Black holes from singlet models

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Motivation

Improve understanding of high temperature phases in higher spin gravity.

Assess status of black holes in such theories purely from a boundary perspective. Which properties of their weakly coupled gravitational ancestors do higher spin theories retain?

Theories dual to bulk higher spin theories undergo large N phase transitions on S^d.

In boundary matrix models, believed to involve black holes. Similar to Hawking-Page, transition at $T \sim \mathcal{O}(1)$

[Sundborg 1999]

For vector models, interpretation unclear. Transition at $T \sim \mathcal{O}(1/\sqrt{G_N})$, thermodynamically stable large AdS black hole seemingly absent [Shenker&Yin 2011]. No explicit candidate solutions known for $D \geq 4$

But studies for flat boundary reveal black hole like properties. [Jevicki&Yoon 2016]

Teaser

Correlation functions as simplest probe of emergent geometry in free large N singlet models reveal emergent localized structure with various black hole like properties at high temperatures.

The way:

- Introduction to free singlet models, Higher Spin / CFT and their phase transitions

- The singlet constraint and correlation functions

- Emergence of (A) localized structure at high T which (B) implies exponential decay of autocorrelations and (C) sustains evanescent modes

Higher Spin / CFT

Singlet sector of free scalar theories with U(N)/O(N) symmetry group

Low energy spectrum very different from semiclassical gravity. Infinite tower of conserved primary singlet currents of the form

$$J^{(s)}_{\mu_1\ldots\mu_s} \sim \mathrm{Tr}\, \Phi \partial_{\mu_1\ldots\mu_s} \Phi$$

Higher spin currents

(e.g. stress tensor at spin 2)

Bulk coupling constant: $\langle JJJ \rangle \sim 1/N^{\gamma}$, $\gamma \geq 1$ theory dependent.

Single trace spectrum depends on the U(N)/O(N) representation:

Fundamental → Only conserved currents and scalar $\mathrm{Tr}\, \Phi^2$ → Dual to Vasiliev's higher spin theories (Klebanov&Polyakov 2002)

Adjoint → Conserved currents and scalar $\mathrm{Tr}\, \Phi^2$, but also infinite number of multilinears $\mathrm{Tr}\, \partial^n \Phi^m$ → Free $\mathcal{N}=4$ at $\lambda = 0$ dual to tensionless strings on AdS (Sundborg 1999, Witten 1999)

Phase transitions in free large N CFTs

Due to relations amongst products of singlet operators.

At sufficiently low temperatures, number of singlets involved in typical states dominating the ensemble is relatively small. Partition sum well described by gas of independent singlets.

At high temperatures, more singlet operators contribute than there are degrees of freedom. Relations become important. Partition sum approaches that of theory without singlet condition.

Enforcing the singlet constraint

Introduce $\delta_{Q^a,0} \sim \int d\lambda^a e^{i\lambda^a Q^a}$ for all U(N) charges into all thermal expectation values. Constraining matrix $\cdot\lambda = \lambda^a T^a_R$

In thermal field theory, acts as additional Lagrange multiplier field in path integral. Time-component of non-dynamical gauge field, gauge holonomy.

Free scalars can be integrated out. All correlation functions then involve effective action for eigenvalues of constraining matrix.

Gauge redundancy unavoidable. Gauge fixing implies Fadeev-Popov determinant giving repulsive contribution to effective action.

[Aharony et al, 2003]

Coupling to scalars provides attractive contributions.

Eigenvalues

Effective action

$$S \approx \underbrace{\sum_{i\neq j} \sum_{k=1}^{\infty} \frac{1}{k} \cos(k(\lambda_i - \lambda_j))}_{\text{Repulsive}} - \underbrace{\sum_i S_s}_{\text{Attractive}}$$

Repulsive

Attractive

Thermal dynamics

For $N \to \infty$: Eigenvalue distribution $\rho(\lambda)$ or Fourier cos transform $\rho_k = \int d\lambda\, \rho(\lambda) \cos k\lambda$

Normalized and positive $\int d\lambda\, \rho(\lambda) = \rho_0 = 1, \quad \rho(\lambda) \geq 0$

Effective action:

$$S_f = N^2 \sum_{k=1}^{\infty} \frac{\rho_k}{k} \left(\rho_k - 2 \frac{1}{N} z_S^d(x^k) \right)$$

Fundamental

$$S_a = N^2 \sum_k \frac{\rho_k^2}{k} \left(1 - z_S^d(x^k)\right)$$

Adjoint

Phase transition

Attractive contributions grow with T. Can overcome repulsion. Eigenvalues bunch up.

With adjoint scalars transition follows Hagedorn like growth for the partition sum and happens at $T \sim \mathcal{O}(1)$. First order transition.

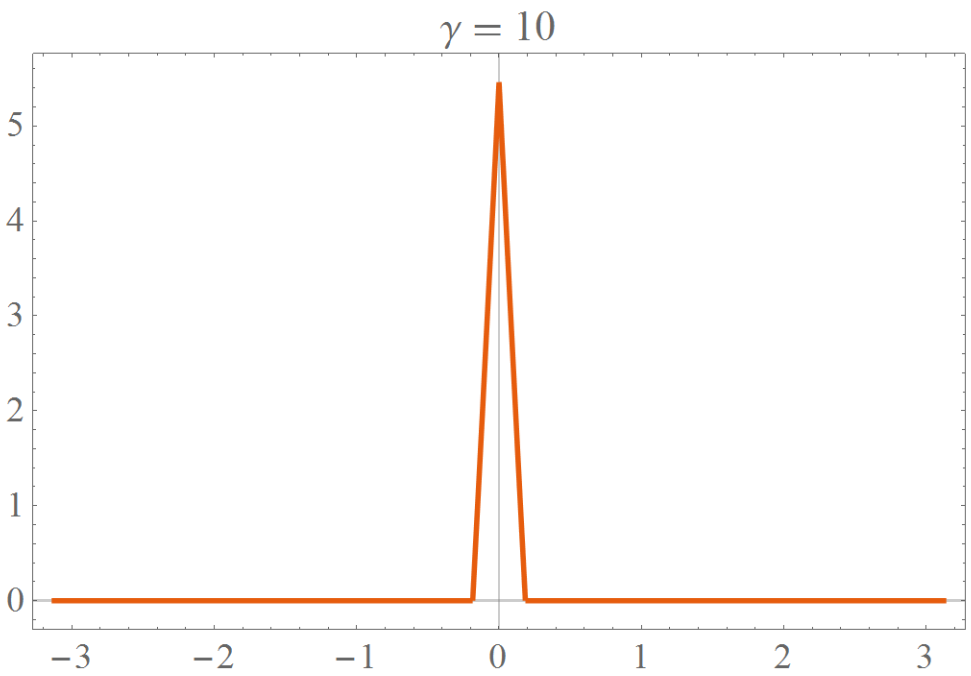
[Sundborg 2000, Aharony et al. 2003]

With only fundamental scalars $T \sim \mathcal{O}(N^{\frac{1}{d-1}})$. Third order.

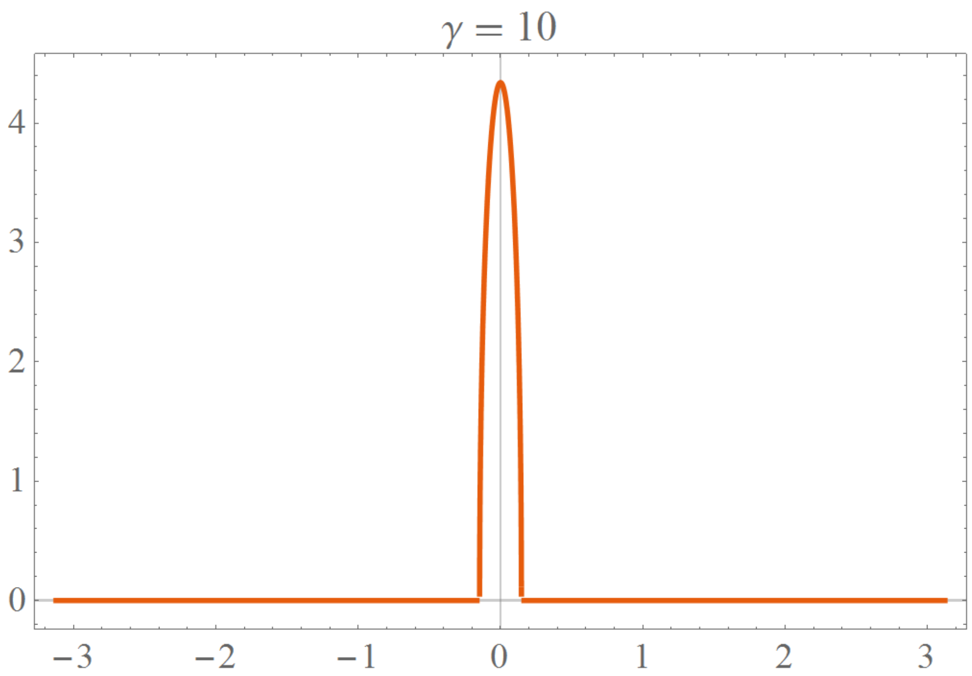
[Gross&Witten 1980, Shenker&Yin 2011]

Saddles

FUNDAMENTAL SCALARS



ADJOINT SCALARS



	Fundamental scalars		Adjoint scalars	
$T \ll T_c$	$\rho_{k\neq 0} = \frac{1}{N} z_S(x_d^k)$	$F \sim \mathcal{O}(1)$	$\rho_{k\neq 0} = 0$	$F \sim \mathcal{O}(1)$
				Hagedorn
$T \gg T_c$	$\rho_k \to 1$	$F \sim \mathcal{O}(N)$	$\rho_k \to 1$	$F \sim \mathcal{O}(N^2)$

Emergence of geometry

For low lying states, interpretation in terms of AdS-space with a few particles by construction (structure fixed by conformal invariance)

But what do typical (highly) excited states correspond to?

In ordinary AdS/CFT, compelling case for black holes. However, the bulk theory is very different from a weakly coupled gravitational theory.

Probe using thermal two point function of lightest scalar singlet $\mathrm{Tr}\,\Phi^2$

Propagator in free singlet models

Euclidean propagator gives rise to all correlation functions of interest by analytic continuation (free theory)

E.g. vector model: $G_{lM}^{AB}(n) = \sum_{j=1}^{N} \frac{\Psi_j^A (\Psi_j^B)^\dagger}{\beta^{-2}\left(2\pi n + \lambda_j\right)^2 + E_\ell^2}$

Analytic continuation, Fourier transformation and Wick contraction yield singlet-singlet correlator

$$\left\langle \mathrm{Tr}|\Phi^2(t,\mathbf{y})|\mathrm{Tr}|\Phi^2(0)|\right\rangle \sim \sum_{j=1}^{N}\left(\sum_{m=-\infty}^{\infty}\frac{e^{im\lambda_j}}{(\cos(t+i\beta m)-\cos\theta)^{\frac{d-2}{2}}}\right)^2$$

$$\sim \int_{-\pi}^{\pi} d\lambda\, \rho(\lambda)\left(\sum_{m=-\infty}^{\infty}\frac{e^{im\lambda}}{(\cos(t+i\beta m)-\cos\theta)^{\frac{d-2}{2}}}\right)^2$$

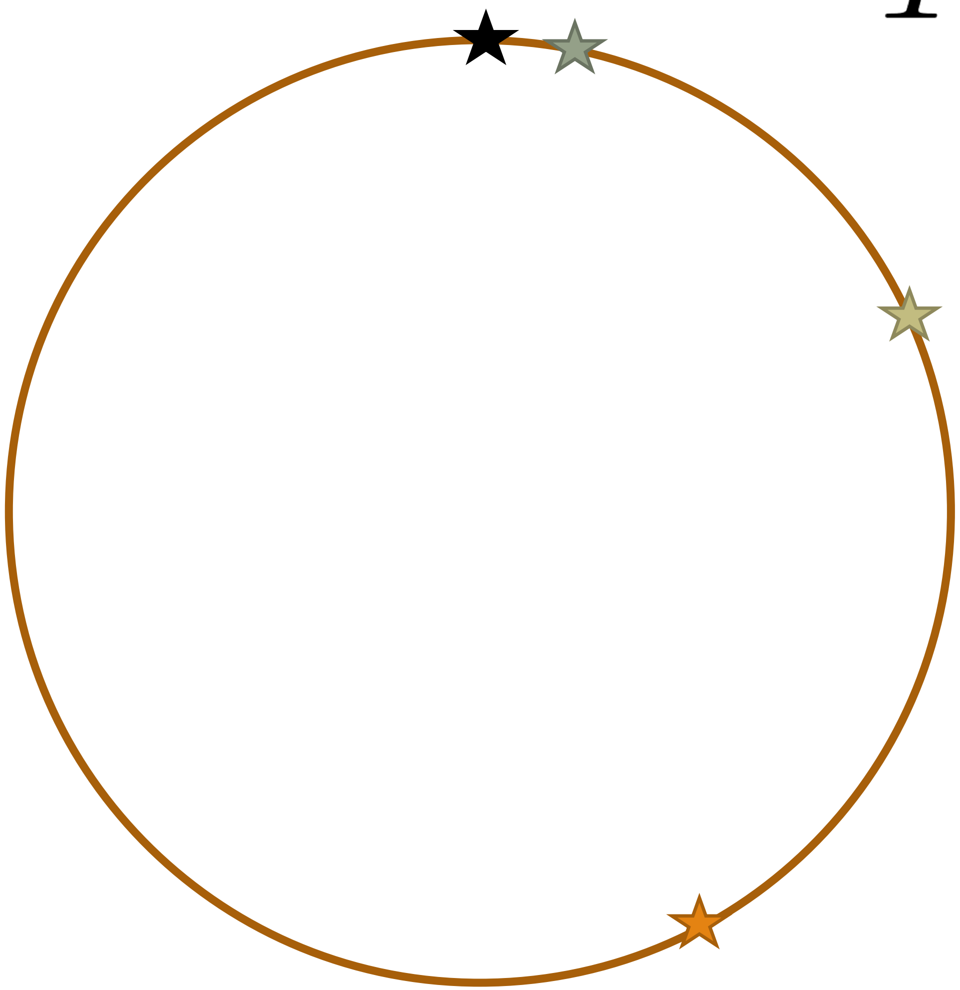
A - Equal time correlators

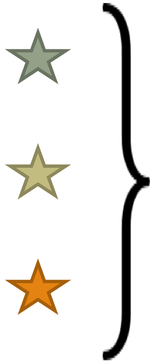
$$\left\langle \mathrm{Tr}\,|\Phi^2(0,\mathbf{y})|\mathrm{Tr}\,|\Phi^2(0)|\right\rangle \sim \begin{cases} \displaystyle\sum_{n=-\infty}^{\infty} \frac{1}{(\cosh n\beta - \cos\theta)^{d-2}} & T \ll T_c \\ \displaystyle\left(\sum_{n=-\infty}^{\infty} \frac{1}{(\cosh n\beta - \cos\theta)^{\sigma}}\right)^2 & T \gg T_c \end{cases}$$

λ - integration projects on diagonal terms in sum, $\rho(\lambda) = \text{const}$

$\rho(\lambda) = \delta(\lambda)$

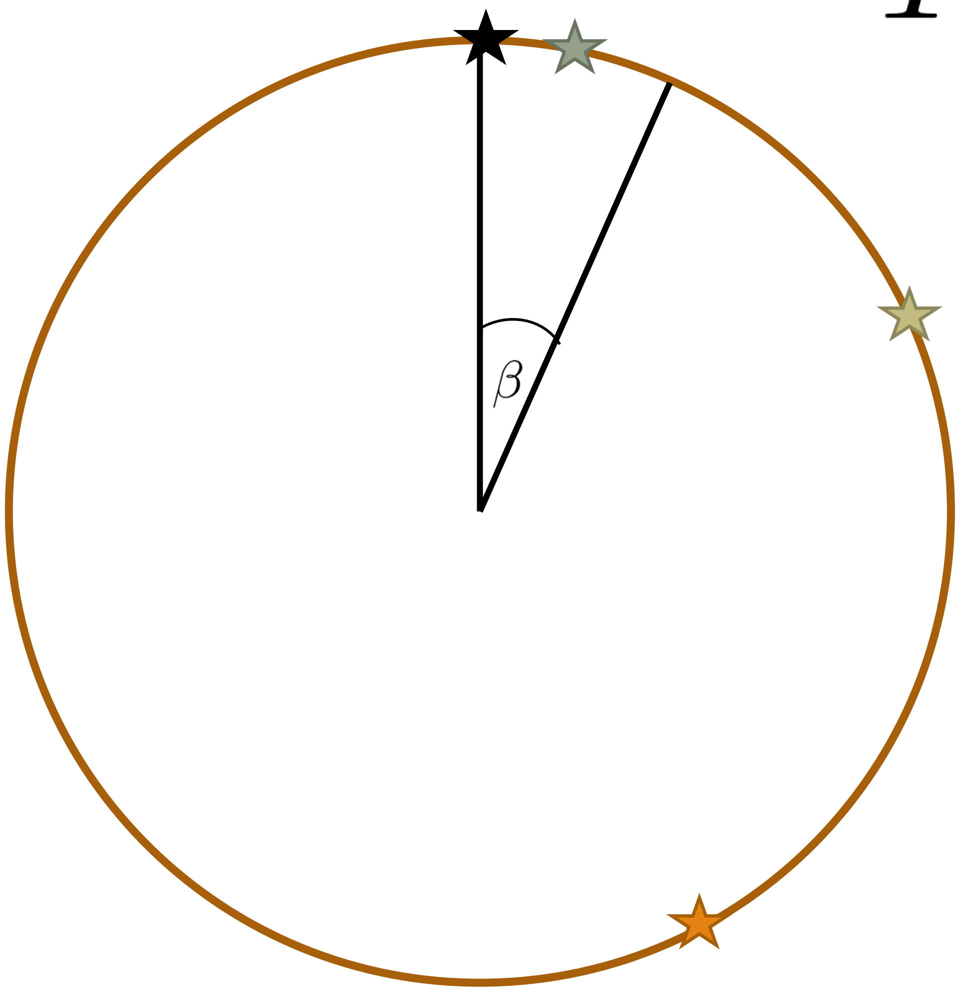
$$T \ll T_c$$





$$\langle \mathrm{Tr}\,|\phi(0)|^2\,\mathrm{Tr}\,|\phi(y)|^2 \rangle = \mathcal{K}^{d+1}_{\mathrm{AdS}}(\beta,\theta)$$

$$T \gg T_c$$



$$\theta \ll \beta : \ \left\langle \mathrm{Tr}\,|\phi(0)|^2\, \mathrm{Tr}\,|\phi(y)|^2 \right\rangle = \mathcal{K}^{d+1}_{\mathrm{AdS}}(\beta,\theta)$$

$$\theta \gg \beta : \ \left\langle \mathrm{Tr}\,|\phi(0)|^2\, \mathrm{Tr}\,|\phi(y)|^2 \right\rangle = \frac{F_d(\theta)}{\beta^2}$$

Angle at which deviations appear of order 1 in adjoint theories. AdS-sized object. In vector models, $\theta \lesssim 1/\sqrt{N}$, implying size parametrically larger than R_{AdS}

Concrete form of correlators suggests deviations from Euclidean geometry with noncontractible imaginary time circle visible at superthermal distances. Consistent with expectation value of Polyakov loop. “Deconfinement”.

In adjoint case may be related to gluing of string bit propagators.

[Gopakumar 2003/04, Furuuchi 2006]

In fundamental case interpretation more subtle, but since analytic form the same, conclusion should carry over.

On the other hand, key differences in the way the phase transition is approached. In vector model ρ_k receives $\mathcal{O}(1)$ - contributions for low k even for $T/T_c \lesssim 1$

B – time dependent autocorrelator

$$G(t) = (-2)^{2-d} \int d\lambda \, \rho(\lambda) \left(\sum_{m=-\infty}^{\infty} \frac{e^{im\lambda} (\text{sgn}(m))^d}{\sin^{d-2}\left(\frac{t+i\beta m}{2}\right)} \right)^2$$

Implies dimensional dependence of high T correlators

Periodic, since free theory on sphere of radius $R = 1$.

Investigate decay behavior for $t \ll R$.

At **low temperatures**, $T \ll T_c$, picks out diagonal part of square

$$G(t) \sim \sum_{m=-\infty}^{\infty} \left| \frac{1}{\sin^{d-2}\left(\frac{t+i\beta m}{2}\right)} \right|^2$$

Power law decay

At **high temperatures**, behavior depends on number of boundary dimensions (cf. Huygen's principle).

Even boundary dimensions: Poisson summation

Higher dimensional correlator obtained by differentiation w.r.t. t

d=4:

$$G(t) \sim \int d\lambda\, \rho(\lambda) \left(\sum_{k=-\infty}^{\infty} \operatorname{csch}\left(\frac{\pi\lambda}{\beta}\right) \partial_t e^{\frac{(\pi-t)\lambda}{\beta}} \Big|_{\lambda+2\pi k} \right)^2$$

Can for $\beta \ll 1$ be approximately resummed

$$G(t) \sim \frac{1}{\beta^2} \int d\lambda\, \rho(\lambda) \left[\partial_t \left(e^{-\frac{\lambda t}{\beta}} \left(\coth\left(\frac{\pi t}{\beta}\right) + \coth\left(\frac{\pi\lambda}{\beta}\right) \right) \right) \right]^2$$

For $\rho(\lambda) \to \delta(\lambda)$

$$G(t) \sim \frac{1}{\beta^2}\left[1 + \frac{\pi^2}{\beta}\text{csch}^2\left(\frac{\pi t}{\beta}\right)\right]^2$$

Exponential decay

Finite volume contribution

-> Exponential decay stops at $t \sim \dfrac{\log T}{T}$, remains approximately constant

Holds even for spatially smeared operators and more complicated single traces.

This exponential decay is not due to dissipation of individual momentum modes (conserved), which in fact decay as power laws [Hartnoll, Kumar, 2005].

Instead due to intricate interference pattern which emerges beyond the critical temperature.

For free singlet models, this is however the only way to retain this feature of theories with weakly coupled gravitational duals.

Odd boundary dimensions: No exponential decay.

Instead, in high temperature limit, summation over Matsubara frequencies yields Q-Polygamma functions that give rise to effective dimensional reduction

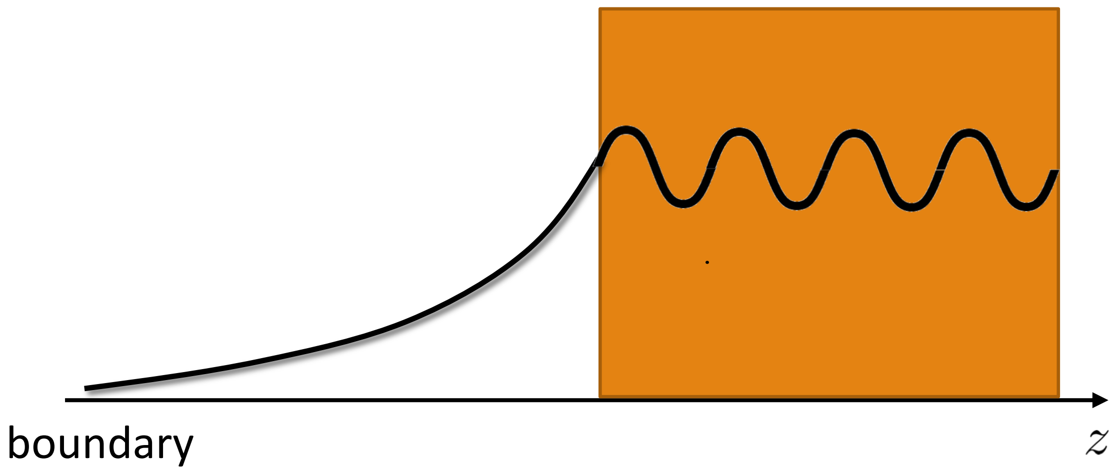
$$G^{(d)}(t) \sim \left(\Pi_{\rm vac}(t) + \frac{i}{\beta} \Pi_T^{(d-1)}(t) \right)^2$$

Logarithmic in d=3. Power law tails in higher dimensions.

Note the support of the retarded propagator $G_R(t) = \Theta(t) {\rm Im} G(t)$ inside the light cone.

C - Evanescent modes...

Evanescent modes with imaginary k_z only supported in bulk spacetimes with nontrivial spatial structure (cf. Suvrat's talk).



Leave exponentially suppressed imprint in spectral density as the imaginary part of retarded Green's function in Fourier space $\mathrm{Im}\, G_R \sim e^{-\#\beta k}$ for $\omega \ll |\mathbf{k}|$, $T \ll |\mathbf{k}|$

[Son&Starinets 2002, Rey&Rosenhaus 2014]

… in higher spin gravity

Argued to be present in planar limit by invoking breaking of $U(N) \times U(N) \to U(N)$ in Keldysh action by thermal effects.

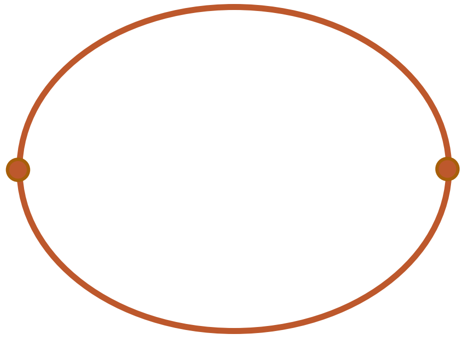
[Jevicki&Yoon, 2016]

Could be expected from $\mathcal{O}(N)/\mathcal{O}(N^2)$ free energy

[cf e.g. Papadodimas&Raju 2012]

Spectral density $\mathrm{Im} G_R(k,\omega)$ with

$$G_R(k,\omega) = \int dt\, d^{d-1}p_1\, d^{d-1}p_2\, e^{i\omega t}\delta^{(d-1)}(\mathbf{k} - \mathbf{p_1} - \mathbf{p_2})\, G_R(p_1,p_2,t)$$



$$G_R(p_1,p_2,t) = \Theta(t)\,\mathrm{Im}\left[\int d\lambda\,\rho(\lambda) G_T(p_1,t,\lambda) G_T(p_2,t,\lambda)\right]$$

$$G_T(p,t,\lambda) = \frac{1}{2p}\left[e^{-ipt}\left(\Theta(t) + n_B(p - i\frac{\lambda}{\beta})\right) + e^{ipt}\left(\Theta(-t) + n_B(p + i\frac{\lambda}{\beta})\right)\right]$$

Time integration gives rise to various δ-functions that kill non-thermal contributions in far off-shell regime and imply support for G_T only for $p \gg 1$. Bose distribution gives Boltzmann suppression.

Final result to lowest order in $\frac{\omega}{k}$

$$\operatorname{Im} G_R(k,\omega) \sim \frac{\omega}{k}\left(\frac{k}{\beta}\right)^{\frac{d-4}{2}} \rho_1 e^{-\frac{k\beta}{2}}$$

Expected contribution from evanescent modes

Directly linked to phase transition

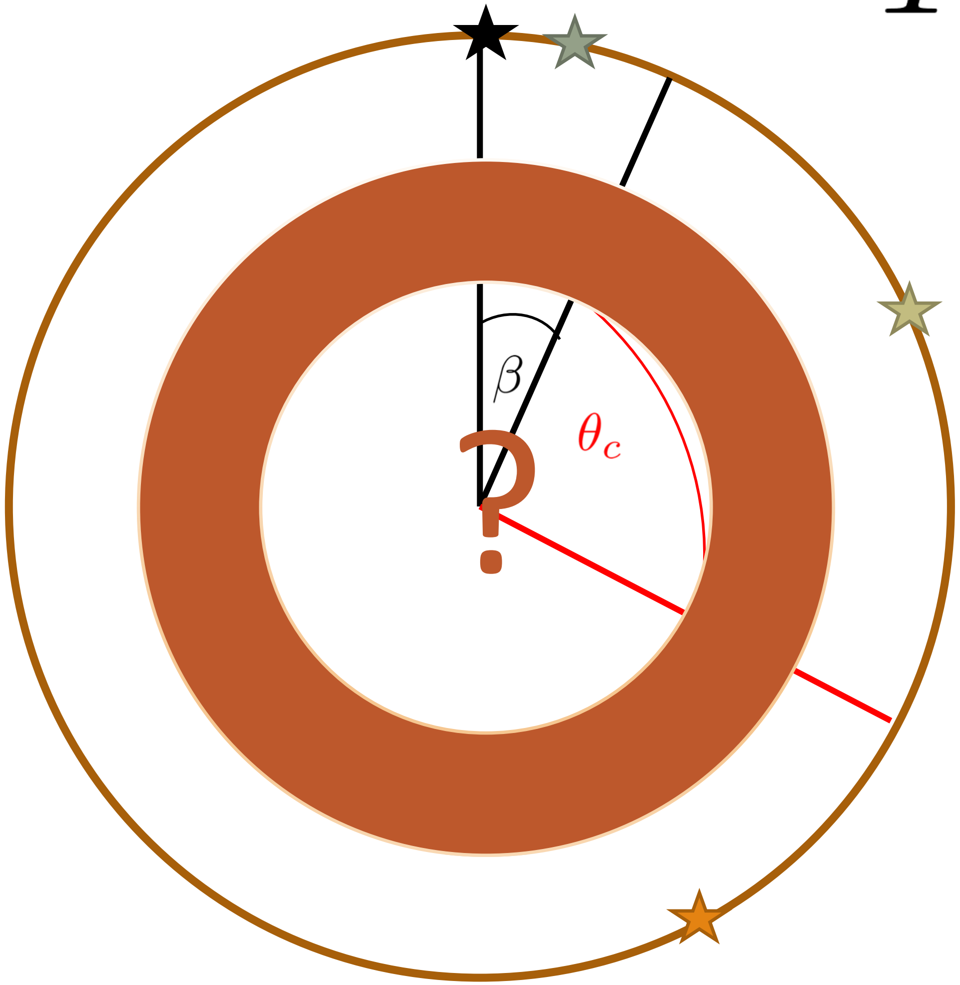
Further observations

Interestingly, there is quite a large temperature range (infinite for d=3) with sensitivity to the central region of the eigenvalue distribution

Manifests itself at large angles / long times. Finite volume effect.

Since central region of distribution approximately constant, correlators return to a renormalized thermal AdS form.

$T \gg T_c$



★ $\theta \ll \beta: \ \left\langle \mathrm{Tr}\,|\phi(0)|^2\, \mathrm{Tr}\,|\phi(y)|^2 \right\rangle = \mathcal{K}^{d+1}_{\mathrm{AdS}}(\beta,\theta)$

★ $\theta_c \gg \theta \gg \beta: \ \left\langle \mathrm{Tr}\,|\phi(0)|^2\, \mathrm{Tr}\,|\phi(y)|^2 \right\rangle = \frac{F_d(\theta)}{\beta^2}$

★ $\theta \gg \theta_c: \ \left\langle \mathrm{Tr}\,|\phi(0)|^2\, \mathrm{Tr}\,|\phi(y)|^2 \right\rangle = f_\beta \mathcal{K}^{d+1}_{\mathrm{AdS}}(\beta,\theta)$

Conclusions

Thermal two-point functions in singlet sector of free U(N) models controlled by eigenvalue distribution.

Thermal AdS at low temperatures, significant deviations at high temperatures. Surprising number of black hole features retained even for higher spin theories.

- Spatial structure suggests link to emergent geometry without noncontractible thermal circle.
- Evanescent modes appear beyond the transition.
- In even boundary dimensions, correlations decay exponentially

Key differences between matrix and vector models arise in the way the phase transition is approached.

What about odd dimensional behavior? Implications for BH in 3+1D Vasiliev theory?

Understand central region. Possible appearance in theories with nonvanishing ‘t Hooft coupling?