Warped Black Holes in Lower-Spin Gravity

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[T. Azeyanagi, S. Detournay, M.R; 1801.07263]

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WAdS$_3$

- What?

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- What?

AdS$_3$ deformation:

$$ds^2_{\mathrm{WAdS}_3} = ds^2_{\mathrm{AdS}_3} - 2H\xi \otimes \xi$$

$WAdS_3$

- What?
- Why?

WAdS$_3$

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NHEK $\rightarrow$ global $\mathfrak{sl}(2,\mathbb{R}) \oplus \mathfrak{u}(1)$ isometry.

WAdS$_3$

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At fixed polar angle $\rightarrow$ (selfdual spacelike) WAdS$_3$.

$WAdS_3$

- What?
- Why?

NHEK $\rightarrow$ global $\mathfrak{sl}(2,\mathbb{R}) \oplus \mathfrak{u}(1)$ isometry.

At fixed polar angle $\rightarrow$ (selfdual spacelike) $WAdS_3$.

$\rightarrow$ study NHEK.

WCFT

- What?

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QFT with global $\mathfrak{sl}(2,\mathbb{R}) \oplus \mathfrak{u}(1)$ symmetries.

WCFT

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QFT with global $\mathfrak{sl}(2,\mathbb{R}) \oplus \mathfrak{u}(1)$ symmetries.

Has ∞ conserved charges but not Lorentz invariant.

WCFT

- What?
- Why?

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Possible WAdS$_3$ dual.

WCFT

- What?
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Possible WAdS$_3$ dual.

$\rightarrow$ better understanding of holography in a realistic setup!

Lower-Spin Gravity

- What?

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$\mathfrak{sl}(2,\mathbb{R}) \oplus \mathfrak{u}(1)$ Chern-Simons theory.

Lower-Spin Gravity

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- What?
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Very simple model of $WAdS_3$ without local DOF.

Lower-Spin Gravity

- What?
- Why?

- Black Holes?

ULB

Lower-Spin Gravity

- What?
- Why?

- Black Holes?
- Higher-Spin $WAdS_3$ black holes?

- Boundary conditions $\rightarrow$ asymptotic symmetries.

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- Determine thermal entropy.

Outline

Describing $WAdS_3$ BHs Using Lower-Spin Gravity

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  - Compare with WCFT results.

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  - Compare with WCFT results.
  - Holographic entanglement entropy.
  - Metric interpretation.

$$I_{\mathrm{CS}} = \frac{k}{4\pi}\int_{\mathcal{M}}\langle\mathcal{A}\wedge\mathrm{d}\mathcal{A}+\frac{2}{3}\mathcal{A}\wedge\mathcal{A}\wedge\mathcal{A}\rangle+\frac{\kappa}{8\pi}\int_{\mathcal{M}}\langle\mathcal{C}\wedge\mathrm{d}\mathcal{C}\rangle$$

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Boundary Conditions

$$\mathcal{A}(\rho,t,\varphi) = b^{-1}(\rho)\left[a(t,\varphi)+\mathrm{d}\right]b(\rho)$$
$$\mathcal{C}(\rho,t,\varphi) = c(t,\varphi)$$

$$a_\varphi = \mathtt{L}_1 - \left(\frac{2\pi}{k}\left(\mathcal{L} - \frac{2\pi}{\kappa}\mathcal{K}^2\right)\right)\mathtt{L}_{-1} \qquad a_t = 0$$

$$c_\varphi = \frac{4\pi}{\kappa}\mathcal{K}\mathtt{S} \qquad c_t = \mathtt{S}$$

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Asymptotic Symmetries

$$[\mathtt{L}_n,\,\mathtt{L}_m] = (n-m)\,\mathtt{L}_{n+m} + \frac{c}{12}n(n^2-1)\delta_{n+m,0}$$
$$[\mathtt{L}_n,\,\mathtt{K}_m] = -m\,\mathtt{K}_{n+m}$$
$$[\mathtt{K}_n,\,\mathtt{K}_m] = \frac{\kappa}{2}\,n\,\delta_{n+m,0}$$

[Hofman, Rollier ’14]

- Determine mass M and angular momentum J.

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Assumption

$$\varepsilon = \xi^{\mu}\mathcal{A}_{\mu}, \qquad \bar{\varepsilon} = \xi^{\mu}\mathcal{C}_{\mu},$$

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$$\varepsilon = \xi^\mu \mathcal{A}_\mu, \qquad \bar{\varepsilon} = \xi^\mu \mathcal{C}_\mu,$$

$$\delta M := \delta Q[\varepsilon\big|_{\partial_t}] + \delta Q[\bar{\varepsilon}\big|_{\partial_t}] = 2\pi\delta\mathcal{K}$$

$$\delta J := \delta Q[\varepsilon\big|_{-\partial_\varphi}] + \delta Q[\bar{\varepsilon}\big|_{-\partial_\varphi}] = -2\pi\delta\mathcal{L}$$

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$$\delta S_{\mathrm{Th}} = \beta\,(\delta M - \Omega\delta J) = k\langle h\,\delta a_\varphi\rangle + \frac{\kappa}{2}\langle\bar{h}\,\delta c_\varphi\rangle$$

where

$$h = \frac{\beta}{2\pi}\oint \mathrm{d}\varphi\,(a_t + \Omega a_\varphi)$$
$$\bar{h} = \frac{\beta}{2\pi}\oint \mathrm{d}\varphi\,(c_t + \Omega c_\varphi)$$

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$$\mathrm{Eigen}[h] = \mathrm{Eigen}[2\pi \mathrm{L}_0] \qquad \mathrm{Eigen}[\bar{h}] = \mathrm{Eigen}[2\pi\gamma \mathtt{S}]$$

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$$\beta = 2\pi\left(\gamma - \frac{M}{\kappa\sqrt{\frac{1}{k}\left(-J - \frac{M^2}{\kappa}\right)}}\right) \qquad \Omega = \frac{1}{2\gamma\sqrt{\frac{1}{k}\left(-J - \frac{M^2}{\kappa}\right)} - \frac{2M}{\kappa}}$$

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Thermal Entropy

$$S_{\mathrm{Th}} = 2\pi\left(M\gamma + \sqrt{\frac{c}{6}\left(-J - \frac{M^2}{\kappa}\right)}\right)$$

$$S_{\text{Th}} = -\frac{4\pi i M M^{\text{v}}}{\kappa} + 4\pi\sqrt{-\left(-J^{\text{v}} - \frac{(M^{\text{v}})^2}{\kappa}\right)\left(-J - \frac{M^2}{\kappa}\right)}$$

[Detournay, Hartman, Hofman '12]

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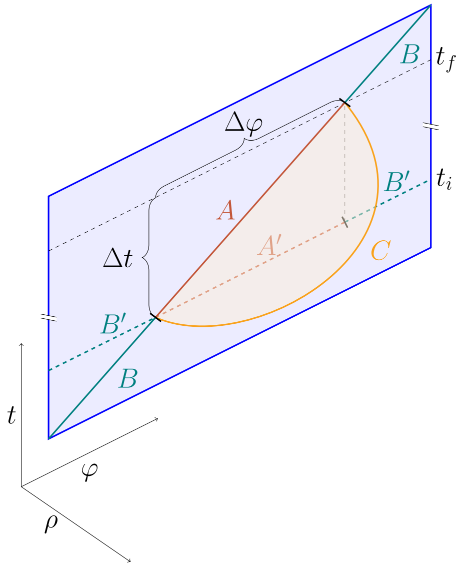
Consistency Checks

HEE and Wilson Lines



$$S_{\mathrm{EE}} = -\log\left[\mathcal{W}_{\mathcal{R}}^{\mathfrak{sl}(2,\mathbb{R})}(C;\mathcal{A})\right] - \log\left[\mathcal{W}_{\mathcal{R}}^{\mathfrak{u}(1)}(C;\mathcal{C})\right]$$

[Castro, Hofman, Iqbal '15]

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Holographic Entanglement Entropy

$$S_{\mathrm{EE}} = \gamma\left(\frac{\kappa}{2}\Delta t + M\Delta\varphi\right) + \frac{c}{6}\log\left[\frac{\beta_\varphi}{\pi\epsilon}\sinh\left[\frac{\pi\Delta\varphi}{\beta_\varphi}\right]\right]$$

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$$S_{\mathrm{EE}} = iM^{\mathrm{v}}\left(-\Delta t + \frac{\beta-\delta}{\beta_\varphi}\Delta\varphi\right) + \left(i\frac{\delta}{\pi}M^{\mathrm{v}} - 4J^{\mathrm{v}}\right)\log\left[\frac{\beta_\varphi}{\pi\epsilon}\sinh\left[\frac{\pi\Delta\varphi}{\beta_\varphi}\right]\right]$$

[Song, Wen, Xu ’16]

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$$\Rightarrow \delta = 2\pi\gamma$$

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$$S_{\mathrm{EE}} \overset{\frac{\Delta\varphi}{\beta_\varphi}\gg 1}{\approx} \frac{S_{\mathrm{Th}}}{2\pi}\Delta\varphi$$

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- Metric:

$$\mathrm{d}s^2 = g_{\mu\nu} dx^\mu dx^\nu = \langle \mathcal{B}, \mathcal{B} \rangle$$

Consistency Checks
Metric Formulation

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WAdS_3 BH Parameters

$$\mathfrak{b}^2 = \frac{\nu^2}{2\ell^2} \qquad \mathfrak{c} = \frac{\nu^2 + 3}{2\ell^2} \qquad \alpha = \frac{16}{\mathfrak{c}^2 k \ell^2}$$

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$\Rightarrow$ spacelike (stretched) warped AdS_3 black hole!

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Initial Assumption

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- Thermodynamics: $\beta(M, J)$, $\Omega(M, J)$ ✓
- $\gamma = \frac{\mathfrak{b}}{2}\sqrt{\frac{k\alpha}{2}} = \frac{2\nu}{\nu^2+3}$

Conclusion and Outlook

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- Near horizon soft hair?

Thank you for your attention!