N=2* Super Yang-Mills on a Lattice

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Focus Week on Quantum Gravity and Holography
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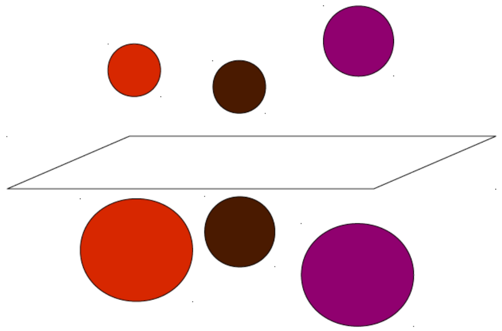
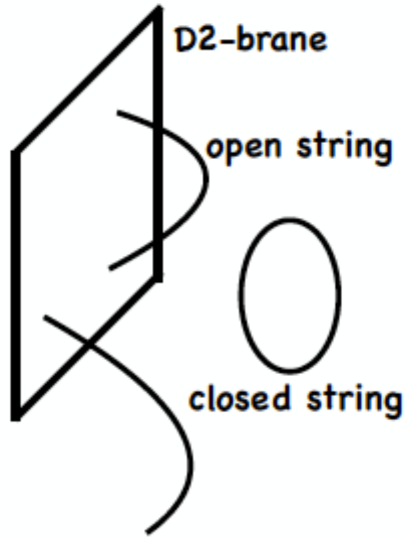
BASED ON

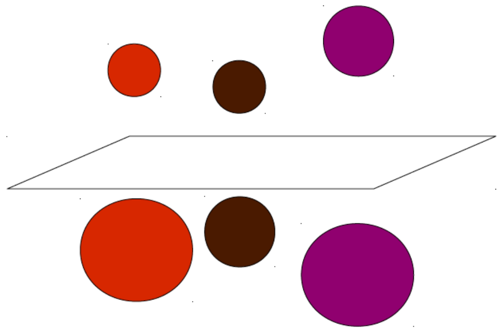
- arXiv: 1710:10172 [hep-lat]
- arXiv: 1710:11390 [hep-lat]

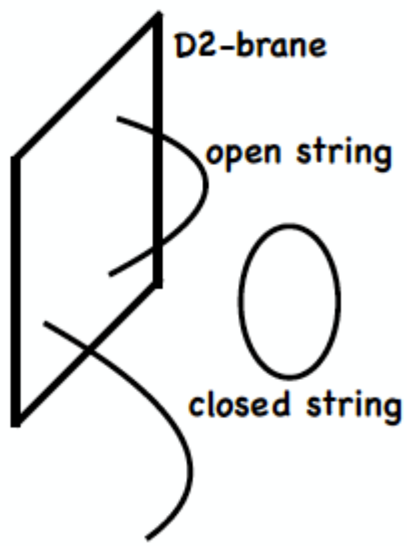




OUTLINE

- N=4 Supersymmetric Yang-Mills
- Operator Deformation and N=2* SYM
- Lattice Formulation
- Gravitational Dual
- Some Results from SUGRA
- Conclusions and Future Directions





Maximally Supersymmetric Yang-Mills

$\mathcal{N} = 4$ SYM — has **largest** possible number of supersymmetries for a 4d theory without gravity

Takes part in the AdS/CFT correspondence

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Phase structure of QFT at finite temperature

Precisely described by black hole geometries

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Phase structure of QFT at finite temperature

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Dual gravitational theory - D3 brane geometry on $AdS_5 \times S^5$

Maximally Supersymmetric Yang-Mills

$\mathcal{N} = 4$ SYM — has **largest** possible
for a 4d theory witho

Strongly coupled

Takes part in the AdS/CFT correspondence

Phase structure of QFT at finite temperature

Weakly coupled

Precisely described by black hole geometries

Dual gravitational theory - D3 brane geometry on $AdS_5 \times S^5$

Maximally Supersymmetric Yang-Mills

Strongly coupled

Phase structure of QFT at finite temperature

Lower dimensional versions - well explored on the lattice

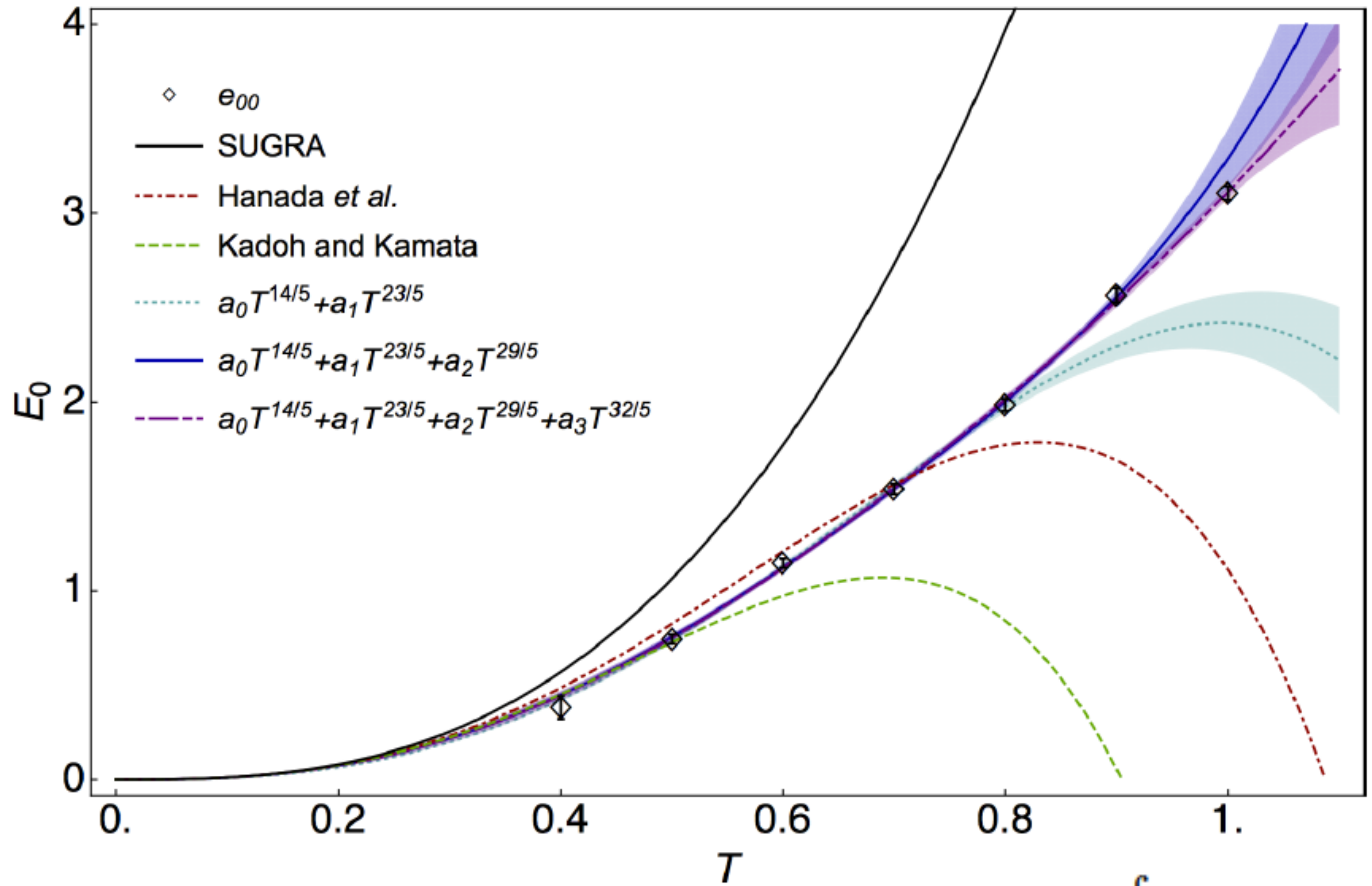
Excellent validations of AdS/CFT

Maximally Supersymmetric Yang-Mills

Simplest case:
D0 brane system

Excellent validations of AdS/CFT

Maximally Supersymmetric Yang-Mills



MCMS Collaboration (2016)

Phys. Rev. D94 (2016) 094501

$$\frac{1}{N^2}E_{gravity} = 7.41\,T^{2.8} - \underbrace{\frac{1}{N^2}5.77\,T^{0.4}}_{\text{quantum gravity}}$$

Fit to **lattice data:**

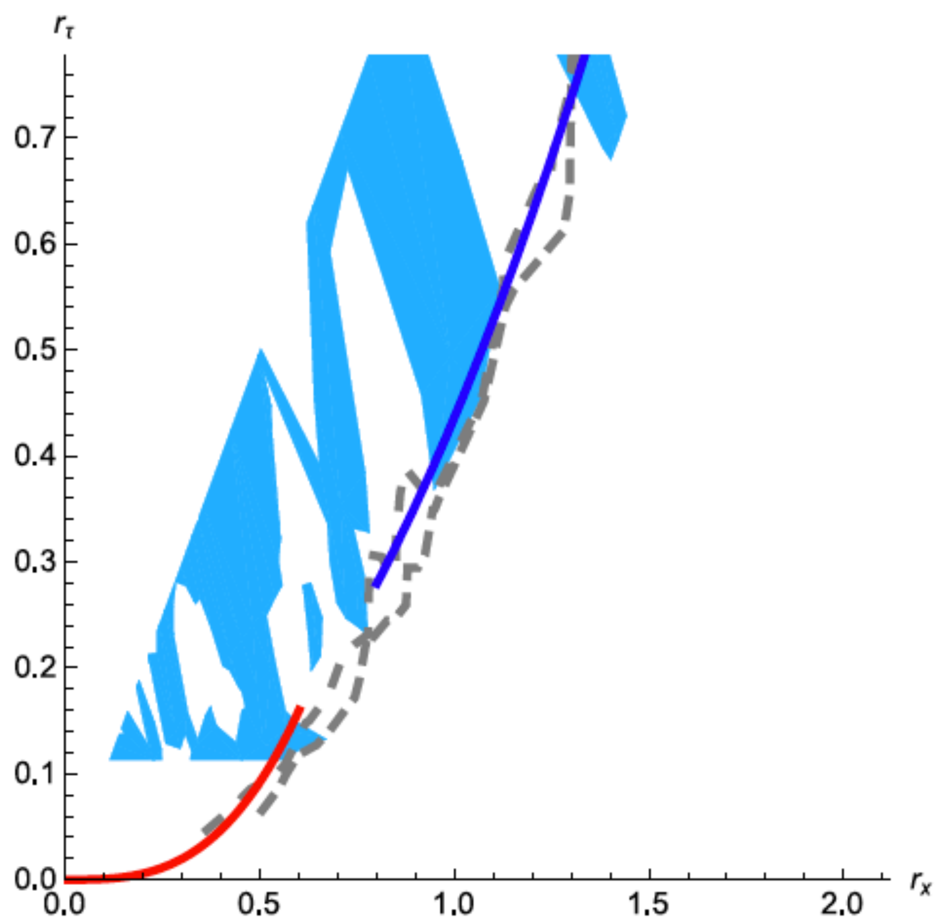
	free	a_0 fixed		free	b_0 fixed
a_0	7.4±0.5	7.41	b_0	−5.8±3.0	−5.77
a_1	−9.7±2.2	−10.0±0.4	b_1	−3.4±5.7	−3.5±2.0
a_2	5.6±1.8	5.8±0.5			

$$\frac{E}{N^2} = \frac{(a_0T^{14/5} + a_1T^{23/5} + a_2T^{29/5} + a_3T^{32/5}\dots)}{N^0} + \frac{(b_0T^{2/5} + b_1T^{11/5} + \dots)}{N^2} + \mathcal{O}\left(\frac{1}{N^4}\right)$$

A bit more involved case:
D1 brane system

Excellent validations of AdS/CFT

Maximally Supersymmetric Yang-Mills



Good agreement with high temperature prediction (red curve)

$$r_x^3 = 1.35 r_\tau$$

Boundary between confined and deconfined phases correspond to:

$$\frac{1}{N}|P_s| = 0.5$$

S. Catterall, A. J., T. Wiseman, [JHEP 1012 (2010) 022]

Good agreement with SUGRA

Fit to lattice data $\boxed{r_x^2 = 2.29 r_\tau}$

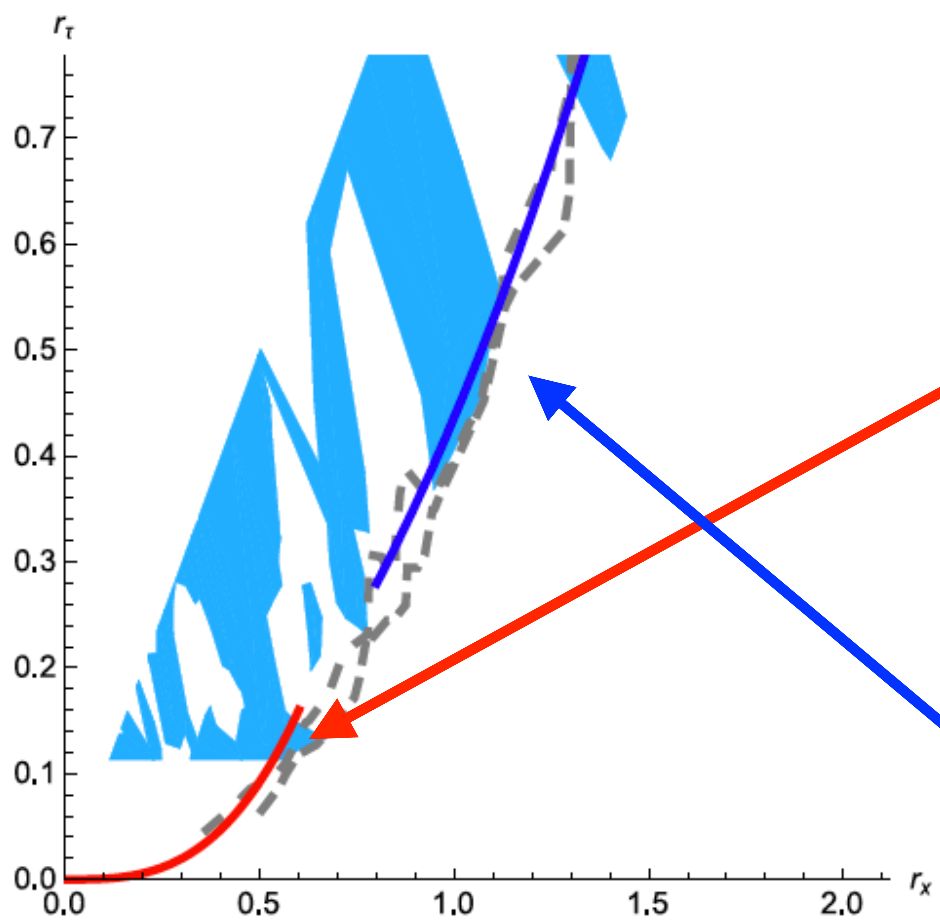
Blue curve: $r_x^2 = c_{crit} r_\tau$

$c_{crit} \simeq 2.29$

Recent studies:

M. Hanada and P. Romatschke, [Phys.Rev. D96 (2017) no.9, 094502]
O. Dias, J. Santos and B. Way, [JHEP 1706 (2017) 029]
S. Catterall, R. Jha, D. Schaich and T. Wiseman, [arXiv: 1709.07025]

Maximally Supersymmetric Yang-Mills



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Maximally Supersymmetric Yang-Mills

Need for testing gauge-gravity duality in higher dimensions

D2 branes?

D3 branes?

Other exotic realizations?

Maximally Supersymmetric Yang-Mills in 4D

Field content: A **vector multiplet** and 3 **chiral multiplets**

$$V \longrightarrow A_\mu,\ \lambda_{4\alpha},\ \overline{\lambda}^4{}_{\dot{\alpha}}$$
$$\Phi_s, \Phi^{\dagger s} \longrightarrow \phi_s,\ \lambda_{s\alpha},\ \phi^{\dagger s},\ \overline{\lambda}^s{}_{\dot{\alpha}}$$

Global symmetry group

$$SU(2)_L \times SU(2)_R \times SU(4)$$

Euclidean Lorentz symmetry

$$SU(2)_L \times SU(2)_R \simeq SO(4)$$

Internal symmetry

$$SO(6) \simeq SU(4)$$

Mass Deformation: $\mathcal{N}=2^*$ SYM Theory

Introduced by **Polchinski and Strassler** (hep-th/0003136)

"The string dual of a confining four-dimensional gauge theory"

A mass deformation of $\mathcal{N}=4$ SYM theory

Combine 2 chiral multiplets → $\mathcal{N}=2$ hypermultiplet

Action

$$S_{\mathcal{N}=2^*} = S_{\mathcal{N}=4} + S_m$$

Mass deformation:

$$S_m = \frac{1}{g^2}\int d^4x \,\mathrm{Tr}\Big(- m\lambda_1{}^{\alpha}\lambda_{2\alpha} - m\overline{\lambda}^1{}_{\dot\alpha}\overline{\lambda}^{2\dot\alpha} + m^2\phi_1\phi_1^\dagger + m^2\phi_2\phi_2^\dagger$$
$$-\sqrt{2}m\phi_3[\phi_1,\phi_1^\dagger] - \sqrt{2}m\phi_3[\phi_2,\phi_2^\dagger]$$
$$-\sqrt{2}m\phi_3^\dagger[\phi_1,\phi_1^\dagger] - \sqrt{2}m\phi_3^\dagger[\phi_2,\phi_2^\dagger]\Big)$$

$\mathcal{N} = 2^*$ SYM Theory

Convenient to express mass deformation using 2 operators

$$\mathcal{O}_2 = \frac{1}{3}\Big(\phi_1\phi_1^\dagger + \phi_2\phi_2^\dagger - 2\phi_3\phi_3^\dagger\Big)$$

dimension 2

$$\begin{aligned}\mathcal{O}_3 = 2\Big(& -\lambda_1{}^\alpha\lambda_{2\alpha} - \overline{\lambda}^1{}_{\dot\alpha}\overline{\lambda}^{2\dot\alpha} \\ & -\sqrt{2}\phi_3[\phi_1,\phi_1^\dagger] - \sqrt{2}\phi_3[\phi_2,\phi_2^\dagger] - \sqrt{2}\phi_3^\dagger[\phi_1,\phi_1^\dagger] - \sqrt{2}\phi_3^\dagger[\phi_2,\phi_2^\dagger]\Big) \\ & +\frac{2}{3}m\Big(\phi_1\phi_1^\dagger + \phi_2\phi_2^\dagger + \phi_3\phi_3^\dagger\Big)\end{aligned}$$

dimension 3

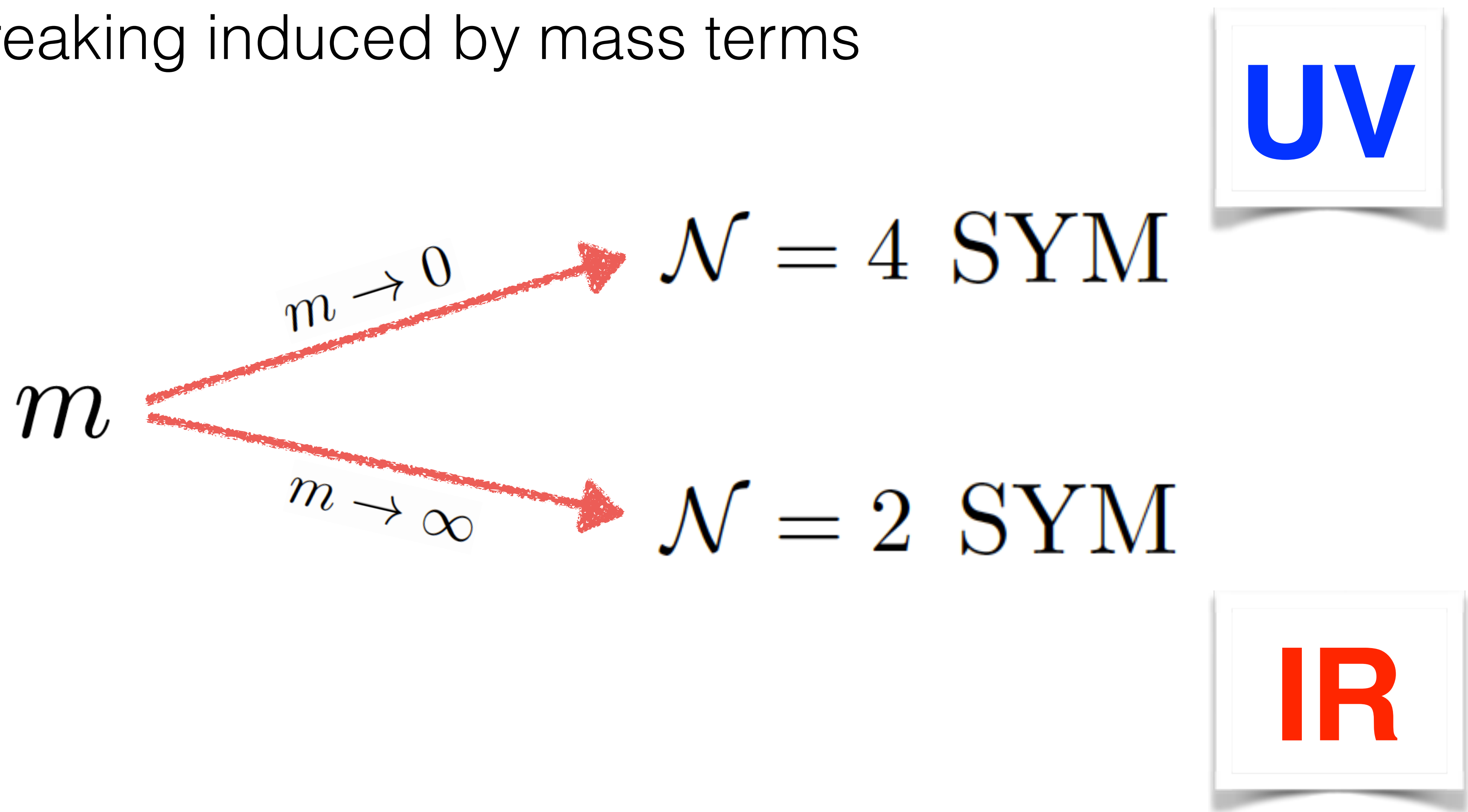
Correspond to turning on ***bosonic*** and ***fermionic*** scalars in dual gravitational theory

$\mathcal{N}=2^*$ SYM Theory

Action

$$S_{\mathcal{N}=2^*} = S_{\mathcal{N}=4} - \frac{1}{2g^2}\int d^4x\ m^2\ \mathrm{Tr}\ \mathcal{O}_2 - \frac{1}{2g^2}\int d^4x\ m\ \mathrm{Tr}\ \mathcal{O}_3$$

Soft SUSY breaking induced by mass terms



Vafa-Witten Twist of $\mathcal{N} = 4$ SYM

$\mathcal{N} = 4$ SYM can be twisted in 3 different ways

1.) Half twist
2.) Vafa-Witten twist
3.) Geometric Langlands twist

Given by how we *embed* $SO(4)$ in $SO(6)$

Twisted and untwisted theories are *equivalent* on flat $\mathbb{R}^4$

We will make use of **Vafa-Witten twist**

E. Witten and C. Vafa, [Nucl. Phys. B431 (1994) 3-77]
"A Strong Coupling Test of S-Duality"

Vafa-Witten Twist of $\mathcal{N} = 4$ SYM

Before twist

$$SU(2)_L \times SU(2)_R \times SU(4)$$

Vafa-Witten Twist of $\mathcal{N} = 4$ SYM

$$(SU(2) \times SU(2) \times U(1))/\mathbb{Z}_2 \subset SU(4)$$

Before twist

$$SU(2)_L \times SU(2)_R \times SU(4)$$

twist

After twist

$$SU(2)'_L \times SU(2)_R \times SU(2)_F$$

$$SU(2)'_L = \text{diag}\Big(SU(2)_L \times SU(2)_I\Big)$$

Vafa-Witten Twist of $\mathcal{N}=4$ SYM

Twisted fields of $\mathcal{N}=4$ SYM

Twisted fields

$$
\begin{aligned}
A_\mu &\longrightarrow A_\mu, \\
\lambda_{u\alpha} &\longrightarrow \eta, \zeta, \chi_{\mu\nu}, \psi_{\mu\nu}, \\
\overline{\lambda}^u{}_{\dot\alpha} &\longrightarrow \psi_\mu, \chi_\mu, \\
\phi_{uv} &\longrightarrow B_{\mu\nu}, \phi, \overline{\phi}, C
\end{aligned}
$$

Twist gives 2 scalar supercharges: Q and $\widetilde{Q}$

$$
\boxed{\begin{aligned} Q^2 A_\mu &= 2\sqrt{2} D_\mu \phi, \\ Q^2 X &= 2\sqrt{2}[X, \phi] \end{aligned}}
\qquad
\boxed{\begin{aligned} \widetilde{Q}^2 A_\mu &= -2\sqrt{2} D_\mu \overline{\phi}, \\ \widetilde{Q}^2 X &= -2\sqrt{2}[X, \overline{\phi}] \end{aligned}}
$$

Vafa-Witten Twist of $\mathcal{N} = 4$ SYM

Q supersymmetry transformations

$$
\begin{aligned}
QA_\mu &= -\psi_\mu, \\
Q\psi_\mu &= -2\sqrt{2}D_\mu\phi, \\
Q\phi &= 0, \\
Q\overline{\phi} &= \sqrt{2}\eta, \\
Q\eta &= -2[\phi, \overline{\phi}], \\
Q\chi_{\mu\nu} &= 2H_{\mu\nu}, \\
QH_{\mu\nu} &= -\sqrt{2}[\phi, \chi_{\mu\nu}],
\end{aligned}
$$

$$
\begin{aligned}
QC &= \sqrt{2}\zeta, \\
Q\zeta &= -2[\phi, C], \\
Q\chi_\mu &= 2H_\mu, \\
QH_\mu &= -\sqrt{2}[\phi, \chi_\mu], \\
QB_{\mu\nu} &= \sqrt{2}\psi_{\mu\nu}, \\
Q\psi_{\mu\nu} &= -2[\phi, B_{\mu\nu}].
\end{aligned}
$$

Similar transformations for $\widetilde{Q}$

Vafa-Witten Twist of $\mathcal{N}=4$ SYM

Action takes following form

$$S_{\mathcal{N}=4} = Q\widetilde{Q}\ \frac{1}{g^2}\int d^4x\ \mathcal{F}$$

Action potential

$$\mathcal{F} = \mathrm{Tr}\left(-\frac{1}{2\sqrt{2}}B_{\mu\nu}F_{\mu\nu} - \frac{1}{24\sqrt{2}}B_{\mu\nu}[B_{\mu\rho}, B_{\nu\rho}] - \frac{1}{8}\chi_{\mu\nu}\psi_{\mu\nu} - \frac{1}{8}\psi_\mu\chi_\mu - \frac{1}{8}\eta\zeta\right)$$

Twist of $\mathcal{N} = 2^*$ SYM

Mass deformation (untwisted theory)

$$S_m = \frac{1}{g^2}\int d^4x \,\mathrm{Tr}\Big(-m\lambda_1{}^{\alpha}\lambda_{2\alpha} - m\bar{\lambda}^1{}_{\dot\alpha}\bar{\lambda}^{2\dot\alpha} + m^2\phi_1\phi_1^\dagger + m^2\phi_2\phi_2^\dagger$$
$$-\sqrt{2}m\phi_3[\phi_1,\phi_1^\dagger] - \sqrt{2}m\phi_3[\phi_2,\phi_2^\dagger]$$
$$-\sqrt{2}m\phi_3^\dagger[\phi_1,\phi_1^\dagger] - \sqrt{2}m\phi_3^\dagger[\phi_2,\phi_2^\dagger]\Big)$$

Mass terms → Need to modify Q transformations

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$$-\sqrt{2}m\phi_3^\dagger[\phi_1,\phi_1^\dagger] - \sqrt{2}m\phi_3^\dagger[\phi_2,\phi_2^\dagger]\Big)$$

Mass terms → Need to modify Q transformations

$$Q \longrightarrow Q^{(m)}$$

Twist of $\mathcal{N} = 2^*$ SYM

Modified Q and $\widetilde{Q}$ transformations

$$
\begin{aligned}
Q^{(m)} A_\mu &= -\psi_\mu, \\
Q^{(m)} \psi_\mu &= -2\sqrt{2} D_\mu \phi, \\
Q^{(m)} \phi &= 0, \\
Q^{(m)} \overline{\phi} &= \sqrt{2}\eta, \\
Q^{(m)} \eta &= -2[\phi, \overline{\phi}], \\
Q^{(m)} C &= \sqrt{2}\zeta, \\
Q^{(m)} \zeta &= -2[\phi, C] + 2mC, \\
Q^{(m)} \chi_\mu &= 2H_\mu, \\
Q^{(m)} H_\mu &= -\sqrt{2}[\phi, \chi_\mu] + \sqrt{2} m \chi_\mu, \\
Q^{(m)} B_{\mu\nu} &= \sqrt{2}\psi_{\mu\nu}, \\
Q^{(m)} \psi_{\mu\nu} &= -2[\phi, B_{\mu\nu}] + 2mB_{\mu\nu}, \\
Q^{(m)} \chi_{\mu\nu} &= 2H_{\mu\nu}, \\
Q^{(m)} H_{\mu\nu} &= -\sqrt{2}[\phi, \chi_{\mu\nu}].
\end{aligned}
$$

$$
(Q^{(m)})^2 A_\mu = 2\sqrt{2} D_\mu \phi,
$$

$$
((Q^{(m)})^2 X = 2\sqrt{2}[X, \phi] + 2\sqrt{2} m \alpha X
$$

$\alpha = 1$ for

$\zeta, \chi_\mu, \psi_{\mu\nu}, C, H_\mu, B_{\mu\nu}$

$\alpha = 0$ for rest of fields

Twist of $\mathcal{N} = 2^*$ SYM

Twisted action of $\mathcal{N} = 2^*$ SYM

$$\boxed{S_{\mathcal{N}=2^*} = \frac{1}{g^2}\int d^4x \ \mathrm{Tr}\ Q^{(m)}\Psi^{(m)}}$$

where

$$\begin{aligned}\Psi^{(m)} = \mathrm{Tr}\Big(&\chi_{\mu\nu}\Big[\frac{1}{2}F_{\mu\nu} - \frac{1}{4}H_{\mu\nu} - \frac{1}{8}[B_{\mu\rho}, B_{\nu\rho}] - \frac{1}{4}[C, B_{\mu\nu}]\Big]\\ &+\frac{1}{2\sqrt{2}}\psi_\mu(D_\mu\overline{\phi}) - \frac{1}{4}\eta[\phi,\overline{\phi}] + (\mathcal{V}+\mathcal{W}+\mathcal{Y})\\ &-\frac{1}{4}\zeta[C,\overline{\phi}] - \frac{1}{4}\psi_{\mu\nu}[B_{\mu\nu},\overline{\phi}] + \mathcal{T}\\ &+\chi_\mu\Big[-\frac{1}{2\sqrt{2}}(D_\mu C) - \frac{1}{2\sqrt{2}}(D_\nu B_{\nu\mu})\Big]\Big)\end{aligned}$$

Twist of $\mathcal{N} = 2^*$ SYM

Twisted action of $\mathcal{N} = 2^*$ SYM

$$\boxed{S_{\mathcal{N}=2^*} = \frac{1}{g^2} \int d^4x \ \mathrm{Tr} \ Q^{(m)} \Psi^{(m)}}$$

$$\mathcal{V} = -\frac{1}{4} m \Big((\psi_{12} - i\psi_{23})(B_{12} + iB_{23}) + (\psi_{13} - i\zeta)(B_{13} + iC) \Big)$$

$$\mathcal{W} = \frac{i}{4} \Big(-\psi_{12}[\overline{\phi}, B_{23}] + \psi_{23}[\overline{\phi}, B_{12}] + \eta[B_{12}, B_{23}] \Big)$$

$$\mathcal{Y} = \frac{i}{4} \Big(-\psi_{13}[\overline{\phi}, C] + \zeta[\overline{\phi}, B_{13}] + \eta[B_{13}, C] \Big)$$

$$\mathcal{T} = \frac{1}{4} \Big((\chi_1 - i\chi_2)(H_1 + iH_2) + (\chi_3 + i\chi_4)(H_3 - iH_4) \Big)$$

Twist of $\mathcal{N} = 2^*$ SYM

Twisted action of $\mathcal{N} = 2^*$ SYM

$$\boxed{S_{\mathcal{N}=2^*} = S_{\mathcal{N}=4} + S_m}$$

$$\begin{aligned}
S_{\mathcal{N}=4} = \frac{1}{g^2}\int d^4x \ \mathrm{Tr}\Big(&H_{\mu\nu}\Big[F_{\mu\nu} + \frac{1}{2}H_{\mu\nu} + \frac{1}{4}[B_{\mu\rho}, B_{\nu\rho}] + \frac{1}{2}[C, B_{\mu\nu}]\Big] \\
&-(D_\mu\phi)(D_\mu\overline{\phi}) + \frac{1}{2}[\phi, \overline{\phi}]^2 - \frac{1}{2}[\overline{\phi}, C][\phi, C] - \frac{1}{2}[\phi, B_{\mu\nu}][\overline{\phi}, B_{\mu\nu}] \\
&+H_\mu\Big[\frac{1}{2}H_\mu - \frac{1}{\sqrt{2}}(D_\mu C) - \frac{1}{\sqrt{2}}(D_\nu B_{\nu\mu})\Big] \\
&+\frac{1}{2}\chi_{\mu\nu}(D_\mu\psi_\nu) + \frac{1}{2}\psi_{\mu\nu}(D_\mu\chi_\nu) - \frac{1}{2}\psi_\mu(D_\mu\eta) + \frac{1}{2}\chi_\mu(D_\mu\zeta) \\
&+\frac{1}{2\sqrt{2}}\eta[\phi, \eta] + \frac{1}{2\sqrt{2}}\chi_\mu[\phi, \chi_\mu] + \frac{1}{2\sqrt{2}}\chi_{\mu\nu}[\phi, \chi_{\mu\nu}] \\
&-\frac{1}{2\sqrt{2}}\zeta[\overline{\phi}, \zeta] - \frac{1}{2\sqrt{2}}\psi_\mu[\overline{\phi}, \psi_\mu] - \frac{1}{2\sqrt{2}}\psi_{\mu\nu}[\overline{\phi}, \psi_{\mu\nu}] \\
&-\frac{1}{2\sqrt{2}}\eta[\zeta, C] - \frac{1}{2\sqrt{2}}\chi_\mu[\psi_\mu, C] + \frac{1}{2\sqrt{2}}\chi_{\mu\nu}[\psi_{\mu\nu}, C] \\
&+\frac{1}{2\sqrt{2}}\psi_\mu[\chi_\nu, B_{\mu\nu}] - \frac{1}{2\sqrt{2}}\chi_{\mu\nu}[\psi_{\mu\rho}, B_{\nu\rho}] \\
&-\frac{1}{2\sqrt{2}}\chi_{\mu\nu}[\zeta, B_{\mu\nu}] - \frac{1}{2\sqrt{2}}\psi_{\mu\nu}[\eta, B_{\mu\nu}]\Big)
\end{aligned}$$

Twist of $\mathcal{N} = 2^*$ SYM

Mass deformation (twisted theory)

$$
\begin{aligned}
S_m = \frac{1}{g^2}\int d^4x\ \mathrm{Tr}\Big[&-\frac{1}{2}m^2 B_{\mu\nu}^2 - \frac{1}{2}m^2 C^2 \\
&-\frac{1}{2}m\phi\Big([B_{\mu\nu}, B_{\mu\nu}] + [C, C]\Big) - \frac{1}{2}m\overline{\phi}\Big([B_{\mu\nu}, B_{\mu\nu}] + [C, C]\Big) \\
&+im\phi\Big([B_{12}, B_{23}] + [B_{13}, C]\Big) + im\overline{\phi}\Big([B_{12}, B_{23}] + [B_{13}, C]\Big) \\
&+\frac{im}{\sqrt{2}}(\psi_{12}\psi_{23} + \psi_{13}\zeta) - \frac{im}{\sqrt{2}}(\chi_1\chi_2 - \chi_3\chi_4)\Big]
\end{aligned}
$$

Mass deformation (untwisted theory)

$$
\begin{aligned}
S_m = \frac{1}{g^2}\int d^4x\ \mathrm{Tr}\Big(&- m{\lambda_1}^{\alpha}\lambda_{2\alpha} - m{\overline{\lambda}^1}_{\dot{\alpha}}\overline{\lambda}^{2\dot{\alpha}} + m^2\phi_1\phi_1^\dagger + m^2\phi_2\phi_2^\dagger \\
&-\sqrt{2}m\phi_3[\phi_1, \phi_1^\dagger] - \sqrt{2}m\phi_3[\phi_2, \phi_2^\dagger] \\
&-\sqrt{2}m\phi_3^\dagger[\phi_1, \phi_1^\dagger] - \sqrt{2}m\phi_3^\dagger[\phi_2, \phi_2^\dagger]\Big)
\end{aligned}
$$

BTFT Form of $\mathcal{N} = 4$ SYM

Appropriate for lattice:

Balanced Topological Field Theory Form (BTFT) of action

R. Dijkgraff and G. Moore [Commun.Math.Phys. 185 (1997) 411-440]

Introduce 3-vectors: $\vec{\Phi},\ \vec{B},\ \vec{H},\ \vec{\psi},\ \vec{\chi}$

$$\Phi_A \equiv 2\Big(F_{A4} + \frac{1}{2}\epsilon_{ABC}F_{BC}\Big)$$

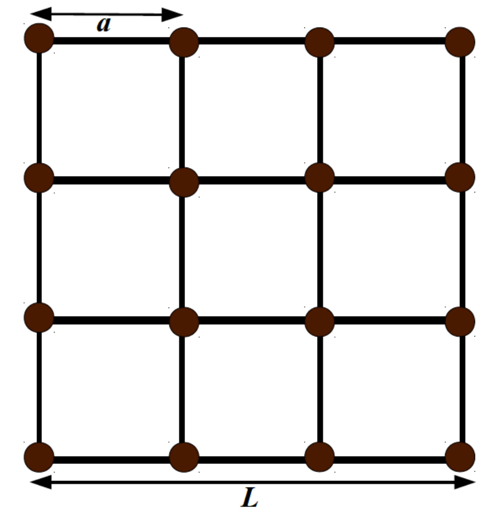
Action potential

$$\mathcal{F} = \Big(-\frac{1}{2\sqrt{2}}B_A\Phi_A - \frac{1}{24\sqrt{2}}\epsilon_{ABC}B_A[B_B, B_C] - \frac{1}{8}\chi_A\psi_A - \frac{1}{8}\psi_\mu\chi_\mu - \frac{1}{8}\eta\zeta\Big)$$

Lattice Formulation of $\mathcal{N} = 2^*$ SYM

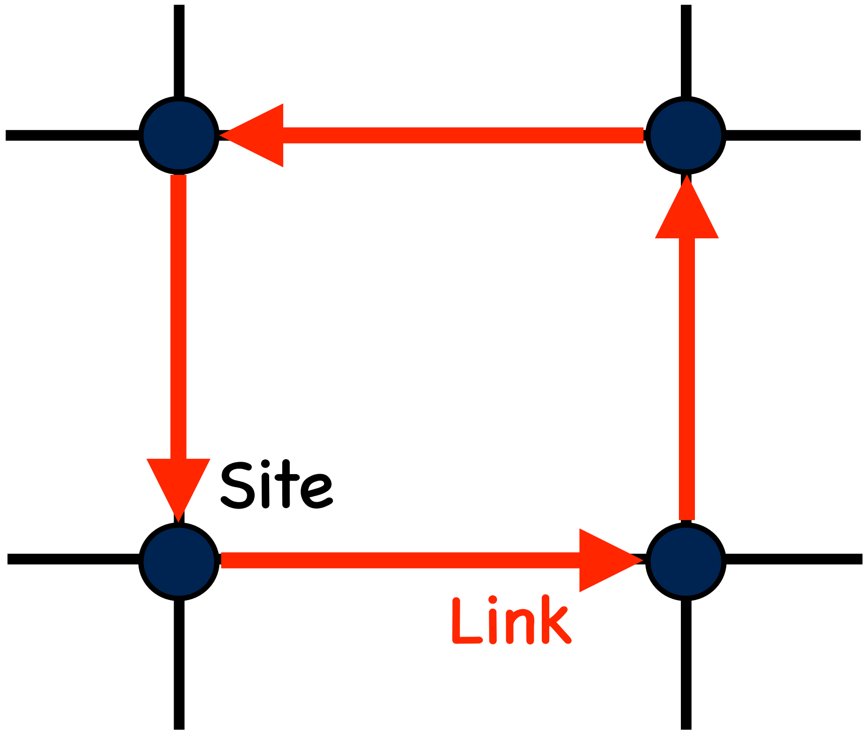
Use discretization prescription by **Sugino** F. Sugino, [JHEP 0401 (2004) 015]

Gauge field on links



$$U_\mu(\mathbf{n}) \equiv U(\mathbf{n}, \mathbf{n}+\mu) = e^{A_\mu(\mathbf{n})},$$
$$U^\dagger_\mu(\mathbf{n}-\mu) \equiv U(\mathbf{n}, \mathbf{n}-\mu) = e^{-A_\mu(\mathbf{n})}$$

All other fields on sites

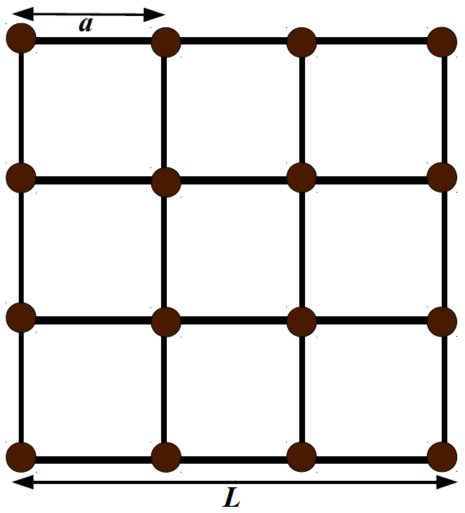


Makes lattice theory local, gauge invariant and doubler free

Lattice Formulation of $\mathcal{N} = 2^*$ SYM

Field strength on lattice - functional of link fields

$$\Phi_A = -\Big(U_{A4}(\mathbf{n}) - U_{4A}(\mathbf{n}) + \frac{1}{2}\sum_{B,C=1}^{3} \epsilon_{ABC}(U_{BC}(\mathbf{n}) - U_{CB}(\mathbf{n}))\Big)$$



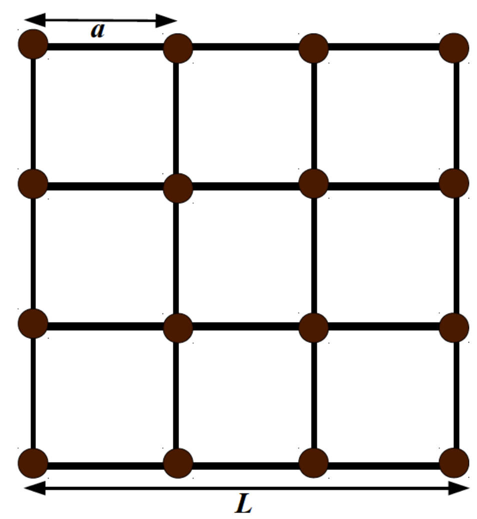
$$\begin{aligned}
&Q^{(m)}U_\mu(\mathbf{n}) = -\psi_\mu U_\mu(\mathbf{n}), && Q^{(m)}\psi_\mu(\mathbf{n}) = \psi_\mu(\mathbf{n})\psi_\mu(\mathbf{n}) - 2\sqrt{2}D_\mu^{(+)}\phi(\mathbf{n}),\\
&Q^{(m)}\phi(\mathbf{n}) = 0, && \\
&Q^{(m)}\overline{\phi}(\mathbf{n}) = \sqrt{2}\eta(\mathbf{n}), && Q^{(m)}\eta(\mathbf{n}) = -2[\phi(\mathbf{n}),\overline{\phi}(\mathbf{n})],\\
&Q^{(m)}C(\mathbf{n}) = \sqrt{2}\zeta(\mathbf{n}), && Q^{(m)}\zeta(\mathbf{n}) = -2[\phi(\mathbf{n}),C(\mathbf{n})] + 2mC(\mathbf{n}),\\
&Q^{(m)}\chi_\mu(\mathbf{n}) = 2H_\mu(\mathbf{n}), && Q^{(m)}H_\mu(\mathbf{n}) = -\sqrt{2}[\phi(\mathbf{n}),\chi_\mu(\mathbf{n})] + \sqrt{2}m\chi_\mu(\mathbf{n}),\\
&Q^{(m)}B_A(\mathbf{n}) = \sqrt{2}\psi_A(\mathbf{n}), && Q^{(m)}\psi_A(\mathbf{n}) = -2[\phi(\mathbf{n}),B_A(\mathbf{n})] + 2mB_A(\mathbf{n}),\\
&Q^{(m)}\chi_A(\mathbf{n}) = 2H_A(\mathbf{n}), && Q^{(m)}H_A(\mathbf{n}) = -\sqrt{2}[\phi(\mathbf{n}),\chi_A(\mathbf{n})].
\end{aligned}$$

Lattice Formulation of $\mathcal{N} = 2^*$ SYM

Covariant difference operators

$$D^{(+)}_\mu f(\mathbf{n}) = U_\mu(\mathbf{n}) f(\mathbf{n}+\mu) U^\dagger_\mu(\mathbf{n}) - f(\mathbf{n})$$

$$D^{(-)}_\mu g_\mu(\mathbf{n}) = g_\mu(\mathbf{n}) - U^\dagger_\mu(\mathbf{n}-\mu) g_\mu(\mathbf{n}-\mu) U_\mu(\mathbf{n}-\mu)$$



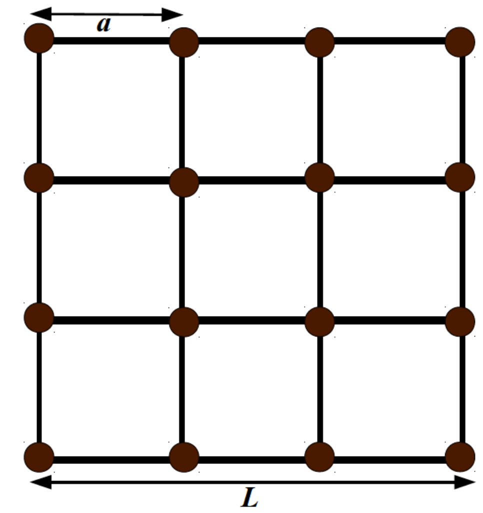
Straightforward to write down lattice action

$$S_{\mathcal{N}=2^*} = \beta_L \sum_{\mathbf{n}} Q^{(m)} \Psi^{(m)}(\mathbf{n})$$

Local, doubler free, gauge invariant, twisted SUSY invariant

Lattice Formulation of $\mathcal{N} = 2^*$ SYM

Issue with vacuum degeneracy - need to be resolved

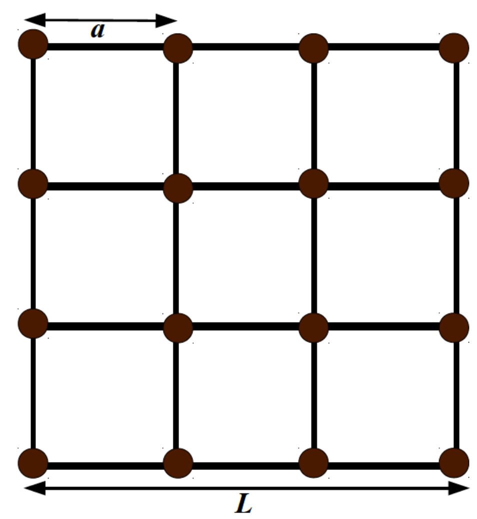


Twisted theory gauge action:

$$\frac{1}{2g_0^2} \sum_{\mathbf{n}} \sum_{\mu<\nu} \mathrm{Tr} \left[- \left(U_{\mu\nu}(\mathbf{n}) - U_{\nu\mu}(\mathbf{n}) \right)^2 \right]$$

Lattice Formulation of $\mathcal{N} = 2^*$ SYM

Issue with vacuum degeneracy - need to be resolved



Twisted theory gauge action:

$$\frac{1}{2g_0^2} \sum_{\mathbf{n}} \sum_{\mu<\nu} \mathrm{Tr}\left[-\left(U_{\mu\nu}(\mathbf{n}) - U_{\nu\mu}(\mathbf{n})\right)^2 \right]$$

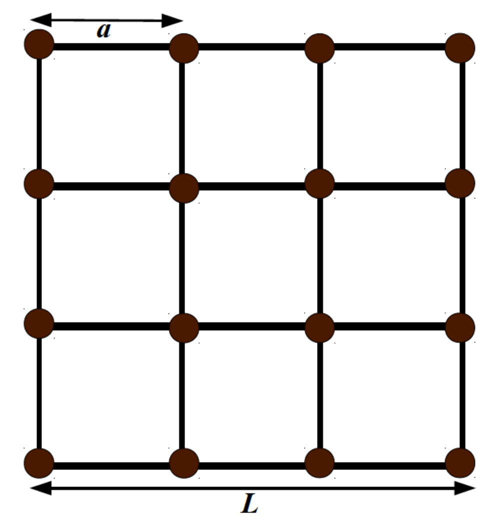
$$U_{\mu\nu} = \mathrm{diag}(\pm 1, \cdots, \pm 1)$$

$$U_{\mu\nu} = z_k \mathbb{I}_N$$ center of group

many classical vacua

Lattice Formulation of $\mathcal{N} = 2^*$ SYM

Issue with vacuum degeneracy - need to be resolved



Twisted theory gauge action:

$$\frac{1}{2g_0^2}\sum_{\mathbf{n}}\sum_{\mu<\nu}\mathrm{Tr}\left[-\left(\mathcal{U}_{\mu\nu}(\mathbf{n})-\mathcal{U}_{\nu\mu}(\mathbf{n})\right)^2\right]$$

$$\mathcal{U}_{\mu\nu} = \mathrm{diag}(\pm 1, \cdots, \pm 1)$$

$$\mathcal{U}_{\mu\nu} = z_k \mathbb{I}_N \quad \text{center of group}$$

many classical vacua

Standard Wilson:

$$\frac{1}{2g_0^2}\sum_{\mathbf{n}}\sum_{\mu<\nu}\mathrm{Tr}\left[2-\mathcal{U}_{\mu\nu}(\mathbf{n})-\mathcal{U}_{\nu\mu}(\mathbf{n})\right]$$

$$\mathcal{U}_{\mu\nu} = \mathbb{I}_N$$

unique minimum

Could resolve by adding standard Wilson term - softly breaks SUSY

Another option of imposing admissibility conditions - preserves SUSY

Gravitational Dual of $\mathcal{N} = 2^*$ SYM

Constructed by **Pilch and Warner** **[Nucl. Phys. B594 (2001) 209-228]**

A product of deformed AdS_5 and a deformed five-sphere

Gravitational Dual of $\mathcal{N} = 2^*$ SYM

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A product of deformed AdS_5 and a deformed five-sphere

isometry of gravity solution

$$\boxed{SU(2) \times U(1)}$$

R-symmetry of $\mathcal{N} = 2^*$ SYM

Gravitational Dual of $\mathcal{N} = 2^*$ SYM

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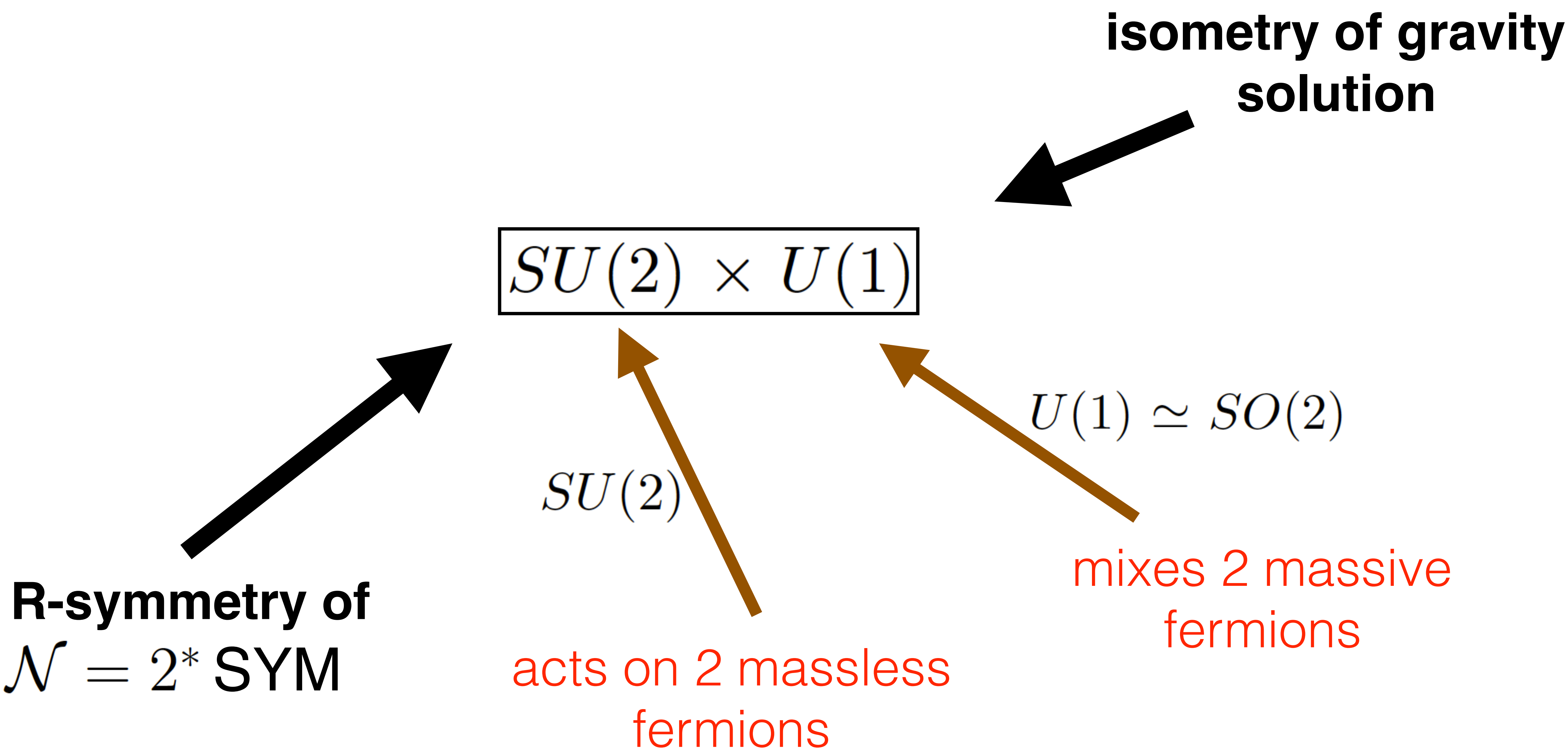
A product of deformed AdS_5 and a deformed five-sphere



Gravitational Dual of $\mathcal{N} = 2^*$ SYM

Constructed by **Pilch and Warner** **[Nucl. Phys. B594 (2001) 209-228]**

A product of deformed AdS_5 and a deformed five-sphere

Supergravity scalars: α and χ

$$\alpha \longleftrightarrow \mathcal{O}_2$$

"bosonic"

$$\chi \longleftrightarrow \mathcal{O}_3$$

"fermionic"

Gravitational Dual of $\mathcal{N} = 2^*$ SYM

Supergravity action

$$I_5 = \frac{1}{4\pi G_5} \int_{\mathcal{M}_5} d\xi^5 \sqrt{-g} \left(\frac{1}{4} R - \mathcal{L}_{\text{matter}} \right)$$

$$G_5 \equiv \frac{G_{10}}{2^5 \ \text{vol}_{S^5}} = \frac{\pi L^3}{2N^2}$$

Gravitational Dual of $\mathcal{N} = 2^*$ SYM

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$$\mathcal{L}_{\rm matter} = -3(\partial\alpha)^2 - (\partial\chi)^2 - \mathcal{P}$$

$$G_5 \equiv \frac{G_{10}}{2^5\ {\rm vol}_{S^5}} = \frac{\pi L^3}{2N^2}$$

Potential

$$\mathcal{P} = \hat{g}^2\left(\frac{1}{16}\left[\frac{1}{3}\left(\frac{\partial W}{\partial\alpha}\right)^2 + \left(\frac{\partial W}{\partial\chi}\right)^2\right] - \frac{1}{3}W^2\right)$$

$$\hat{g}^2 = \left(\frac{2}{L}\right)^2$$

Gravitational Dual of $\mathcal{N} = 2^*$ SYM

Supergravity action

$$I_5 = \frac{1}{4\pi G_5}\int_{\mathcal{M}_5} d\xi^5 \sqrt{-g}\left(\frac{1}{4}R - \mathcal{L}_{\text{matter}}\right)$$

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Superpotential

$$\widehat{g}^2 = \left(\frac{2}{L}\right)^2$$

$$W = -e^{2\alpha} - \frac{1}{2}e^{4\alpha}\cosh(2\chi)$$

Results from SUGRA

How can we use lattice $\mathcal{N} = 2^*$ SYM construction to test duality?

Pilch-Warner solution at finite temperature

Constructed by Buchel, Peet and Polchinski

[Phys. Rev. D63 (2001) 044009]

SUGRA geometry was numerically solved

Buchel, Deakin, Kerner and Liu, [Nucl. Phys. B784 (2007) 72]

Pressure

$$p = \frac{\pi^2}{8} N_c^2 T^4 f(m/T)$$

Entropy density

$$S/\mathcal{V} = \frac{\pi^2}{2} N_c^2 T^3 \sigma(m/T)$$

scaling function

Results from SUGRA

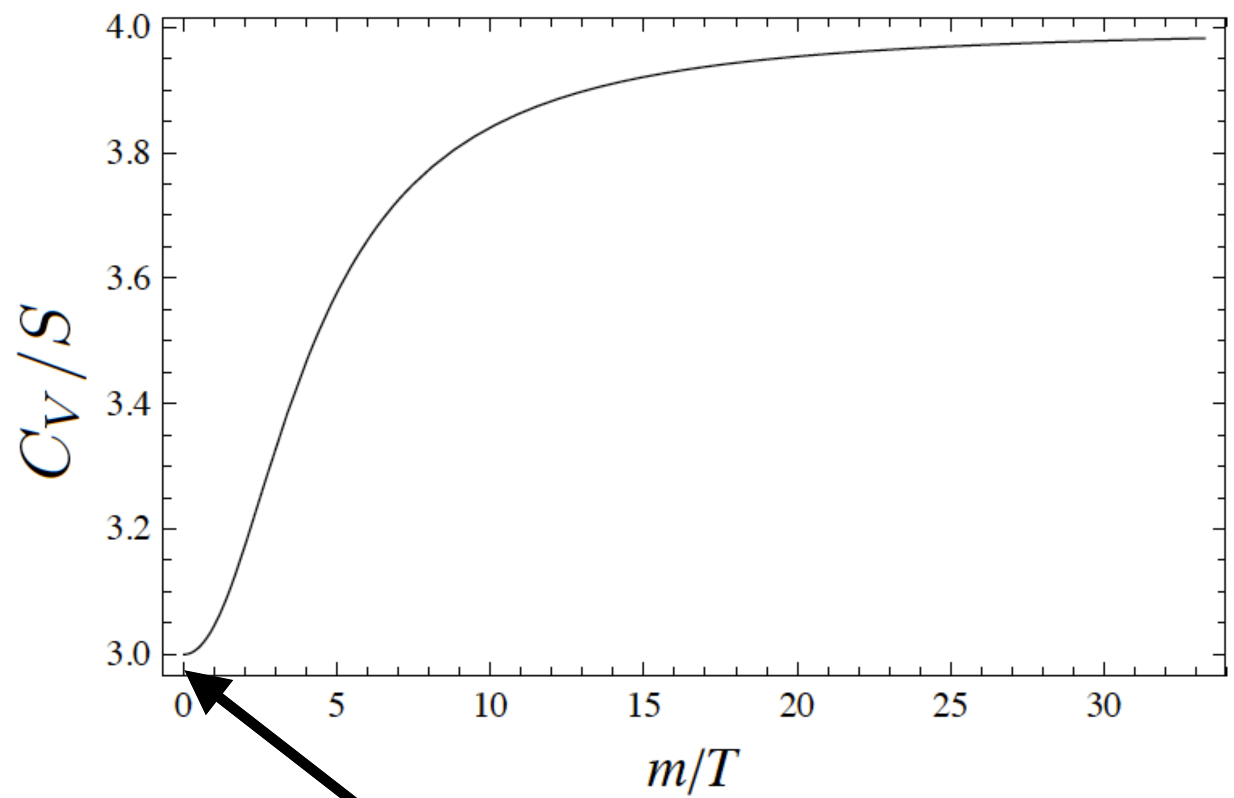
Equation of state

$$\epsilon - 3p = N_c^2 T^4\, \omega(m/T)$$

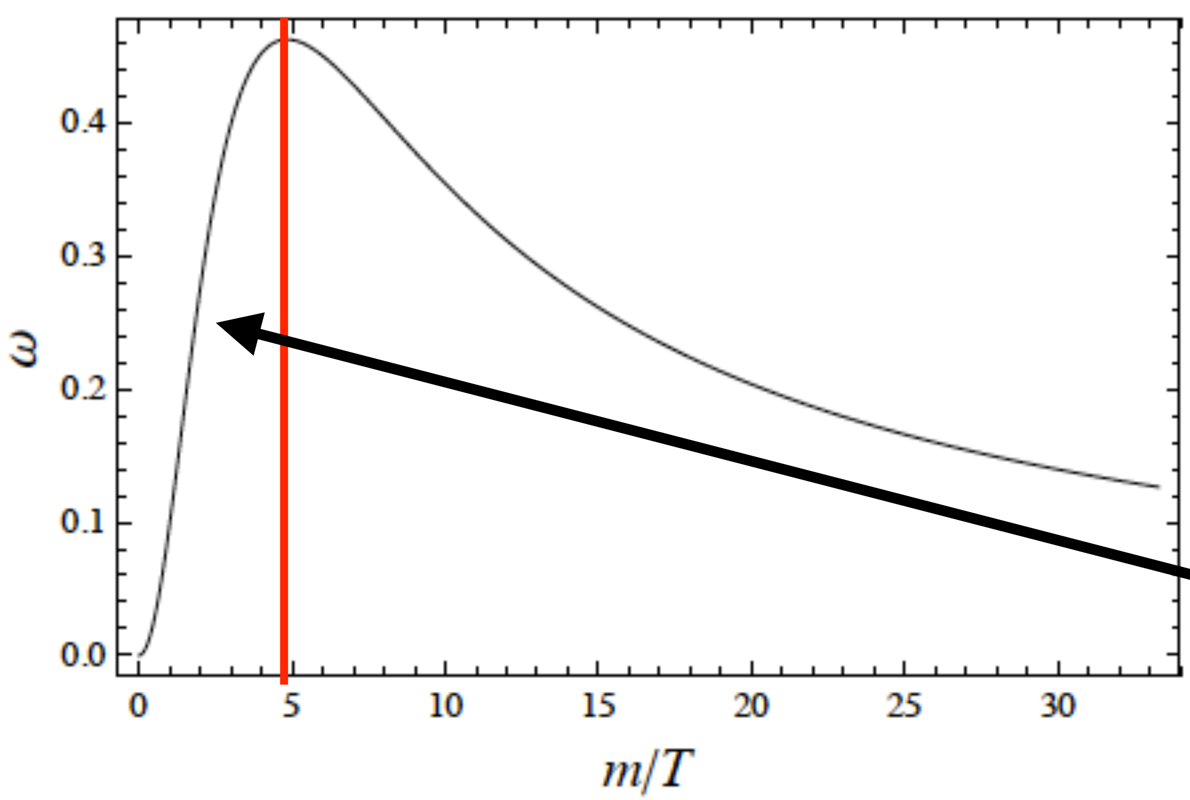
Hoyos, Paik and Yaffe, **[JHEP 1110 (2011) 062]**

$$\omega = \frac{\pi^2}{2}\,(\sigma - f) = -\frac{\pi^2}{8}\,\frac{m}{T}\,f'$$

Related to **speed of sound**



Required by **conformal invariance**



Plasma is cool. Low probability for massive fields excitation

Conclusions and Future Directions

- Presented lattice formulation of N=2* super Yang-Mills
  - Could simulate thermodynamics
  - Could test gauge-gravity duality in non-conformal setting
- Study finite temperature properties
  - First law of thermodynamics in AdS/CFT setting
  - Thermal phases in the theory
- Need to enumerate symmetries of lattice theory
- Study renormalization and fine tuning
- Sign problem in lattice simulations?

More things to do!

THANK YOU!