Symposium “Why does the Universe accelerate ?”, 10-12 Feb 2018 @ Tohoku Univ.
公募研究「宇宙マイクロ波背景放射における揺らぎの非線形進化」

Reconstruction of primordial tensor power spectrum from B-mode observations

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CMB B-mode

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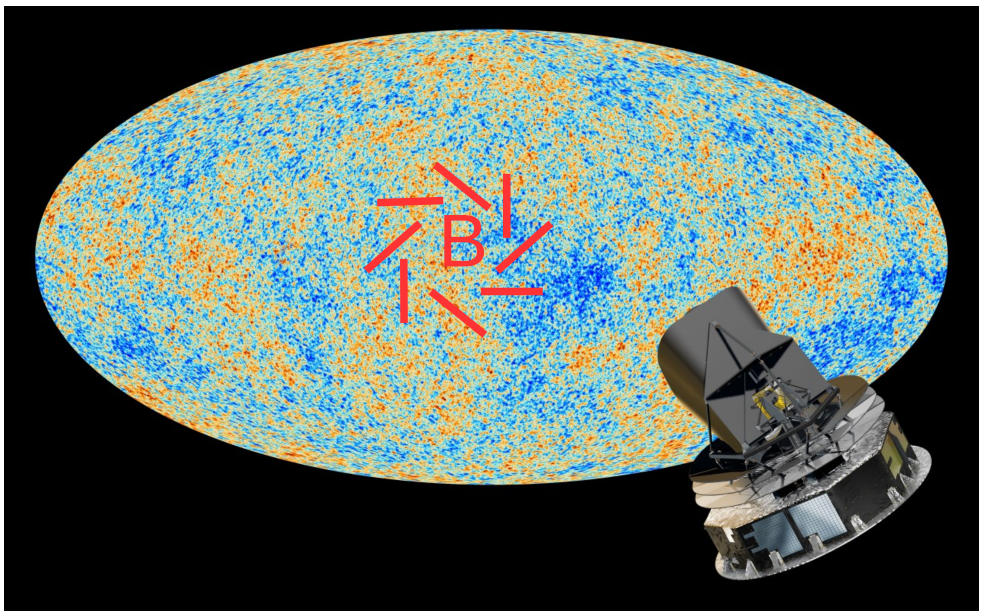
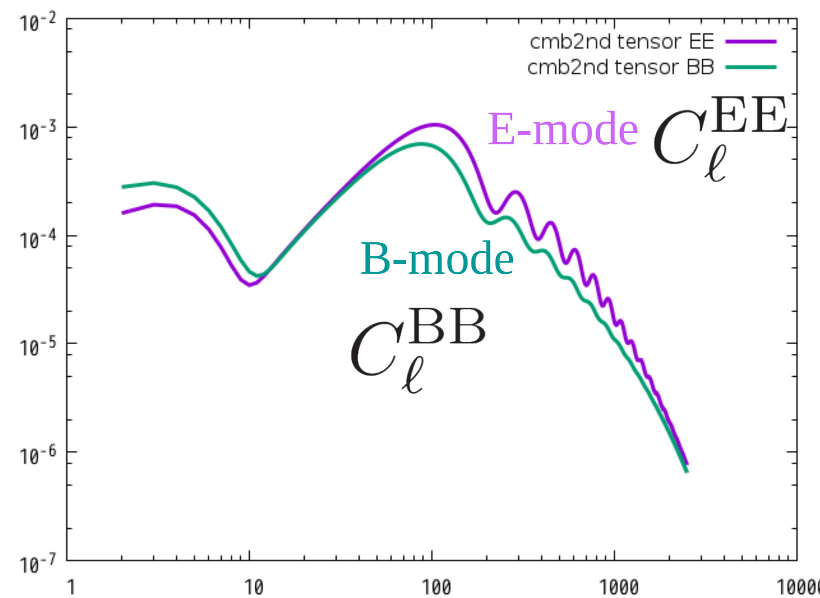
- A polarisation mode of photons $(Q, U) \to (E_{\ell m}, B_{\ell m})$
- Generated by gravitational waves (tensor perturbations)
- No detections so far, but possibly to be done in the (near) future (e.g. LiteBIRD)





Gives fruitful information on inflation and the early Universe.

How well can we distinguish primordial power spectra predicted in various models from the fiducial one (slow-roll inflation) ?

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How accurately can we measure the primordial spectrum under the expected observational noises ?

$$P^{(T)}(k) + \delta P^{(T)}(k) \qquad \longleftarrow \qquad C_\ell^{(T)BB} + \mathcal{N}_\ell$$

Fisher matrix

Fisher Information Matrix for CMB

$$F_{ij} = \frac{1}{2}\sum_{\ell=2}^{\ell_{\rm max}}(2\ell+1)\frac{1}{\mathcal{N}_\ell^2}\left(\frac{\partial C_\ell^{(T)BB}}{\partial\theta_i}\right)\left(\frac{\partial C_\ell^{(T)BB}}{\partial\theta_j}\right)$$

$$\mathcal{N}_\ell = C_\ell^{(T)BB} + C_\ell^{(S)\rm lens} + N_\ell$$ "noise"

θ_i : characterising the primordial spectrum $P^{(T)}(k)$

Modelling tensor power spectrum

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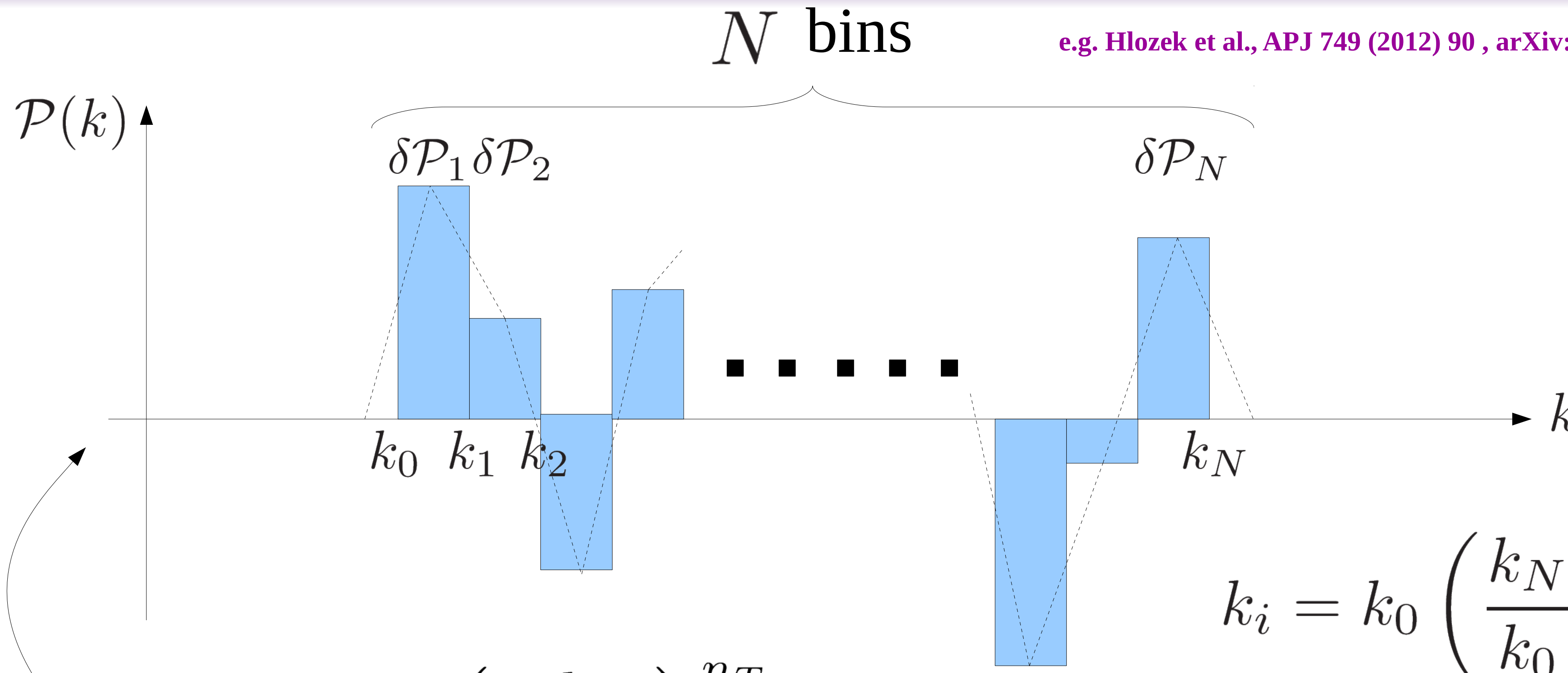
e.g. Hlozek et al., APJ 749 (2012) 90 , arXiv:1105.4887



$$k_i = k_0 \left(\frac{k_N}{k_0}\right)^{i/N}$$

$$\mathcal{P}_h^{\rm fid}(k) = r\mathcal{P}_{\mathcal{R}0}\left(\frac{k}{k_{\rm pivot}}\right)^{n_T}$$ following the Planck 2015 results

$$\mathcal{P}_h^{\rm fid}(k) + \delta\mathcal{P}_i \qquad C_\ell^{(T)BB} + \mathcal{N}_\ell$$

Fisher matrix

Fisher Information Matrix for CMB

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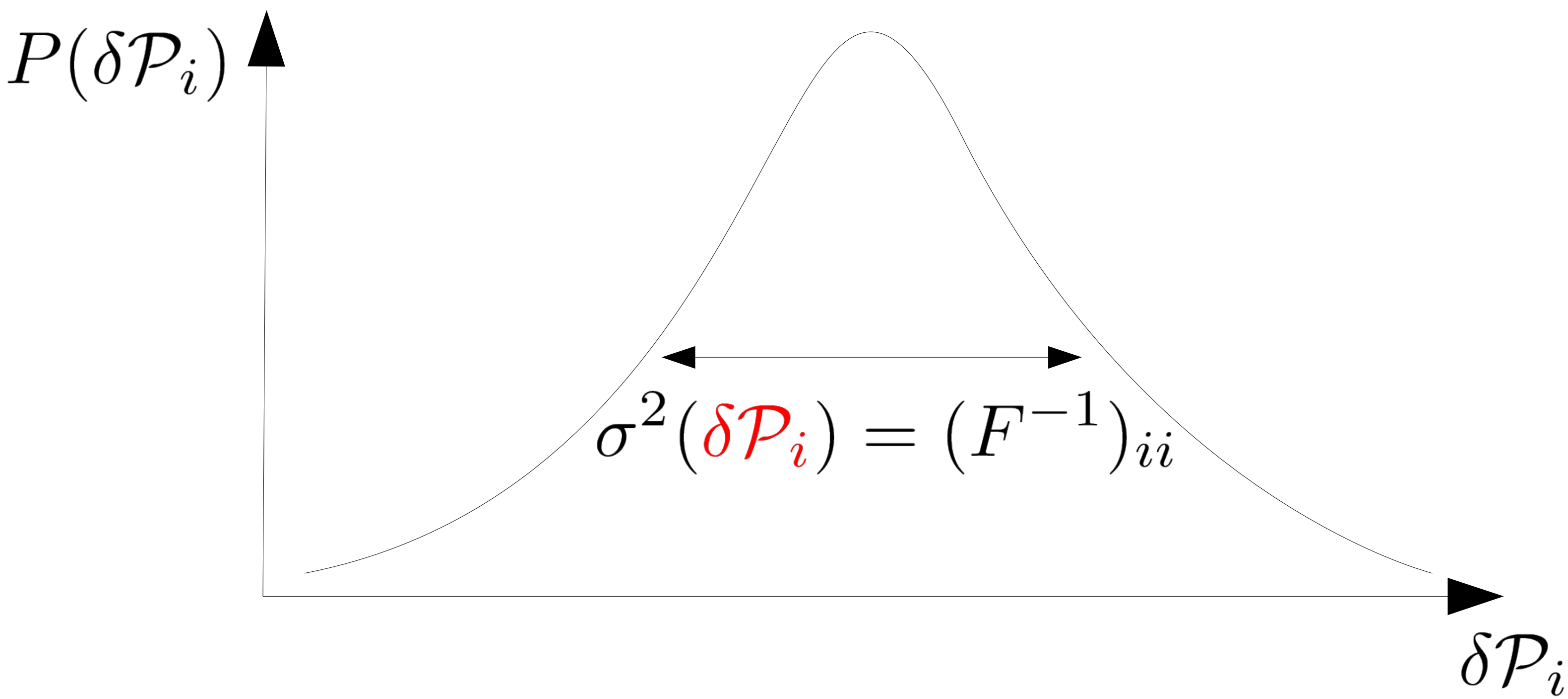
$$F_{ij} = \frac{1}{2}\sum_{\ell=2}^{\ell_{\max}}(2\ell+1)\frac{1}{\mathcal{N}_\ell^2}\left(\frac{\partial C_\ell^{(T)BB}}{\partial \delta\mathcal{P}_i}\right)\left(\frac{\partial C_\ell^{(T)BB}}{\partial \delta\mathcal{P}_j}\right)$$

$$\mathcal{N}_\ell = C_\ell^{(T)BB} + C_\ell^{(S)\mathrm{lens}} + N_\ell$$



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Angular power spectrum of B-mode fluctuations

$$C_\ell^{(T)BB} = 4\pi \int T_{B\ell}^{(T)2}(k)\mathcal{P}_h(k)\frac{dk}{k}$$

(ℓ is an index of $Y_{\ell m}$)

$$\mathcal{P}_h(k) = \frac{k^3}{2\pi^2}P_h(k)$$

Transfer function given by solving Boltzmann equations with parameters from Planck 2015 results :

$$h = 0.6774 \qquad T_{\gamma,0} = 2.7255\ \mathrm{K}$$
$$h^2\Omega_{\mathrm{CDM}} = 0.1188 \qquad \tau = 0.066$$
$$h^2\Omega_{\mathrm{b}} = 0.02230 \qquad Y_p = 0.24667$$
$$N_{\mathrm{eff}} = 3.046$$

Building block 2/3 : Lensing B-mode

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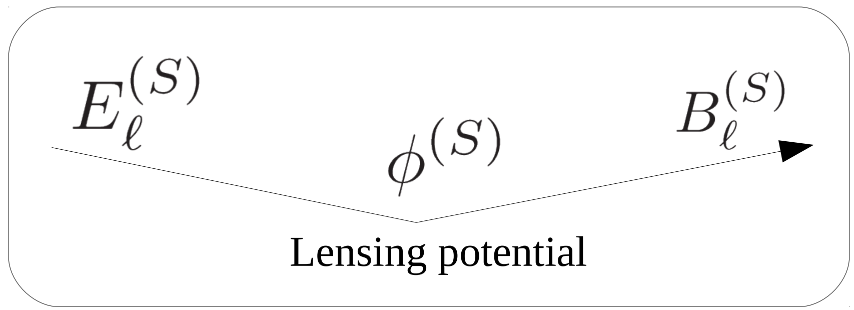
B-mode induced by E → B conversion of lensing

Smith et al., JCAP 1206 (2012) 014, arXiv:1010.0048



$$C_\ell^{(S)\mathrm{lens}} \approx \frac{1}{2\ell+1}\sum_{\ell' L}^{\ell'_{\mathrm{max}}} \mathcal{S}^2_{\ell\ell' L} C_{\ell'}^{(S)EE} C_L^{(S)\phi\phi}$$

(ℓ'_{max} ← 2000)

$$\begin{cases} C_\ell^{(S)\phi\phi} \approx 16\pi \displaystyle\int \frac{dk}{k}\mathcal{P}_\mathcal{R}(k)\left[\int_0^{\chi_{\mathrm{LSS}}} d\chi\, T_\Psi(k,\eta_0-\chi) j_\ell(k\chi)\left(\frac{\chi_{\mathrm{LSS}}-\chi}{\chi_{\mathrm{LSS}}\chi}\right)\right]^2 \\ C_\ell^{(S)EE} = 4\pi \displaystyle\int \frac{dk}{k}\mathcal{P}_\mathcal{R}(k) T_{E\ell}^{(S)2}(k) \end{cases}$$

Building block 3/3 : Detector noise

For simplicity, here we consider a white noise (independent to ℓ)

Katayama & Komatsu, APJ 737 (2011) 78, arXiv:1101.5210

$$N_\ell = \left(\frac{\pi}{10800} \frac{w_p^{-1/2}}{\mu\mathrm{K} \cdot \mathrm{arcmin}} \right)^2 \mu K^2 \cdot \mathrm{str}$$

$$w_p^{-1/2} = 63.1\ \mu K \cdot \mathrm{arcmin}$$ (Planck, averaged over 3 bands)

Zaldarriaga et al. arXiv:0811.3918

↓

$$w_p^{-1/2} = \mathcal{O}(1) \mu\mathrm{K} \cdot \mathrm{arcmin}$$ (Future experiments)

Fisher Information Matrix for CMB

$$F_{ij} = \frac{1}{2}\sum_{\ell=2}^{\ell_{\max}}(2\ell+1)\frac{1}{\mathcal{N}_\ell^2}\left(\frac{\partial C_\ell^{(T)BB}}{\partial \delta\mathcal{P}_i}\right)\left(\frac{\partial C_\ell^{(T)BB}}{\partial \delta\mathcal{P}_j}\right)$$

($\ell_{\max}$ → 500)

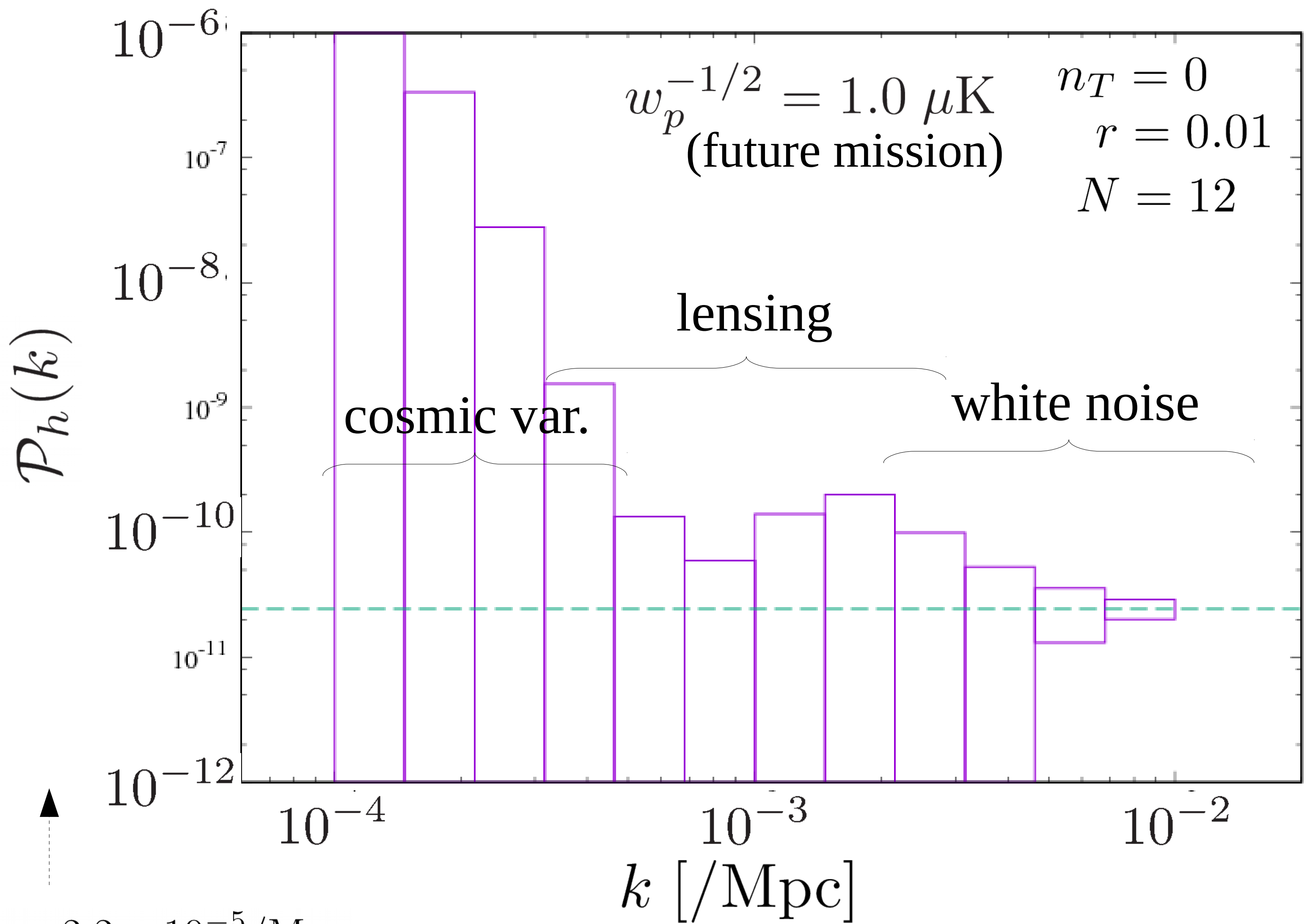
Numerator : $\frac{\partial}{\partial\theta_i}C_\ell^{(T)BB} = 4\pi\int_{k_{i-1}}^{k_i} T_{B\ell}^{(T)2}(k)\frac{dk}{k}$

Denominator : $\mathcal{N}_\ell = C_\ell^{(T)BB} + C_\ell^{\rm lens} + N_\ell$

Uncertainty to measure $\delta\mathcal{P}_i$: $\sigma(\delta\mathcal{P}_i)^2 = (F^{-1})_{ii}$

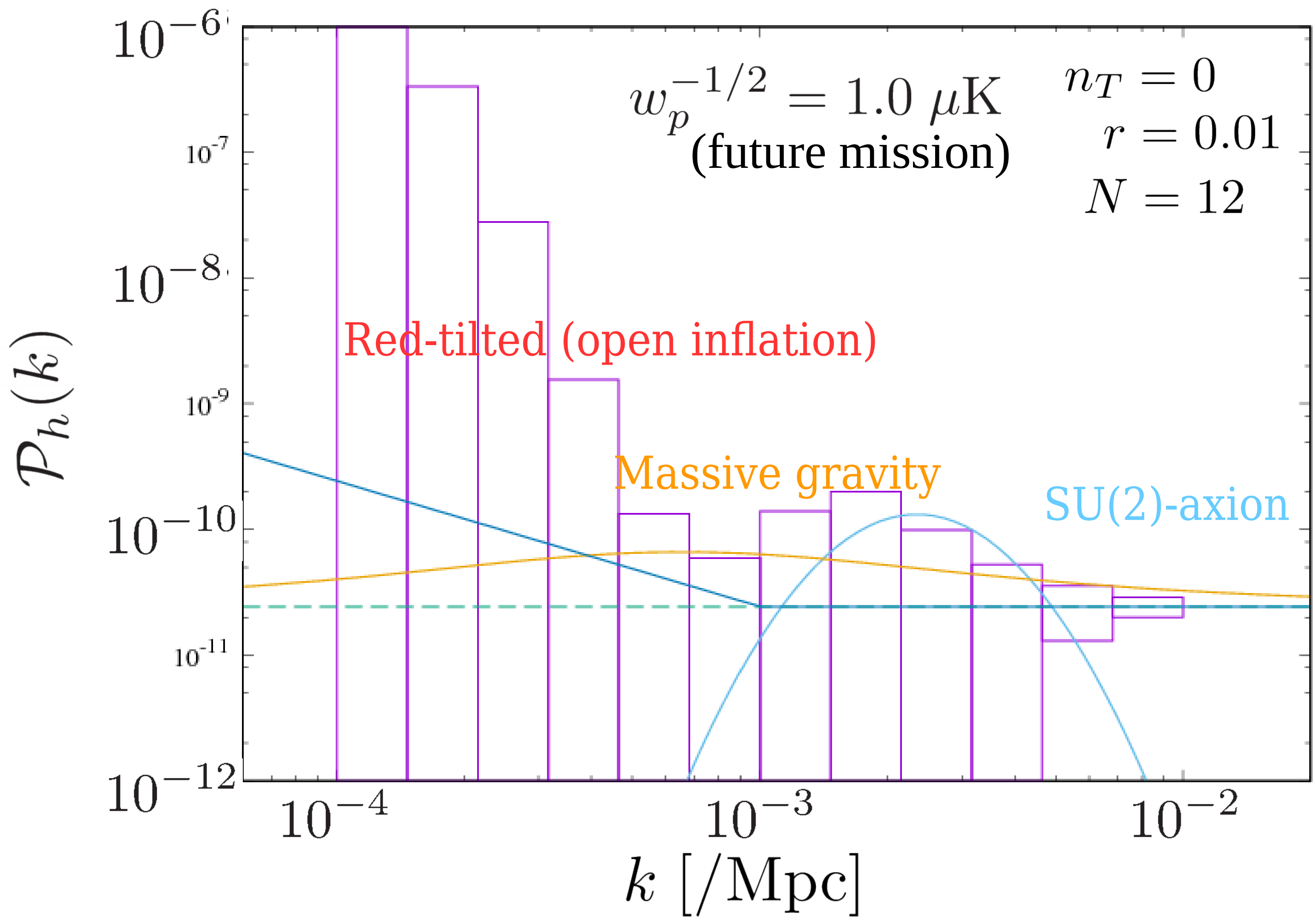
Results : 1-σ error of binned spectrum

Contributions from the noise sources



$k_H = 2.2 \times 10^{-5}/\mathrm{Mpc}$

Results : demonstration



Red-tilted	**Yamauchi et al., PRD 84 (2011) 043513**
SU(2)-axion	**Thorne et al., arXiv:1707.03240; Dimastrogiovanni et al., JCAP 01 (2017) 019**
Massive gravity	**Sasaki, private communication; Domenech et al., JCAP 1705 (2017) 034**

Results : demonstration

More quantitatively, we should estimate χ^2.

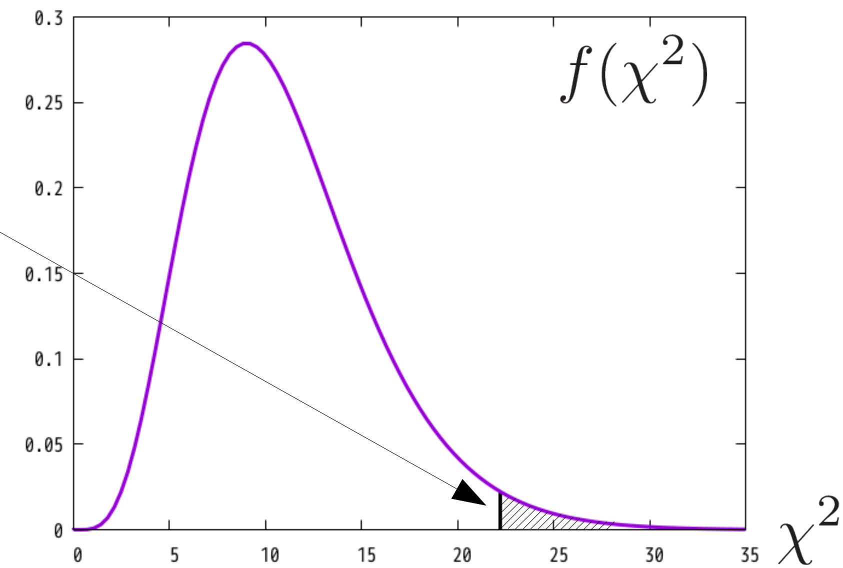
$$\chi^2 = \sum_{ij} \Delta\mathcal{P}_i(k) F_{ij} \Delta\mathcal{P}_j(k)$$

$$\Delta\mathcal{P}_i(k) = \mathcal{P}_i^{\rm model}(k) - \mathcal{P}_i^{\rm fid}(k)$$

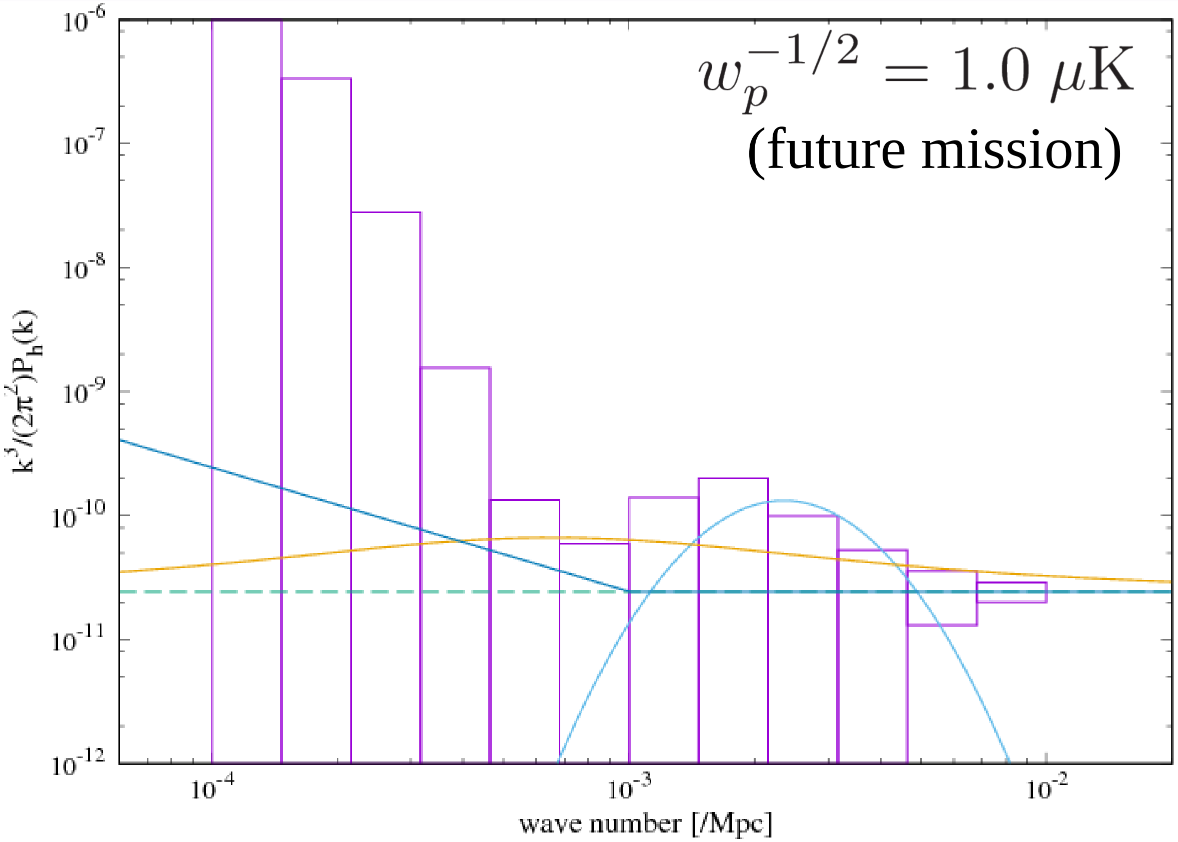
Probability to exceed (PTE) = Probability to confuse a model spectrum with the fiducial one.

$$P(\chi^2 > a) = \int_a^\infty f(\chi^2)\, d\chi^2$$

$$f(\chi^2) = \frac{x^{k/2-1} e^{-x/2}}{2^{k/2}\Gamma(k/2)}$$



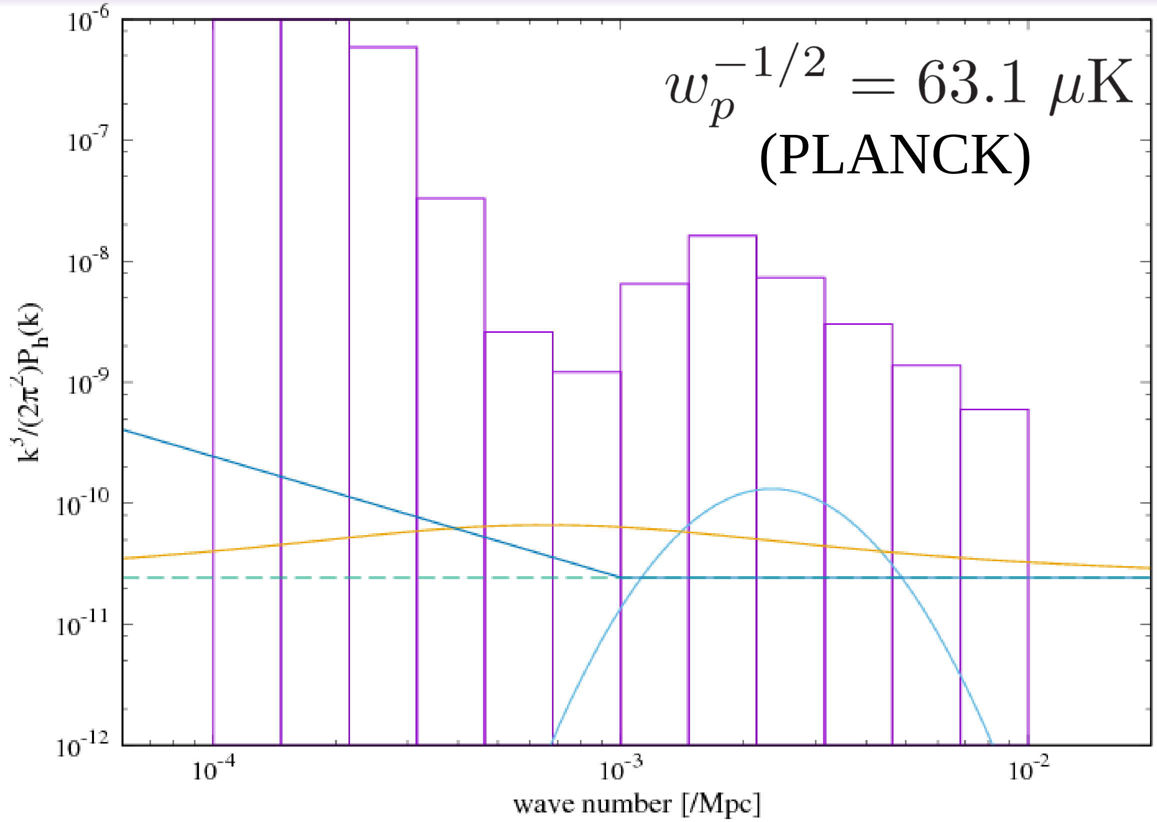
Results : demonstration



	χ^2	PTE
SU(2)	1.20×10^2	1.75×10^{-20}
Massive	1.47×10^2	7.74×10^{-26}
Red	2.17×10^1	2.68×10^{-2}

Distinguishable from fiducial spectrum with a high significance !

Results : demonstration

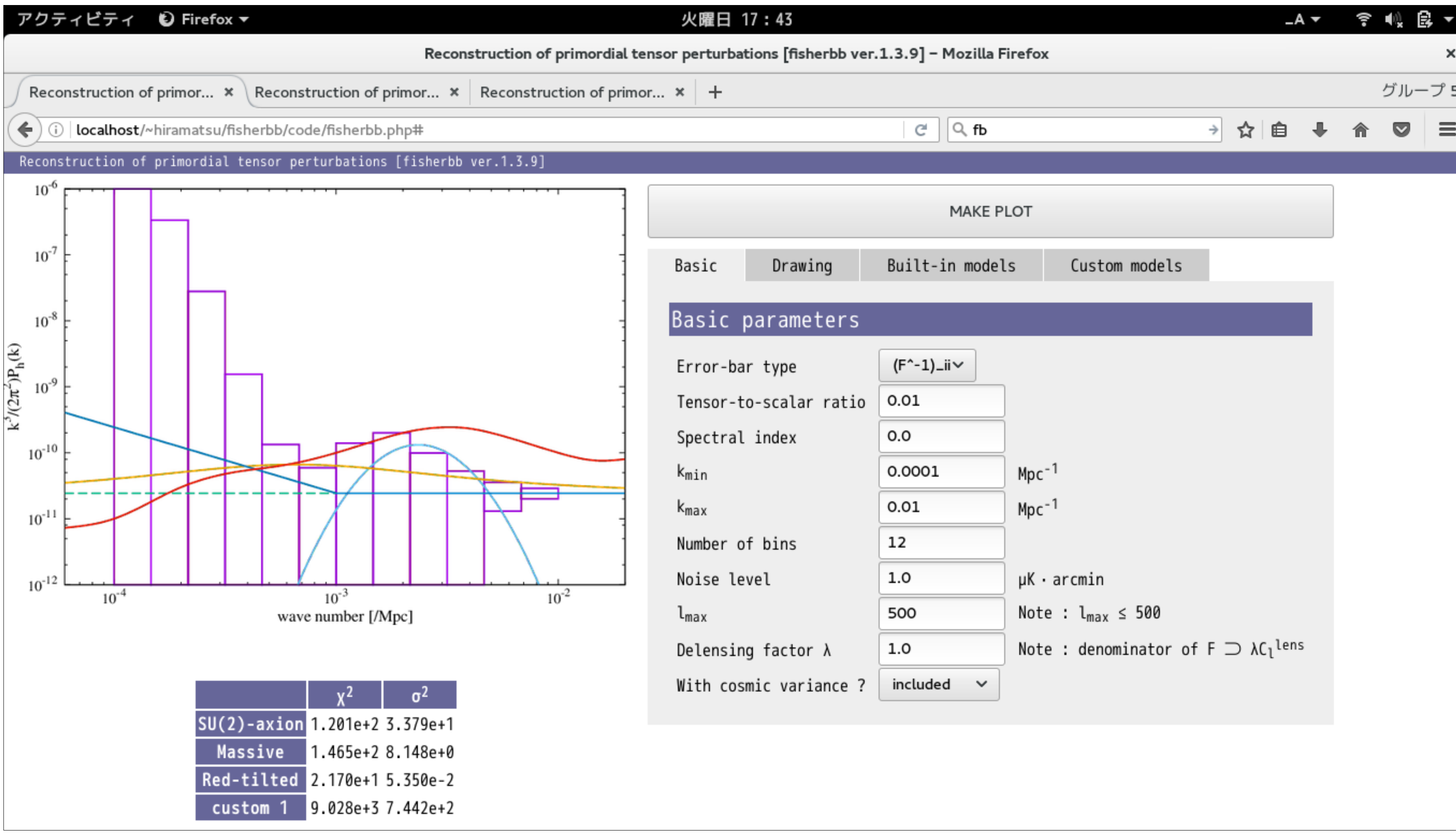


	χ^2	PTE
SU(2)	6.48×10^{-1}	1.00
Massive	1.74	9.99×10^{-1}
Red	1.86	9.99×10^{-1}

All models cannot be distinguished from the fiducial one.

Simulator is to be in public

- To be available on your web browser
- Compute from numerical data of spectrum as well as in built-in models



Summary

How accurately can we measure the primordial spectrum under the expected observational noises ?

$$\mathcal{P}_h^{\rm fid}(k) + \delta\mathcal{P}_i \qquad\qquad C_\ell^{(T)BB} + \mathcal{N}_\ell$$

Fisher matrix

Detectable ?

$$\mathcal{P}_h^{\rm model}(k)$$

(with a simple detector noise model + no other foreground noises)

Current status of CMB2ND

Achievement

- Linear power spectra of Scalar/Vector/Tensor Θ/E/B-modes
- Linear power spectra of Gradient/Curl-modes induced by S/V/T perturbations
- Lensed power spectra and bispectra of all possible combinations up to 2×2 and $2 \times 1 \times 1$ (50 C_ℓ's and 128 $B_{\ell_1 \ell_2 \ell_3}$'s)
- $f_{\rm NL}$ estimator for Local/Equilateral/Orthogonal/Folded templates
- Scalar/Tensor bispectra based on 'curve'-of-sight formula

Current status of CMB2ND

Lensed signal

$$\widetilde{X}_{LM} \approx X_{LM} + \sum \mathcal{M}^{L\ell\ell';x;X\overline{X}}_{Mmm'} x_{\ell m} \overline{X}_{\ell' m'}$$

$$X = \Theta, E, B$$

$$x = \phi, \varpi$$

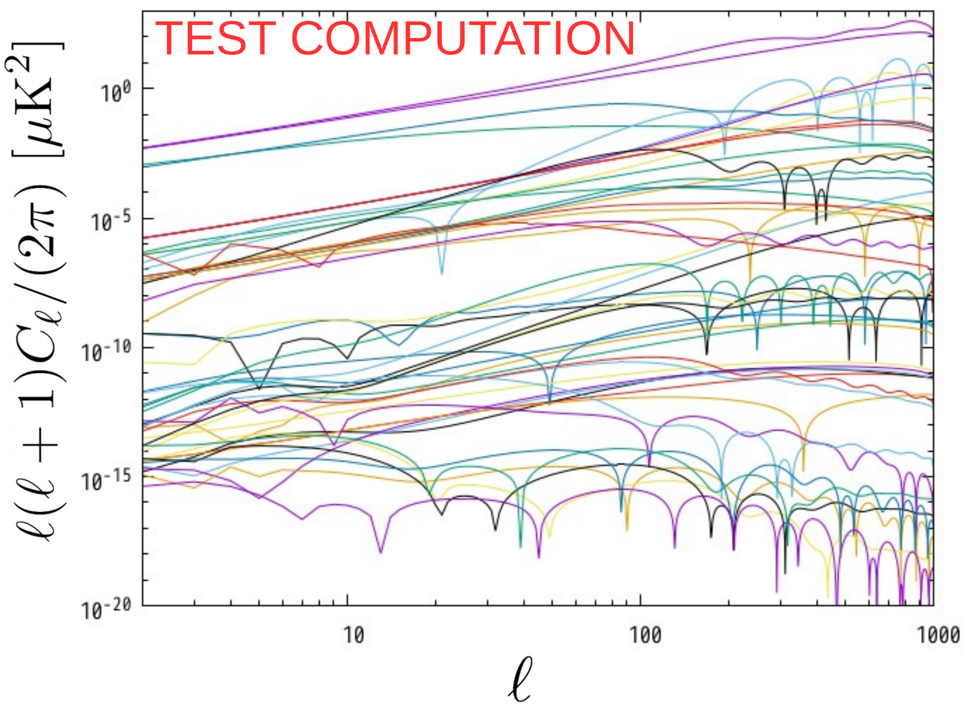
Lensed spectra

$$\Delta C_L^{\widetilde{X}\widetilde{Y}(22)} = \frac{1}{2L+1} \sum_{\ell\ell'} \sum_{xy\overline{XY}} M^{X\overline{X},x}_{L\ell\ell'} \left(M^{Y\overline{Y},y}_{L\ell\ell'} C^{\overline{XY}}_{\ell'} C^{xy}_{\ell} + (-1)^{L+\ell+\ell'} M^{Y\overline{Y},y}_{L\ell'\ell} C^{\overline{X}y}_{\ell'} C^{\overline{Y}x}_{\ell} \right)$$

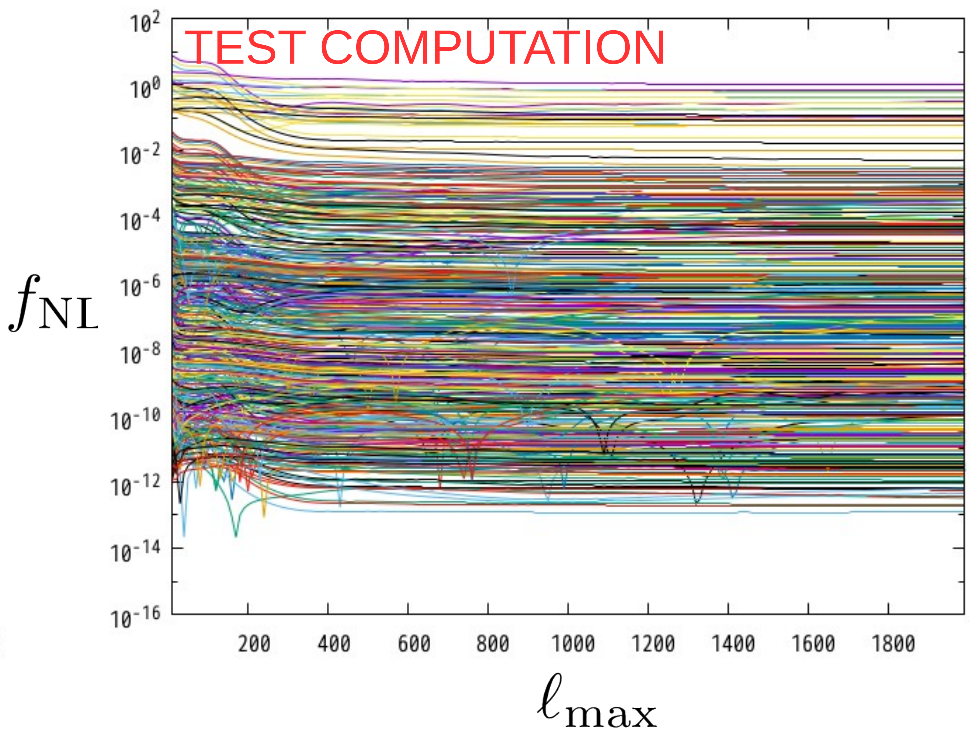
$$\begin{aligned} \widehat{B}^{XYZ,s_1s_2s_3(211)}_{L_1L_2L_3} = & \sum_{\overline{X}x} \left[M^{X\overline{X},x}_{L_1L_3L_2} C^{\overline{X}Y(s_2)}_{L_2} C^{xZ(s_3)}_{L_3} \delta_{s_1s_2} + (Y \leftrightarrow Z) \right] \\ & + \sum_{\overline{X}x} \left[M^{Y\overline{X},x}_{L_2L_1L_3} C^{\overline{X}Z(s_3)}_{L_3} C^{xX(s_1)}_{L_1} \delta_{s_2s_3} + (X \leftrightarrow Z) \right] \\ & + \sum_{\overline{X}x} \left[M^{Z\overline{X},x}_{L_3L_2L_1} C^{\overline{X}X(s_1)}_{L_1} C^{xY(s_2)}_{L_2} \delta_{s_1s_3} + (X \leftrightarrow Y) \right] \end{aligned}$$

Current status of CMB2ND

50 power spectra



128 bispectra × 4 templates



Using CMB2ND, we study...

- Cosmic strings, inducing the unequal-time correlation $\mathcal{P}(k, \eta_1, \eta_2)$

  e.g. Daveiro et al., PRD93 (2016) 085014, arXiv:1510.05006

- Statistical anisotropy, inducing $\ell, \ell+1$ correlation $C_{\ell,\ell+1}$

  e.g. Fujita et al., arXiv:1801.02778