

研究課題名：新しい重力理論の探求と修正重力理論の観測的検証に関する理論研究 (公募研究)

Stable cosmological solutions in *extended vector-tensor* theory

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“extended vector-tensor” theory

II

vector field theory

on **curved** spacetime in 4D

with **degenerate** kinetic matrix

Kimura, **AN**, Yoshida [1608.07066 (2016)]

motivation ①

- ✓ to explain primordial/current accelerated expansion of the universe
 - Λ ?? (why so small ? why that value ?)
 - Λ [ϕ] ?? = **scalar**-tensor theory
 - e.g. canonical, k-essence, Horndeski, more & more...
 - $g_{\mu\nu}$?? (change of gravity law) = **tensor** theory
 - e.g. (dRGT) massive gravity, bi-gravity...
 - ⇒ decoupling limit of massive gravity (or bi-gravity)
 - = described by **scalar** & **vector** fields

D01

motivation ②

[HEP](#)

2 records found

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1. Finding the chiral gravitational wave background of an axion-SU(2) inflationary model using CMB observations and laser interferometers

Ben Thorne (Tokyo U., IPMU & Oxford U.), Tomohiro Fujita (Stanford U., TTP & Stanford U., Phys. Dept. & Kyoto U.), Iwan W. G. M. Long (Stanford U., Phys. Dept. & JAXA, Sagamihara), Nobuhiko Katayama (Tokyo U., IPMU), Eiichiro Komatsu (Tokyo U., IPMU & Graduate U. & JAXA, Sagamihara), and Garret A. Morgan (Tokyo U., IPMU & Graduate U. & JAXA, Sagamihara). Jul 11, 2017. 24 pp.

e-Print: [arXiv:1707.03240](#) [astro-ph.CO] | [PDF](#)

[References](#) | [BibTeX](#) | [LaTeX\(US\)](#) | [LaTeX\(EU\)](#) | [Harvmac](#) | [EndNote](#)

[ADS Abstract Service](#)

[Detailed record](#) - [Cited by 5 records](#)

2. Large Tensor Non-Gaussianity from Axion-Gauge Fields Dynamics

Aniket Agrawal (Stanford U., Phys. Dept.), Tomohiro Fujita (Tokyo U., IPMU & Tokyo U., RESCEU), Eiichiro Komatsu (Tokyo U., IPMU & Graduate U. & JAXA, Sagamihara), and Garret A. Morgan (Tokyo U., IPMU & Graduate U. & JAXA, Sagamihara).

e-Print: [arXiv:1707.03023](#) [astro-ph.CO] | [PDF](#)

[References](#) | [BibTeX](#) | [LaTeX\(US\)](#) | [LaTeX\(EU\)](#) | [Harvmac](#) | [EndNote](#)

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concrete title :

Finding the chiral gravitational wave background of an axion-SU(2) inflationary model using CMB observations and laser interferometers [134 characters]

degenerate theory
or
degenerate kinetic matrix

⇔ **magic** to introduce the kinetic term
for *non-dynamical* d.o.f.(s)

Maxwell & Proca

- ✓ A_μ has 4 components = (in maximum) **4 d.o.f.s**
- ✓ In Maxwell theory, **A_0 is non-dynamical** (no kinetic term)

$$-F_{\mu\nu}F^{\mu\nu} \sim \vec{E}^2 - \vec{B}^2 \sim \dot{A}_i^2 - A_{[i,j]}^2 \not\sim \dot{A}_0^2$$

(gauge sym. kills longitudinal mode \rightarrow 2 d.o.f.s)

- ✓ **Proca theory** ($+m^2A^2$, no gauge sym.) \Leftrightarrow 3 d.o.f.s

In Maxwell & Proca, **A_0 is non-dynamical = no kinetic term**

\rightarrow **with magic (degeneracy)**, kinetic term for **A_0 ??**

Extended vector-tensor

- ✓ action with two first derivative of A_μ & **4D general covariance**

$$\mathcal{L} = f(Y) R + C^{\mu\nu\rho\sigma} (\nabla_\mu A_\nu) (\nabla_\rho A_\sigma)$$

$$C^{\mu\nu\rho\sigma} = \alpha_1 g^{\mu(\rho} g^{\sigma)\nu} + \alpha_2 g^{\mu\nu} g^{\rho\sigma} + \frac{1}{2} \alpha_3 (A^\mu A^\nu g^{\rho\sigma} + A^\rho A^\sigma g^{\mu\nu})$$

— sym.

$$+ \frac{1}{2} \alpha_4 (A^\mu A^{(\rho} g^{\sigma)\nu} + A^\nu A^{(\rho} g^{\sigma)\mu}) + \alpha_5 A^\mu A^\nu A^\rho A^\sigma + \alpha_6 g^{\mu[\rho} g^{\sigma]\nu}$$

— asym.

$$+ \frac{1}{2} \alpha_7 (A^\mu A^{[\rho} g^{\sigma]\nu} - A^\nu A^{[\rho} g^{\sigma]\mu}) + \frac{1}{4} \alpha_8 (A^\mu A^\rho g^{\nu\sigma} - A^\nu A^\sigma g^{\mu\rho}) + \frac{1}{2} \alpha_9 \epsilon^{\mu\nu\rho\sigma}.$$

f, α_i : functions of $Y = A_\mu A^\mu$

☝ not symmetric under $\mu \leftrightarrow \nu$ ($\rho \leftrightarrow \sigma$) [cf. $\nabla_\mu A_\nu \rightarrow \nabla_\mu \nabla_\nu \phi$]

☝ for general α_i , **6** = **1 (A0)** + 3 (Ai) + 2 (GW) d.o.f.s

degeneracy condition

✓ Action after ADM decomposition (separate time & space) :

$$\mathcal{L}_{\text{kin}} = \mathcal{A} \dot{A}_*^2 + 2\mathcal{B}^i \dot{A}_* \dot{A}_\mu + 2\mathcal{C}^{\mu\nu} \dot{A}_* K_{\mu\nu} + \mathcal{D}^{\mu\nu} \dot{A}_\mu \dot{A}_\nu + 2\mathcal{E}^{\mu\nu\rho} \dot{A}_\mu K_{\nu\rho} + \mathcal{F}^{\mu\nu\rho\sigma} K_{\mu\nu} K_{\rho\sigma},$$

kinetic term for A0

$$A_* (= n^\mu A_\mu) \sim A_0$$

✓ **degeneracy** cond. \Leftrightarrow making **A0 non-dynamical** :

$$0 = |\mathcal{M}_{\text{kin}}| = \mathcal{D}_0 + \mathcal{D}_2 A_*^2 + \mathcal{D}_4 A_*^4$$

$$0 = \mathcal{D}_0 \propto (\alpha_1 + \alpha_2) F(\alpha_i, f)$$

→ **case A** : $\alpha_1 + \alpha_2 = 0$
→ **case B** : $F = 0$ ($f \neq 0$)
→ **case C** : $F = 0$ ($f = 0$)

an example

asym. parts $\neq 0$!!
 no scalar counterpart !!
 only exist in V-T theory !!

$$\mathcal{L} = f(Y) R + C^{\mu\nu\rho\sigma} (\nabla_\mu A_\nu) (\nabla_\rho A_\sigma)$$

$$C^{\mu\nu\rho\sigma} = \alpha_1 g^{\mu(\rho} g^{\sigma)\nu} + \alpha_2 g^{\mu\nu} g^{\rho\sigma} + \frac{1}{2} \alpha_3 (A^\mu A^\nu g^{\rho\sigma} + A^\rho A^\sigma g^{\mu\nu})$$

— sym.
 — asym.

$$+ \frac{1}{2} \alpha_4 (A^\mu A^{(\rho} g^{\sigma)\nu} + A^\nu A^{(\rho} g^{\sigma)\mu}) + \alpha_5 A^\mu A^\nu A^\rho A^\sigma + \alpha_6 g^{\mu[\rho} g^{\sigma]\nu}$$

$$+ \frac{1}{2} \alpha_7 (A^\mu A^{[\rho} g^{\sigma]\nu} - A^\nu A^{[\rho} g^{\sigma]\mu}) + \alpha_8 (A^\mu A^\rho g^{\nu\sigma} - A^\nu A^\sigma g^{\mu\rho}) + \frac{1}{2} \alpha_9 \epsilon^{\mu\nu\rho\sigma}.$$

$$f = 1 \quad 2\alpha_6 + Y\alpha_7 = 0 \quad \alpha_1 = \frac{-8(2\alpha_2 + Y\alpha_3) - Y(4 + 4Y\alpha_2 - Y^2\alpha_3)\alpha_8}{2Y^2\alpha_8}$$

$$\alpha_4 = \frac{4(1 + Y\alpha_2)}{Y^2} - \alpha_3 + \frac{8(2\alpha_2 + Y\alpha_3)}{Y^3\alpha_8} + \alpha_8 - \frac{Y^2\alpha_8^2}{8}$$

$$\alpha_5 = \frac{-2 + Y^2\alpha_3}{Y^3} - \frac{4(2\alpha_2 + Y\alpha_3)}{Y^4\alpha_8} - \frac{\alpha_8}{Y} + \frac{Y\alpha_8^2}{8} + \frac{12(2\alpha_2 + Y\alpha_3)}{Y^2(Y^2\alpha_8 - 8)}$$

application to
cosmology / dark energy ??

no-go for degenerate theory ??

✓ By construction, there is no Ostrogradsky instability.
However, there could be **another instabilities...**

✓ de Rham investigated cosmologies in *S-T theory*
finding **ghost/gradient instability in perturbations**

de Rham & Matas (2016)

$$\mathcal{L}_\Phi = 2 \frac{a^3}{\dot{\phi}_0^2} (G - X A_1) \left(3\dot{\phi}_0^2 - 2 \frac{G - X A_1}{A_1 + A_2} \right) \dot{\Phi}^2 - 2aG\Phi \nabla^2 \Phi, \quad (\text{B.6})$$

and that branch of solutions always exhibits a gradient instability (if $G > 0$, the instability is in the scalar mode and had we taken $G < 0$, the instability would have been in the tensor modes).

✓ instability in **vector-tensor** theory as well ??

cosmology in EVT

✓ The future will be **probably** bright !!

✓ cosmology in extended vector-tensor theory

$$\text{BG : FLRW} + \bar{A}_\mu = (\bar{A}_0(t), \mathbf{0})$$

✓ There is (could be) parameter space in which
all the perturbations does not suffer from **ANY instabilities**.

✓ Bianchi cosmology ? inflationary cosmology ??
generation of primordial magnetic fields, GWs ???

summary

- ✓ We have constructed degenerate vector-tensor theory that includes two first derivative of vector field.
- ✓ New theory for massive vector field includes
5 d.o.f.s = 3 massive vector in A_μ & 2 GW in $g_{\mu\nu}$
- ✓ We have investigated an application to cosmology finding that there exist healthy cosmological solutions where there is no instabilities in perturbations
⇔ scalar theory

Thank you
for your attention

&

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