An integrable Lorentz-breaking deformation of 2d CFTs

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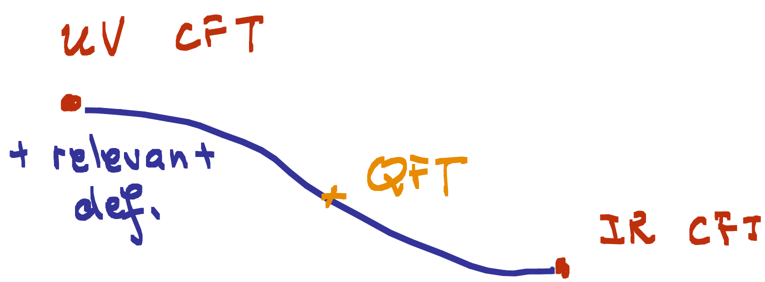
Motivation

- usual, local QFT framework



- non-local, UV complete QFTs ?

  - quantum gravity

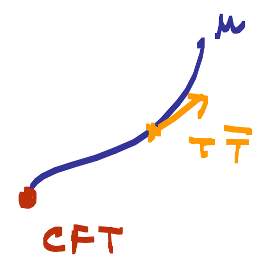
  - holography (not asymptotically AdS)

$T\bar{T}$-deformed CFT_2

Smirnov, Zamolodchikov '16
Cavaglià, Negro, Szecsenyi, Tateo '16
Dubovski, Flauger, Gorbenko '12-'17
+ others

- universal deformation of 2d QFTs

$$S_\mu = S_{CFT} + \mu \int d^2z \, (T\bar{T} - \Theta^2)$$



- deformation irrelevant but integrable

  - finite-size spectrum $E(\mu, R)$, thermodyn, KdV charges
    (universal!)

- theory non-local (scale μ), but well-defined UV

S-matrix : $$S_\mu = e^{\frac{is\mu}{4}} S_0$$

not polynomially bounded

$T\bar{T}$-deformed CFT_2

Applications:

- $CFT_2 = 24$ free bosons $\rightarrow$ worldsheet bosonic string $(\mu = \ell_s^2)$

  (2d quantum gravity)

  Dubovski et al, Tateo et. al.

- CFT_2 = large N, large gap, $\mu < 0$ $\rightarrow$ holographically dual to

  AdS_3 gravity w/ finite bulk cutoff $\quad r_c = 1/\sqrt{-\mu}$

  McGough, Mezei, Verlinde '16

- $CFT = Sym(CFT_2)^N$, $\mathcal{O}_{T\bar{T}} = \sum_{i=1}^{N} T_i \bar{T}_i$ $\rightarrow$ holographic dual to

  an asymptotically linear dilaton backgnd

  Giveon, Itzhaki, Kutasov '17

This talk

- another universal deformation of 2d CFTs w/ a $U(1)$ current

$$S_\mu = S_{CFT} + \mu \int d^2z \, (J\bar{T} - \bar{J}\Theta)$$

$(1,2)$

- breaks Lorentz invariance
- deformation irrelevant but integrable
- preserves $SL(2,\mathbb{R})_L \times U(1)_R$ invariance

$\downarrow$

interesting symmetry enhancement

Plan

- definition of the $J\bar{T}$ deformation
- the finite-size spectrum & thermodynamics
- holographic interpretation & dictionary
- match to the field theory prediction
- symmetry enhancement
- conclusions & future directions

The $J\bar{T}$ deformation in field theory

The $J\bar{T}$ operator

- assume deformed CFT can be treated as a quasi-local QFT below some scale
- translation & u(1) invariance → local conserved currents

$$\bar\partial \underbrace{T_{zz}}_{T} + \partial \underbrace{T_{\bar z z}}_{0 \text{ by } SL(2,\mathbb{R})_L} = 0 \qquad \bar\partial \underbrace{T_{z\bar z}}_{\Theta} + \partial \underbrace{T_{\bar z\bar z}}_{\bar T} = 0 \qquad \bar\partial J + \partial \bar J = 0$$

- the OPE

$$\mathscr{C}(z,z') = J(z)\bar T(z') - \bar J(z)\Theta(z') \sim O_{J\bar T}(z') + \text{derivatives}$$

b/c. $\partial_z \mathscr{C}$, $\partial_{\bar z} \mathscr{C}$ $\sim$ $\partial_z + \partial_{z'}$ and $\partial_{\bar z} + \partial_{\bar z'}$

Zamolodchikov '04

Finding the finite-size spectrum

- cylinder $\quad z = \varphi + i\tau \quad , \quad \varphi \sim \varphi + R$

- consider eigenstates of the energy (H), momentum (P) & charge (Q)

$$H|n\rangle = E_n|n\rangle \qquad P|n\rangle = P_n|n\rangle \qquad Q|n\rangle = Q_n|n\rangle$$

- the deformed energy levels satisfy

$$\frac{\partial E_n}{\partial \mu} = R\,\langle n|\mathcal{O}_{J\bar{T}}|n\rangle = R\,\langle n|J|n\rangle\langle n|\bar{T}|n\rangle - R\,\langle J\rangle\langle \bar{T}\rangle$$

- re-expressing the one-point f. in terms of E_n, P_n, Q_n

  $\Rightarrow$ eqn. for the spectrum

Details of the calculation

- possible to show that $\langle n| \mathcal{C}(z,z') |n\rangle$ is independent of z, z'
  - as $z' \to z$ $\quad \langle n| \mathcal{C}(z,z') |n\rangle = \langle n| O_{J\bar{T}} |n\rangle$
  - factorization $\quad \langle n| \mathcal{C}(z,z') |n\rangle = \sum_{n'} \langle n|J(z)|\overset{=n}{n'}\rangle \overset{\sim e^{-i(n-n')(z'-z)}}{\langle n'|\bar{T}(z')|n\rangle} - \ldots$
- express $J, \bar{T}, \bar{J}, \Theta$ in terms of z, φ components : $\langle n| J_z |n\rangle = \dfrac{iQ_n}{R}$

$$\langle n| T_{zz} |n\rangle = -\frac{E_n}{R} \qquad \langle n| T_{z\varphi} |n\rangle = \frac{iP_n}{R} \qquad \langle n| T_{\varphi\varphi} |n\rangle = -\frac{\partial E_n}{\partial R}$$

- $T_{\varphi z}$ $(\neq T_{z\varphi})$ is det. by $SL(2,\mathbb{R})_L$ symmetry $\quad T_{\bar{z}\bar{z}} = 0$
- $J_\varphi = ?$ $\quad \rightarrow$ restrict to purely 1. chiral $(J_{\bar{z}} = 0)$ or
  2. antichiral $(J_z = 0)$ current $\quad (J_\varphi = \mp i J_z)$

Deformed spectrum (J chiral)

$$\frac{\partial E_n}{\partial \mu} = R \langle n | O_{J\bar{T}} | n \rangle = -\frac{Q_n}{2}\left(\frac{\partial E_n}{\partial R} + \frac{P_n}{R} \right)$$

- P_n is quantized $\quad P_n R \in \mathbb{Z}$

- let $\quad E_n = E_L + E_R \qquad P_n = E_L - E_R$

CFT:

$$E_R(\mu, R) = E_R\left(R - \frac{\mu Q_n}{2}\right) \qquad E_R = \frac{h_R - \frac{c}{24}}{R}$$

- deformed finite-size spectrum

$$E_R = \frac{h_R - c/24}{R - \frac{\mu Q}{2}} \qquad E_L = E_R + P = E_R + \frac{h_L - h_R}{R}$$

- breaks down for $R < \frac{\mu Q}{2}$ for both signs of μ.

Deformed spectrum (J antichiral)

$$\frac{\partial E_n}{\partial \mu} = R \langle n | \mathcal{O}_{J\bar{T}} | n \rangle = -\frac{Q_n}{2} \left(\frac{\partial E_n}{\partial R} + \frac{E_n}{R} \right)$$

- letting $\mathcal{E}_n(\mu, R) = E_n(\mu, R) \cdot R$ we find

$$\mathcal{E}_n(\mu, R) = \mathcal{E}_n\left(R - \frac{\mu Q_n}{2}\right) \overset{\text{CFT}}{=} h_L + h_R - \frac{c}{12} = \text{const}$$

$\Rightarrow$ energy spectrum is undeformed

- likely due to $T_{\bar{z}\bar{z}} = 0$ in the original CFT ($\mathcal{O} = J_{\bar{z}} T_{\bar{z}\bar{z}}$)

Thermodynamics

- in the original CFT $(\bar{J}=0)$

$$S = 2\pi\sqrt{\frac{c}{6}\left(h_L - \frac{c}{24} - \frac{Q^2}{4k}\right)} + 2\pi\sqrt{\frac{c}{6}\left(h_R - \frac{c}{24}\right)}$$

- $+\mu$ → energy levels continuously deformed

$$E = E_L + E_R = \frac{2(h_R - c/24)}{R - \mu Q/2} + \frac{h_L - h_R}{R} \qquad J = E_L - E_R = \frac{h_L - h_R}{R}$$

- $S(E) = S(h(E,J))$

- thermodyn. quantities $T = \left(\frac{\partial S}{\partial E}\right)^{-1}_{R,Q}$; $p = T\left(\frac{\partial S}{\partial R}\right)_{Q,E}$; $\Phi = -T\left(\frac{\partial S}{\partial Q}\right)_{R,E}$

  all diverge as $\frac{1}{R - \frac{\mu Q}{2}}$ as $R \to \frac{\mu Q}{2}$

Superluminal propagation

- $J\bar{T}$ $\leftarrow$ field-dependent diffeomorphism $\left(\delta S = -\int T^{\lambda}{}_a \partial_\lambda \xi^a\right)$

$$z \to z' = z \qquad \bar{z} \to \bar{z}' = \bar{z} - \frac{\mu}{2}\int^{z} J(w)\, dw$$

- CFT on space w/ coord. identifications $z' \sim z' + R$, $\bar{z}' \sim \bar{z}' + \underbrace{R - \frac{\mu Q}{2}}_{\text{modified radius!}}$

- $dz'\, d\bar{z}' = dz\left(d\bar{z} - \frac{\mu}{2} J\, dz\right) = d\varphi^2 - dt^2 - \frac{\mu J}{2}(d\varphi + dt)^2$

- signal propagation $\left(\frac{d\varphi}{dt}\right)_L = 1$, $\left(\frac{\partial\varphi}{\partial t}\right)_R = \frac{R + \frac{\mu Q}{2}}{R - \frac{\mu Q}{2}} \approx 1 + \frac{\mu Q}{R} > 1$ for $\mu Q > 0$!

  (ok, b/c no Lorentz invariance)

- CTCs for $R < \mu Q/2$

- $\bar{J}\, ☺$ $\to$ field-dependent diffeo $\bar{z} \to \bar{z}' = \bar{z} + \frac{\mu}{2}\int^{\bar{z}} \bar{J}(\bar{w})\, d\bar{w}$ no effect

Partial summary

- the $J\bar{T}$-deformation of 2d CFTs is integrable and can be studied à la Smirnov-Zamolodchikov $\rightarrow$ universal prediction for the deformed spectrum for J chiral / antichiral

- J chiral : deformed energy spectrum $R \rightarrow R - \frac{\mu Q}{2}$

  - thermodynamic quantities diverge @ $R = \frac{\mu Q}{2}$, CTCs

  - superluminal propagation for $\mu Q > 0$ (no problem as $R \to \infty$)

- J antichiral : no apparent effect

- checked in explicit examples involving deformed free fermions

The holographic interpretation

of $J\bar{T}$-deformed CFTs

Double-trace deformations in AdS/CFT

- single-trace operator $\leftrightarrow$ bulk field
- double-trace operator $\leftrightarrow$ mixed bnd. cond. for bulk field
- $S_\mu = S_{CFT} + \frac{\mu}{2}\int O^2 \underset{\text{large } N}{\Rightarrow} W_\mu[\underbrace{\tilde{J}}_{J-\mu\langle O\rangle}] = W[J] - \frac{\mu}{2}\int \langle O\rangle^2$

  $\underbrace{}_{\text{mixed bnd. cond.}}$

- variational principle

$$\delta S_\mu = \int O\,\delta J - \frac{\mu}{2}\int \delta O^2$$

$$= \int O\,\delta\underbrace{(J-\mu O)}_{\tilde{J}}$$

Double-trace deformations in AdS/CFT

- single-trace operator $\leftrightarrow$ bulk field
- double-trace operator $\leftrightarrow$ mixed bnd. cond. for bulk field
- $S_\mu = S_{CFT} + \frac{\mu}{2}\int O^2 \underset{\text{large } N}{\Longrightarrow} W_\mu[\underbrace{\tilde{J}}_{J-\mu\langle O\rangle}] = W[J] - \frac{\mu}{2}\int \langle O\rangle^2$

  $J-\mu\langle O\rangle$: mixed bnd. cond.
- variational principle

$$\delta S_\mu = \int\sqrt{\gamma}\,O\,\delta J - \frac{\mu}{2}\int \delta(O^2\sqrt{\gamma}) + \int\sqrt{\gamma}\,T_{\alpha\beta}\,\delta\gamma^{\alpha\beta}$$

$$= \int\sqrt{\gamma}\,O\,\delta\,(\underbrace{J-\mu O}_{\tilde{J}}) + \int\sqrt{\gamma}\,(\underbrace{T_{\alpha\beta}+\frac{\mu}{4}\gamma_{\alpha\beta}O^2}_{\tilde{T}_{\alpha\beta}})\,\delta\gamma^{\alpha\beta}$$

Effect of the $J\bar{T}$ deformation

- $S_\mu = S_{CFT} + \int d^2x \, \underbrace{\mu J\bar{T}}_{\mu_a T^a{}_\alpha J^\alpha \times e} \to$ covariantize !

- $\mu_a \to$ fixed null vect. on tangent space $\quad \mu_a = \mu \delta_a^+$

- assume J chiral $\quad \mu_a J^a = 0$

- $\delta S_\mu = \delta S_{CFT} - \delta S_{J\bar{T}} = \int d^2x \left[e T^a{}_\alpha \delta e^\alpha{}_a + e J^\alpha \delta a_\alpha - \delta(e \mu_a T^a{}_\alpha J^\alpha) \right]$

$$= \int d^2x \, e \Big[\big(\underbrace{T^a{}_\alpha + (e^a{}_\alpha + \mu_a J^a) \mu_b T^b{}_\beta}_{\tilde{T}^a{}_\alpha \text{ new vev}} J^\beta \big) \delta (\underbrace{e^\alpha{}_a - \mu_a J^\alpha}_{\tilde{e}^\alpha{}_a})$$

$$+ J^\alpha \delta (\underbrace{a_\alpha - \mu_a T^a{}_\alpha}_{\tilde{a}_\alpha}) \Big]$$

new source

Holographic dictionary for the $J\bar{T}$-deformed CFT

- the deformed CFT in presence of sources $\tilde{e}^{\alpha}{}_{a}$ and $\tilde{a}_{\alpha}$ can be mapped to the original CFT w/ sources $\begin{cases} e^{\alpha}{}_{a} = \tilde{e}^{\alpha}{}_{a} + \mu_a J^{\alpha} \\ a_{\alpha} = \tilde{a}_{\alpha} + \mu_a T^{a}{}_{\alpha} \end{cases}$

  (purely field th.)

- in particular, the asymptotic bulk solution is given by the Fefferman-Graham expansion in AdS_3 w/ the above boundary sources (needs a few iterations)

- the vevs $\tilde{J}^{\alpha} = J^{\alpha}$; $\tilde{T}^{a}{}_{\alpha} = T^{a}{}_{\alpha} + (e^{a}{}_{\alpha} + \mu_{\alpha} J^{a}) \mu_b T^{b}{}_{\beta} J^{\beta}$ are simply obtained by plugging in the vevs in the original CFT in presence of these sources

- purely field-theoretical → holography: original vevs are easy to compute

The AdS_3/CFT_2 holographic dictionary

- 3d Einstein gravity (T) coupled to $u(1)$ Chern-Simons (J)

- metric given by the Fefferman-Graham expansion

source vev $\propto T^{grav.}_{\alpha\beta}$

$$ds^2 = \ell^2 \frac{dz^2}{z^2} + \left(\frac{g^{(0)}_{\alpha\beta}}{z^2} + g^{(2)}_{\alpha\beta} + z^2 g^{(4)}_{\alpha\beta} \right) dx^\alpha dx^\beta$$

- gauge field ($F_{\mu\nu}=0$ + radial gauge) $A = A_+ dx^+ + A_- dx^-$

  - $A_\pm$ canonically conjugate $\mapsto$ fix only one on the bnd. (A_-)

  - well-defined variational principle: add $S_{ct} = \frac{k}{16\pi}\int d^2x \sqrt{\gamma}\, \gamma^{\alpha\beta} A_\alpha A_\beta$

    contributes to stress tensor!

$$T^{CS.}_{\alpha\beta} = \frac{k}{8\pi}\left(A_\alpha A_\beta - \frac{1}{2} A^2 \gamma_{\alpha\beta}\right) \qquad J^\alpha = \frac{k}{8\pi}\left(A^\alpha - \varepsilon^{\alpha\beta} A_\beta\right) \propto A_+$$

The asymptotic bulk solution

- fix sources in the deformed theory $\tilde{e}^\alpha{}_a = \delta^\alpha{}_a$, $\tilde{a}_\alpha = 0$

- equivalent to CFT w/ sources $e^{\alpha(0)}_a = \delta^\alpha{}_a + \mu_a J^\alpha(x^+)$, $a_\alpha = \mu_a \underbrace{T^a{}_\alpha}_{\text{grav+CS.}}$

$$\boxed{g^{(0)}_{++} = -2\mu J(x^+) \equiv P'(x^+) \qquad g^{(0)}_{+-} = \frac{1}{2}}$$

Compère - Song - Strominger bnd. cond.

$$\boxed{g^{(2)}_{--} = \bar{\mathcal{L}}(x^- + P(x^+)) \qquad g^{(2)}_{+-} = \bar{\mathcal{L}} P' \qquad g^{(2)}_{++} = \mathcal{L}(x^+) + P'^2 \bar{\mathcal{L}}}$$

$(AdS_3 + x^- \to x^- + P(x^+))$

- gauge field: $A_\alpha = \overset{\propto J^\alpha}{A^{vev}_\alpha} + \overset{\text{source}}{a_\alpha} = \Big(\underbrace{\frac{4\pi}{k} J(x^+) + P' \mathcal{A}}_{a_+}\Big) dx^+ + \underbrace{\mathcal{A}(x^- + P(x^+))}_{a_-} dx^-$

- boundary cond: $a_- = \mathcal{A} = 2\mu T_{--} = \frac{\bar{\mathcal{L}}}{8\pi G \ell} + \frac{k}{8\pi} \underbrace{\mathcal{A}^2}_{\text{cs}}$ → correct ?

The current sector contribution: a proposal

- $U(1)$ Chern - Simons $\langle - \rangle$ chiral boson $\langle - \rangle$ complex chiral fermion

  "on" deformed backgnd

- model as chiral fermions: $S = \int d^2x \; i\psi^*(\partial_- - i\, a_-)\psi$

- e.o.m.: $\psi(x^+, x^-) = e^{i\lambda(x^+,x^-)}\, \psi^{(0)}(x^+)$ $\qquad \partial_-\lambda = a_-(x^- + P(x^+))$

  $\Rightarrow \lambda = a_-^0 (x^- - x^+) +$ non-zero modes

- stress tensor (canonical $+ J_a a_\alpha$ for gauge invariance)

$$T_{++} = \psi^*(\partial_+ - i a_+)\psi = \underbrace{\psi^{*(0)}\partial_+\psi^{(0)}}_{T_{++}^{(0)}} + \underbrace{\psi^*\psi}_{J(x^+)}\,(\underbrace{a_+ - \partial_+\lambda}_{\text{only zero mode contributes!}})$$

$$T_{+-} = T_{-+} = T_{--} = 0 \quad \Rightarrow \quad \mathcal{R} = \frac{2\mu\bar{\mathcal{L}}}{8\pi G\ell} \qquad \frac{2\mu\bar{\mathcal{L}}_0(1+P')}{8\pi G\ell}$$

Summary of the holographic dictionary

- flat bnd. sources $\leftrightarrow$ AdS_3 F.G. expansion w/ $\begin{cases} e^{(0)\alpha}_{a} = \delta^{\alpha}_{a} + \mu_a J^{\alpha} \\ a_\alpha = \mu_a T^a{}_\alpha \end{cases}$

- $T_{a\alpha} = \overset{\propto g^{(2)}_{\alpha\beta}}{T^{\text{grav}}_{a\alpha}} + T^{CS}_{a\alpha}$ CFT vev in presence of sources J

$$\begin{cases} T_{++} = \dfrac{\mathcal{L}(x^+)}{8\pi G\ell} + \dfrac{2\pi J^2(x^+)}{k} + \dfrac{2\mu J \mathcal{L}_0(1+P')}{8\pi G\ell} & \text{"fit"} \qquad T_{+-} = 0 \\ T_{-+} = \dfrac{\bar{\mathcal{L}}\, P'}{8\pi G\ell} \qquad\qquad T_{--} = \dfrac{\bar{\mathcal{L}}(x^- + P(x^+))}{8\pi G\ell} \end{cases}$$

hologr.

- expectation values in deformed theory $\tilde{T}^a_\alpha = T^a{}_\alpha + \tilde{e}^a_\alpha \mu_b T^b{}_\beta J^\beta$

$$\left\{ \tilde{T}_{++} = T_{++} \qquad \tilde{T}_{-+} = 0 \qquad \tilde{T}_{+-} = -\frac{\bar{\mathcal{L}}\, P'}{8\pi G\ell} \qquad \tilde{T}_{--} = \frac{\bar{\mathcal{L}}}{8\pi G\ell} \right\}$$

- Ward id ✓ $\qquad SL(2,\mathbb{R})$

Match to field theory

$E_R = \frac{h_R}{R - \mu Q/2}$

- in field theory, energy eigenstates deformed, but entropy (h_L, h_R) same

- energy eigenstates $\to$ black holes w/ new asymptotics = BTZ + coord. transf

  $x^- \to x^{-\prime} = x^- - 2\mu \int J(x^+)$

- BTZ : $ds^2 = \ell^2 \frac{dz^2}{z^2} + \frac{d\hat{x}^+ d\hat{x}^-}{z^2} + 4G\ell\, h_L (d\hat{x}^+)^2 + 4G\ell\, h_R (d\hat{x}^-)^2 + 16 G^2 \ell^2 h_L h_R z^2 d\hat{x}^+ d\hat{x}^-$

  $\hat{x}^\pm \sim \hat{x}^\pm + 2\pi$ rescaled bnd. coord. $\hat{x}^\pm = \frac{x^\pm}{R} = \frac{\varphi \pm t}{R}$

- same entropy $\Rightarrow$ deformed b.h. same in hatted coord $\hat{x}^{+\prime} = \frac{x^+}{R}$; $\hat{x}^{-\prime} = \frac{x^{-\prime}}{R - \frac{\mu Q}{\pi}}$

- reproduces correct left/right temperatures from $\hat{x}^\pm \sim \hat{x}^\pm \pm \frac{i}{\tilde{T}_\pm}$

- holographic revs reproduce the energies $E_R = \frac{h_R}{R - \mu Q/\pi}$ $E_L = \frac{h_L}{R} + \frac{\mu Q h_R}{\pi R (R - \frac{\mu Q}{\pi})}$

Symmetry enhancement

- Ward id $\Rightarrow$ infinite family of conserved charges

$$Q_{\chi_L} = \int d\varphi\ \tilde{T}_{++}(x^+)\,\chi_L(x^+) \qquad Q_\lambda = \int d\varphi\ J(x^+)\,\lambda(x^+)$$

$$Q_{\chi_R} = \int d\varphi\ \Big(\tilde{T}_{--}(x^- + P(x^+)) - \underbrace{\tilde{T}_{+-}}_{-P'\,\tilde{T}_{--}}\Big)\,\chi_R(x^- + P(x^+))$$

- correspond to asymptotic symmetry pairs (ξ_I, Λ_I)

- the asymptotic symmetry algebra of the associated Fourier modes is

$$\boxed{\text{Virasoro}_L \times \text{Virasoro}_R \times U(1)\ \text{Kač-Moody}}$$

non-local def. of original Vir_R

"state-dependent"

same c, k

Summary

- spectrum & thermodyn. of $J\bar{T}$-deformed CFTs → non-trivial for J chiral
- superluminal propagation for $\mu Q > 0$, effective th. breakdown for $R < \mu Q/2$ (CTC, divergent T)
- holographically dual to AdS_3 w/ mixed bnd. cond
- energies computed holographically match perfectly the f.t. answer provided the current contribution to $T_{\alpha\beta}$ is that of a chiral fermion
- ASG analysis shows that Vir_R is not broken, but rather deformed to a non-local Virasoro

Future directions

- is the $J\bar{T}$-deformed CFT UV-complete? (S-matrix defn)
- how non-local is it? $\big(\underset{\text{local}}{\overset{x^+}{SL(2,\mathbb{R})_L}} \times \underset{\text{non-local but lightlike}}{\overset{x^-}{U(1)_R}} \big)$
- general correlation functions from generating functional
- can one see the Vir_R directly from field theory? Implications?
- same methods for $T\bar{T}$ generating functional?
- first tractable example of "flow up the RG" for a (1,2) operator
  - → lessons for Kerr/"CFT"?

Thank you!

Examples

(deformed free fermions)

Purely left-moving current

$$S = \frac{i}{2}\int dt\,d\varphi\left(\underset{\uparrow\ \text{LM}}{\bar\partial\psi_L}\,\psi_L^* - \psi_L\bar\partial\psi_L^* + \underset{\uparrow\ \text{RM}}{\psi_R}\,\partial\psi_R^* - \partial\psi_R\,\psi_R^* + \frac{\mu}{2}\underbrace{\psi_L\psi_L^*}_{J_L}\underbrace{\left(\psi_R\bar\partial\psi_R^* - \bar\partial\psi_R\,\psi_R^*\right)}_{T_R}\right)$$

- e.o.m :

$$\begin{cases} \bar\partial\psi_L = i\mu\,\psi_L\,T_R & \qquad \psi_L = \underbrace{e^{i\mu S}}_{\text{field-dependent gauge transf}}\ \underbrace{\psi_L^{(0)}(z)}_{\text{free fermion sol'n}} \qquad \bar\partial S = T_R \\[2ex] \partial\psi_R + \frac{\mu}{2}J_L\,\bar\partial\psi_R = 0 & \qquad \psi_R = \psi_R\Big(\bar z - \underbrace{\frac{\mu}{2}\int^z J_L}_{\text{field-dependent diffeomorphism}}\Big) \end{cases}$$

- $J_L(z)\ ,\quad T_R\left(\bar z - \frac{\mu}{2}\int^z J_L\right)$

The right-moving energy spectrum

- mode expansion: $\psi_R = \sum_{\substack{n\in\mathbb{Z} \\ \text{or } \mathbb{Z}+\frac{1}{2}}} \gamma_n b_n \exp\left(\frac{2\pi i n\left(\bar{z} - \mu/2 \int J_L\right)}{R - \mu\frac{Q}{2}} \right)$

- commutation relations

$$\{\psi_R(\varphi), \Pi_R(\varphi')\} = i\delta(\varphi-\varphi') \qquad \Pi_R = \frac{i}{2}\left(1-\frac{\mu J_L}{2}\right)\psi_R^*$$

$$\Rightarrow \quad \{b_m, b_n^+\} = \delta_{m,n} \qquad |\gamma_m| = \sqrt{\frac{2}{R-\frac{\mu Q}{2}}}$$

- therefore, the right-moving energy

only zero-modes contribute

$$E_R = \frac{1}{2}(E-P) = \int_0^R d\varphi\,\left(T_{\bar{z}\bar{z}} - T_{z\bar{z}}\right) = \int_0^R d\varphi\, T_R\left(1-\frac{\mu J_L}{2}\right)$$

$$= \frac{E_R^{(0)}\cdot R}{R-\frac{\mu Q}{2}} \quad \text{perfect match!}$$

The left-moving energy spectrum

- $T_{zz} = T^{(0)}_{zz}(z) - \frac{\mu}{2} J_L \partial S - \frac{\mu^2}{4} J_L T_R$ $\qquad \bar\partial S = T_R\left(\bar z - \frac{\mu}{2}\int^{z} J_L\right)$

- for non-zero modes $\quad \partial S = -\frac{\mu}{2} J_L \bar\partial S = -\frac{\mu}{2} J_L T_R \quad$ cancel

- for zero modes $\left(T_R = T_R^{(0)} = \text{const.}\right)$ $\qquad S = T_R^{(0)} \underbrace{(\bar z - z)}_{\text{no winding}}$

- $$E_L = \frac{1}{2}(E+P) = \int_0^R d\varphi \, T_{zz} = E_L^{(0)} + \frac{\mu Q}{2} T_R^{(0)} \left(1 - \frac{\mu J_L}{2}\right)$$

$$\sim \frac{1}{\left(R - \frac{\mu Q}{2}\right)^2}$$

$$= E_L^{(0)} + \frac{\mu Q}{2} \frac{E_R^{(0)}}{R - \frac{\mu Q}{2}}$$

perfect match!

Purely right-moving current

$$S = i\int dt\,d\varphi \left[\underset{\substack{\uparrow \\ \text{R.M. real}}}{\psi_1} \partial \psi_1 \left(1 + \frac{\mu}{2} \underbrace{\psi_2 \psi_2^*}_{\bar{J}}\right) + \frac{1}{2}\left(\underset{\substack{\uparrow \\ \text{R.M. complex}}}{\psi_2} \partial \psi_2^* - \partial \psi_2 \psi_2^*\right)\right]$$

- $\partial_\mu \mathcal{L} = -\bar{J}\, T_{\bar{z}\bar{z}}$; differs from free action by coord. transf.

$$\bar{z} \to \bar{z}' = \bar{z} + \frac{\mu}{2} \int^{\bar{z}} d\bar{w}\, \bar{J}(\bar{w}) \qquad \text{(exact)}$$

- solutions purely anti-holomorphic

$$\psi_1(\bar{z}) = \frac{1}{\sqrt{R'}} \sum_m b_m\, e^{-\frac{2\pi i m \bar{z}'}{R'}} \qquad R' = R - \frac{\mu Q}{2}$$

- $$E_R^{(\psi_1)} = \int_0^R d\varphi\, T_{\bar{z}\bar{z}}^{(\psi_1)} = \frac{i}{2}\int_0^R d\varphi\, \psi_1 \bar{\partial} \psi_1 \left(1 + \frac{\mu}{2}\bar{J}\right) = \frac{i}{2}\int_0^R d\varphi\, \psi_1 \bar{\partial}' \psi_1 \left(1 + \frac{\mu}{2}\bar{J}\right)^2$$

perfect match ! $\Leftarrow$ energy unmodified $\Leftarrow$ $R \times \frac{1}{R'} \times \frac{1}{R'} \times \left(\frac{R'}{R}\right)^2 \sim \frac{1}{R}$

Symmetry enhancement

Asymptotic symmetries

· bnd. conditions left invariant by

$$\begin{cases} \xi_L = \chi_L(x^+)\,\partial_+ - \left(\chi_L P' + \frac{z^2\ell^2}{2}\chi_L''\right)\partial_- + \frac{z}{2}\chi_L'\,\partial_z + \dots \\ \xi_R = \left(\chi_R(x^- + P(x^+))\right) + \frac{z^2\ell^2}{2} P'\chi_R''\big)\,\partial_- - \frac{z^2\ell^2}{2}\chi_R''\,\partial_+ + \frac{z}{2}\chi_R'\,\partial_z \\ \lambda = \Lambda(x^+) + 2\mu\bar{\mathcal{L}}\chi_R - \mu\ell^2\chi_R'' + \dots \end{cases}$$

preserving $\mathcal{A} = 2\mu\bar{\mathcal{L}}$

$$\delta\mathcal{L} = 2\mathcal{L}\chi_L' - \frac{\ell^2}{2}\chi_L''' \qquad \delta\bar{\mathcal{L}} = 2\bar{\mathcal{L}}\chi_R' - \frac{\ell^2}{2}\chi_R''' \qquad \delta J = \partial_+\left(\chi_L J + \Lambda(x^+)\right)$$

Virasoro$_L$ × Virasoro$_R$ × U(1) Kač-Moody

non-local def. of original Vir$_R$

"state-dependent"

Enhancement of the $SL(2,\mathbb{R})_L \times U(1)_R$ global symmetry

Hofman, Strominger '11

- 2d local QFT w/ $\underbrace{SL(2,\mathbb{R})}_{\text{Virasoro}_L}{}_L \times U(1)_R$ global symm

- $U(1)_R \rightarrow T_{\bar{z}\bar{z}},\ T_{z\bar{z}}$ improve $\ni$ $\partial T_{\bar{z}\bar{z}} = \bar{\partial} T_{\bar{z}\bar{z}} = 0$

- if $T_{z\bar{z}}(z) \neq 0$ $\quad U(1)_R \longrightarrow U(1)_L$ Kač-Moody warped CFT

  Hofman et.al.

- if $T_{\bar{z}\bar{z}}(\bar{z}) \neq 0$ $\quad U(1)_R \rightarrow \text{Virasoro}_R$ (μ continuous)

Which enhancement (if any) happens for $J\bar{T}$-def. CFTs?

- test : conformal perturbation theory; holography (asymptotic symmetries)

Holographic realisation of J & T

- 2d CFT large c, large gap

- low energies → 3d Einstein gravity + U(1) Chern-Simons

$$S_{bulk} = \int d^3x \sqrt{g} \left[\frac{1}{16\pi G} \left(R + \frac{2}{\ell^2} \right) + \frac{k}{8\pi} \varepsilon^{\mu\nu\rho} A_\mu \partial_\nu A_\rho \right]$$

+ Dirichlet boundary cond. on $g_{\mu\nu}$, A_μ

chiral, $k>0$

$$S_{bnd} = \int d^2x \sqrt{\gamma} \left(\frac{1}{8\pi G} K \pm \frac{k}{16\pi} \gamma^{\mu\nu} A_\mu A_\nu \right) + S_{ct}[\gamma, A]$$

$$\delta(S_{bulk}^{\text{on-shell}} + S_{bnd}) = \int d^2x \sqrt{\gamma} \left(\frac{1}{2} T_{\alpha\beta} \delta\gamma^{\alpha\beta} \pm \underbrace{\frac{k}{4} (A^\alpha \mp \varepsilon^{\alpha\beta} A_\beta)}_{J^\alpha} \right) \delta A_\alpha$$