Bulk reconstruction in the AdS black holes

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Focus Week on Quantum Gravity and Holography
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Based on arXiv:1704.00053 JHEP(2017)
collaboration with Tadashi Takayanagi (YITP, Kyoto)

- Motivation
- The construction in the pure AdS spacetime
- The construction in the AdS black holes
- Conclusion and Prospective

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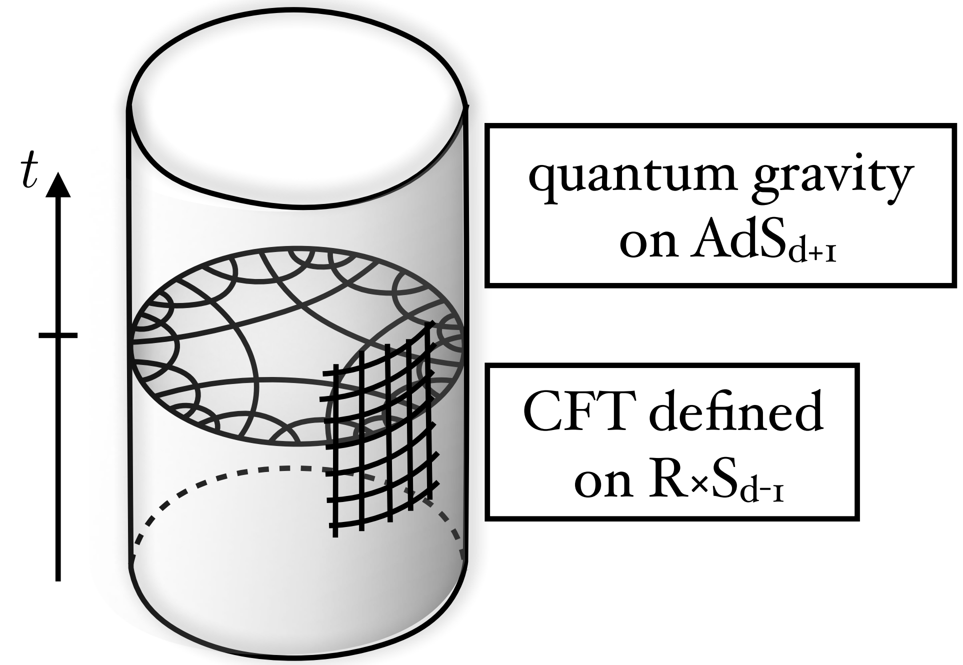
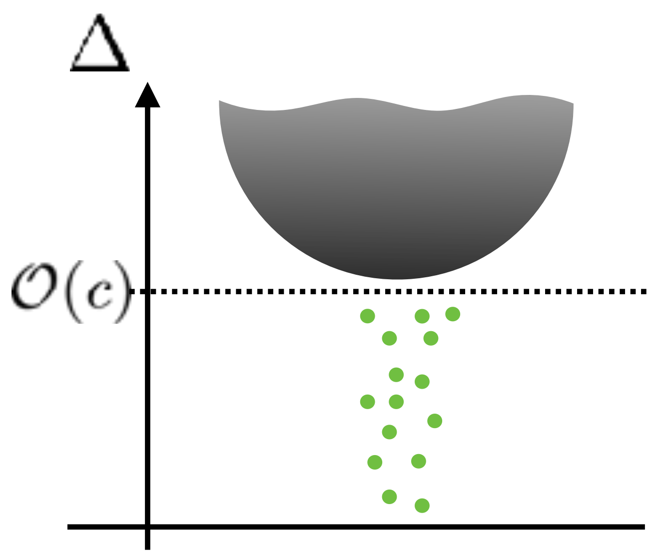
Motivation



- AdS/CFT gives the UV complete description of the quantum gravity on the AdS spacetime in terms of conformal field theories defined on its boundary.

- ***What kind of CFTs have good spacetime descriptions in AdS side?***

[Heemskerk-Penedones-Polchinski-Sully, Papadodimas-El-Showk, Hartman, Fitzpatrick-Kaplan-Walters...]



- CFT has large degrees of freedom $c \to \infty$

↔weakly coupled (semi-classical) gravity in the dual side

- Sparse spectrum in the low energy sector

↔few light matter fields in the low energy effective theory of gravity (Einstein gravity)

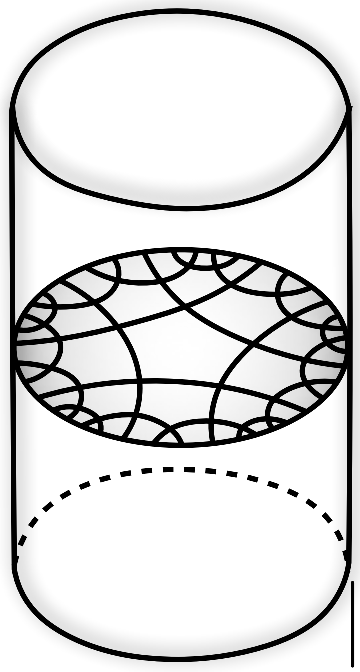
- Dense spectrum in the high energy sector $\Delta \sim \mathcal{O}(c)$

↔ corresponds to the black hole microstates

Is it sufficient for CFT to describe classical spacetime?

How about black hole spacetime?

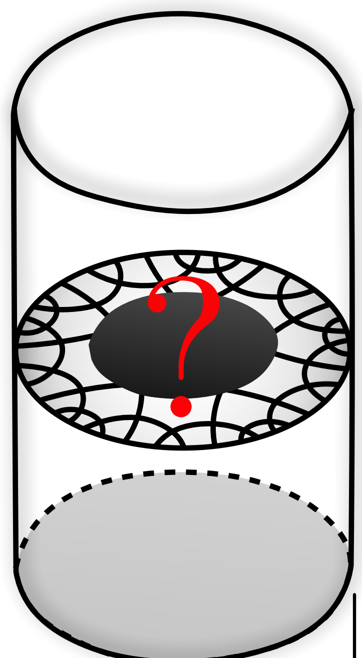
What spacetime do the high energy CFT states describe?



- The CFT vacuum state describes the pure AdS spacetime.

→Checked by enormous number of tests including
the 1/c (quantum gravity corrections) corrections…

- On the other hand, the spacetime structures dual to the high energy excited states are less known.



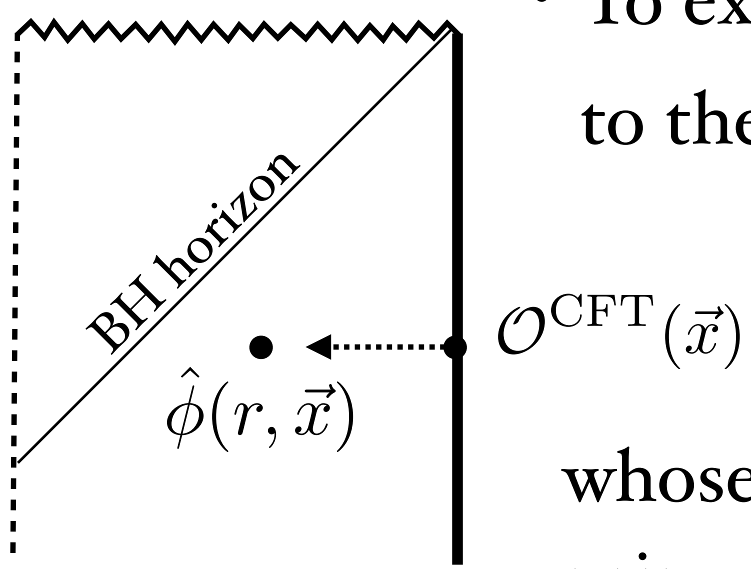
- If we assume the Eigenstate Thermalization Hypothesis (ETH) holds for large c CFTs, the high energy eigenstate behaves as a black hole (or thermal AdS).

$$\langle E|\mathcal{O}_1 \dots \mathcal{O}_n|E\rangle = \frac{1}{Z_\beta}\mathrm{Tr}[e^{\beta H}\mathcal{O}_1 \dots \mathcal{O}_n] + \mathcal{O}(1/c)$$

- They really have black hole spacetime structures in the dual side?
Good semi-classical spacetime descriptions for the black hole interior?

c.f. Firewall problem [Almheiri-Marolf-Polchinski-Sully-(Stanford)]

The bulk local operator acting on "the black hole states" in CFT



- To explore the bulk interior from CFT, consider the CFT duals to the local field operators acting on the black hole states

$$\hat{\phi}^{\rm CFT}(r,\vec{x})|\Psi_{\rm BH}\rangle \in \mathcal{H}^{\rm CFT}$$

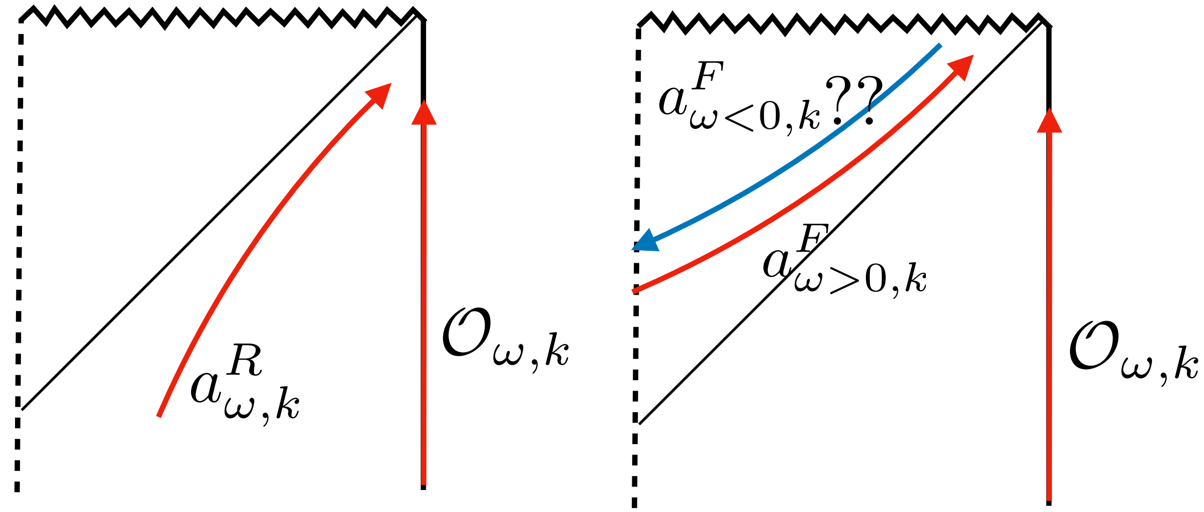
whose boundary limits are the CFT states locally exited by primary operators

$$\lim_{r\to\infty} r^{\Delta}\hat{\phi}^{\rm CFT}(r,\vec{x})|\Psi_{\rm BH}\rangle = \mathcal{O}(\vec{x})|\Psi_{\rm BH}\rangle$$

- ``The black hole states" in CFT have good semi-classical descriptions of black hole spacetimes?

→Can we reconstruct ones that reproduces the two point function on the classical black hole spacetime (thermal correlator) including its interior?

$$\langle\Psi_{\rm BH}|\hat{\phi}^{\rm CFT}(r,\vec{x})\hat{\phi}^{\rm CFT}(r',\vec{x}')|\Psi_{\rm BH}\rangle = G_{\beta}(r,\vec{x};r',\vec{x}')$$



t : timelike $\quad$ t : spacelike

Attempts to reconstruct them including its interior *[Hamilton-Kabat-Lifshetz-Lowe,…]* have some difficulty for the black hole having a single asymptotic boundary (=the BH described by a single CFT).

- Consider expanding the bulk local fields in the Rindler modes

-Outside the horizon: only positive frequency modes moving in the future time direction. → Can be constructed from Fourier modes of the CFT operator

-Inside the horizon: ``negative” frequency modes are allowed since the Schwarzschild time is a spacelike coordinate inside the horizon and its conjugate momentum can take negative values.

→ It seems that CFT doesn’t contain operators corresponding to these modes.

- A remarkable proposal resolving this problem was given by Papadodimas & Raju. They "doubled" the CFT operator using Tomita-Takesaki theory for the operator algebra. (Their construction is still valid for the black hole states with some small excitations.)

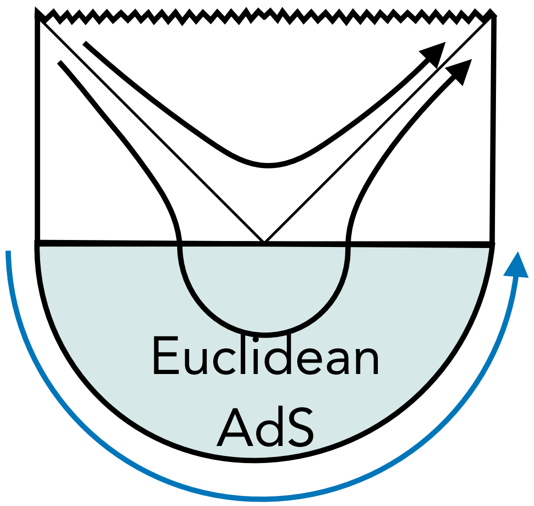
- We propose a completely different approach: "Euclidean path integral approach" to the construction of bulk local operators on the black hole states.

- This construction is an application of the method for the pure AdS spacetime proposed by Takayanagi and his collaborators & Nakayama-Ooguri 15' to the black hole states.

- Our approach can be applied for a single-sided black holes as well as double-sided ones!

→ First give a rough sketch of our construction for the double-sided black holes

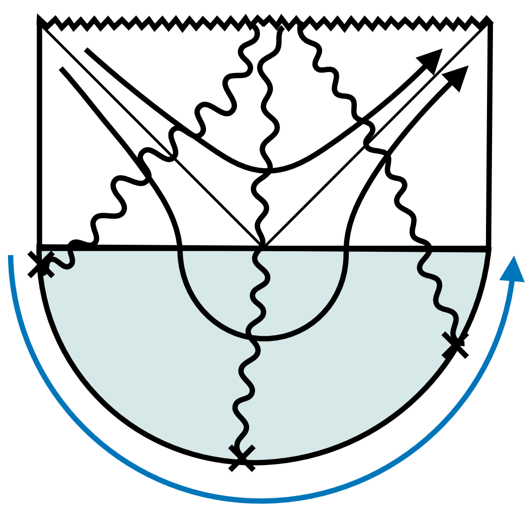
A sketch of our construction



Hartle-Hawking construction of the double-sided BH

(Double-sided) BH state is given by the path-integral over the Euclidean geometry attached below half the Lorentzian one.

$$|\Psi_{\text{H-H}}\rangle \Leftrightarrow |\text{TFD}\rangle_{\text{CFT}}$$



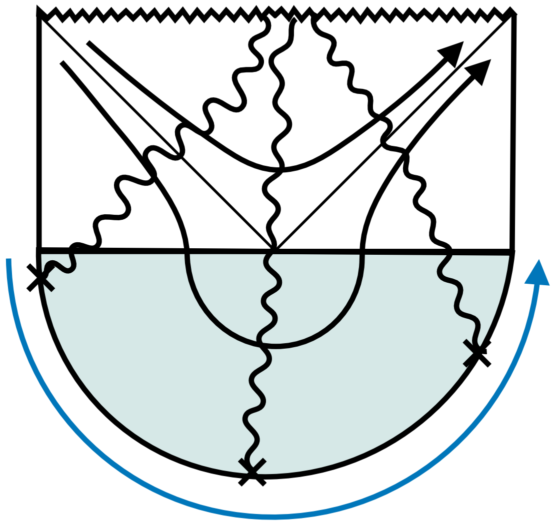
Hartle-Hawking state + some particles

We can add particles to the Hartle-Hawking state by inserting CFT operators during the Euclidean path-integral.
[Maldacena '03 Maldacena-Kourkoulou '17]

$$\mathcal{O}(z)|\text{TFD}\rangle_{\text{CFT}}$$

Boundary position of CFT operators⇔ Bulk position of the particles

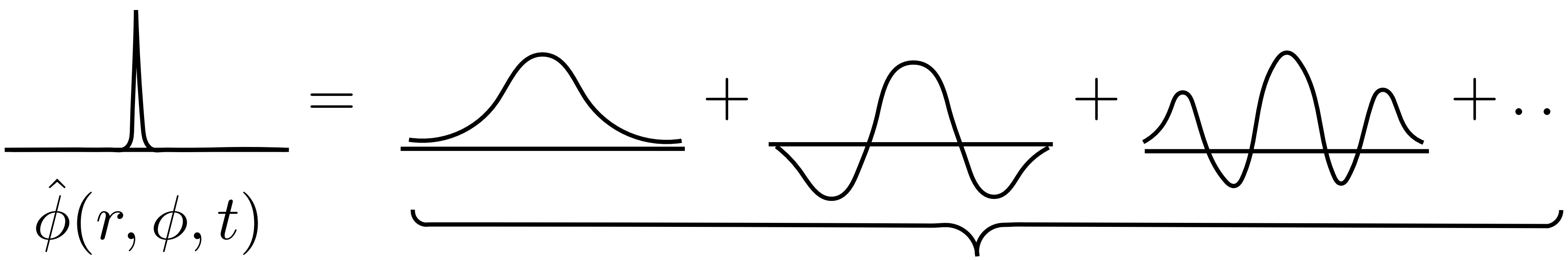
Such particles can enter the horizon, thus we can add particles even in the black hole interior.



Hartle-Hawking state +excitations by bulk fields??

$$\hat{\phi}^{\rm CFT}(r,x,t)|{\rm TFD}\rangle\,??$$

$\rightarrow$ taking a suitable superposition of one particle states created by inserting CFT operators at a point on the Euclidean boundary corresponding to (r,x,t).



$\hat{\phi}(r,\phi,t)$

One particle states (←primary+descendant operators)

Since particles can enter the horizon, we can also reconstruct the excitations by bulk local operators in the black hole interior!

In AdS3/CFT2 case, the black holes are locally equivalent to the pure AdS spacetime and we can obtain the explicit form of the bulk local operators acting on the black hole states making use of the expression for the pure AdS case.

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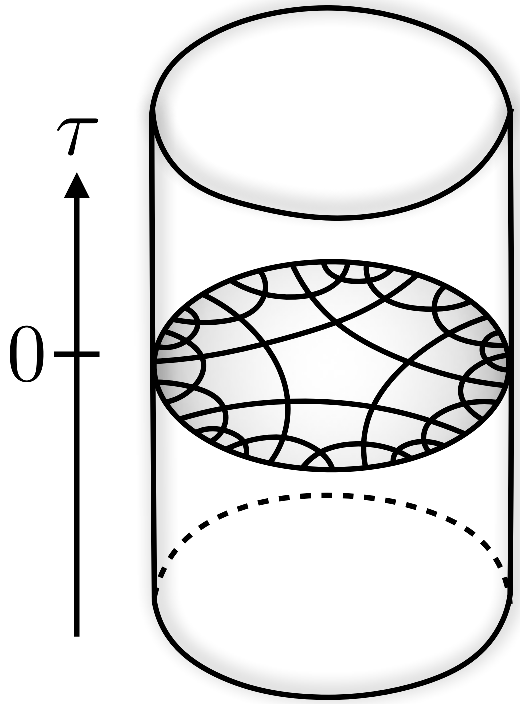
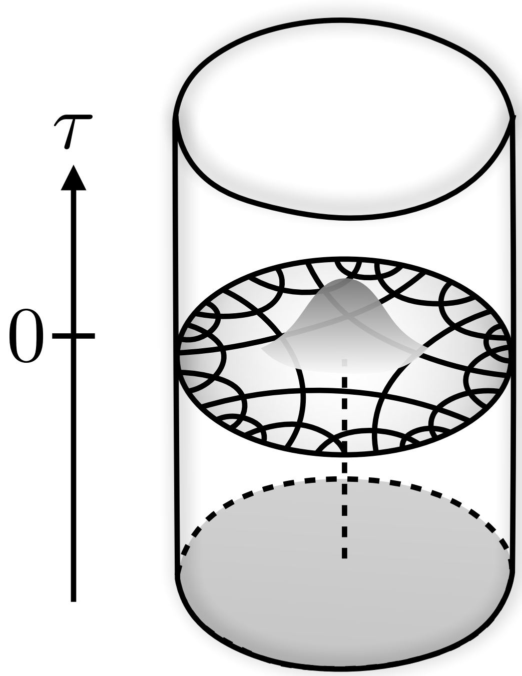
The construction in the pure AdS3 spacetime



$|0\rangle_{\rm CFT}$



$\mathcal{O}(0)|0\rangle_{\rm CFT}$

- We demonstrate "path-integral approach" to the construction of bulk scalar fields living in the pure AdS3 spacetime.

$$\hat{\phi}^{\rm CFT}(r,\vec{x})|0\rangle_{\rm CFT}$$

- The vacuum state dual to the pure AdS can be obtained by the Euclidean path-integral from $\tau=-\infty$ to $\tau=0$ with no insertion.

Vacuum state + one particle excitations around the AdS centre

- Correlators of local operators in a large c CFT factorize in the leading order of the large c expansion: "generalized free fields"

$\rightarrow$ single trace operator behave as a "particle" with $m^2=\Delta(\Delta-1)$

- One particle states can be obtained by the Euclidean path integral with a single trace operator insertion at $\tau=-\infty$

Primary states $\mathcal{O}(0)|0\rangle_{\rm CFT}$ $\leftrightarrow$ lowest energy particle states

Descendant states $L^{h}_{-1}\bar{L}^{\bar{h}}_{-1}\mathcal{O}(0)|0\rangle_{\rm CFT}$ $\leftrightarrow$ higher energy states

Vacuum state + excitations around the AdS centre by bulk local fields

$$\hat{\phi}^{\mathrm{CFT}}(0)|0\rangle_{\mathrm{CFT}}$$

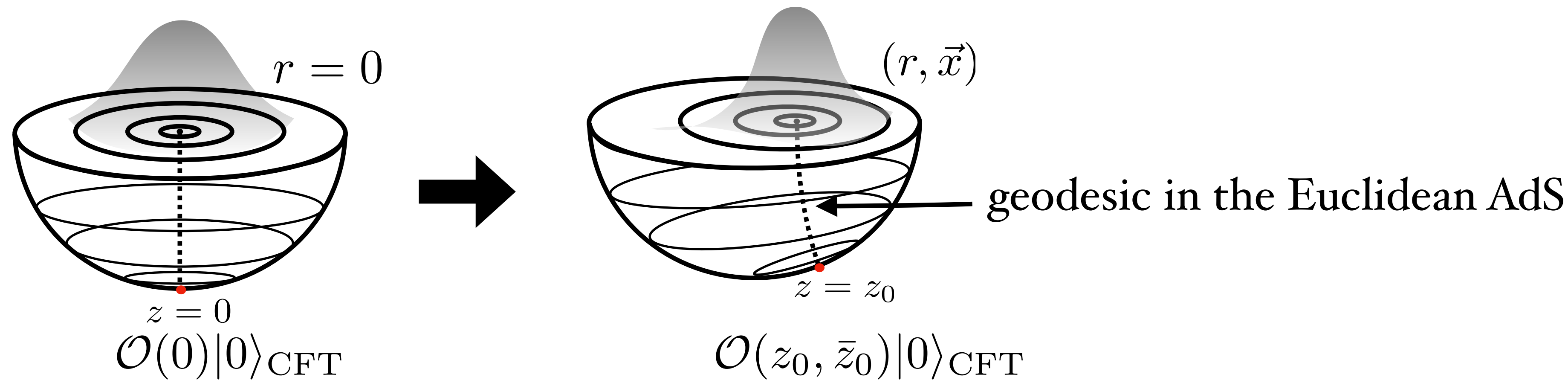
- The scalar field in AdS can be expanded as a superposition of creation operators
- Superposing descendant states in a suitable way, we get the bulk local operator acting on the vacuum state.
- We can fix the liner combination of the descendants by the "geometrical" condition → the excitations around the AdS centre by bulk local fields are invariant under the generators of isotropy group which fixes the point.

[c.f. Kaplan's talk]

$$[L_0 - \bar{L}_0, \hat{\phi}^{\mathrm{CFT}}(0)]|0\rangle_{\mathrm{CFT}} = [L_{\pm 1} + \bar{L}_{\mp 1}, \hat{\phi}^{\mathrm{CFT}}(0)]|0\rangle_{\mathrm{CFT}} = 0$$

- The solution can be expressed by Ishibashi states with respect to the global conformal group [*Miyaji-Numasawa-Shiba-Takayanagi-Watanabe, Nakayama-Ooguri'15*]

$$\hat{\phi}^{\mathrm{CFT}}(0)|0\rangle_{\mathrm{CFT}} = \sum_{k=0}^{\infty}(-1)^k \frac{\Gamma(\Delta)}{k!\Gamma(\Delta+k)} L_{-1}^k \bar{L}_{-1}^k \mathcal{O}(0)|0\rangle_{\mathrm{CFT}}$$



- The bulk local operator at a point $(r,\vec{x})$ can be obtained by a conformal map $g(r,\vec{x})$ parametrized by $(r,\vec{x})$ and is expressed as a superposition of CFT operators inserted at a boundary point $(z_0,\bar{z}_0)$ which related to $(0,0)$ by the map $g(r,\vec{x})$. [*KG-Takayanagi*]

$$\hat{\phi}^{\rm CFT}(r,\vec{x})|0\rangle_{\rm CFT} = \sum_{k=0}^{\infty}(-1)^k \frac{\Gamma(\Delta)}{k!\Gamma(\Delta+k)} L'^{k}_{-1}\bar{L}'^{k}_{-1}\mathcal{O}(z_0,\bar{z}_0)|0\rangle_{\rm CFT}$$

- We "geometrically" (=via geodesics) specified the bulk points, so this method can be applied to the various coordinate systems such as the global AdS, Poincare and the Rindler coordinate in a similar way. [*Verlinde*, *KG-Takayanagi*]
- Two point function reproduces the bulk-to-bulk propagator in AdS.

$$\langle 0|\hat{\phi}^{\rm CFT}(r,\vec{x})\hat{\phi}^{\rm CFT}(r',\vec{x}')|0\rangle = \frac{1}{2\sqrt{\sigma^2-1}(\sigma+\sqrt{\sigma^2-1})^{\Delta-1}}$$

σ : AdS invariant distance

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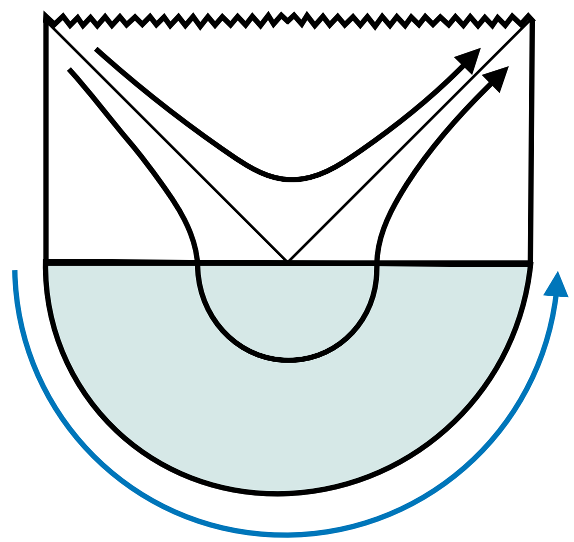
The construction in the AdS black holes



- Consider the AdS_3 (BTZ) black holes which is locally equivalent to the pure AdS spacetime.

$$ds^2 = -(r^2 - R^2)dt^2 + \frac{R^2}{r^2 - R^2}dr^2 + r^2 d\phi^2$$

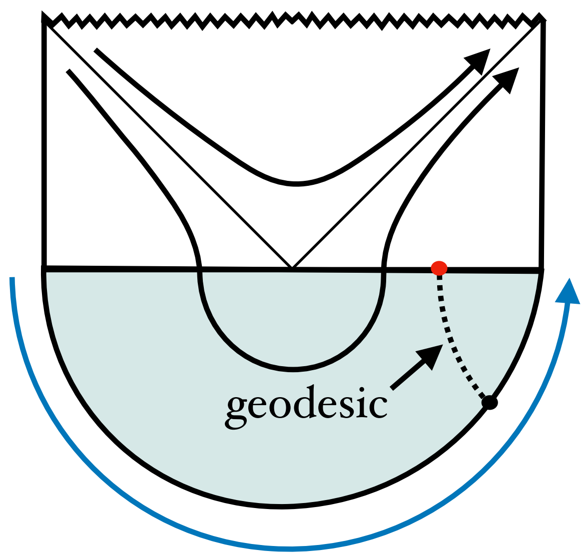
$$0 < \phi < 2\pi L$$

- First consider the double-sided ones where two BHs are connected via a worm hole which is expressed by two equivalent CFTs.

- The AdS-Schwarzschild coordinate in double-sided BTZ nicely fits the Rindler coordinate in the pure AdS but with an identification $\phi \sim \phi + 2\pi L$.
(while for the Rindler, the spatial coordinate ϕ ranges $-\infty < \phi < \infty$)

- The H-H vacuum corresponds to the thermofield double states in CFT.

$$|\mathrm{TFD}\rangle = \sum_E e^{-\beta E/2} |E\rangle_1 \otimes |E\rangle_2$$



- Our bulk local operator is expressed locally around a boundary point, thus the same expression can be used for BTZ.

$$\hat{\phi}_{\mathrm{BTZ}}(r,\vec{x})|\mathrm{TFD}\rangle_{\mathrm{CFT}} \Rightarrow \text{the same expression as the Rindler}$$

- Topological difference between the pure AdS and the BTZ affects the behavior of the correlator: in the semi-classical limit $c \to \infty$, local operators in CFT behave as generalized free fields and the effect of the identification can be correctly represented by placing mirror images: the method of mirror images.

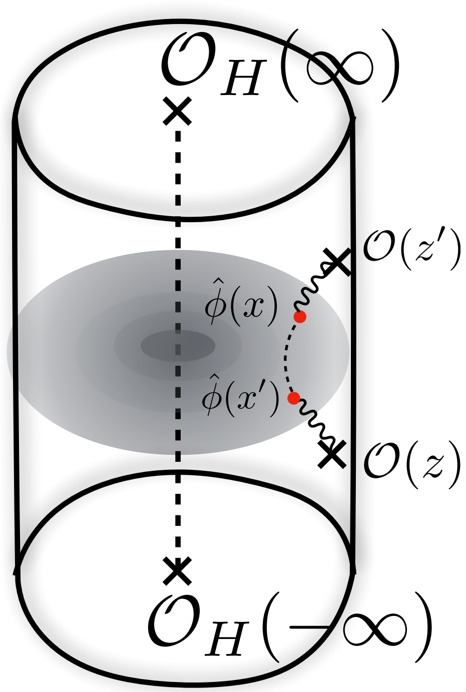
$$\langle \mathrm{TFD}|\hat{\phi}^{\mathrm{CFT}}(r,\vec{x})\hat{\phi}^{\mathrm{CFT}}(r',\vec{x}')|\mathrm{TFD}\rangle = \sum_{m,n} G^{\mathrm{Rindler}}(r,\phi+2\pi n,t;r,\phi+2\pi m,t)$$

- This reproduces the result from the calculations in the gravity side even in the BH interior →The BH singularity: the mirror images give the same contributions to the correlator when $r \to 0$ and the correlator diverges.

- Next we consider a high energy eigenstate in a single CFT created by a heavy primary operator with dimension $h_H = \bar{h}_H > \frac{c}{24}$ which is believed to describe the single-sided black hole. *[c.f. Guica '16]*

$$\mathcal{O}_H(0)|0\rangle \overset{?}{\Leftrightarrow} \text{Single-sided BH with } T_H = \frac{1}{2\pi}\sqrt{\frac{24h_H}{c} - 1}$$

- We formally construct bulk local operator acting on this primary state in the same way as TFD state and see the behavior of the two point function on this background → the classical spacetime is correctly reproduced?

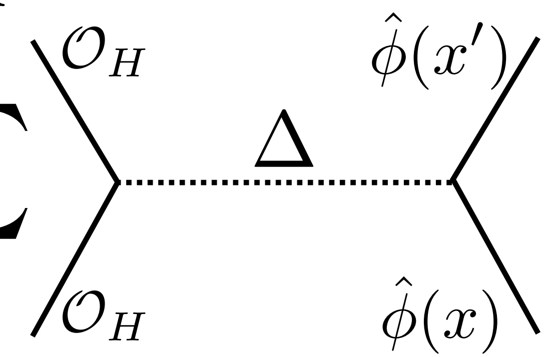


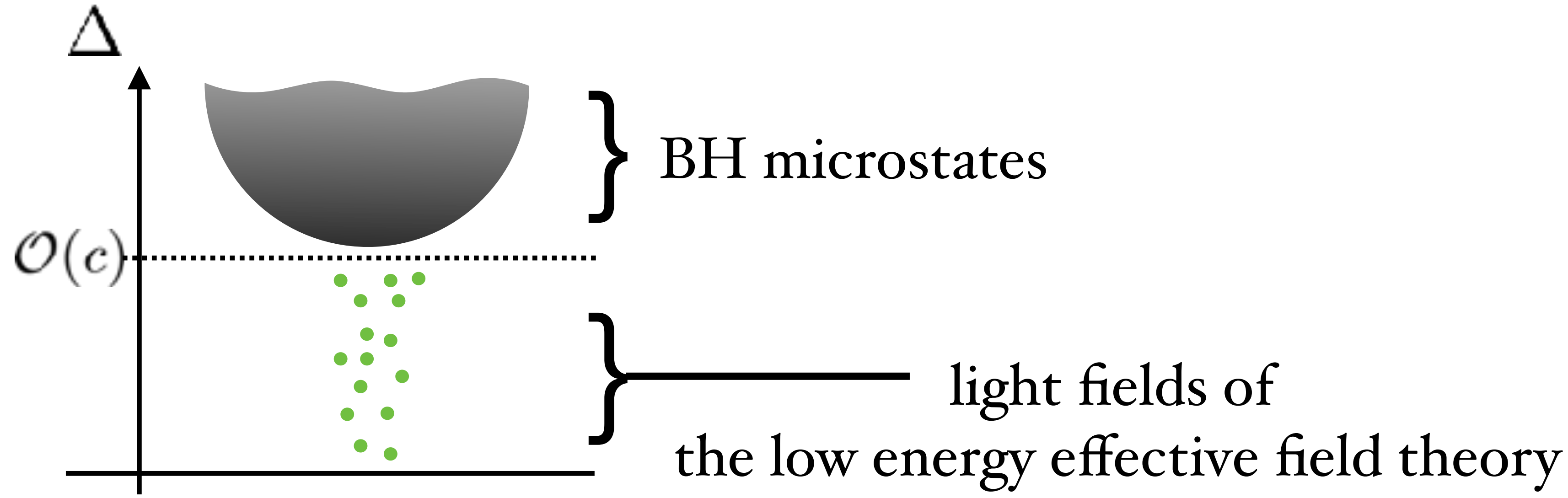
- The bulk-to-bulk two point function on this background is expressed as heavy-heavy-light-light correlator

$$\langle\hat{\phi}(r,\phi,t)\hat{\phi}(r',\phi',t')\rangle_{\text{BH}} = \langle\mathcal{O}_H|\hat{\phi}^{\text{CFT}}(r,\phi,t)\hat{\phi}^{\text{CFT}}(r',\phi',t')|\mathcal{O}_H\rangle$$

- Notice that when we probe near the boundary, we place light CFT operators far from the heavy operator.

$$\langle\mathcal{O}_H|\hat{\phi}^{\text{CFT}}(x)\hat{\phi}^{\text{CFT}}(x')|\mathcal{O}_H\rangle = \sum_{\Delta}$$

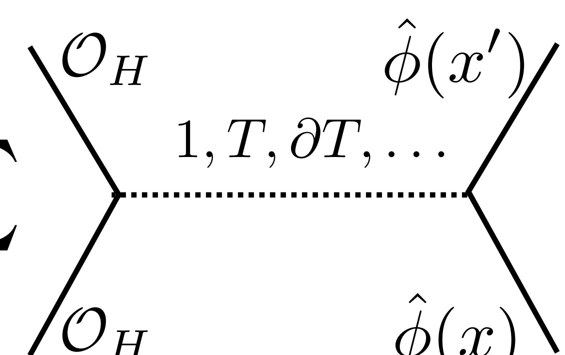


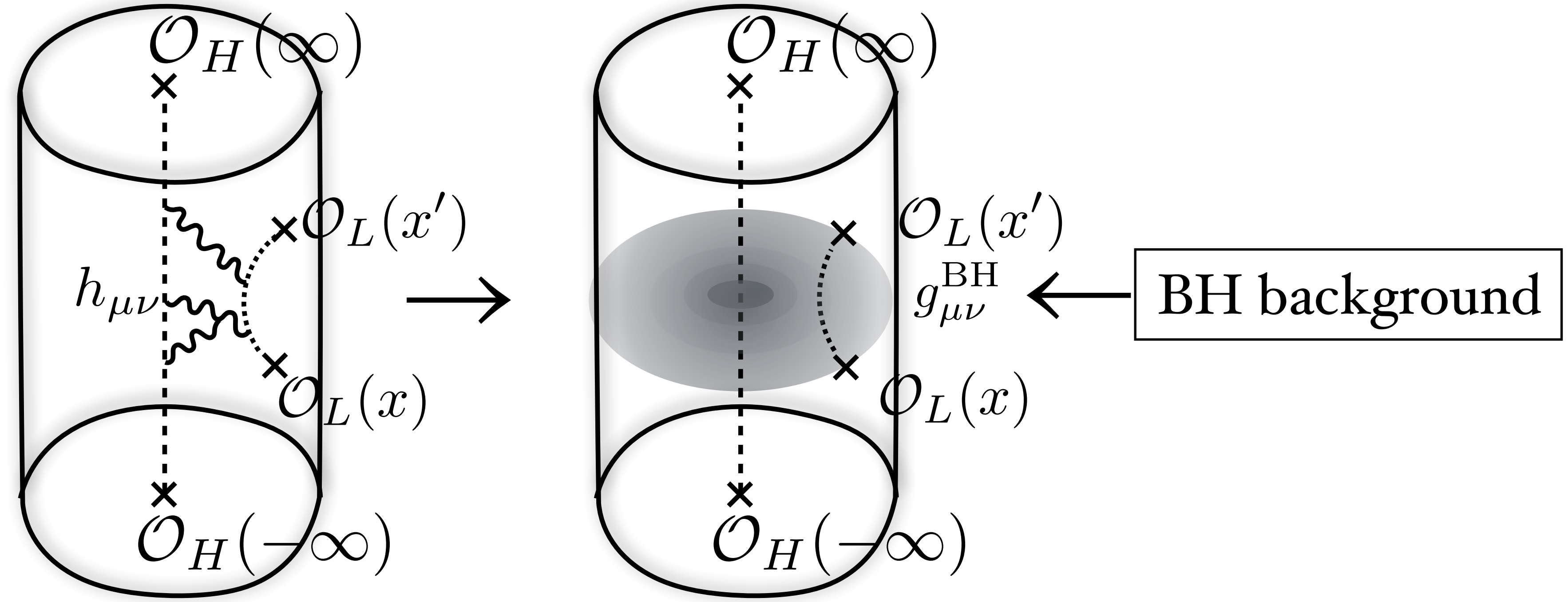


- We make an assumption: the CFT has a sparse spectrum of primary operators in the low energy sector in the large c limit.

- For such CFTs, at least light operators are close with each other, the vacuum contribution of the Virasoro block well approximates the full correlator. *[Hartman '13]*

$$\langle \mathcal{O}_H | \hat{\phi}^{\rm CFT}(x) \hat{\phi}^{\rm CFT}(x') | \mathcal{O}_H \rangle \sim \sum$$

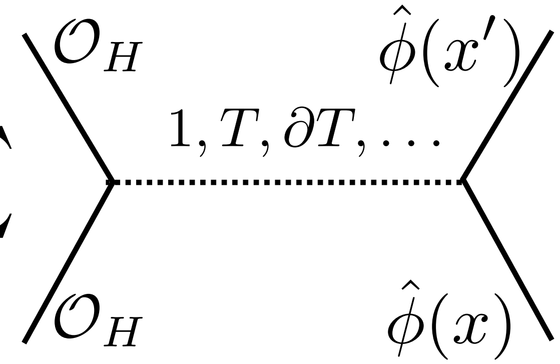




- The vacuum Virasoro block corresponds to the graviton exchanges in the bulk.
- Resuming the graviton exchange, we get in the leading order of 1/c

[Hartman '13, Kaplan-Fitzpatrick-Walters '14 '15]

$$\langle \mathcal{O}_H | \hat{\phi}^{\rm CFT}(x) \hat{\phi}^{\rm CFT}(x') | \mathcal{O}_H \rangle \sim \sum$$



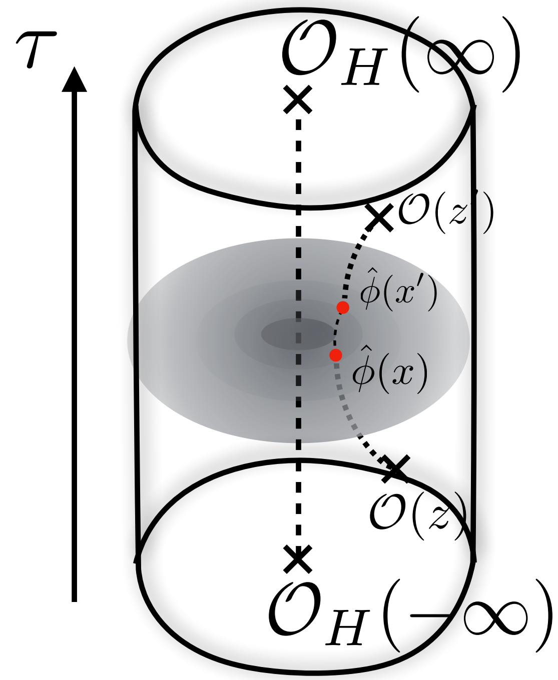
$$\sim \langle {\rm TFD} | \hat{\phi}^{\rm CFT}(x) \hat{\phi}^{\rm CFT}(x') | {\rm TFD} \rangle + \mathcal{O}(1/c)$$

- We can approximate the two function for the high energy eigenstate to the two point function on the thermal background without assuming ETH.

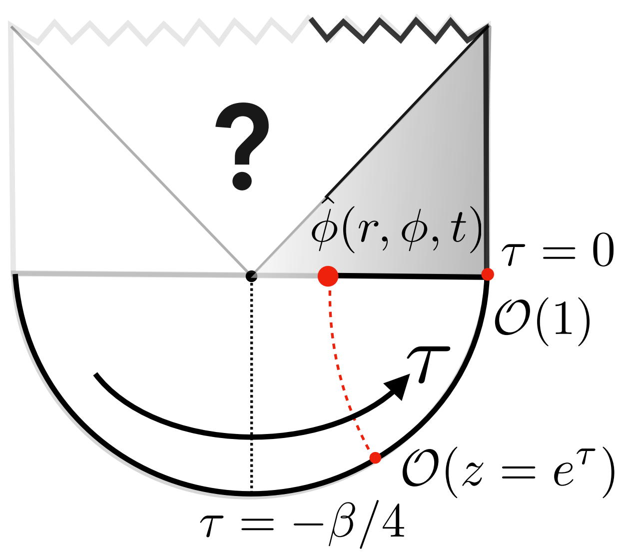
→At least near the boundary, the classical BH spacetime is correctly reproduced.

Comments & Discussion

The possible breakdown of classical geometry in the bulk interior?



$\langle \mathcal{O}_H(\infty)\mathcal{O}(z)\mathcal{O}(z')\mathcal{O}_H(0)\rangle$



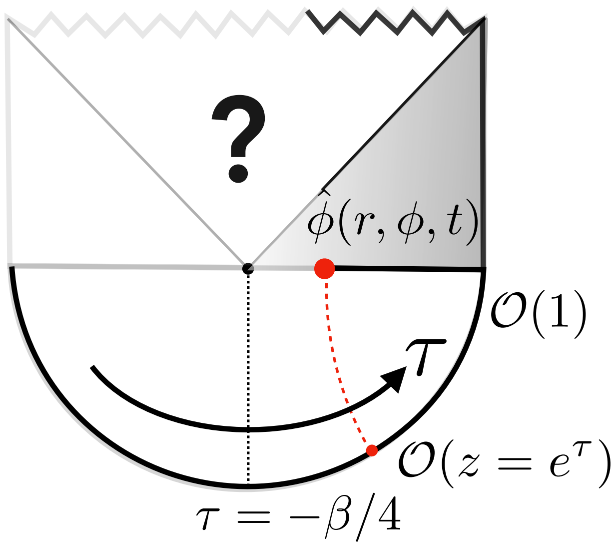
$\langle O_H(\infty)\mathcal{O}_L(1)\hat{\phi}(r,\phi,t)O_H(0)\rangle$

- When we probe the deep interior in the bulk, we need place a light operator closer to the heavy operator and there is a possibility that our semi-classical approximation breaks down.

Semi-classical block v.s. Exact block

- There is a possibility that non-perturbative quantum gravity effect spoils the classical description of the black hole interior.
- Numerical calculations imply that the non-perturbative effect for a single HHLL block is not large even when a primary operator is inserted at the position which can probes the black hole interior (and even for the second asymptotic region.) and the semi-classical block does not deviate so much from the exact one. *[Chen-Hussong-Kaplan-Li]*

Semi-classical block v.s. Exact block



$\langle O_H(\infty)\mathcal{O}_L(1)\hat{\phi}(r,\phi,t)O_H(0)\rangle$

- However, the bulk local operator is constructed as a superposition of light primary/descendant operators. To locally probe the bulk interior we need to sum over infinitely many HHLL blocks and there is still a possibility that the deviation is amplified…
- We may be able to cut off "high energy modes" (say, trans-Plankian modes) at the expense of the exact locality of the bulk "local" operator so that non-perturbative effect is well suppressed. → Locality v.s. Classicality of the spacetime?

Vacuum block v.s. Other blocks

- There is also a possibility that the vacuum block approximation breaks down and the contributions from the other block become important when we probe the deep interior in the bulk.
- We only assumed that the sparse spectrum of a large CFT, but this problem is related to more details of CFT, i.e, behavior of the OPE coefficients & spectrum.

Thank you