Julius-Maximilians-UNIVERSITÄT WÜRZBURG

Quantum Information and Topological Complexity in AdS/CFT

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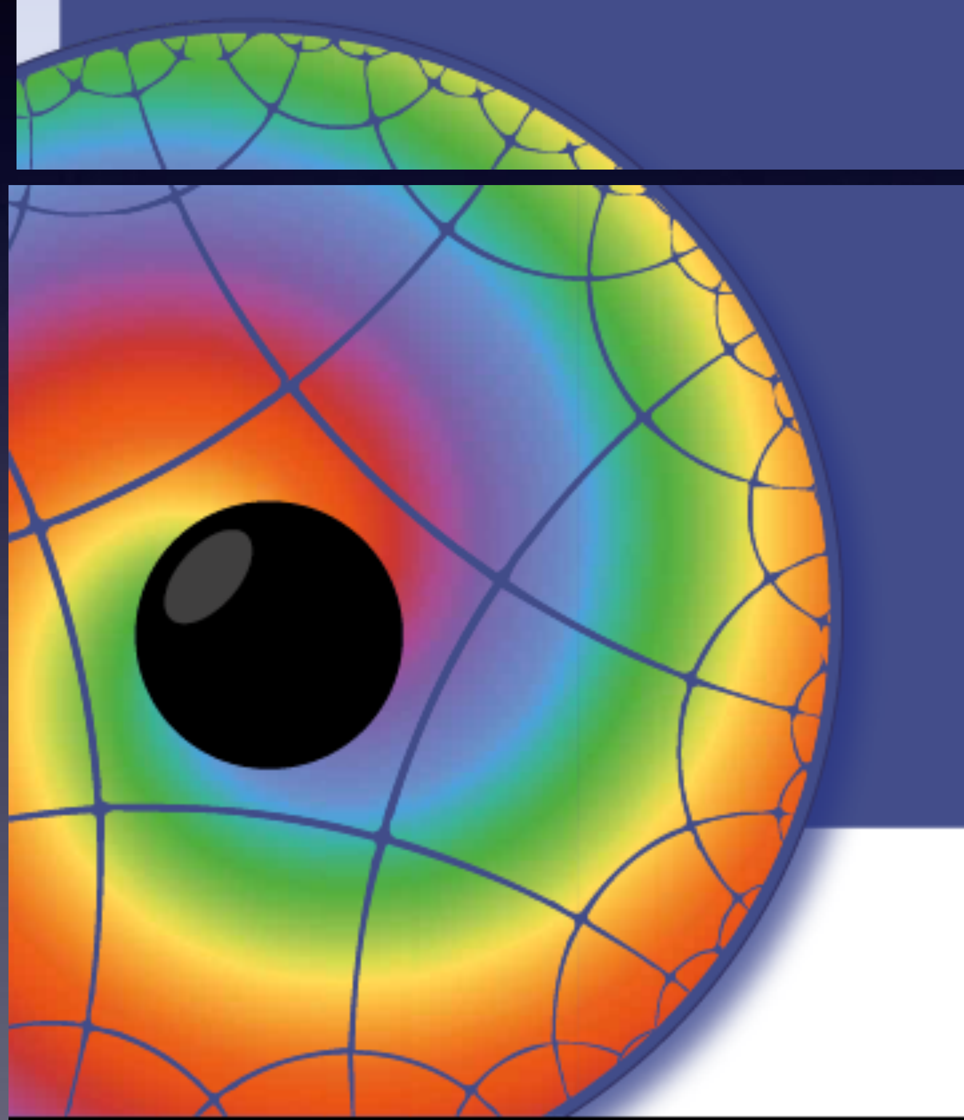
Abt, Erdmenger, Hinrichsen, Melby-Thompson, RM, Northe, Reyes, 1710.01327

GAUGE/GRAVITY DUALITY



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People behind the research

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Quantum Complexity

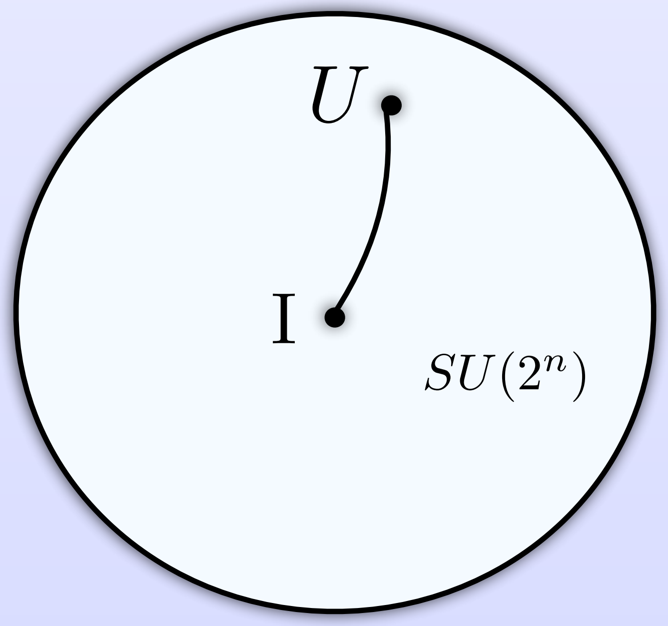
How to build states given a set of unitary operators?

Hilbert Space with $\dim(\mathcal{H}) = 2^n$ Gates $\{U_i\}$

Complexity: Optimal Quantum Circuit $|\Psi\rangle = U|\Psi_0\rangle$

Minimal Number of Gates to build $U \in SU(2^n)$

Nielsen: Riem. Geometry & Hamiltonian Control Theory



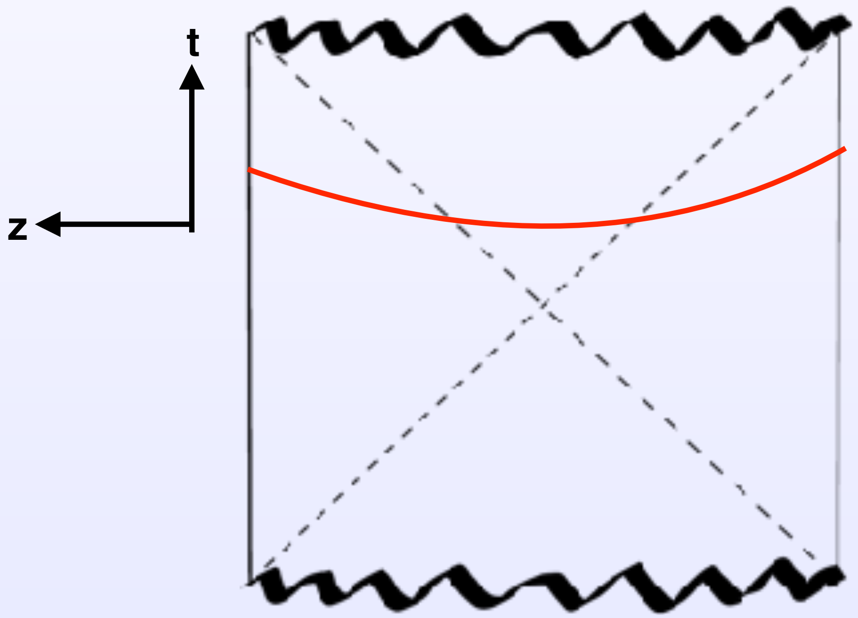
for $U(t) = e^{-iHt}$

$$\mathcal{C}(t) \sim nt$$

Nielsen (et.al.) 2005-2007

Complexity in AdS/CFT

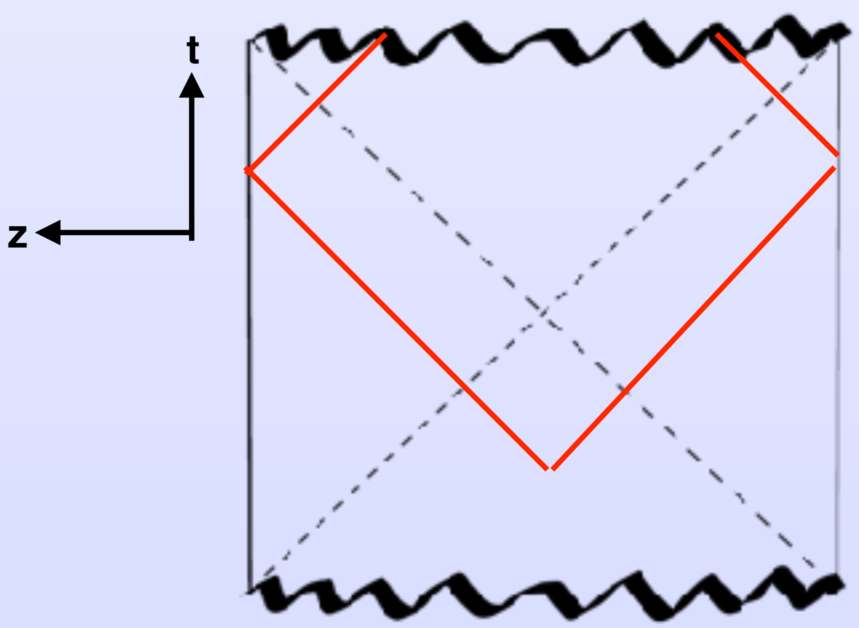
Complexity = Volume



$$\mathcal{C} = \frac{\mathrm{Vol}(\Sigma)}{L_{AdS} G_N}$$

Stanford & Susskind 2014

Complexity = Action

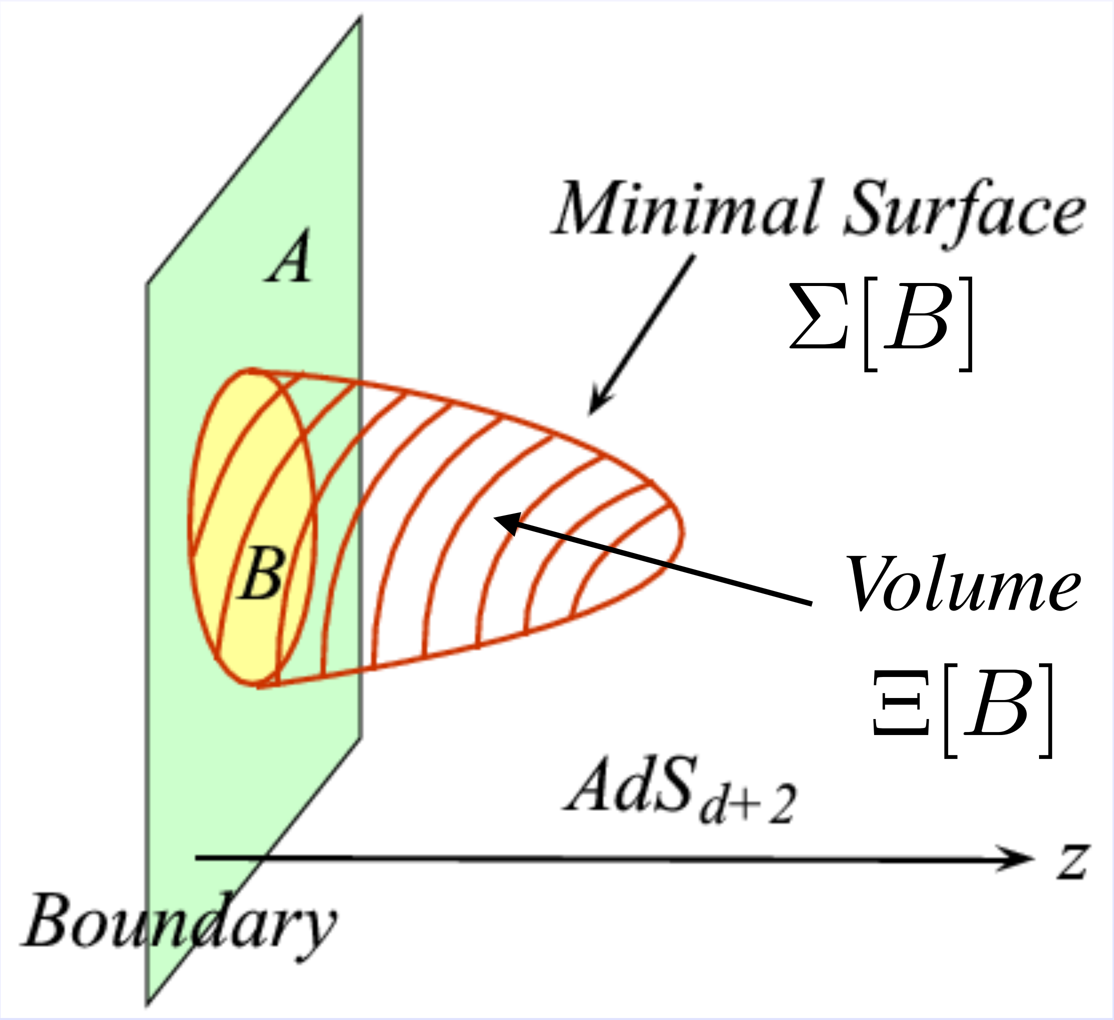


$$\mathcal{C} = S_{grav}\big|_{\mathrm{WdW\ Patch}}$$

Brown-Roberts-Susskind-Swingle-Zhao 2015

Holographic Subregion complexity

Volume under the Ryu-Takayanagi surface



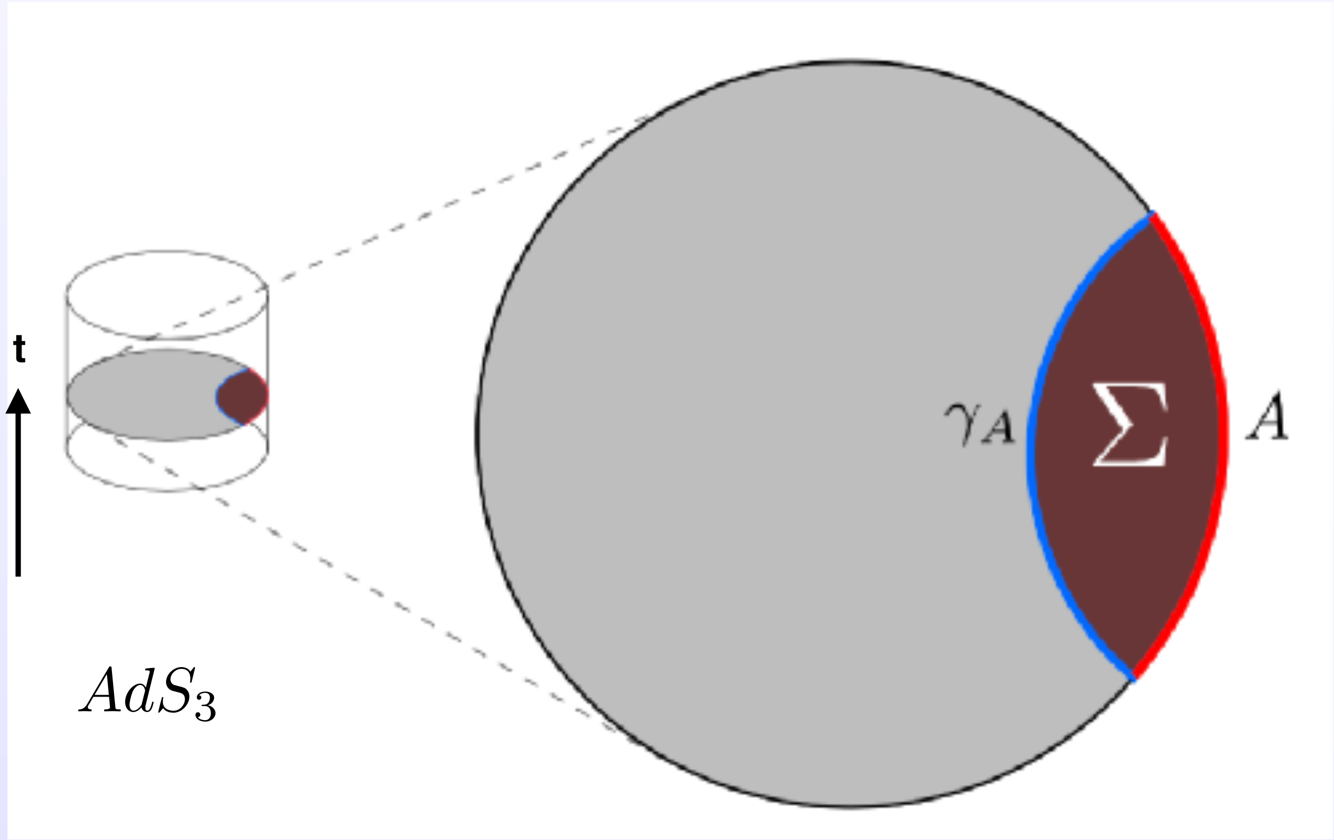
$$\mathcal{C}[\rho_B] = \frac{\mathrm{Vol}(\Xi[B])}{L_{AdS} G_N}$$

$$\partial\Xi[B] = B \cup \Sigma[B]$$

Alishahiha 2015

AdS3 complexity and Gauss-Bonnet

Topology yields universal terms



$$\mathcal{C} \equiv -\frac{1}{2} \int d^2x \sqrt{|g|} R$$

$$\chi(\Sigma) = \frac{1}{4\pi} \int_{\Sigma} d^2x \sqrt{|g|} R + \frac{1}{2\pi} \int_{\partial\Sigma} ds k + \sum_i \alpha_i$$

Geodesic curvature

Corners

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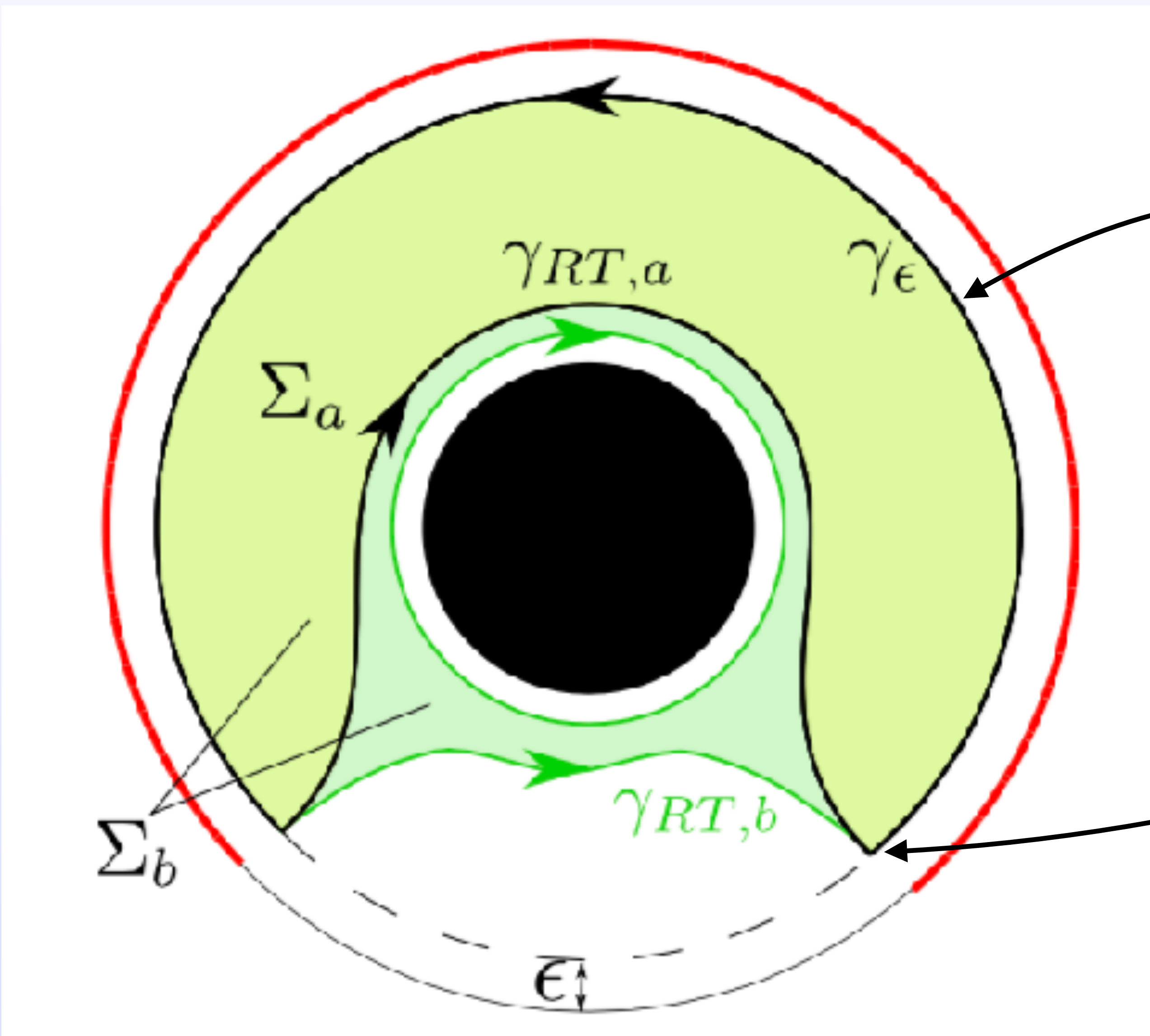
Holographic Topological Complexity

Example: BTZ Black Hole



$$\mathcal{C}_{a,b} = \frac{x}{\epsilon} + \pi - 2\pi\chi_{a,b}$$

Euler characteristic

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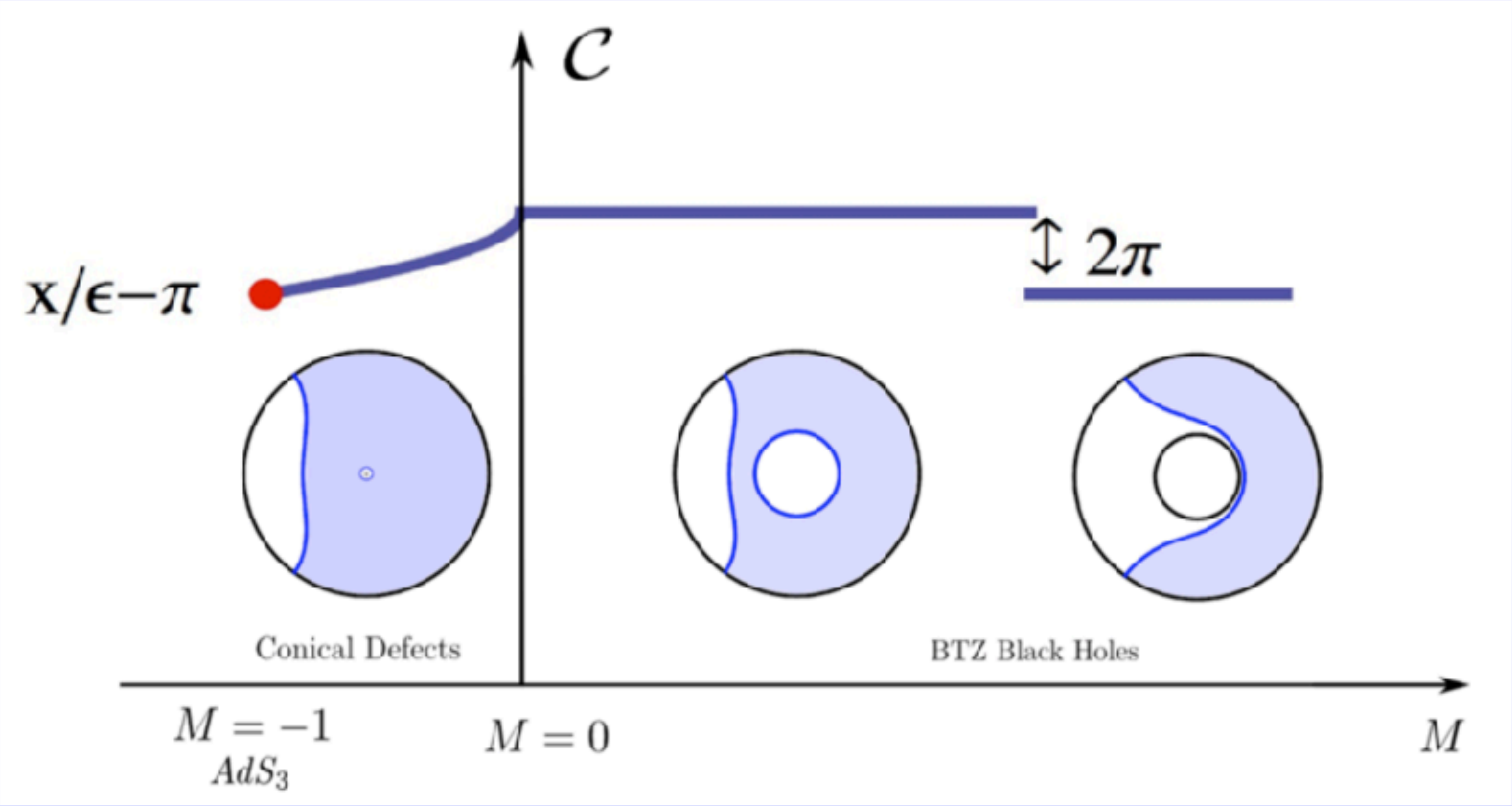
Complexity transitions

Euler characteristic jumps



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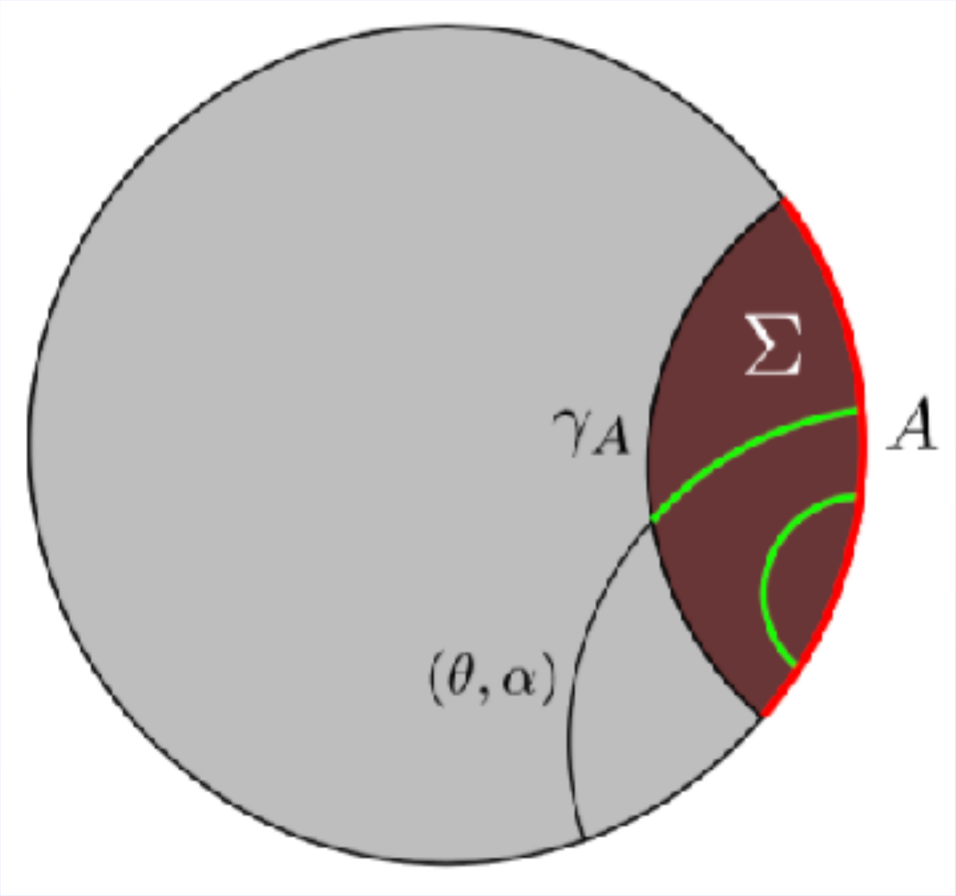
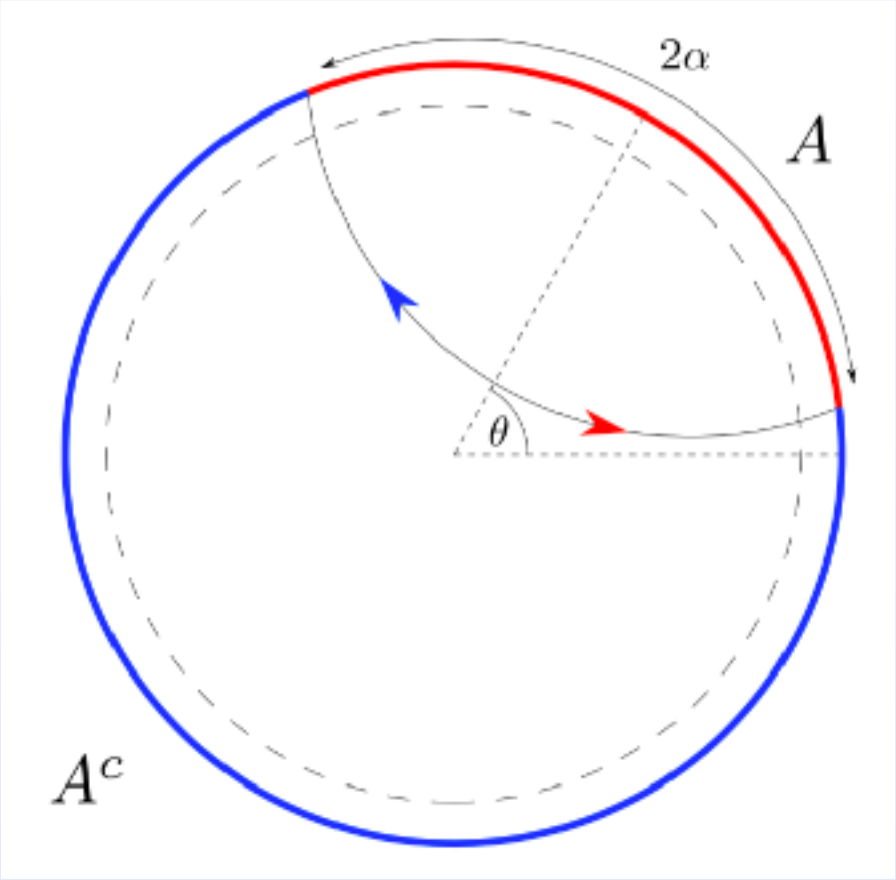
Complexity and Entanglement Entropy

Volume in Vacuum AdS3 from Kinematic Space





Czech et.al. 2015-16

$$\mathcal{C}(A) \sim \int d\theta d\alpha \int d\theta' d\alpha' \partial_\alpha^2 S(\alpha) \partial_{\alpha'}^2 S(\alpha')$$

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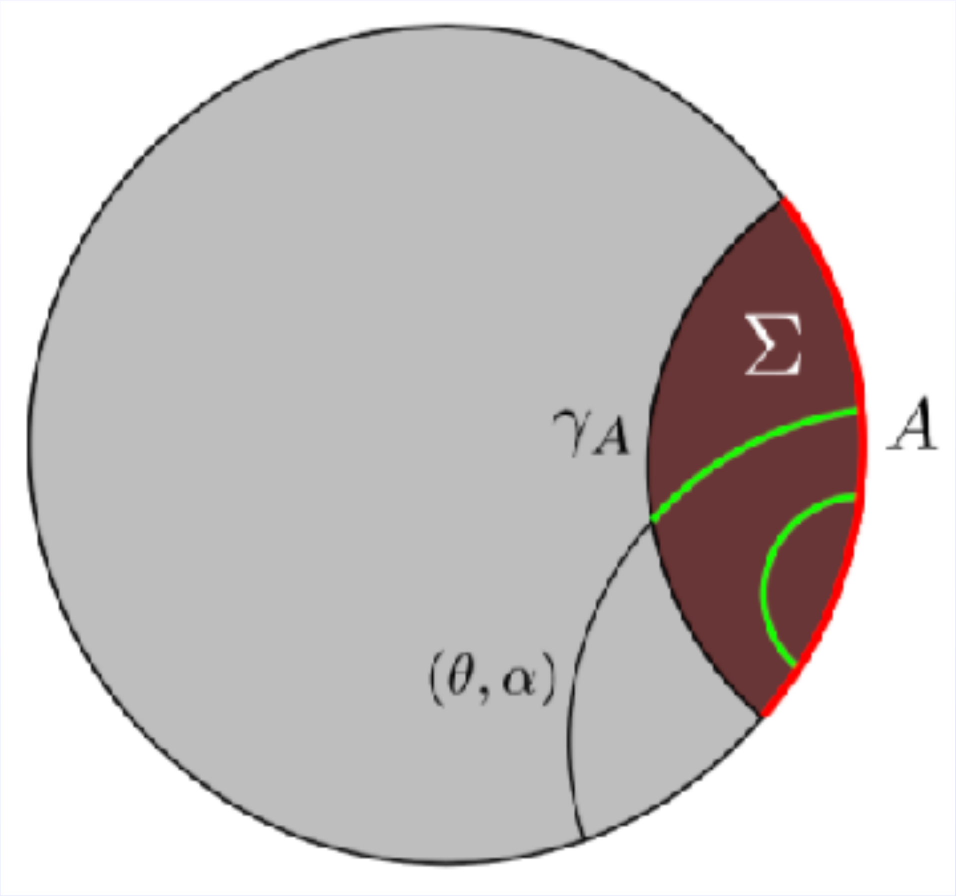
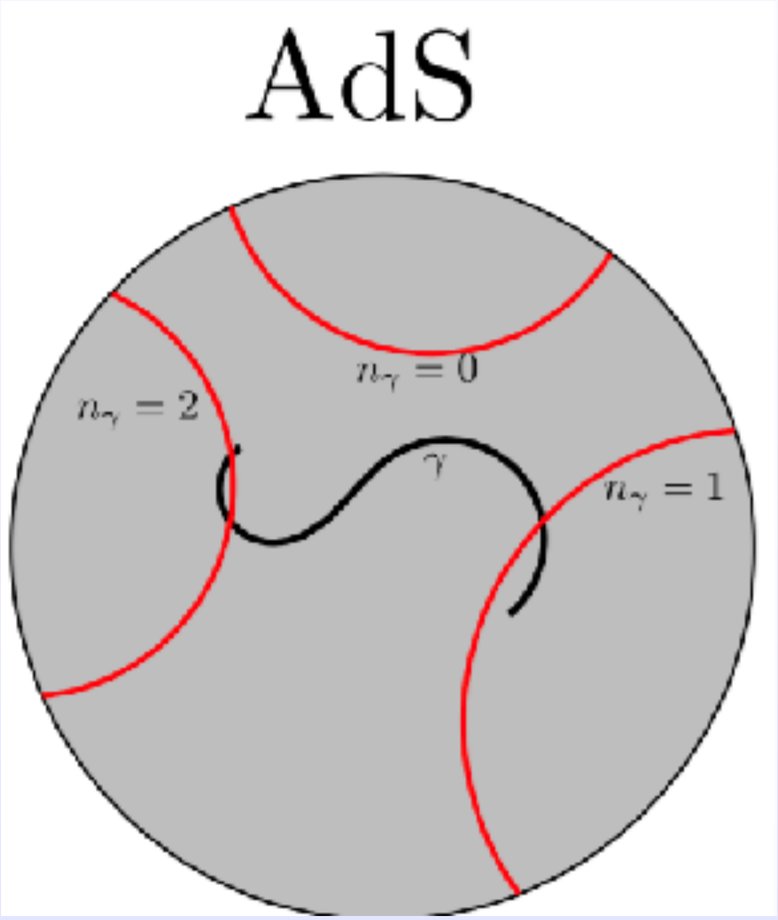
Volumes from Kinematic Space

Volume as integral over geodesics





$$Vol(\Sigma) \sim \int_{\mathcal{K}} \omega \lambda_\Sigma$$

Czech et.al. 2015-16

$$\lambda_\gamma \sim \int_{\mathcal{K}} \omega n_\gamma(\alpha, \theta)$$

$$\omega \sim \partial_\alpha^2 S \, d\alpha \wedge d\theta$$

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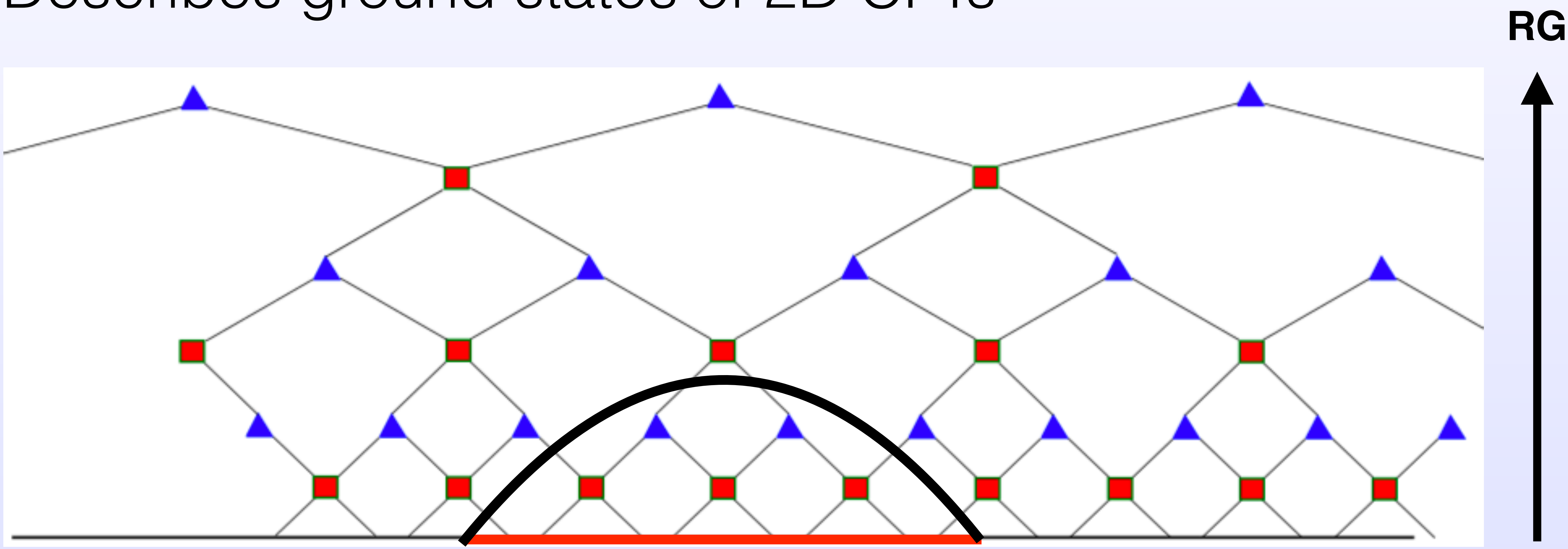
Tensor Networks & AdS/CFT

Multiscale Entanglement Renormalization Ansatz

Describes ground states of 2D CFTs



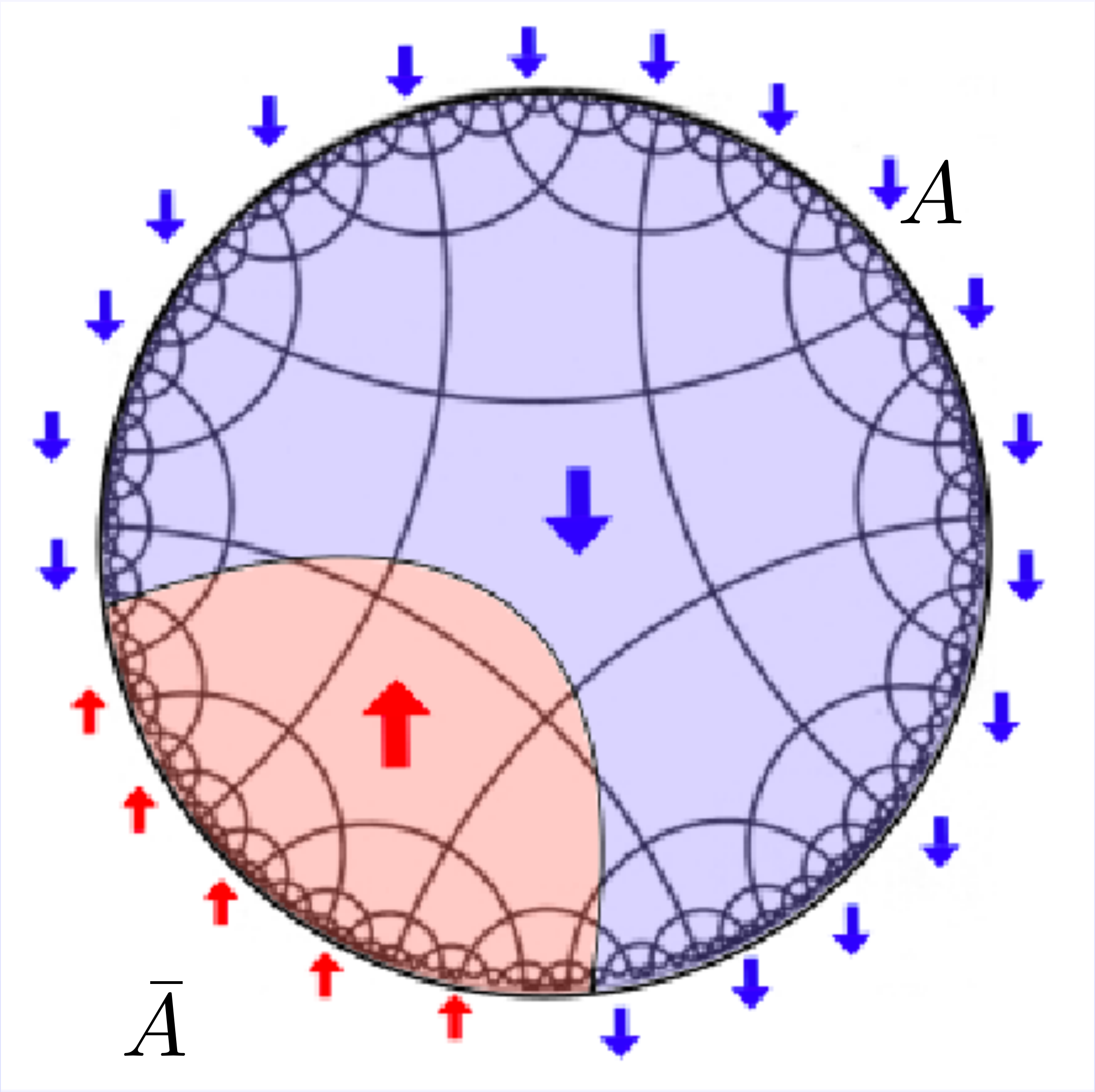
$$S \leq \log D \log n$$

Reproduce the subregion complexity?

Vidal 2005, Swingle 2009

Random Tensor Networks and Ising Model

2nd Renyí Entropy as a nearest neighbor Ising Model



$$\overline{S_2(A)} = -\log \frac{\overline{Z_1}}{\overline{Z_0}}$$

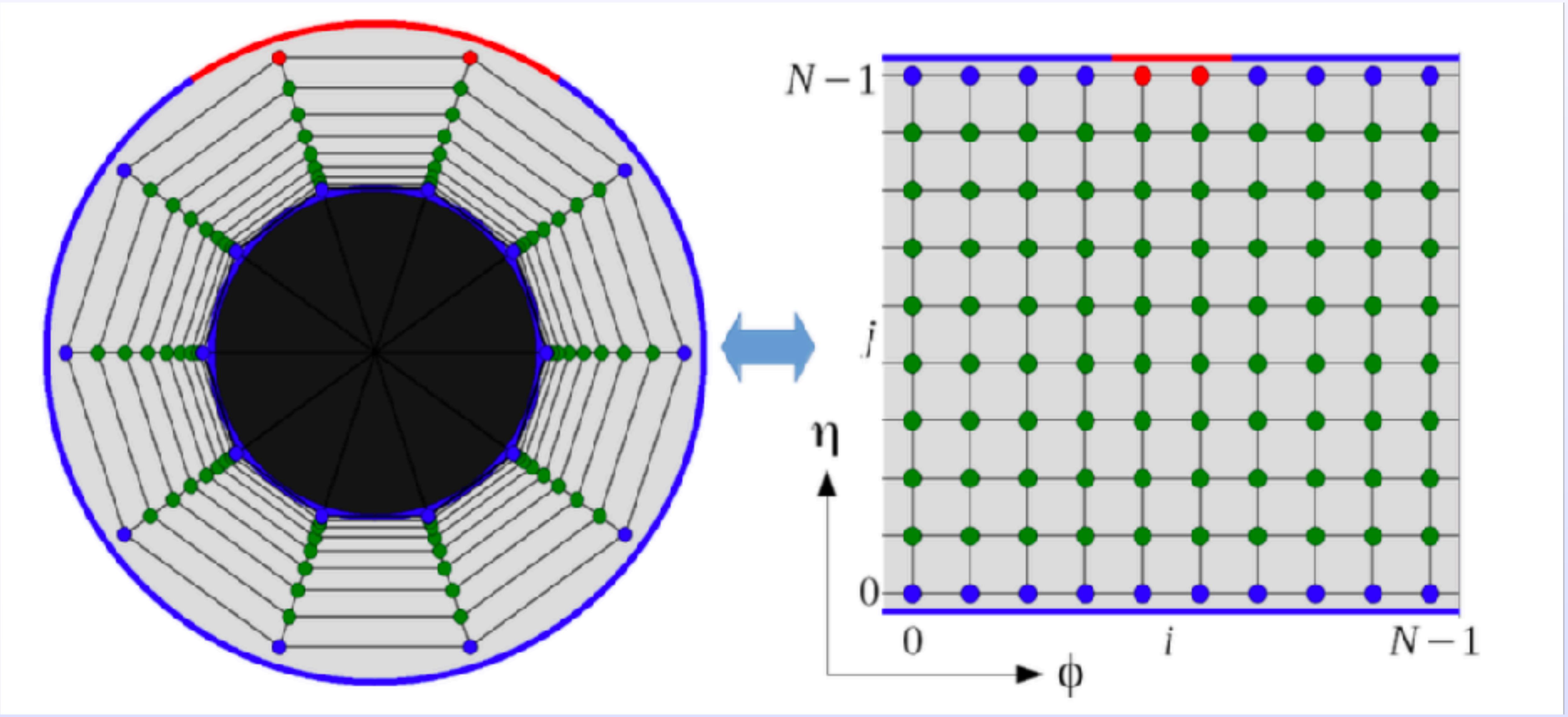
$$\overline{Z_1} = \sum_{\{s_x\}} e^{-\mathcal{A}[\{s_x\},\{h_x\}]}$$

$$\mathcal{A}[\{s_x\}] = -\frac{1}{2}\log D\left(\sum_{\langle xy\rangle} s_x s_y + \sum_{x\in\partial\Gamma} s_x h_x\right)$$

Hayden-Nezami-Qi-Thomas-Walther-Yang 2016

Ising Model Setup

Square lattice on with BTZ geometry



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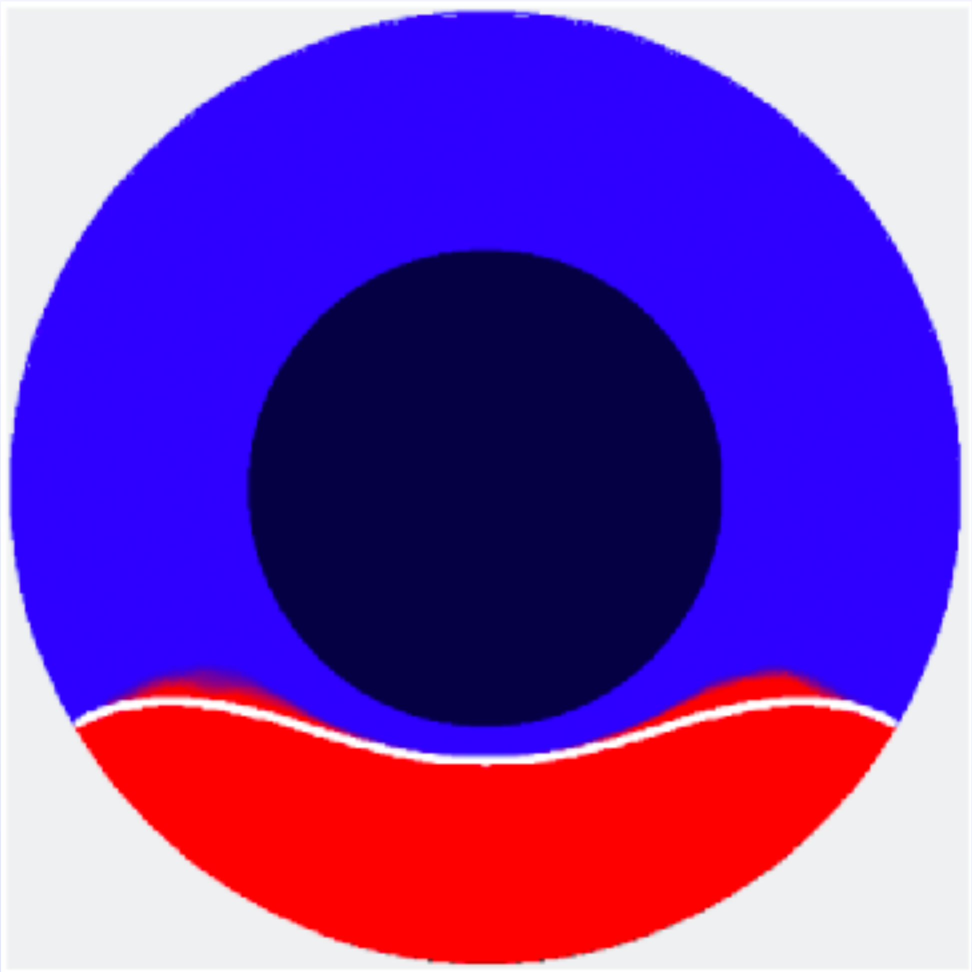
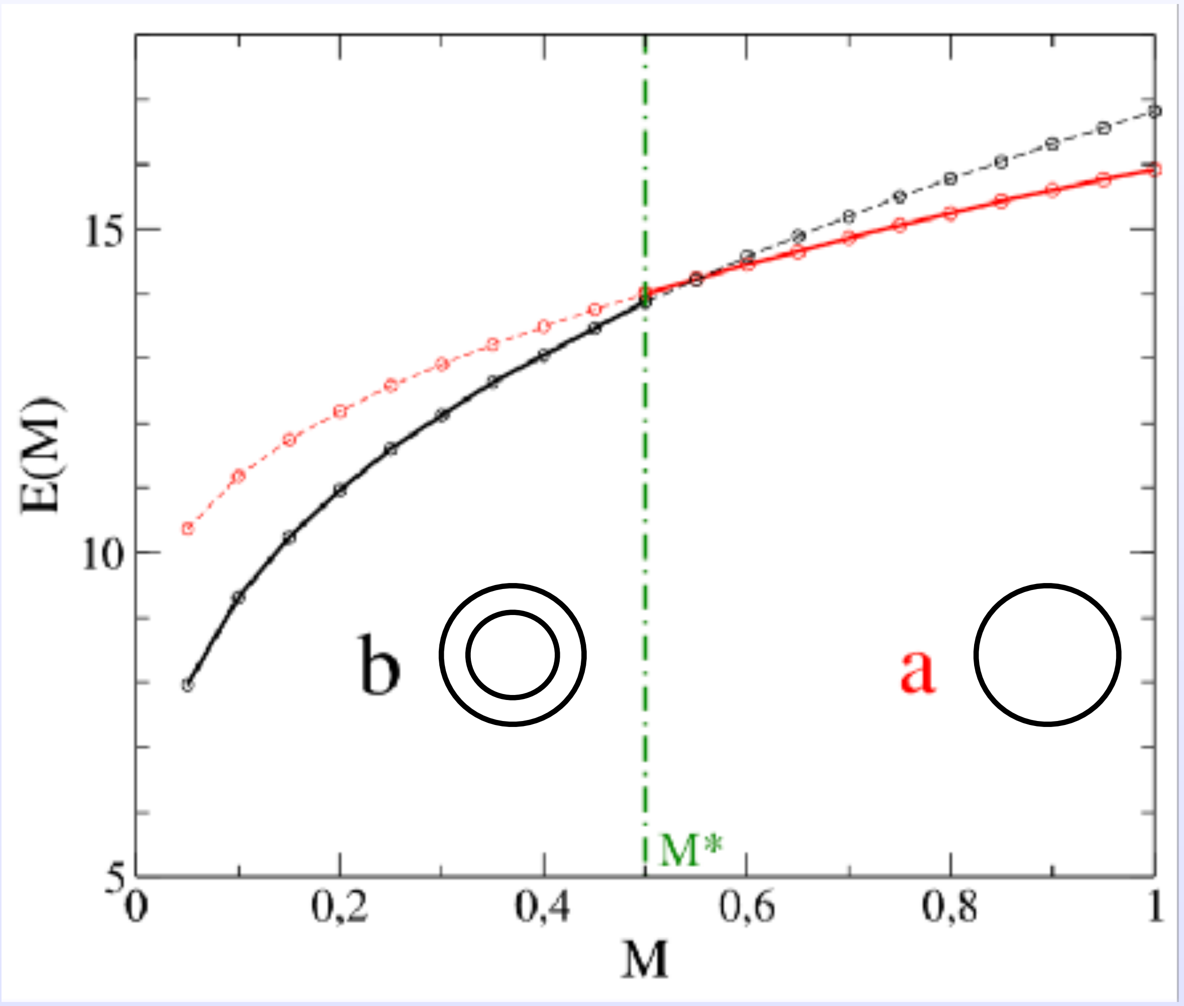
Simulations by Haye Hinrichsen

Ising Model Results

Ryu-Takayanagi Surface as a domain wall



Averaged Magnetization



Entanglement Entropy

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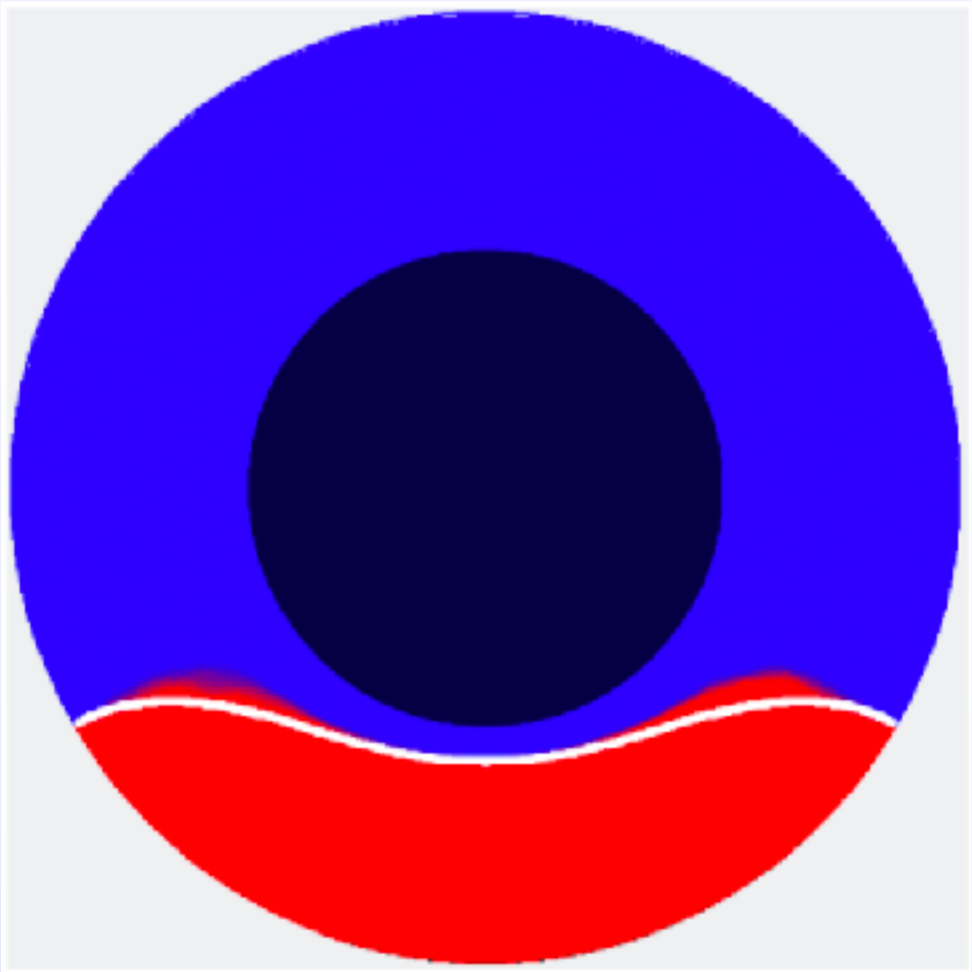
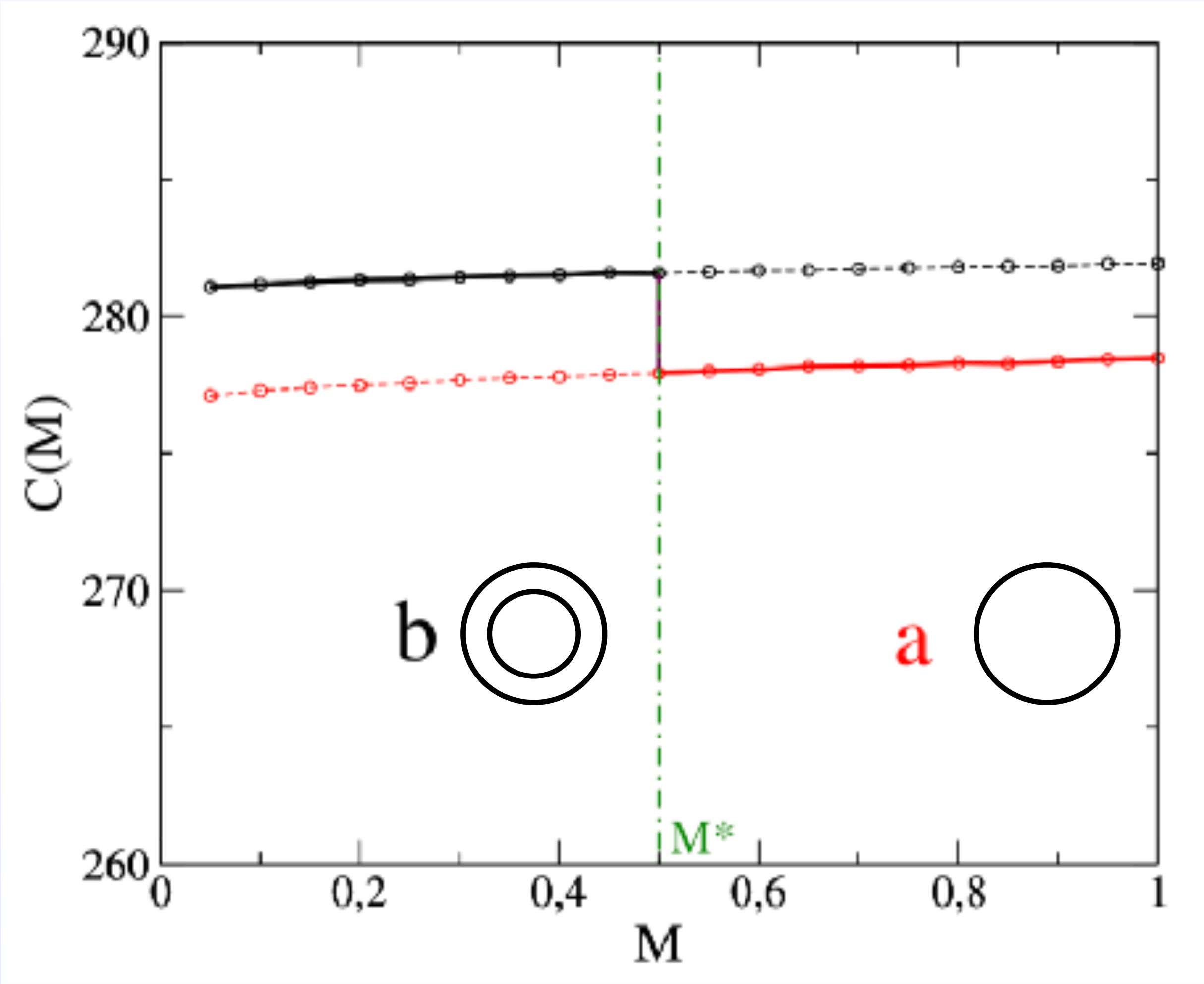
Ising Model Results

Complexity jumps



Averaged Magnetization



Subregion Complexity

$$\Delta\mathcal{C} = 3.8 \pm 0.3$$

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Simulations by Haye Hinrichsen

Conclusions & Outlook

- Subregion complexity:
  Volume under the Ryu-Takayanagi surface

- Gauss-Bonnet in AdS3:
  Finite terms gain topological interpretation

- Kinematic Space:
  Volume in terms of CFT quantities

- Qualitative features
  supported by random tensor network calculations

- Outlook: Complexity of 2D CFT states?
  Better CFT understanding of the volume formula
  as well as of the topological contributions.
  Generalizations of kinematic space