 and 143x143 autospectra
- Use multiple spectral estimator techniques
  - Pseudo-Cl (PCL) - *Lollipop*
  - Quadratic Maximum Likelihood (QML)
- Use instrumental simulations to compute and subtract bias due to systematics (very small), and to construct pixel-pixel noise covariance
- Simulation based Likelihood - *SimBal*

Black lines - Model for t=0.05 (dotted), 0.07(solid) and 0.09 (dashed)

QML computed with two different simulation sets

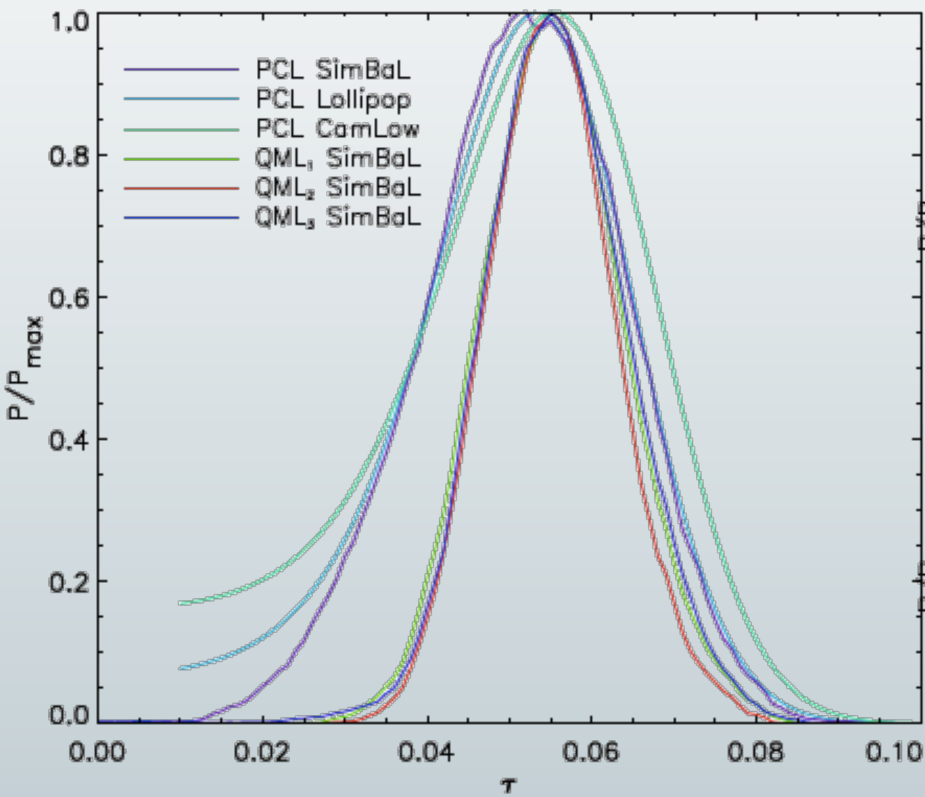


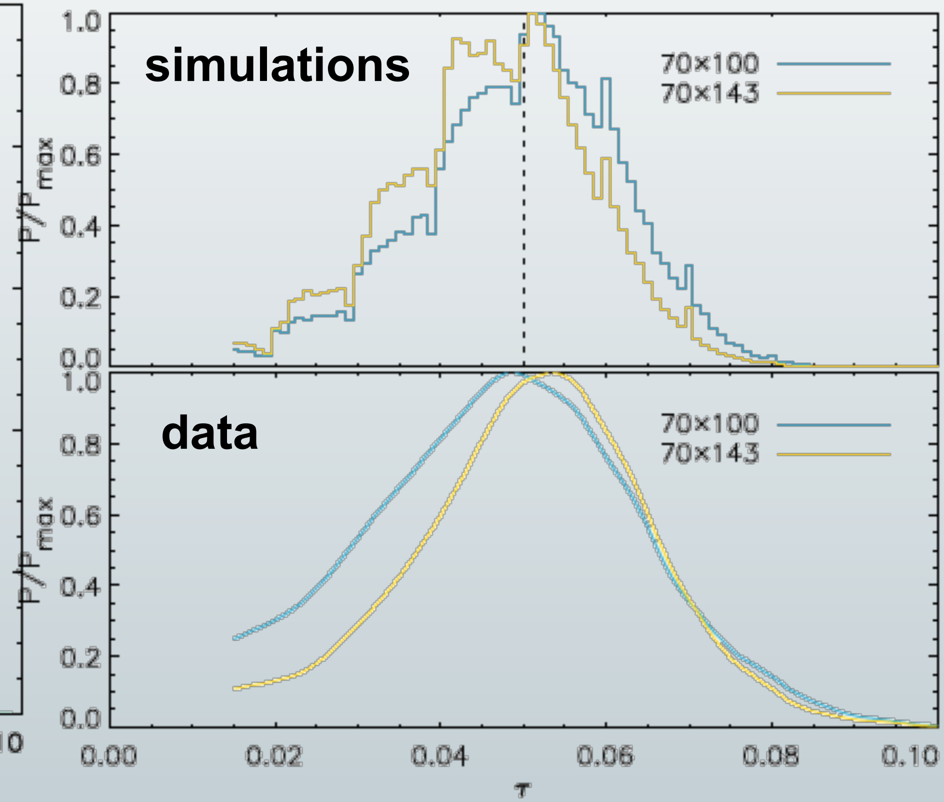
Tau results

Instrumental cross-check: HFI x LFI

100x143

70x100 70x143





QML spectra : $0.055^{+0.009}_{-0.009}$

$\tau = 0.049^{+0.015}_{-0.019}$ for the 70×100 cross-spectra

$\tau = 0.053^{+0.012}_{-0.016}$ for the 70×143 cross-spectra

Summary