Exact Bulk Operators and the Fate of Locality

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based on work with
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Towards the Black Hole Information Paradox…

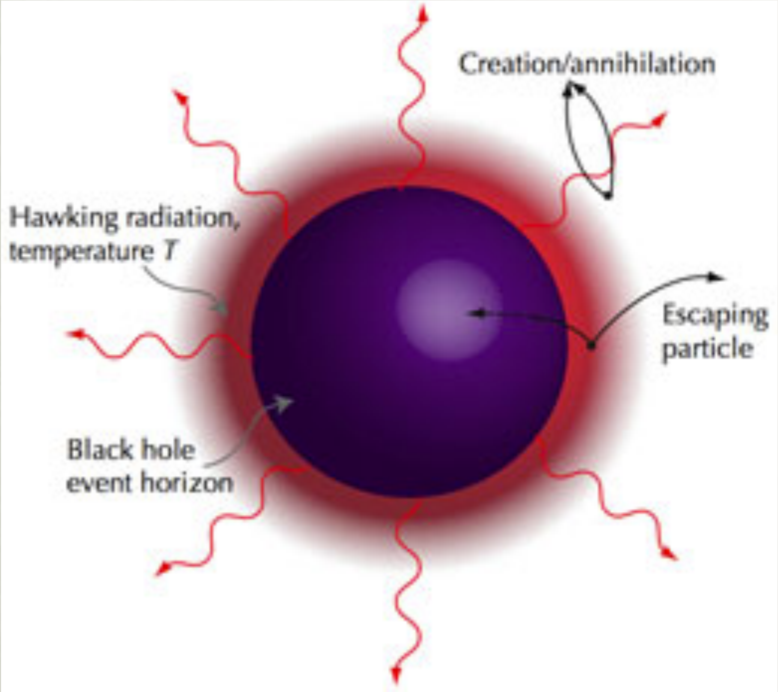
Interested in **qualitative disagreements** between AdS gravitational field theory and CFT.

As **CFT defines AdS**, have two categories of problems:

Easier = unambiguous discrepancies in CFT observables

Hard = potentially ambiguous questions about AdS observables (this talk)

Ultimate Goal: The Paradox



VS

Hawking Radiation

Unitary CFT

Tension between **local bulk** effective field theory and unitarity of exact definition via CFT.
Resolve by understanding **non-perturbative effects**.

"Hard" = Ambiguous (?) Problems

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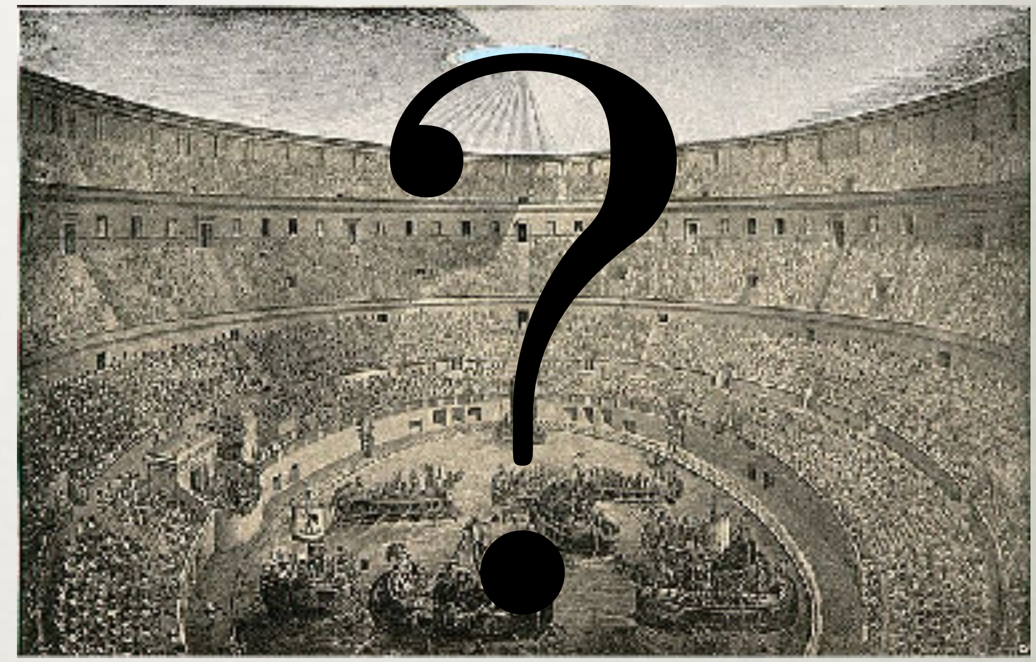


I. Is bulk reconstruction well-defined? What are the **quantitative limitations** on local observables in quantum gravity?

II. Eventually… what do observers see near and across black hole horizons?

The Reconstruction Problem

Given a CFT…

Is there an exact prescription for local field operators in a dual AdS spacetime? It should:

1. specify where in the bulk the field is located
2. match bulk perturbation theory about any semiclassical gravitational background
3. make **quantitative predictions** about its own regime of validity (if locality breaks down)

Plan for the Talk

We will show that we can solve the problem using Virasoro symmetry in $\mathrm{AdS}_3/\mathrm{CFT}_2$. Result is an **exact** bulk “proto-field”.

Equivalent to a **free field coupled to gravity**.

We’ll compute its **propagator** and discover non-perturbative gravity effects that indicate the breakdown of bulk locality.

Let's Solve a Warm-up Problem: Reconstruction Without Gravity

Bulk Reconstruction Without Gravity

We work in the Euclidean AdS metric

$$ds^2 = \frac{dy^2 + dzd\bar{z}}{y^2}$$

Write our scalar field as a sum over descendants

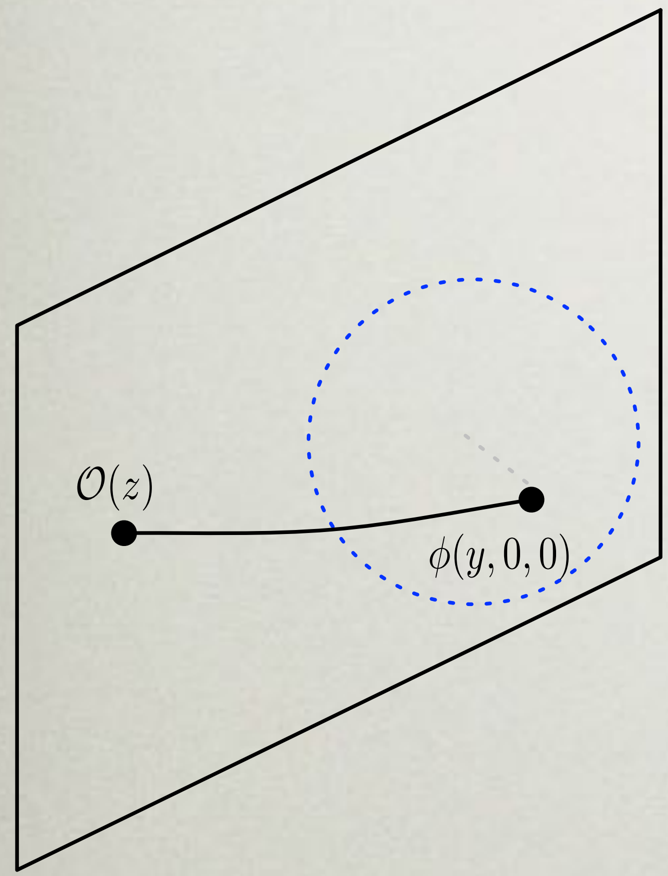
$$\phi(y,0,0) = \sum_N \lambda_N y^{2h+2N} L_{-1}^N \bar{L}_{-1}^N \mathcal{O}(0)$$

mimicking BOE. Demand that conformal symmetries act as AdS isometries on the scalar field:

$$L_{-1} = \partial_z$$
$$L_0 = z\partial_z + \frac{1}{2} y\partial_y \qquad \Longrightarrow \qquad \lambda_N = \frac{(-1)^N}{N!(2h)_N}$$
$$L_1 = z^2\partial_z + zy\partial_y - y^2\partial_{\bar{z}}$$

Bulk Reconstruction Without Gravity

Equivalently… simplest AdS/CFT observable of all:



$$\langle \phi(X)\mathcal{O}(z)\rangle = \frac{y^{2h}}{(y^2+z\bar{z})^{2h}}$$

But this correlator immediately defines the components of the field involving $\mathcal{O}$ and global descendants.

Just expand the correlator in the bulk radial direction and match coefficients, as with the OPE.

Lessons

Symmetries can directly determine bulk proto-fields…

or

We can define the bulk field using radial quantization **if** we know **bulk-boundary correlators**.

Note that we've defined a **proto-field.** Full fields are a linear combination of these primitive objects:

$$\Phi(X) = \sum_\alpha \kappa_\alpha \phi_\alpha(X)$$

Gravitational Proto-Fields from Virasoro

Virasoro and Quantum Gravity

In AdS/CFT we interpret: $g_{\mu\nu}(X) \leftrightarrow T_{\mu\nu}(x)$

and in 2d CFTs: $$T(z) = \sum_n z^{-2-n} L_n$$

Virasoro blocks know about gravity, e.g.

$$\mathcal{V}(t) = \left(\frac{\pi T_H}{\sin(\pi T_H t)} \right)^{2h_L} \quad \text{with} \quad T_H = \frac{1}{2\pi}\sqrt{24\frac{h_H}{c} - 1}$$

We will use Virasoro to define bulk proto-fields…
both methods from the warm-up can be generalized.

Proto-Fields from Symmetry

In $\mathrm{AdS}_3/\mathrm{CFT}_2$ Virasoro acts as asymptotic symmetry.

But this only identifies Virasoro transformations with an **equivalence class** of diffeomorphisms.

However, if we fix a gauge = coordinate system:

$$ds^2 = \frac{dy^2 + dzd\bar{z}}{y^2} - \frac{S(z)}{2}dz^2 - \frac{\bar{S}(\bar{z})}{2}d\bar{z}^2 + y^2\frac{S(z)\bar{S}(\bar{z})}{4}dzd\bar{z}$$

then the asymptotic symmetries are fixed to be **diffeomorphisms that preserve this gauge choice**.

Proto-Fields from Symmetry

We found the transformation rules in our metric

$$L_m\phi(z,\bar{z},y) = (\delta_m y\partial_y + \delta_m z\partial + \delta_m \bar{z}\bar{\partial})\phi(z,\bar{z},y)$$

with

$$\delta_m y = \frac{1}{2}(m+1)yz^m$$

$$\delta_m z = \frac{z^{m-1}\left((m^2+m+z^2S(z))\,\bar{S}\left(\bar{z}\right)y^4 - 4z^2\right)}{y^4 S(z)\bar{S}\left(\bar{z}\right) - 4}$$

$$\delta_m \bar{z} = \frac{2m(m+1)y^2 z^{m-1}}{y^4 S(z)\bar{S}\left(\bar{z}\right) - 4}$$

Notice that all vanish for $m \geq 2$ as $z \to 0$.

$L_{m\geq 2}$ **doesn't move the origin, even in the bulk!**

Bulk Primary Condition

In our gauge, all $L_{m\geq 2}$ preserve the origin, even in the bulk. This leads to a definition:

$$\phi(y,0,0) = \sum_N y^{2h+2N}\lambda_N \mathcal{L}_N \bar{\mathcal{L}}_N \mathcal{O}(0)$$

with "bulk primary" condition

$$L_m \phi(y,0,0)|0\rangle = 0, \quad \bar{L}_m \phi(y,0,0)|0\rangle = 0, \qquad m \geq 2$$

We also normalize our proto-field so we recover

$$\phi^{\text{global}}(y,0,0)|0\rangle = \sum_{N=0}^{\infty} y^{2h+2N}\lambda_N L_{-1}^N \overline{L}_{-1}^N |\mathcal{O}\rangle$$

in the infinite central charge limit. These conditions uniquely determine the proto-field in our gauge!

Algebraic Conditions for the Bulk Proto-Field

One can solve explicitly at finite order, for example

$$\mathcal{L}_{-2} = \frac{(2h+1)(c+8h)}{(2h+1)\,c + 2h(8h-5)} \left(L_{-1}^2 - \frac{12h}{c+8h} L_{-2} \right)$$

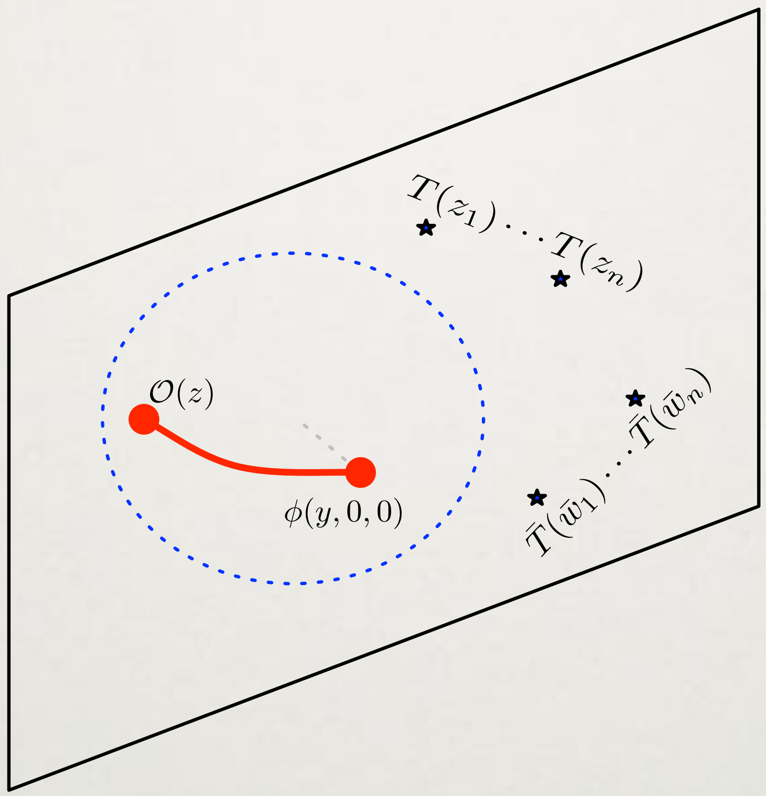
in our expression

$$\phi(y,0,0) = \sum_N y^{2h+2N} \lambda_N \mathcal{L}_N \bar{\mathcal{L}}_N \mathcal{O}(0)$$

So this is an **exact** function of primary operator dimension and central charge.

We can explore non-perturbative gravity effects.

Proto-Field from Correlators



Specification of the proto-field is equivalent to all

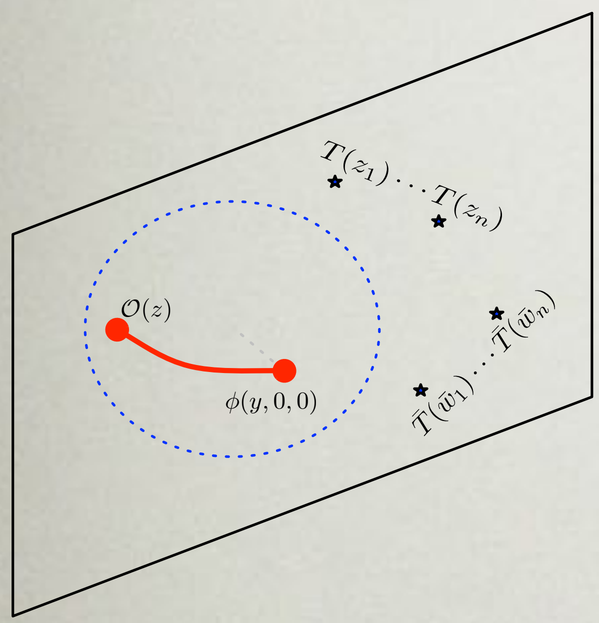
$$\langle \phi(X)\mathcal{O}(z,\bar{z})T(z_1)\cdots T(z_n)\bar{T}(\bar{w}_1)\cdots\bar{T}(\bar{w}_m)\rangle$$

as we can just decompose into descendants of $\mathcal{O}$

Compute correlators and verify?

Bulk-Boundary OPE Block

Can simply use the 1st quantized description



$$\phi(X)\mathcal{O}(0)|_{\text{vac}} = \int \mathcal{D}Y(\tau)\, e^{-m\int_0^X d\tau \sqrt{g_{\mu\nu}\dot{Y}^\mu \dot{Y}^\nu}}$$

where we interpret $T(z)$ in the metric

$$ds^2 = \frac{dy^2 + dz d\bar{z}}{y^2} - \frac{6T(z)}{c}dz^2 - \frac{6\bar{T}(\bar{z})}{c}d\bar{z}^2 + y^2\frac{36T(z)\bar{T}(\bar{z})}{c^2}dz d\bar{z}$$

as a **quantum** CFT stress tensor operator.

This is what guarantees that our operator has correct correlators in **any vacuum geometry**, including e.g. BTZ black hole geometries.

Example Correlator and Algebraic Conditions

The simplest non-trivial correlator is

$$\frac{\langle \phi(y,0,0)\mathcal{O}(z,\bar{z})T(z_1)\rangle}{\langle \phi(y,0,0)\mathcal{O}(z,\bar{z})\rangle} = \frac{hz^2}{z_1^3\,(z_1-z)^2}\left(z_1+\frac{2y^2(z_1-z)}{y^2+z\bar{z}}\right)$$

Pole at origin indicates presence of bulk field, but most singular term z_1^{-3}. This is direct confirmation of our algebraic condition:

$$L_{m\geq 2}\phi(y,0,0)|0\rangle = 0$$

We also derived a recursion relation for adding stress tensors, generalizing Virasoro Ward identity.

The Exact Propagator and the Fate of Bulk Locality

Exact Propagator

We can now study the simplest local bulk observable

$$\langle \phi(X)\phi(Y) \rangle$$

It should be **real**, as quanta cannot decay,
and **non-singular** at spacelike separation.

Because proto-field has a simple algebraic definition based on Virasoro, we can adapt many techniques from the study of Virasoro blocks and apply them to compute the propagator to very high precision in various limits…

Propagator Kinematics

Some useful kinematic variables for $\langle\phi(X)\phi(Y)\rangle$

$\sigma(X,Y)$ is geodesic distance

We will often use long-distance expansion in

$$\rho \equiv e^{-2\sigma(X,Y)}$$

Our gauge choice made the radial direction special, with the consequence that the full propagator is **not spherically symmetric**! So propagator can also depend on angle with radial direction.

For simplicity we will focus on the $(z,\bar{z})$ plane.

Semiclassical Limit

In the semiclassical limit of large central charge:

$$\langle \phi \phi \rangle \approx e^{c\, g\left(\frac{h}{c}, \rho\right)}$$

and we can compute the exponent via a generalization of the 'monodromy method'. For example:

$$\langle \phi \phi \rangle \approx \frac{\rho^h}{1-\rho} e^{\frac{12h^2}{c}\left(\frac{\rho}{(1-\rho)^2} + \log(1-\rho)\right) + O\left(\frac{h^3}{c^2}\right)}$$

But really this is useful non-perturbatively…

Limit of Large Mass for the Bulk Field

We can compute at large dimension h, corresponding to trans-Planckian bulk mass:

$$\lim_{h\to\infty}\langle\phi\phi\rangle = q^{h-\frac{c-1}{24}}\left(\frac{s}{8}\right)^{\frac{c-1}{12}}(1-s)^{\frac{c-13}{144}}\left(\frac{2E(s)}{\pi}\right)^{\frac{19-7c}{36}}$$

with

$$q \equiv 4e^{2\pi\frac{E(1-s)-K(1-s)}{E(s)}-4} \qquad \text{and} \qquad s = \frac{8\sqrt{\rho}}{\rho+4\sqrt{\rho}+1}$$

This result is interesting for two reasons…

Limit of Large Mass for the Bulk Field

1) It's a **seed** for a generalization of the Zamolodchikov recursion relations for the exact propagator.

2) The new q variable has a **branch cut** starting at

$$\rho = 7 - 4\sqrt{3} \approx 0.0718$$

which corresponds to a physical separation of

$$\frac{\sigma_*}{R_{AdS}} \approx 1.32$$

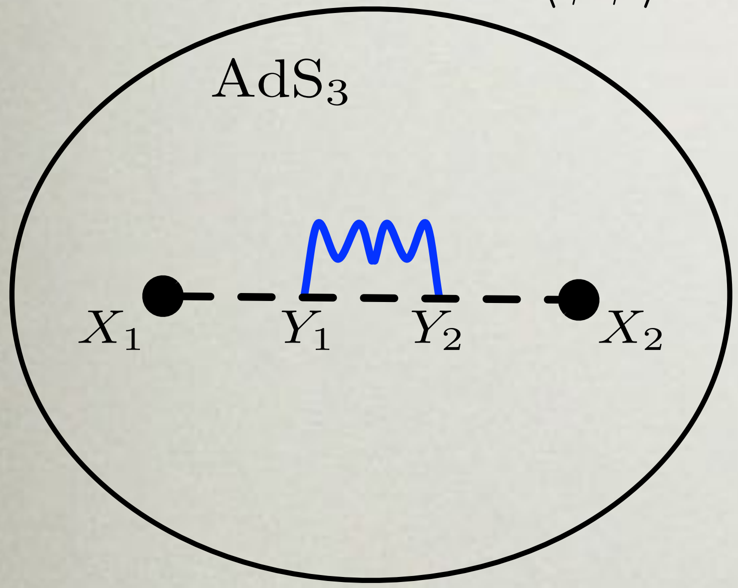
Violates unitarity, as our proto-field cannot decay. So locality has broken down at the AdS scale.

Propagator for Fields with Small Bulk Masses...

Perturbation Theory in Central Charge

Let's look at first perturbative correction.
In the short-distance limit, restoring scales

$$\langle \phi\phi \rangle \approx \frac{1}{\sigma}\left(\frac{3G_N R^3}{4\sigma^4} - \frac{G_N R(10+m^2R^2)}{8\sigma^2} + 2G_N m^2 R \log\left(\frac{\sigma}{R}\right)\right)$$



A **new length scale** has emerged:

$$\sigma_* \sim \sqrt[4]{G_N R^3}$$

This should certainly surprise field theorists…
indicative of UV/IR mixing and absence of
a good flat space limit. But it's finite in AdS.

Non-Perturbative Fate of Locality…

Let's study the propagator to all orders and focus on the short distance limit.

We find a divergent expansion at short distances:

$$\langle \phi(X)\phi(Y)\rangle \approx \sum_n \frac{(4n-1)!!}{n!}\left(\frac{12}{c\,\sigma^4}\right)^n$$

This can be Borel resummed, but there is a branch cut on the positive real axis. **Ambiguous**, and generically the resummation has **imaginary** piece. But we can do better via numerics…

Zamolodchikov Recursion Relations

Algebraic simplicity of the bulk primary propagator means it has much in common with Virasoro blocks. Can also define an exact recursion relation for it.

Organizes the result as an expansion:

$$\langle \phi\phi \rangle = \frac{\rho^h}{1-\rho} \sum_n a_n \rho^n$$

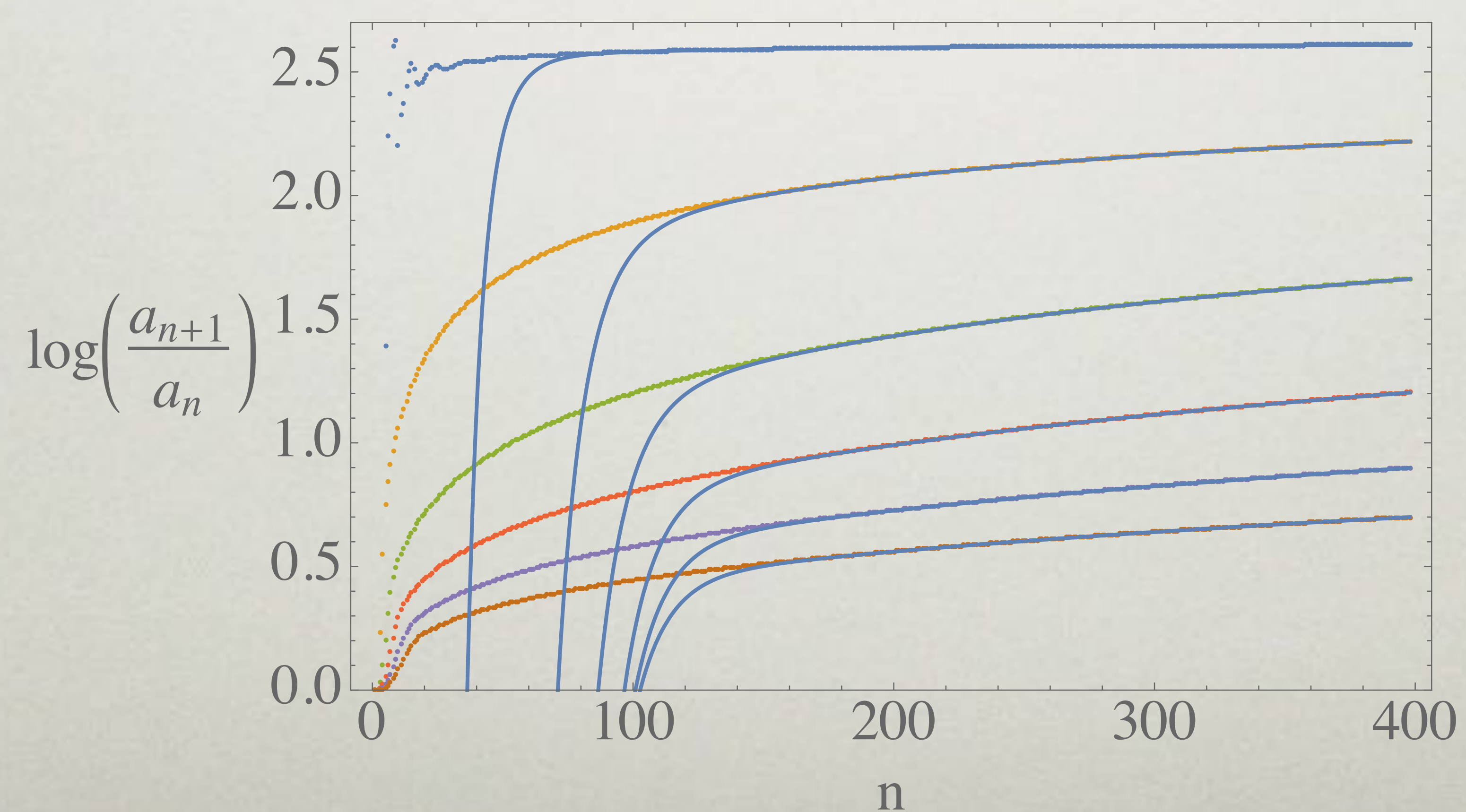
Recall that this is a long-distance expansion as

$$\rho \equiv e^{-2\sigma(X,Y)}$$

Numerical Results

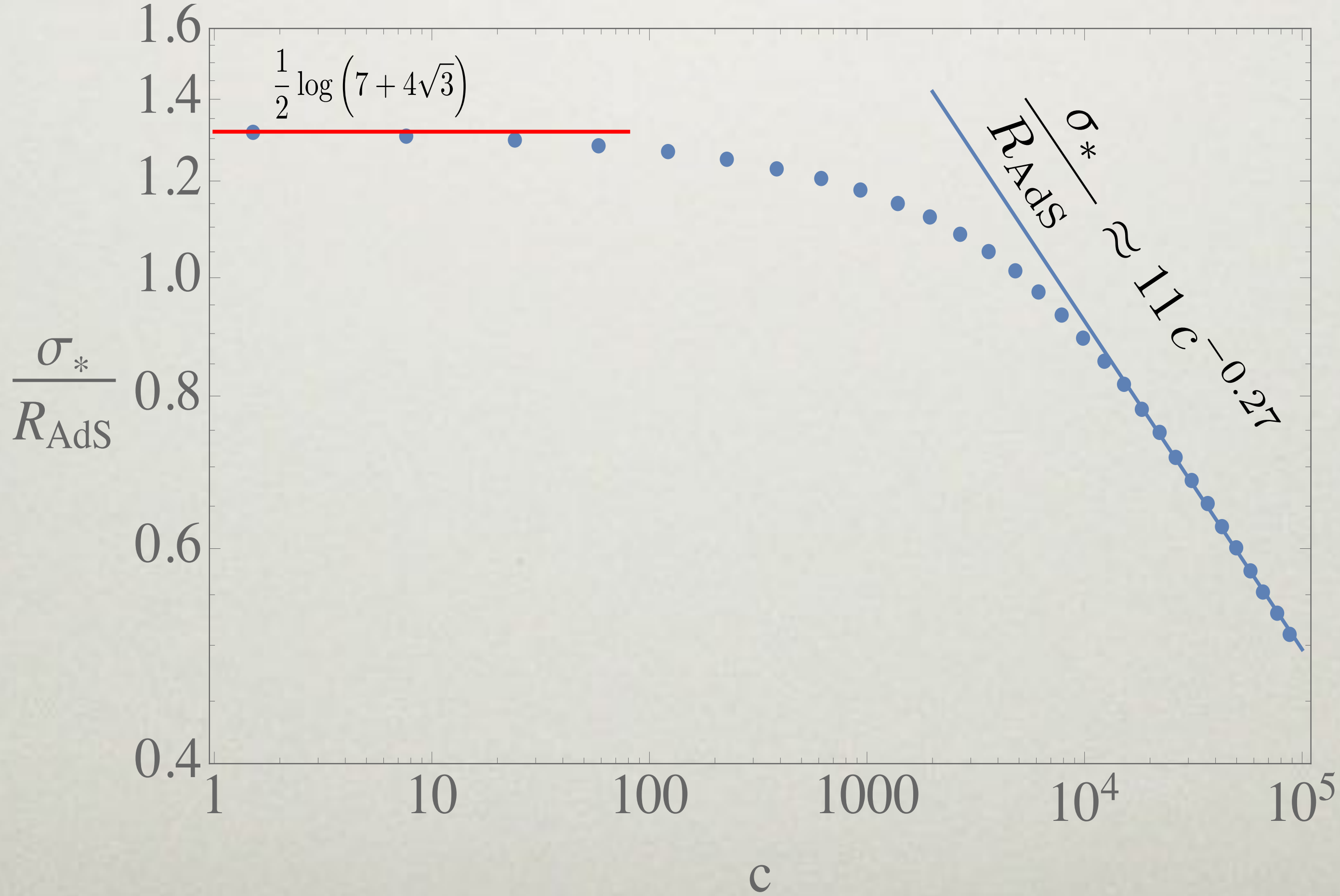
We have the exact coefficients in a ρ^n expansion.

This expansion **diverges** at $\rho_* \equiv e^{-2\sigma_*}$ which can be determined by coefficient growth rates:



Numerical Results

This expansion **diverges** at $\rho_* \equiv e^{-2\sigma_*}$ where dependence on the central charge can be extracted:



Breakdown of Bulk Locality

Thus all-orders perturbative and numerical results seem to imply that bulk locality breaks down at

$$\sigma_* \sim \sqrt[4]{G_N R^3}$$

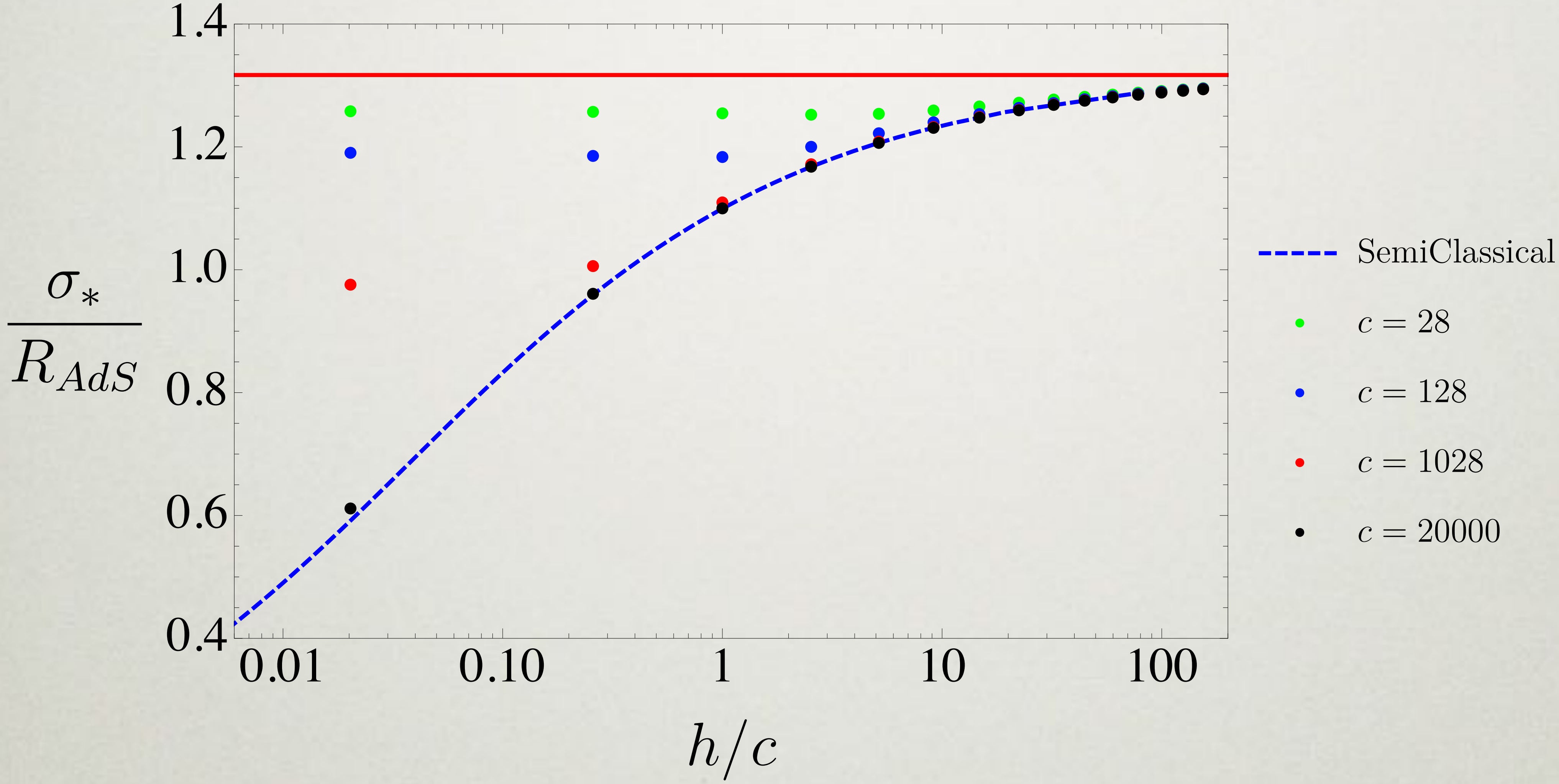
Unexpectedly **intermediate** between Planck and AdS.

It seems that string compactifications all have

$$\ell_s \gtrsim \frac{1}{c^{1/4}}$$

so maybe the string scale cannot be smaller than this new scale? Coincidence?

Semiclassical Interpolation



The scale at which bulk locality breaks down for general central charge and bulk mass.

Summary

I. It's possible to define what we should mean by an exact local bulk observable. It requires and depends importantly on a bulk gauge choice.

II. Although we defined a precise operator, its correlators violate locality and unitarity at short-distances in the bulk due to non-perturbative effects at a new intermediate scale.

Future Directions

I. It appears as though a unitary CFT can produce bulk correlators that violate locality (only) due to non-perturbative effects — implications?

II. What does this mean for other claims about bulk locality? How gauge-dependent is the physics?

III. Now we can compute correlators in a BTZ black hole background, study behavior near the horizon, and dependence on CFT data details…

Thank You!