10th-12th Feb. 2018

Innovative area symposium
@ Tohoku Univ.

Nonlinear structure formation in cold dark matter cosmology

~ Shell-crossing & multi-stream flows ~

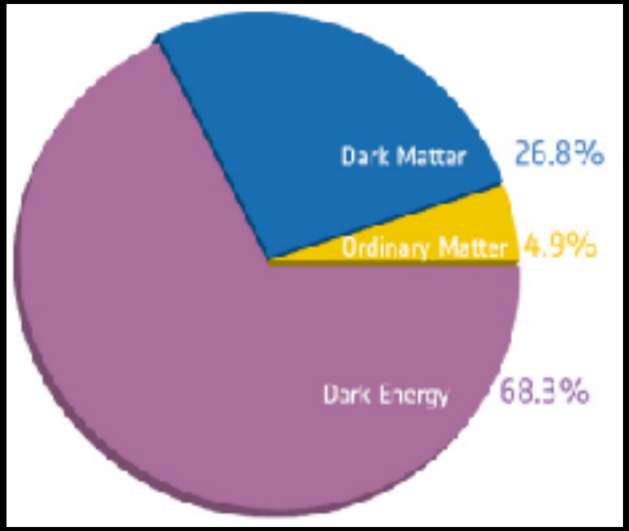
Atsushi Taruya

(Yukawa Institute for Theoretical Physics)

Dark matter & structure formation

Dark matter (DM)

- Hypothetical *invisible* massive particles
- ~30 % of the energy density of the Universe
- Unknown microscopic origin (though many candidates)



Observational evidences:

Flat rotation curves

Weak lensing observations (e.g., Bullet clusters)

CMB & large-scale structure

DM is an important building block for cosmic structure formation

Nature of dark matter

In structure formation,

of particular importance is *cold* nature of DM

velocity distribution was virtually null at an early stage of structure formation

⟶ Cold dark matter (CDM)

- Early growth of CDM fluctuations   Baryon "catch up"
- Hierarchical growth of structure formation

Irrespective of microscopic origin,

Such a system is macroscopically described by Vlasov-Poisson equation starting with cold initial condition

Cosmological Vlasov-Poisson system

Vlasov-Poisson system in a cosmological background:

Distribution function

$$\left[\frac{\partial}{\partial t}+\frac{\boldsymbol{p}}{ma^2}\frac{\partial}{\partial \boldsymbol{x}}-m\frac{\partial \Phi}{\partial \boldsymbol{x}}\frac{\partial}{\partial \boldsymbol{p}}\right]f(\boldsymbol{x},\boldsymbol{p})=0,$$

Collisionless Boltzmann eq.

$$\nabla^2\Phi(\boldsymbol{x})=4\pi\,G\,a^2\left[\frac{m}{a^3}\int d^3\boldsymbol{p}\,f(\boldsymbol{x},\boldsymbol{p})-\rho_{\rm m}\right]$$

Newton potential

a(t) : scale factor of the Universe

Cold initial flow (or single-stream flow):

Dirac's delta function

$$f(\boldsymbol{x},\boldsymbol{p})=\overline{n}\,a^3\,\{1+\delta_{\rm m}(\boldsymbol{x})\}\,\delta_{\rm D}[\boldsymbol{p}-m\,a\,\boldsymbol{v}(\boldsymbol{x})]$$

Mass density field

Velocity field

System at an early phase is reduced to pressureless fluid system

Cosmic fluid and perturbation theory

Assuming single-stream flow,

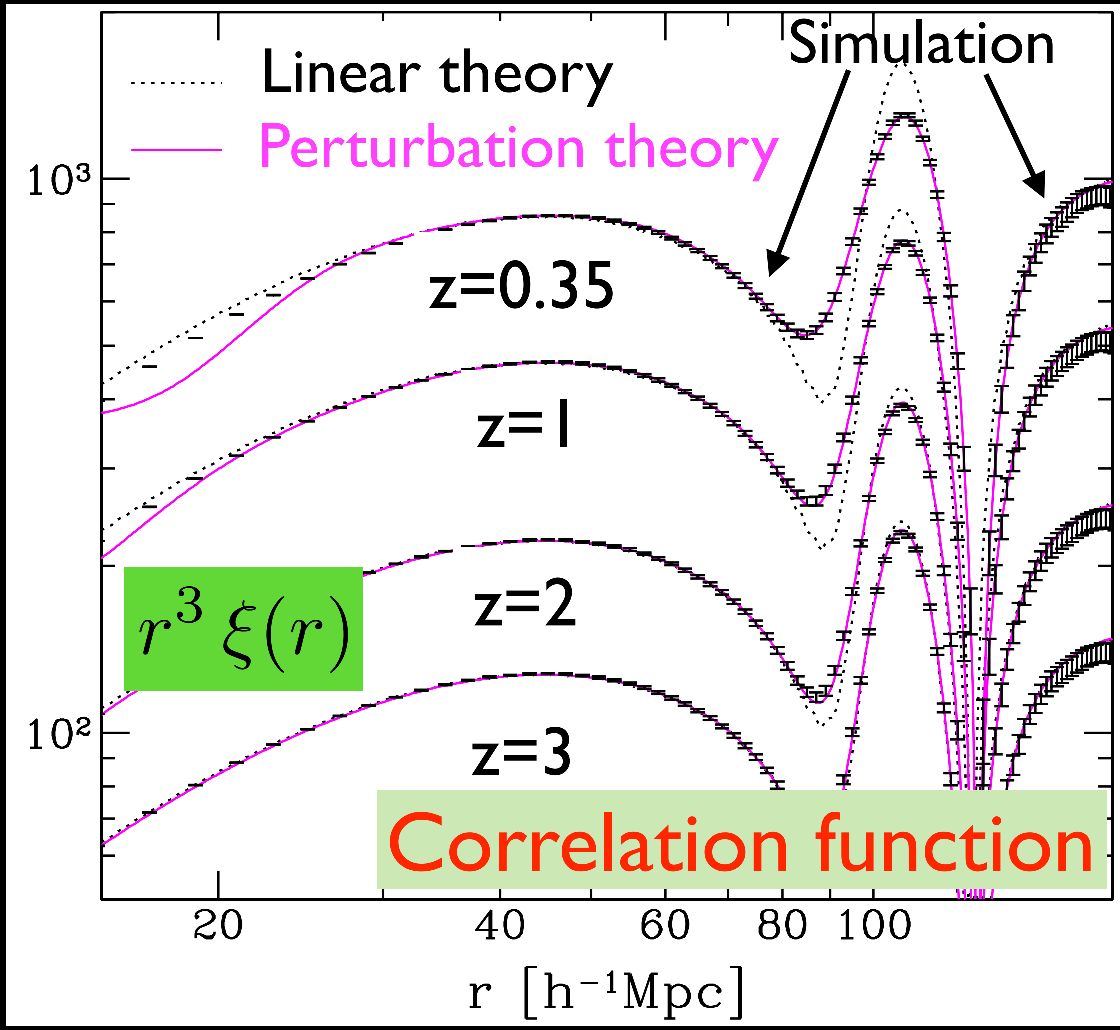
cosmological Vlasov-Poisson system is reduced to fluid system

$$\frac{\partial \delta_{\rm m}}{\partial t} + \frac{1}{a}\nabla\left[(1+\delta_{\rm m})\boldsymbol{v}\right] = 0,$$

$$\frac{\partial \boldsymbol{v}}{\partial t} + \frac{1}{a}(\boldsymbol{v}\cdot\nabla)\,\boldsymbol{v} = -\frac{1}{a}\frac{\partial \Phi}{\partial \boldsymbol{x}},$$

$$\frac{1}{a^2}\nabla^2\Phi = 4\pi\, G\,\rho_{\rm m}\,\delta_{\rm m}.$$

+ resummation technique



AT et al. ('12)

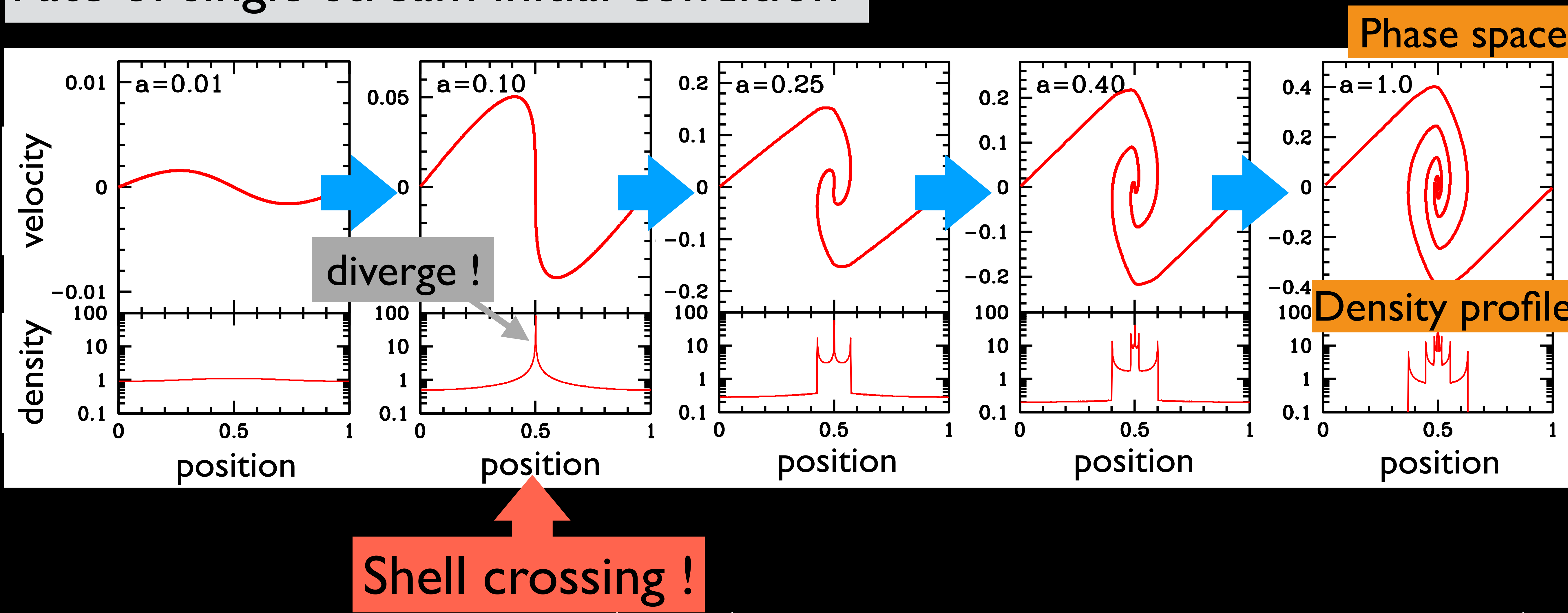
Perturbative expansion: $|\delta| \ll 1$

$$\delta = \delta^{(1)} + \delta^{(2)} + \delta^{(3)} + \cdots$$

Single-stream flow is, however, eventually violated, later followed by *shell-crossing & multi-stream flow* distinctive properties CDM cosmology

Example: 1D cosmology

Fate of single-stream initial condition



Single-stream

Multi-stream flow
(= formation of dark halo)

Boundary between single- & multi-stream → Splashback radius

Nonlinear structure formation

Shell crossing and multi-stream flows are natural outcome of nonlinear structure formation in CDM cosmology

⟶ Test for CDM paradigm

Quantitative understanding of their properties:

- Describing shell-crossing structure *with S. Saga & S. Colombi*

  A first detailed comparison between Lagrangian PT & Vlasov-Poisson simulation

- Characterizing multi-stream flows *with H. Sugiura & Y. Rasera*

  Confrontation of self-similar solution against dark halos from *N*-body simulations

Describing shell-crossing structure

with Shohei Saga & Stéphane Colombi
(YITP) (Institut d'Astrophysique de Paris)

Motivation

6D

cosmological Vlasov-Poisson simulation for initially cold systems is now made available

ColDICE: A parallel Vlasov–Poisson solver using moving adaptive simplicial tessellation

Thierry Sousbie [a,b,c,*], Stéphane Colombi [a,d] 2016

(see also, Yoshikawa et al. '13; Hahn & Angulo '16)

Distribution function (3D hyper-sheet) in 6D phase space represented with moving adaptive simplical tessellation

- Exact projection onto grid to get density field $\xrightarrow[\text{by FFT}]{\text{Poisson solver}}$ Force
- Lagrangian EoM for vertices by standard leapfrog method

→ A first detailed comparison with analytic treatment (Lagrangian perturbation theory)

Lagrangian perturbation theory *(LPT)*

Perturbative description for motion of mass element via Lagrangian picture

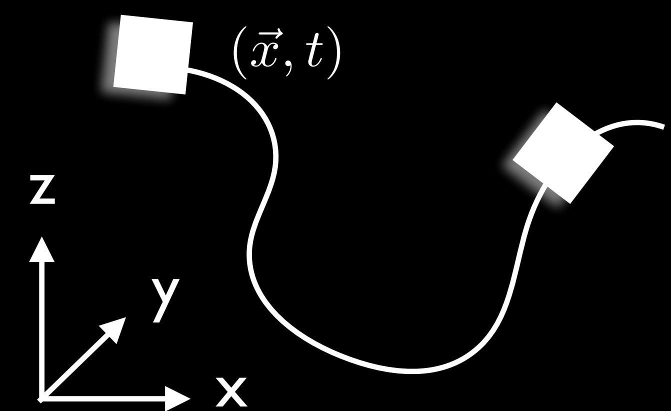
Moutarde et al. ('91); Bouchet et al. ('92); Buchert ('92); Buchert & Ehlers ('93); Bouchet et al. ('95), …, Matsubara ('15), Rampf & Frisch ('17)

Position & velocity of each mass element:

$$\boldsymbol{x}(\boldsymbol{q},t) = \boldsymbol{q} + \boldsymbol{\Psi}(\boldsymbol{q},t), \qquad \boldsymbol{v}(\boldsymbol{q},\,t) = \frac{d\boldsymbol{\Psi}(\boldsymbol{q},\,t)}{dt}$$

$\boldsymbol{q}$: Lagrangian coordinate (initial position)

$\boldsymbol{\Psi}$: displacement field $(\boldsymbol{\Psi} \stackrel{t\to 0}{\longrightarrow} 0)$



Basic eqs.

$$\ddot{\boldsymbol{x}} + 2H\dot{\boldsymbol{x}} = -\frac{1}{a^2}\nabla_x\,\phi(\boldsymbol{x})$$

$$\nabla_x^2\,\phi(\boldsymbol{x}) = 4\pi\,Ga^2\overline{\rho}_{\rm m}\,\delta(\boldsymbol{x})$$

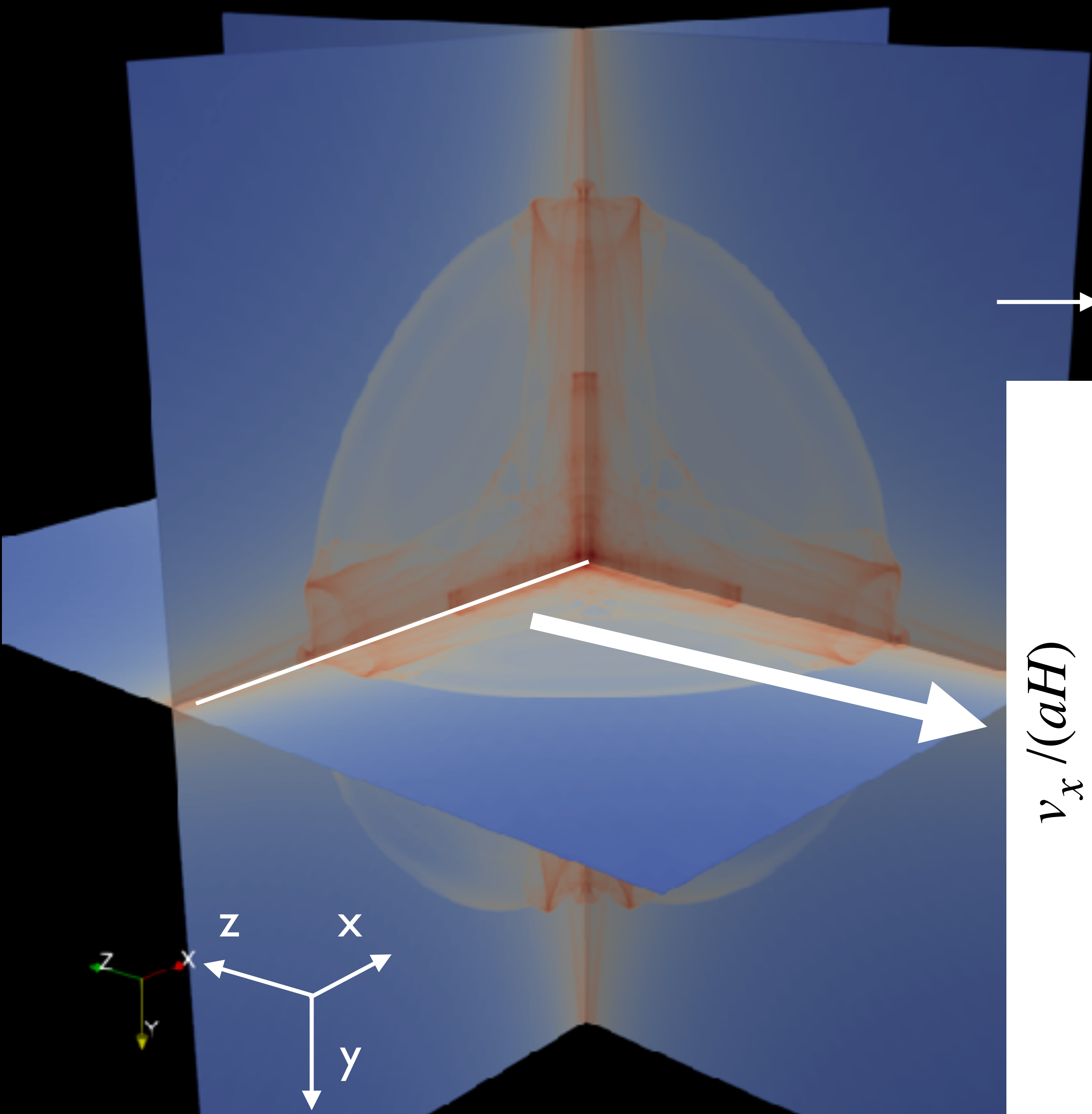
$$\boldsymbol{\Psi}(\boldsymbol{q},t) = \boldsymbol{\Psi}^{(1)}(\boldsymbol{q},t) + \boldsymbol{\Psi}^{(2)}(\boldsymbol{q},t) + \boldsymbol{\Psi}^{(3)}(\boldsymbol{q},t) + \cdots$$

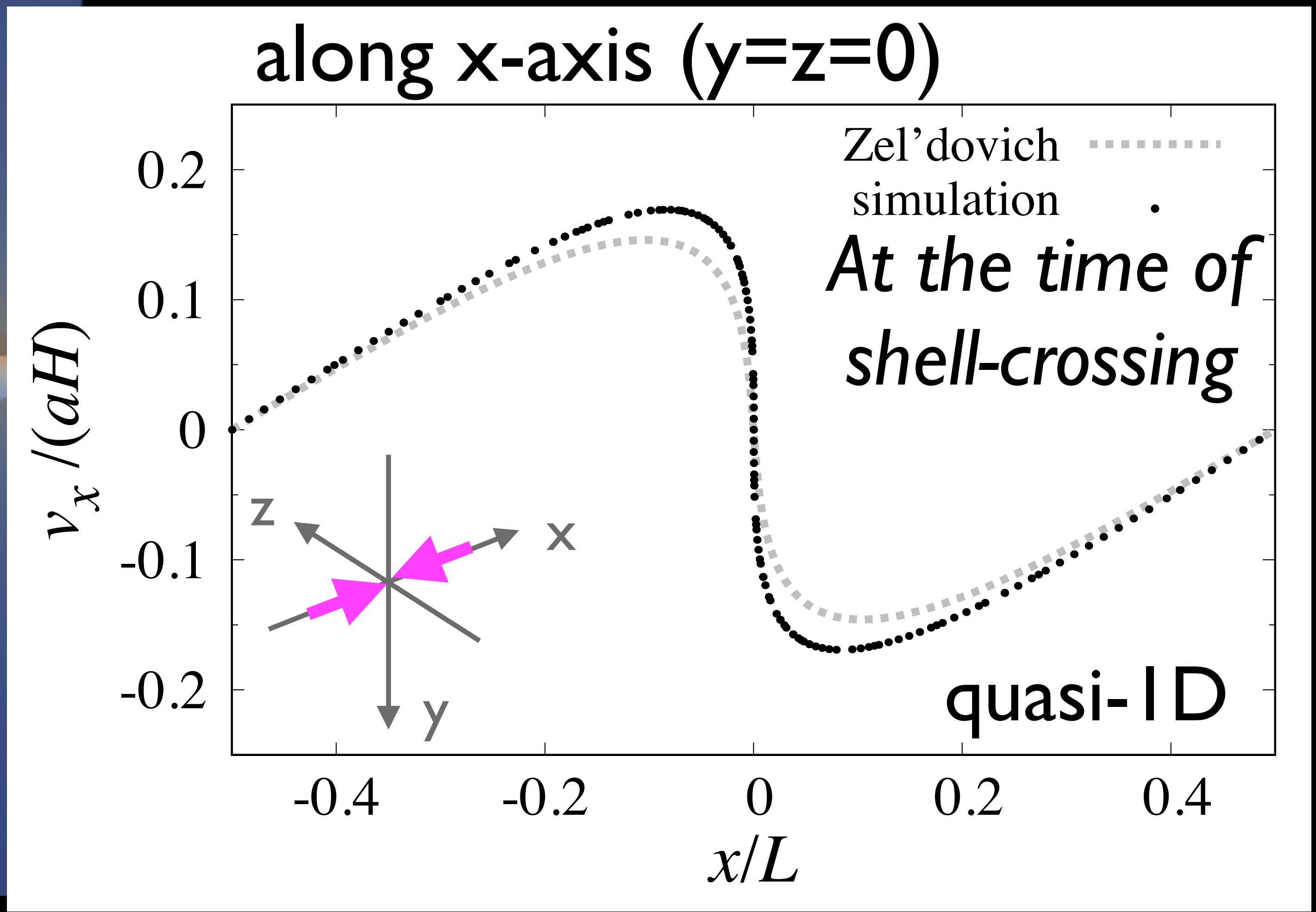
Results

Single density peak in a periodic boundary box

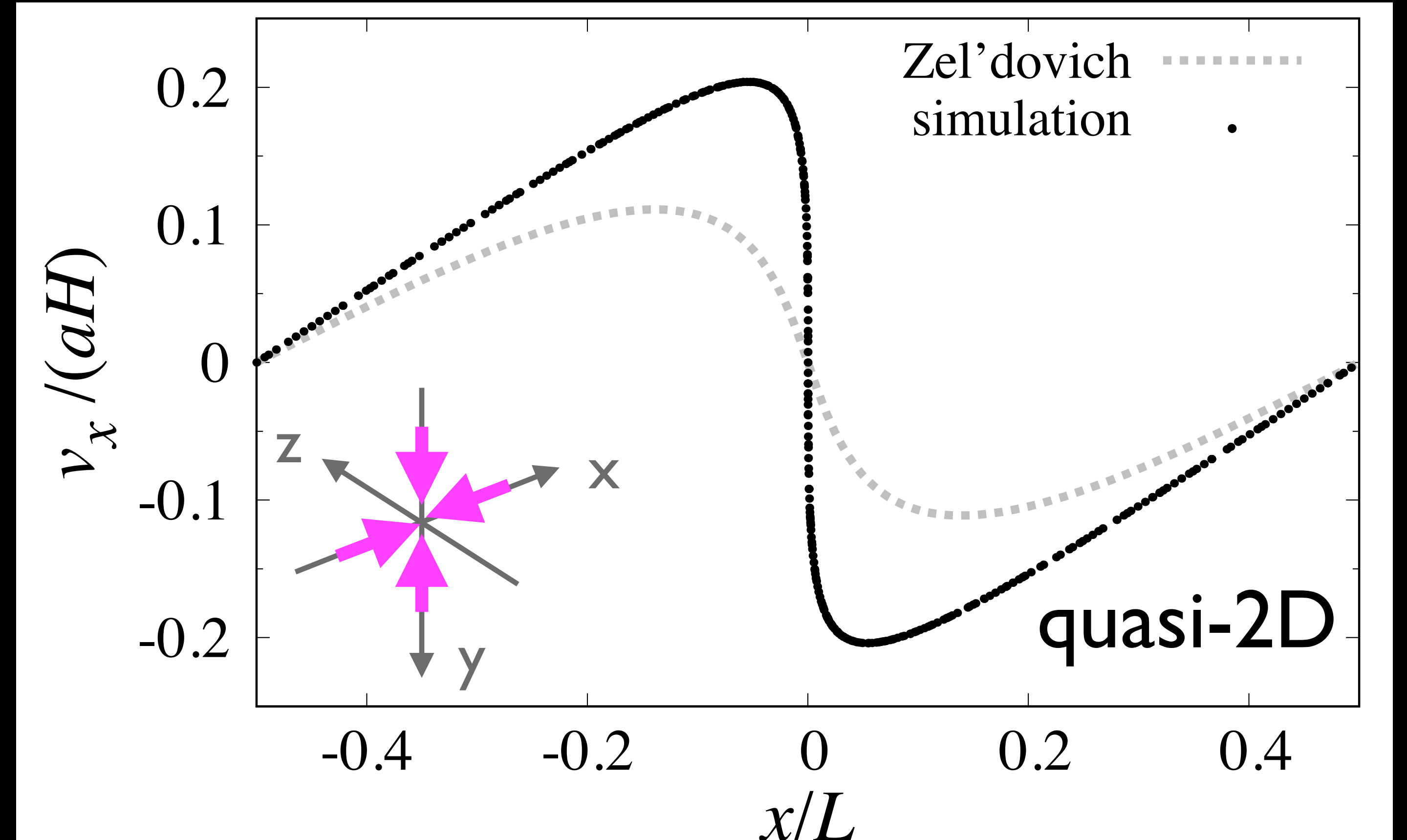
(sinusoidal function)

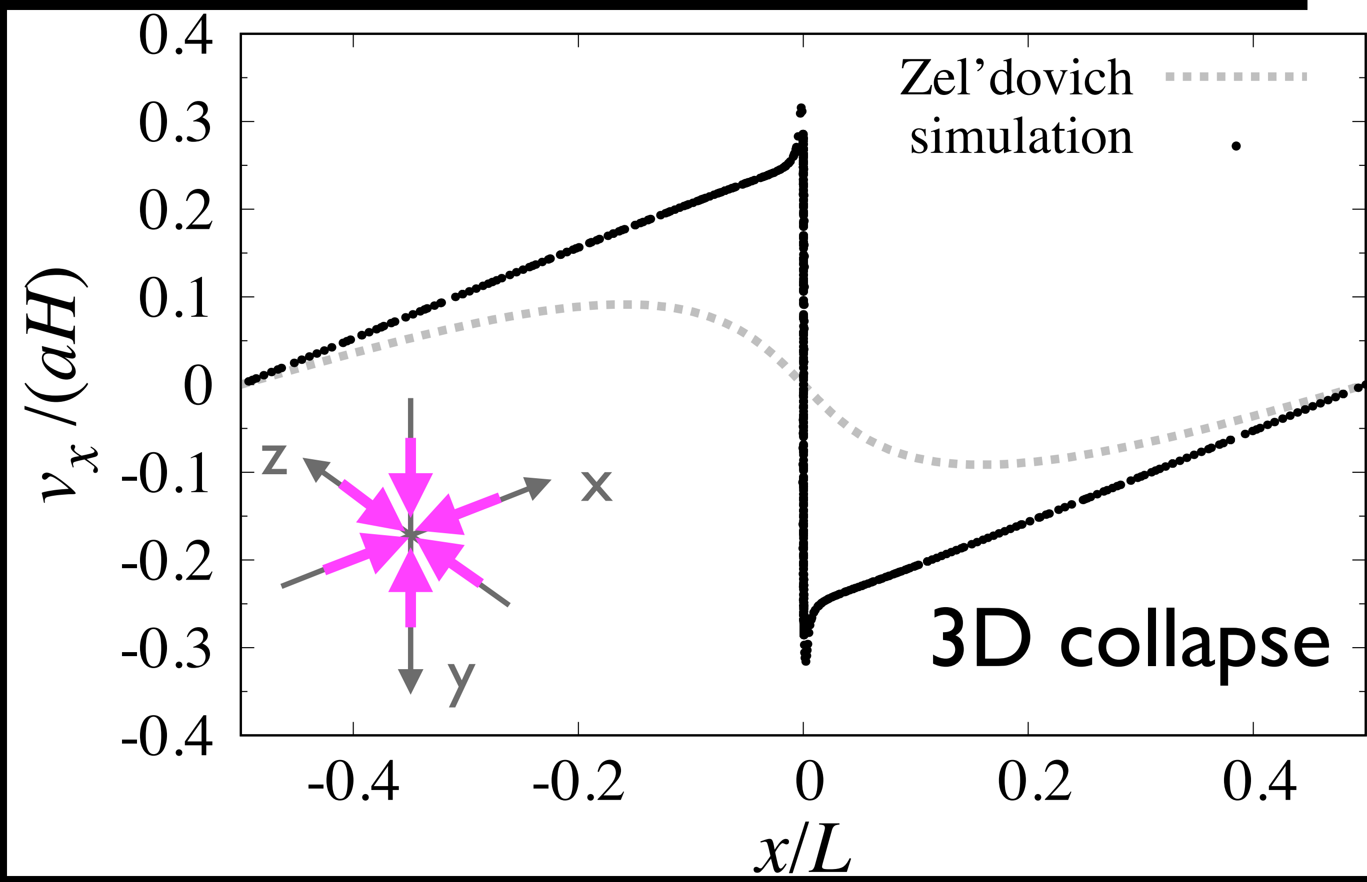
→ Shell-crossing happens at origin

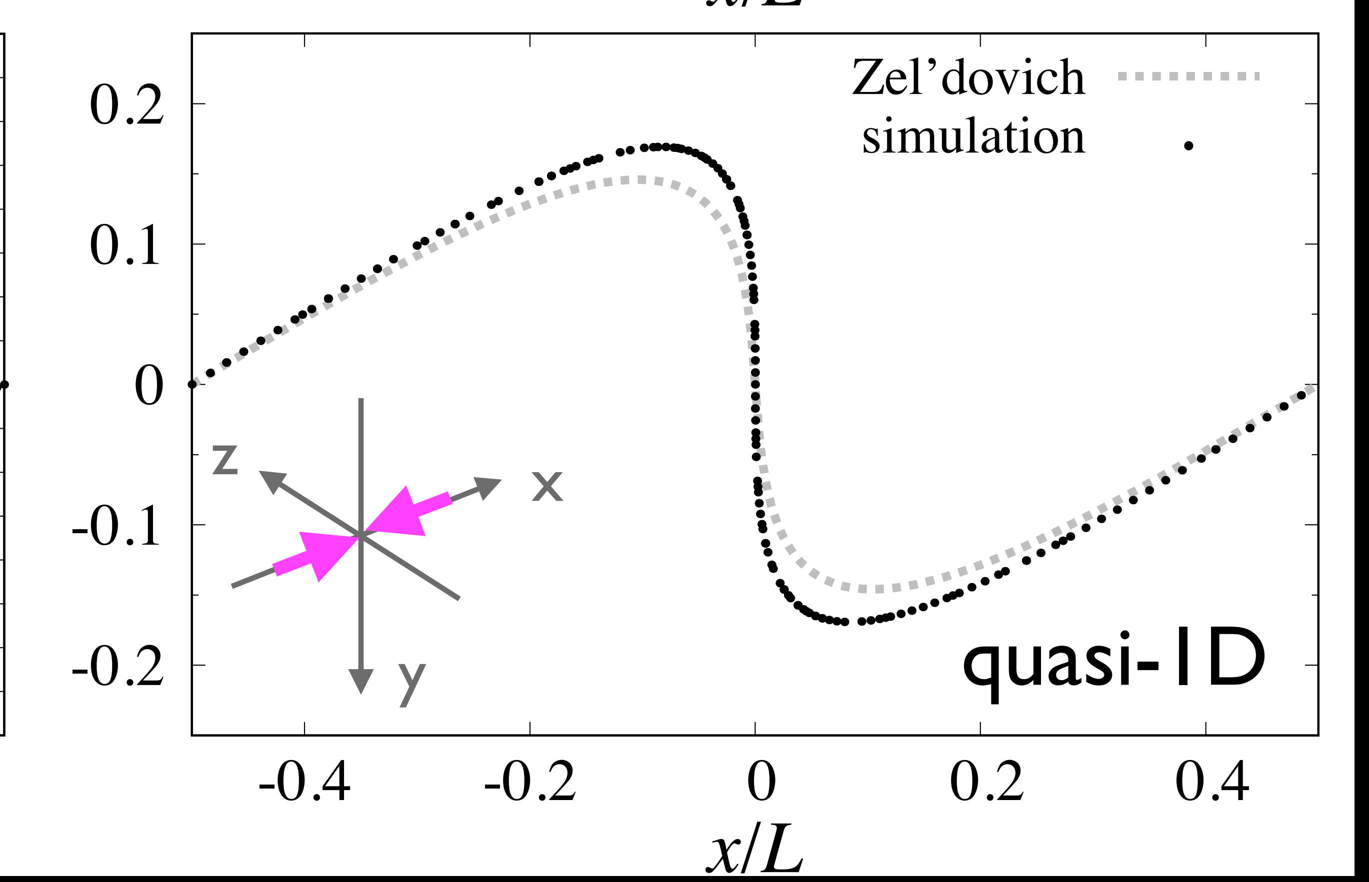




Vlasov simulation vs LPT

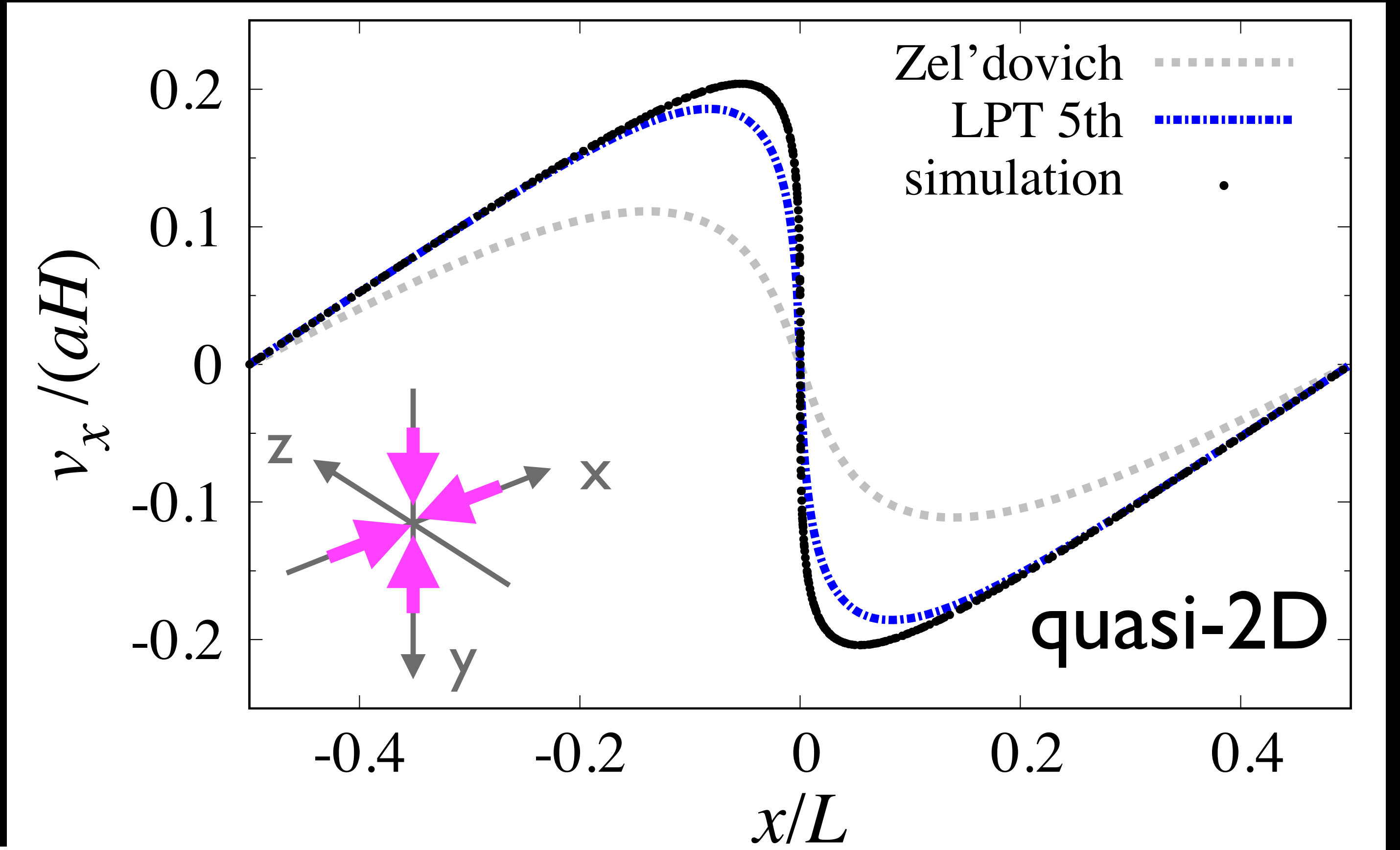


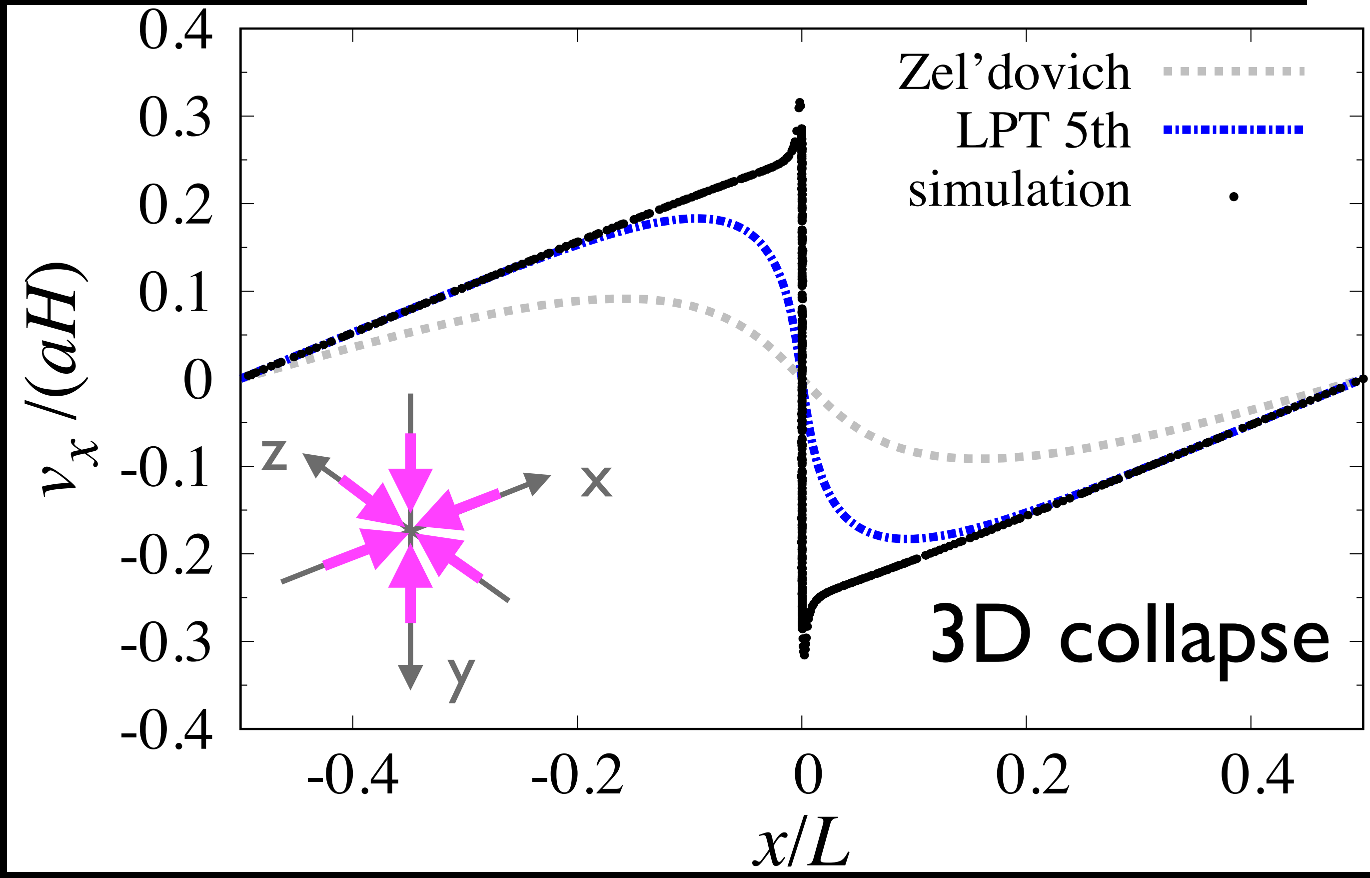


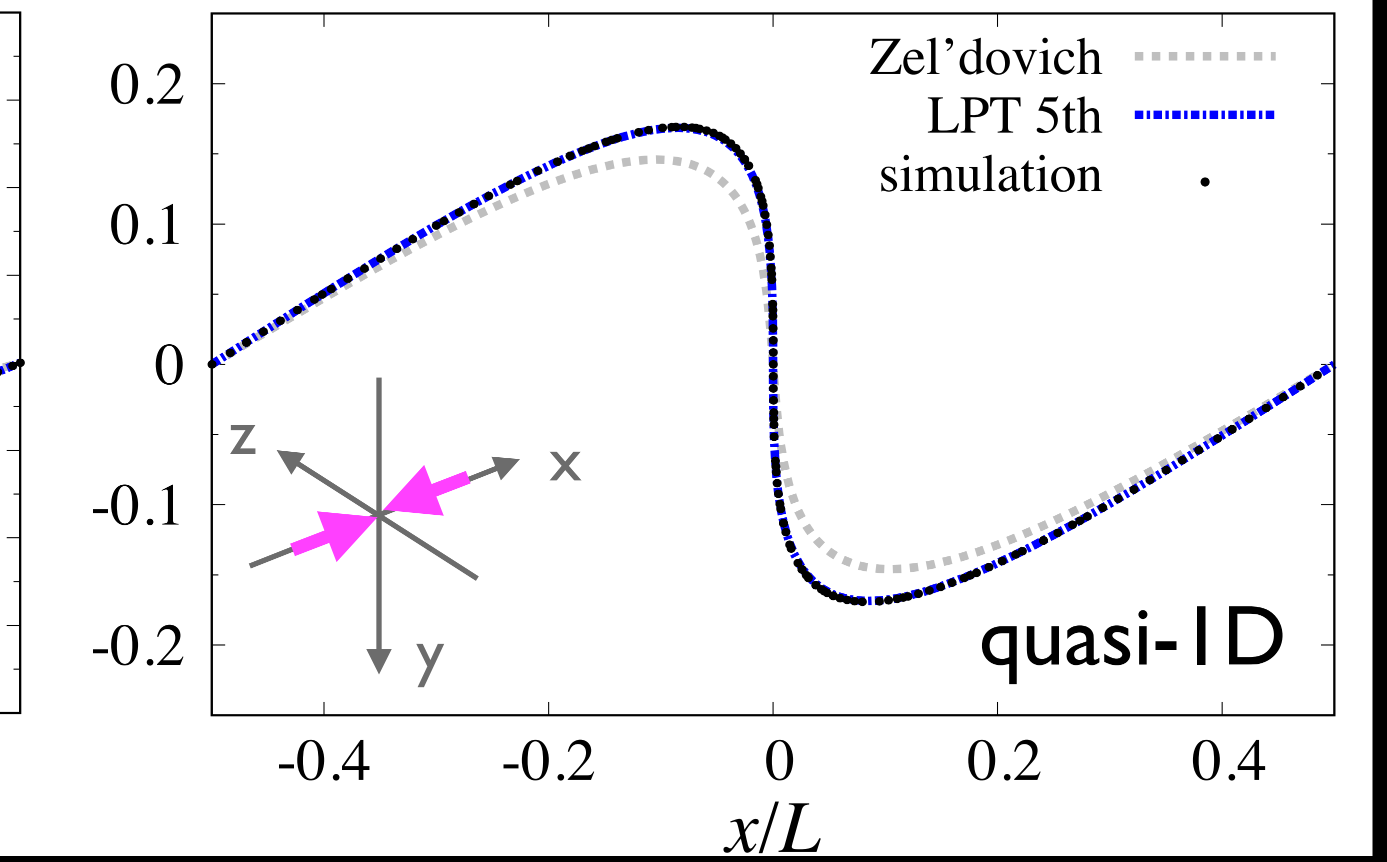


Vlasov simulation vs LPT

5th-order Lagrangian PT
(by Shohei Saga)

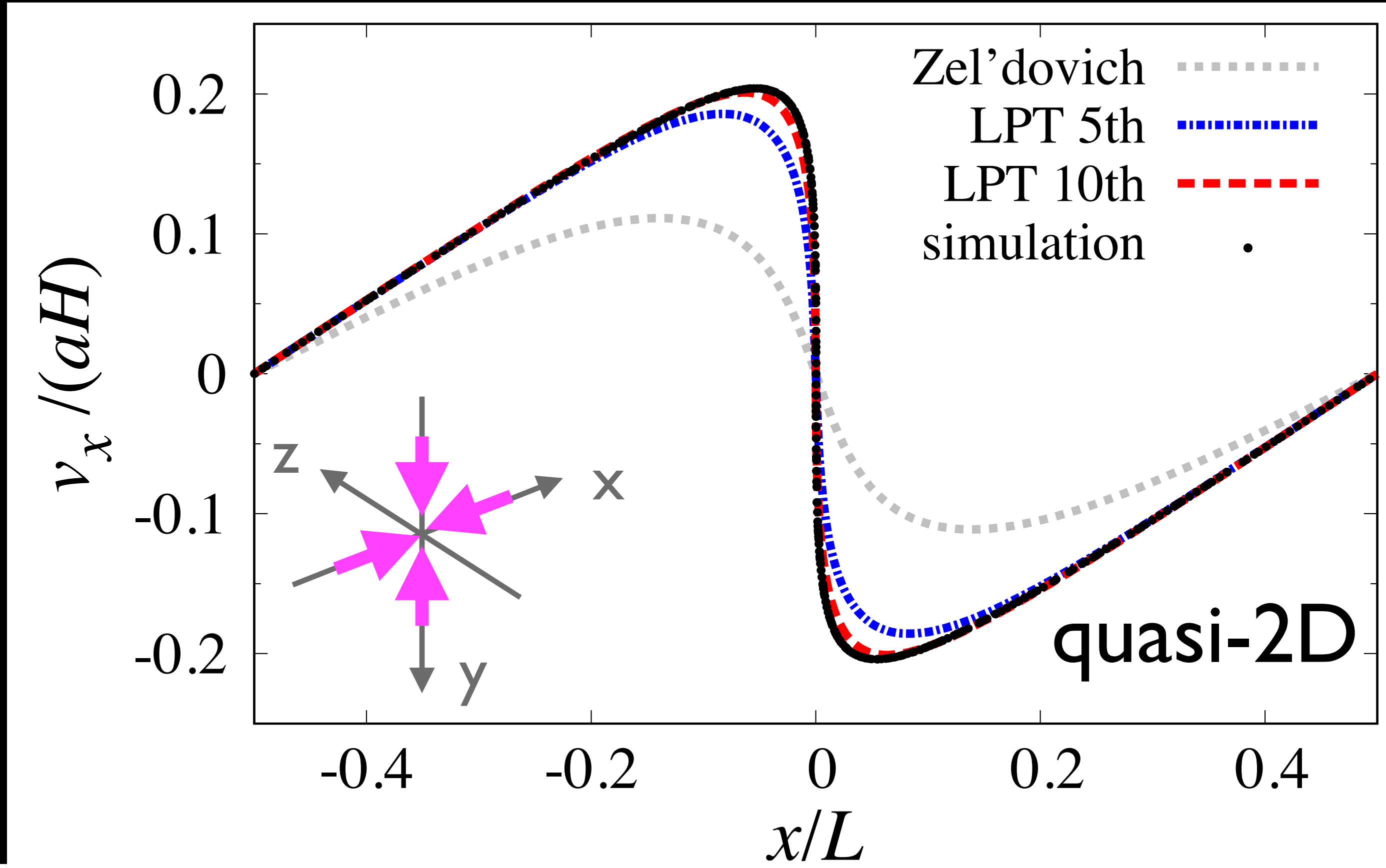


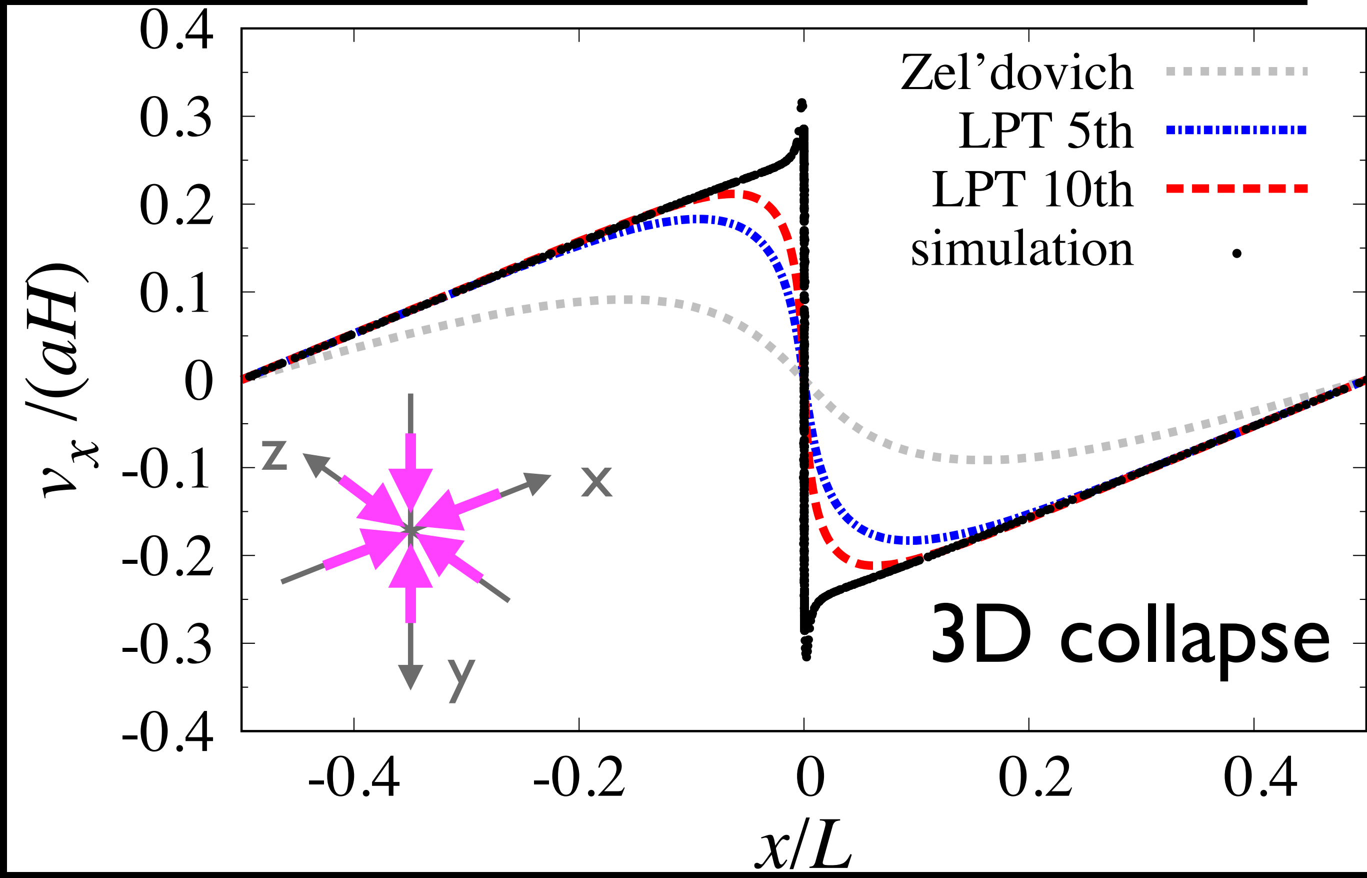


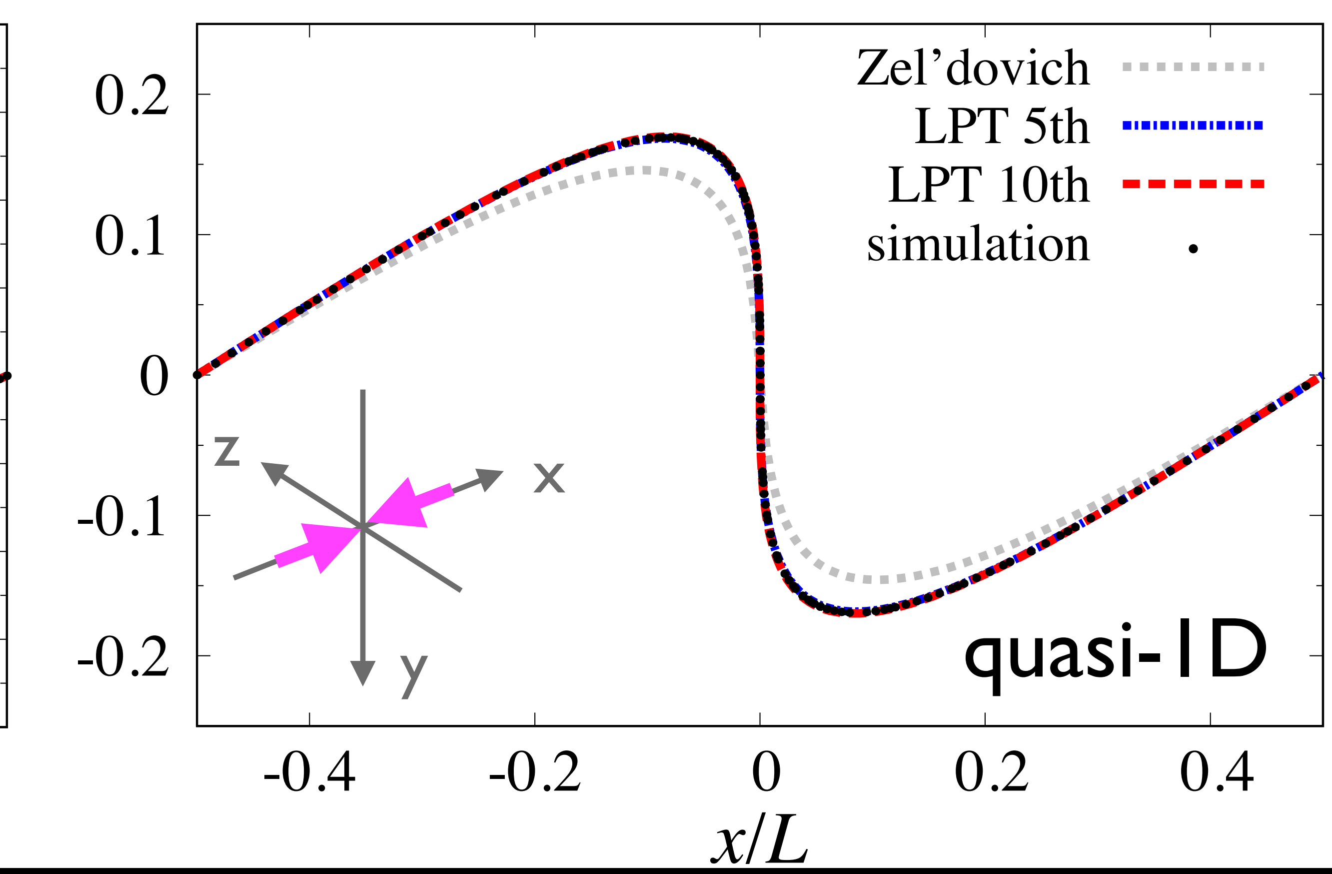


Vlasov simulation vs LPT

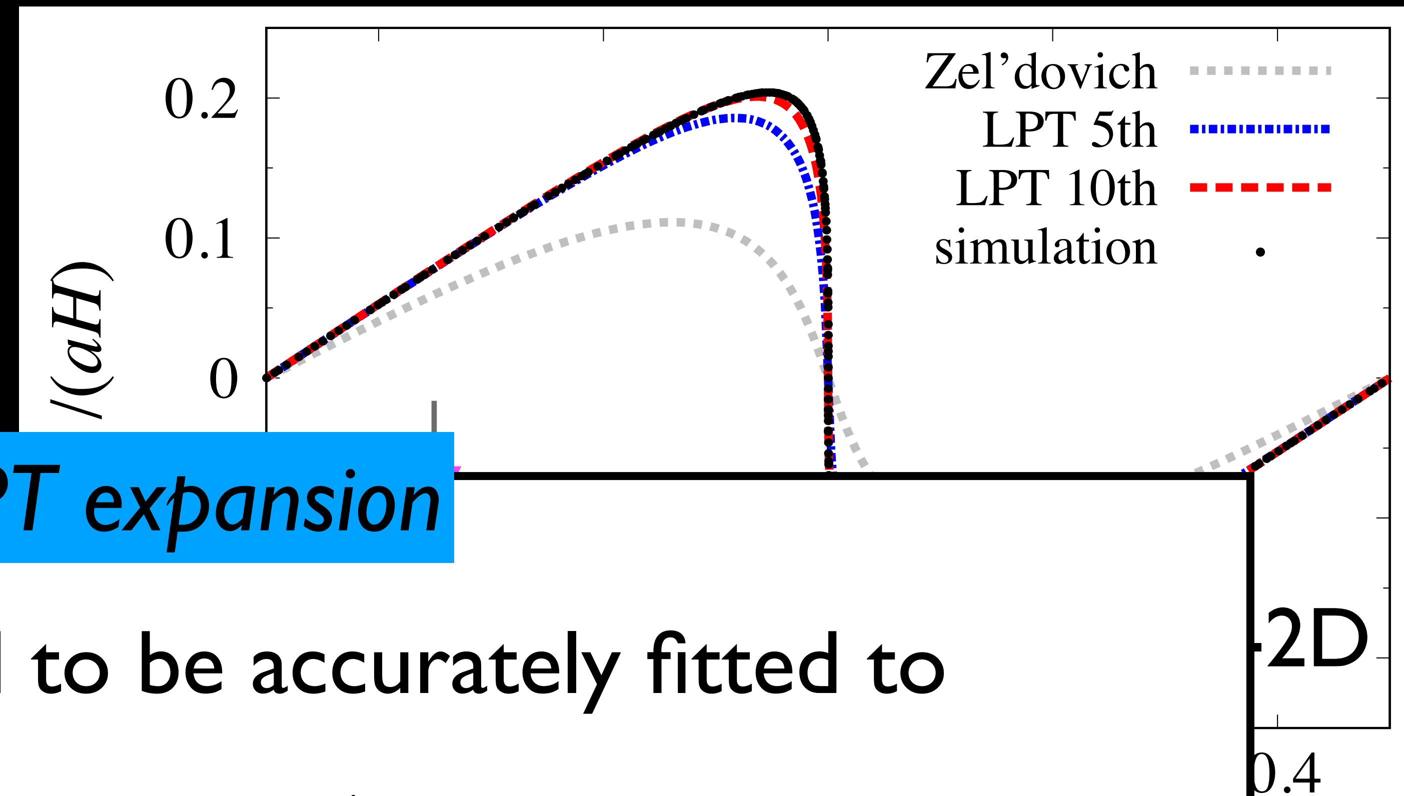
10th-order Lagrangian PT
(by Shohei Saga)







Vlasov simulation vs LPT



Making use of convergence of LPT expansion

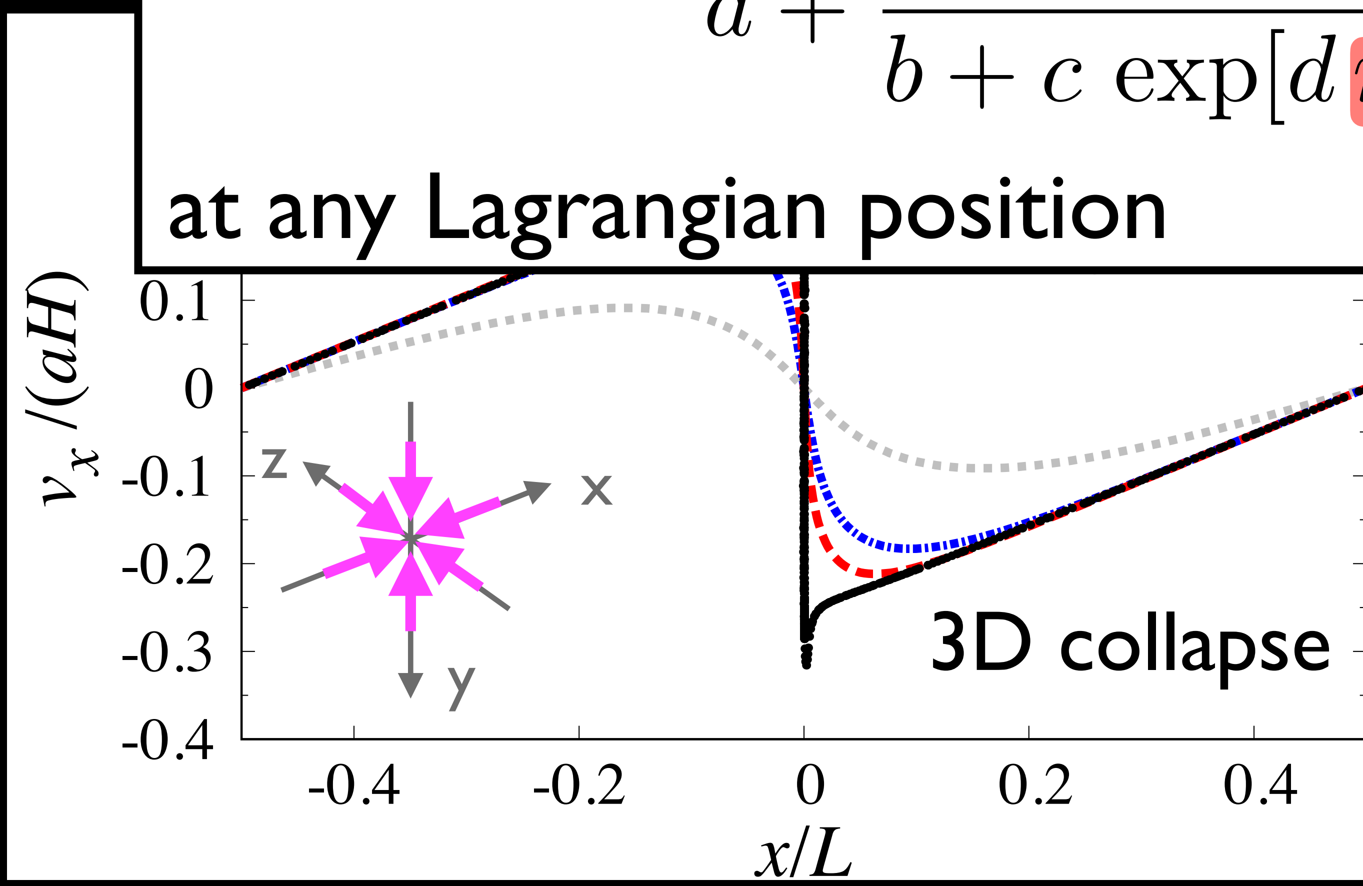
$x_{\rm LPT}(\boldsymbol{q})$ at n-th order is found to be accurately fitted to

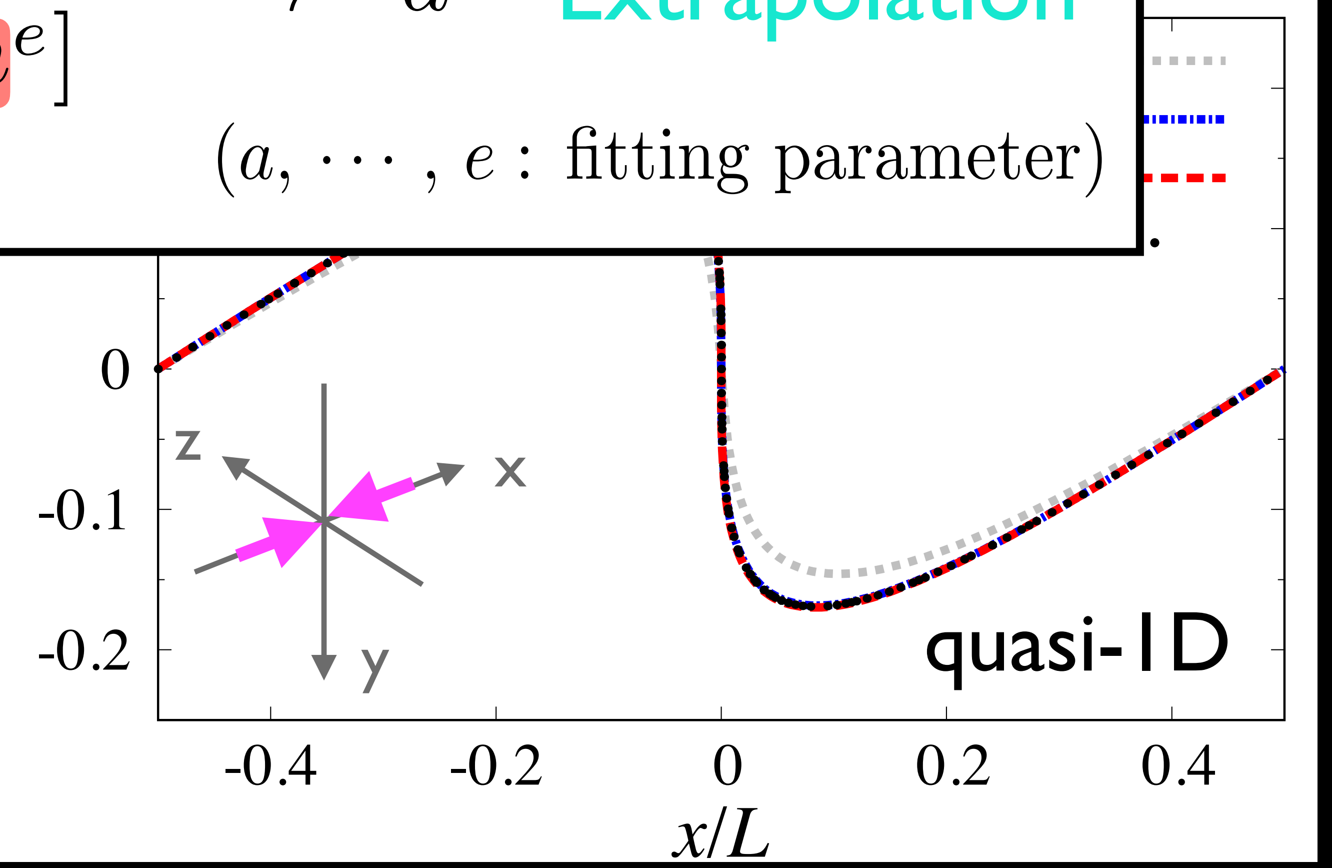
$$a+\frac{1}{b+c\,\exp[d\,n^{e}]}\quad\overset{n\to\infty}{\longrightarrow}\quad a$$

Extrapolation

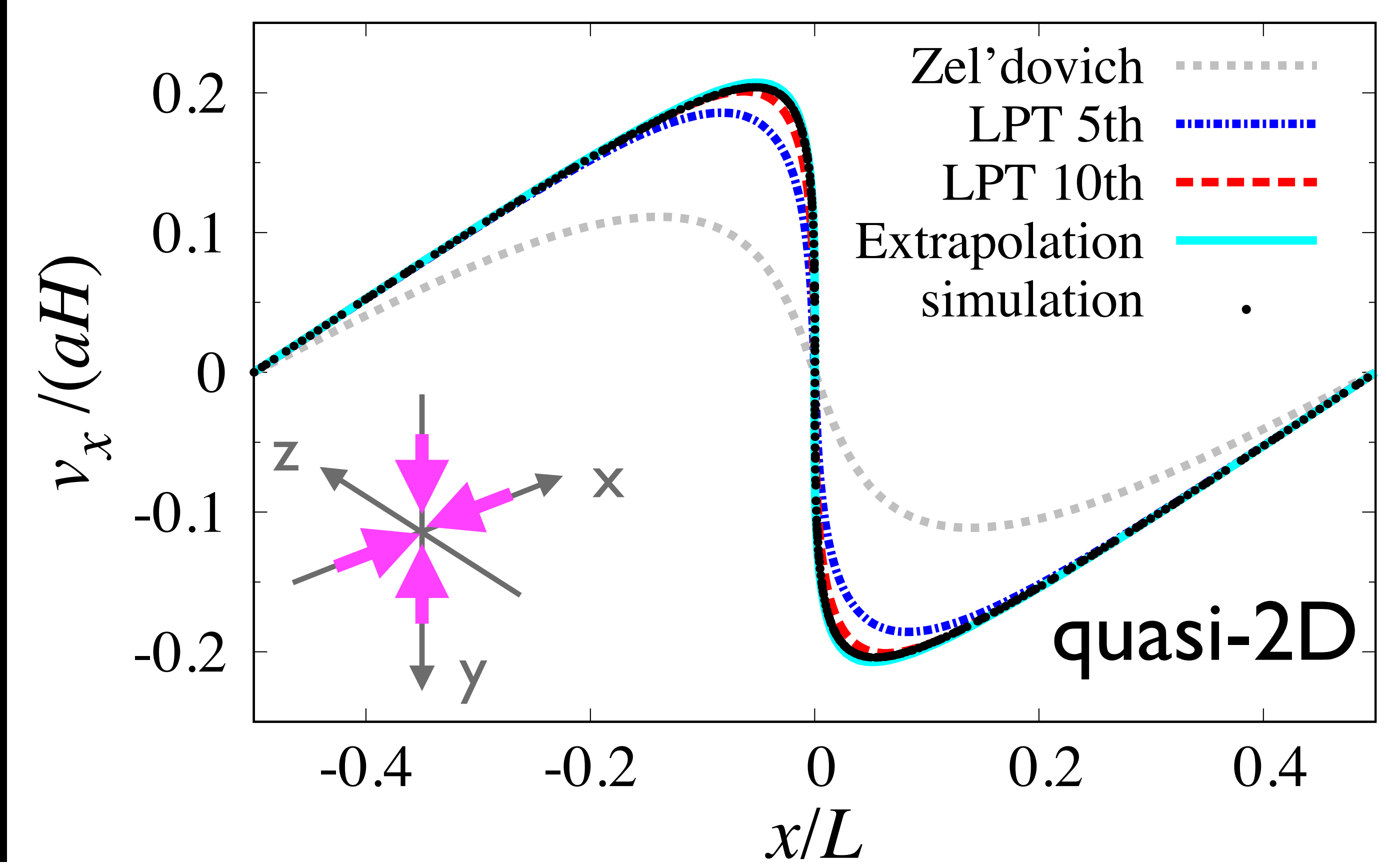
$(a,\cdots,e:\ \text{fitting parameter})$

at any Lagrangian position

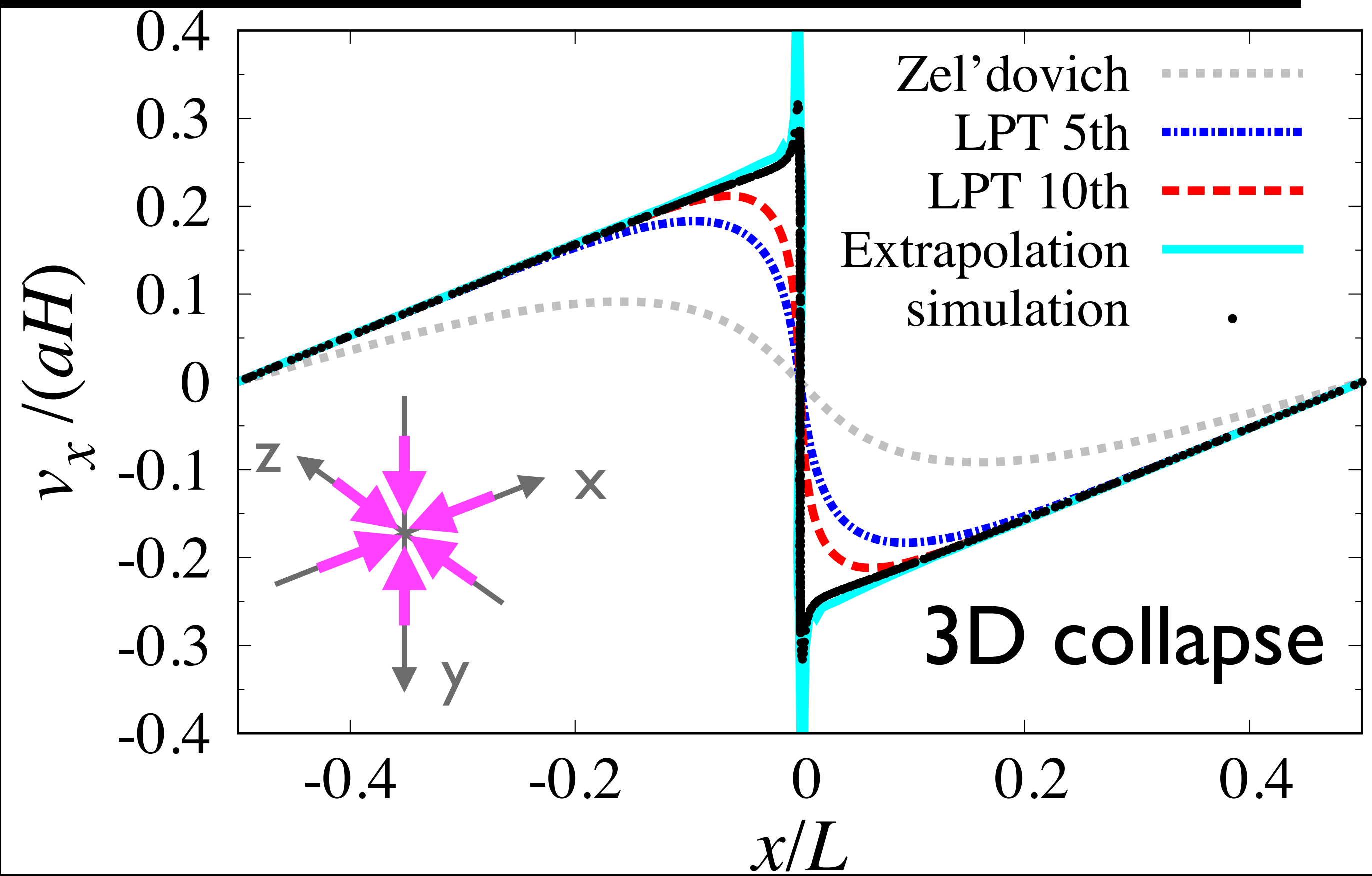


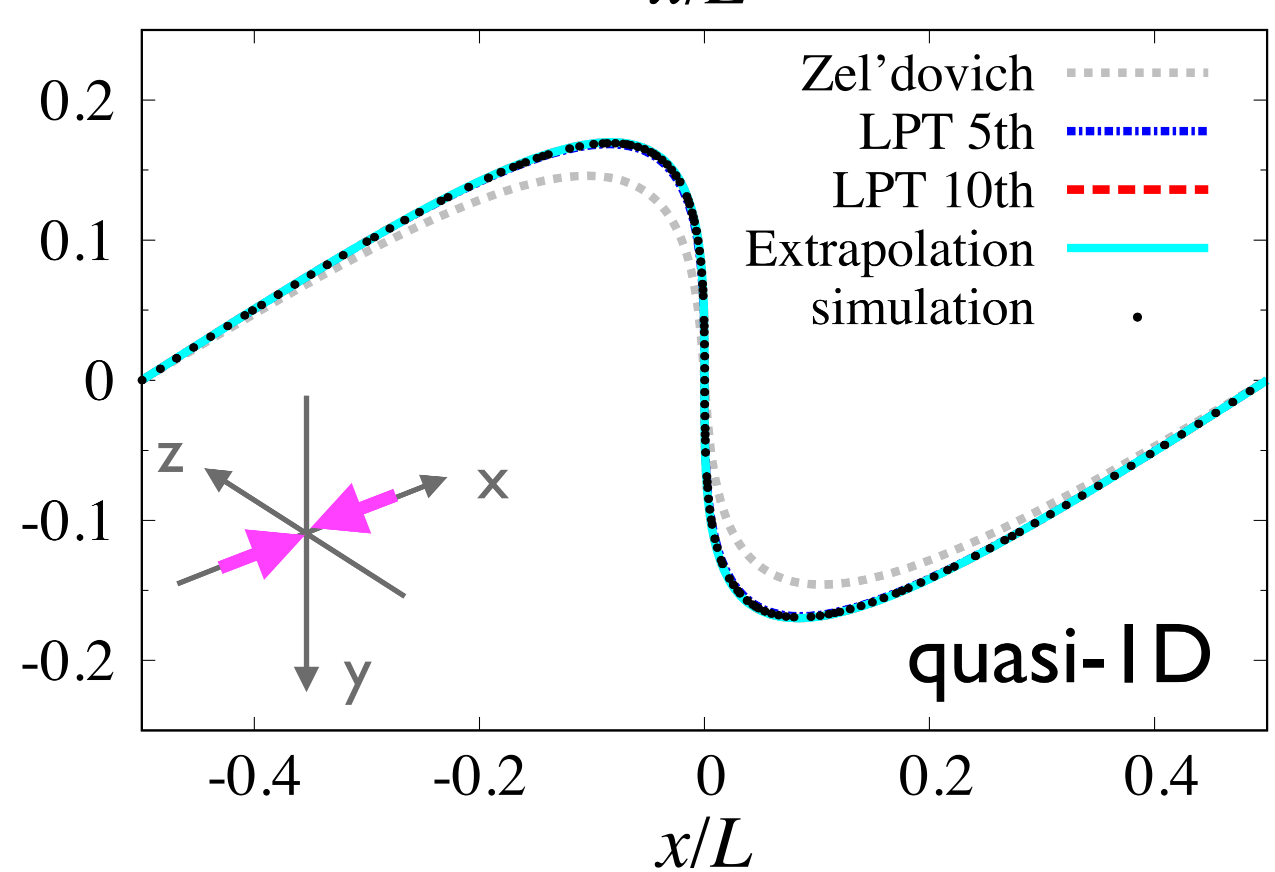


Vlasov simulation vs LPT



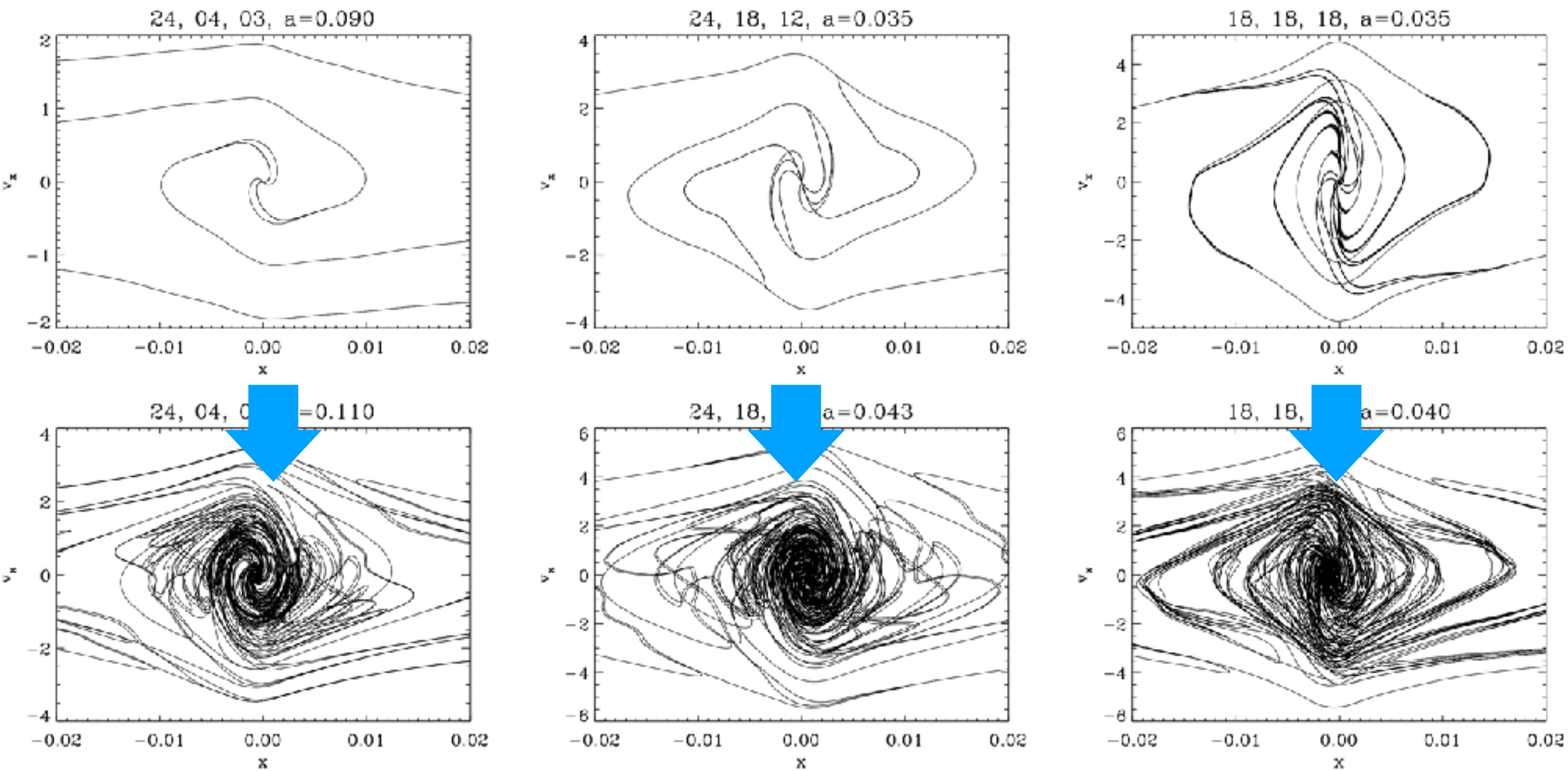
Extrapolation based on LPT up to 10th order





After shell-crossing,

quasi-1D collapse　　quasi-2D collapse　　3D collapse



In reality, what is the nature of multi-stream flows in CDM halos ?

Characterizing multi-stream flows

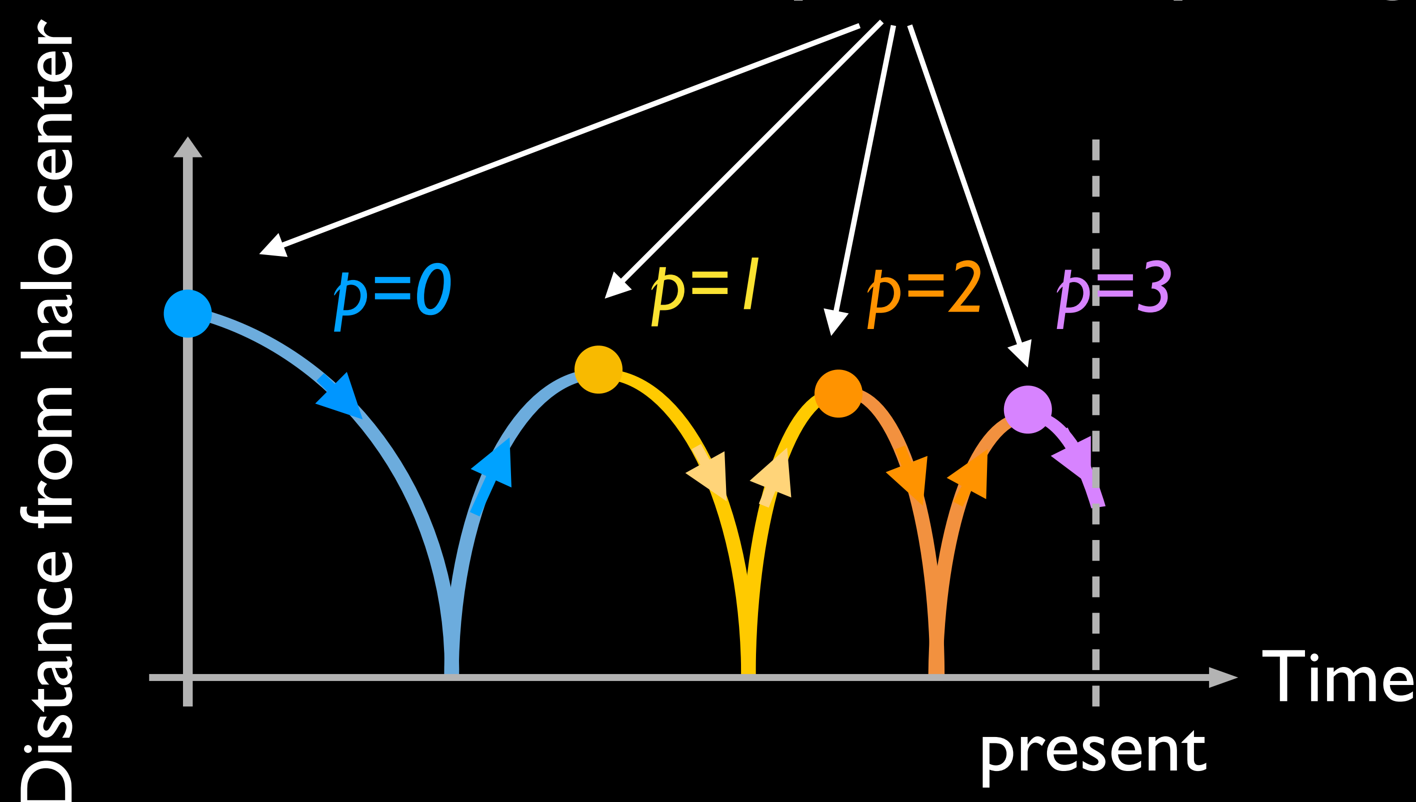
with Hiromu Sugiura & Yann Rasera

(Kyoto Univ.) (Observatoire de Paris)

Tracing multi-stream flow with particle trajectories in *N*-body simulation

Keeping track of apocenter passage(s) for particle trajectories,

number of apocenter passages, *p*, is stored for each particle



= SPARTA algorithm + α

(Diemer'17; Diemer et al.'17)

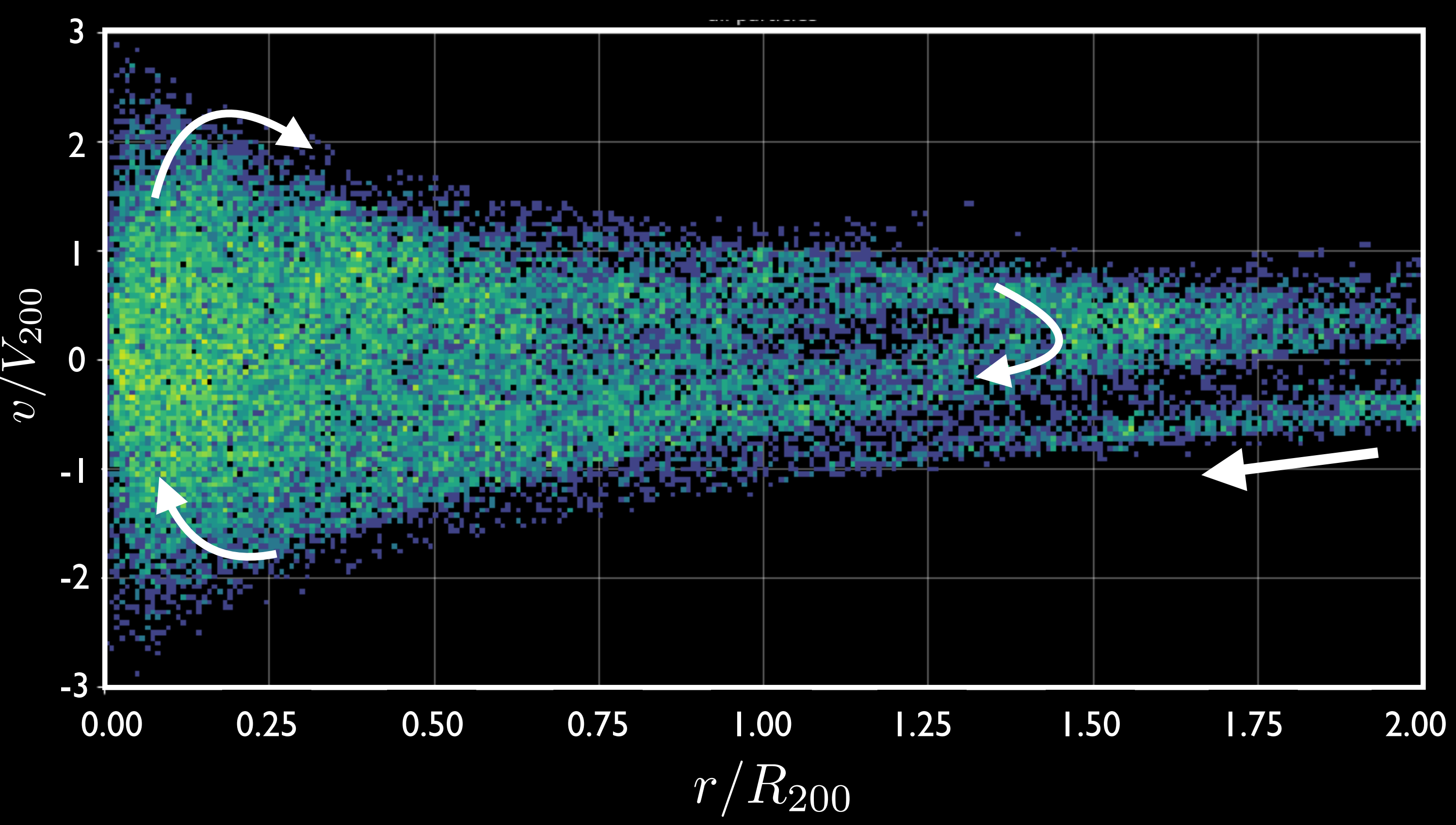
Tiling phase-space streams with *p*

N-body simulation (Y. Rasera)

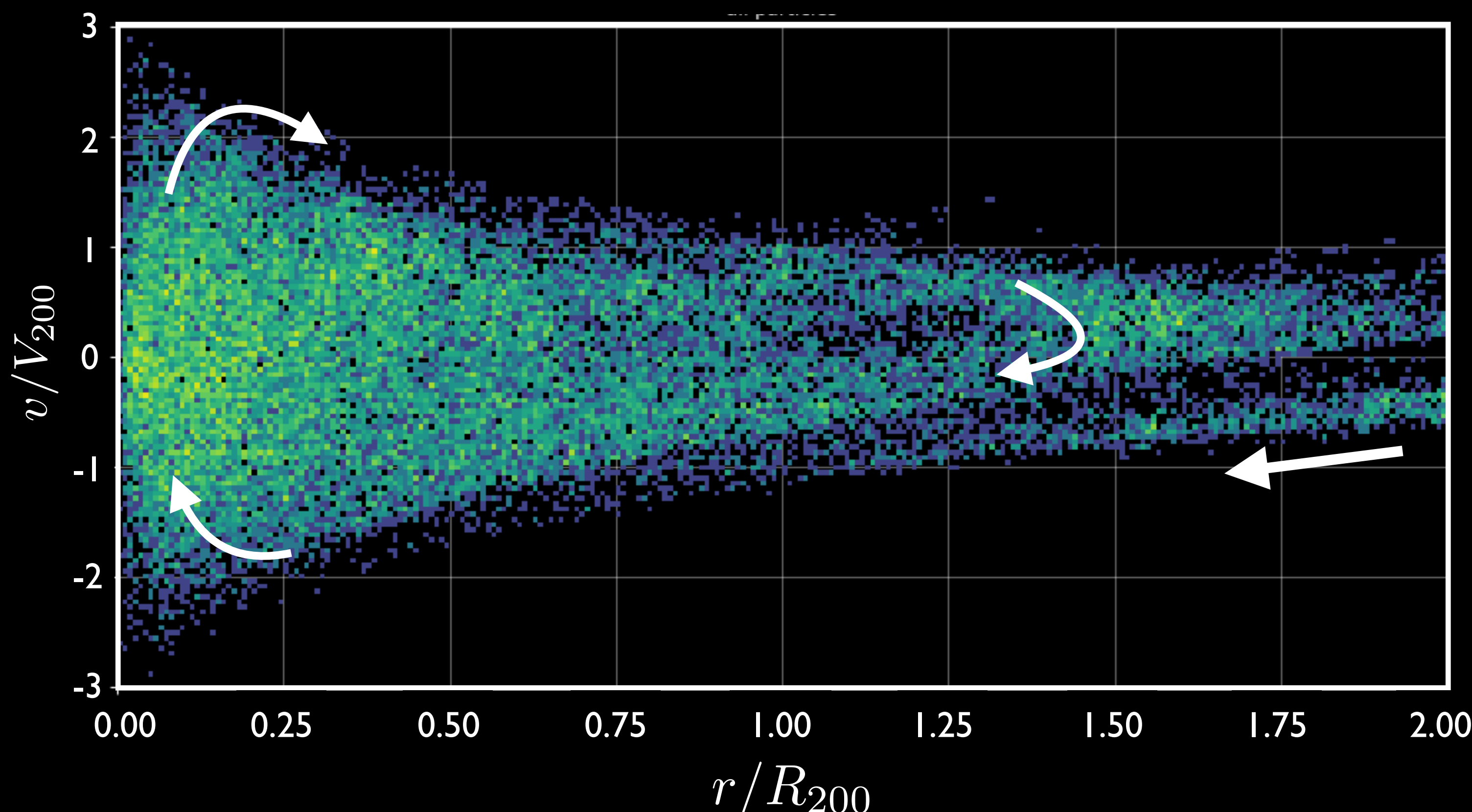
- L=316Mpc/h, N=512^3
- 60 snapshots at 0<z<1.43
- Einstein-de Sitter universe $(\Omega_{\rm m} = 1, \Omega_\Lambda = 0)$

11,000 halos $(M_{200} \geq 10^{13} M_\odot)$

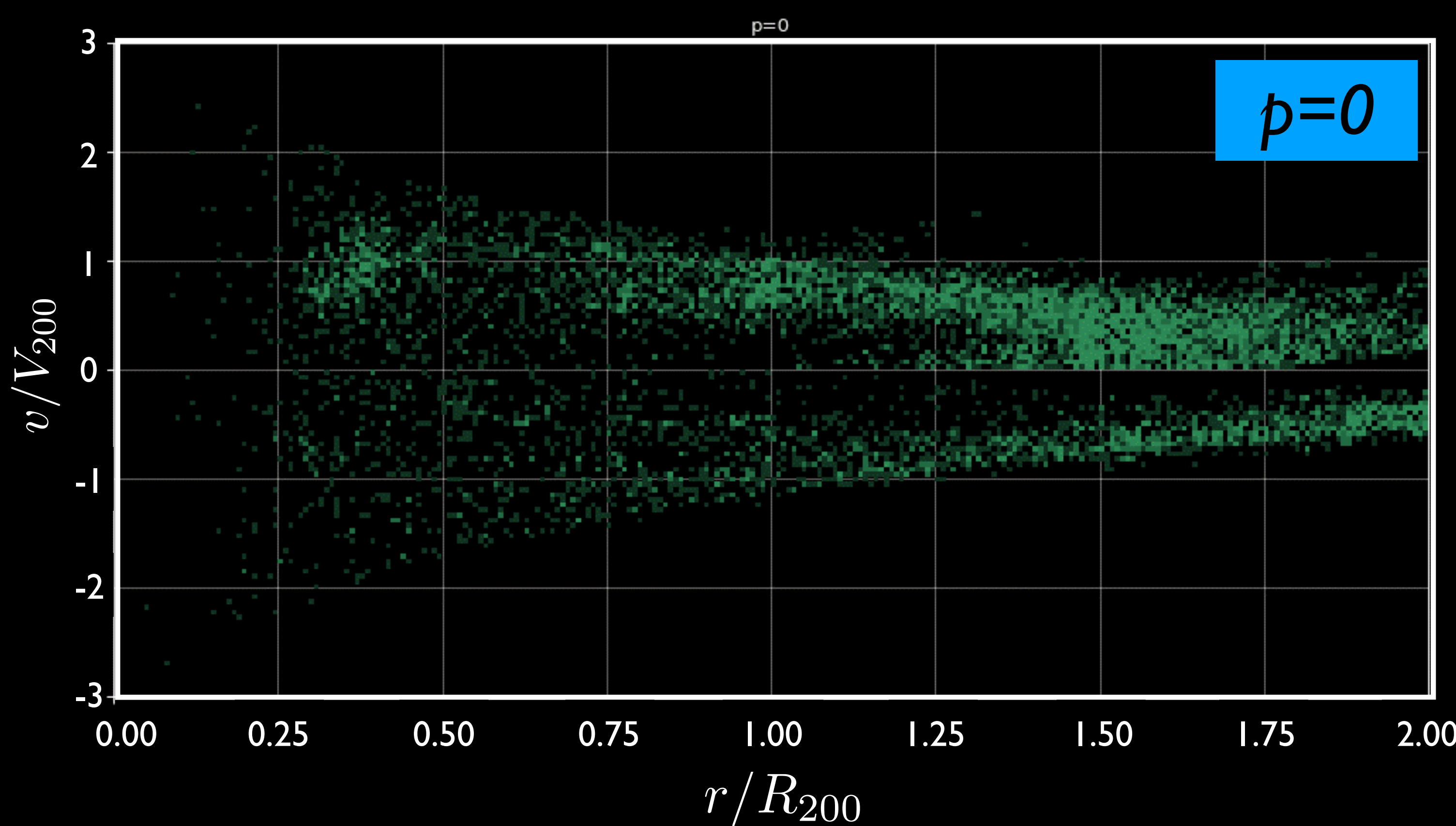
Result



Result



3
2
1
0
-1
-2
-3
v/V_{200}
0.00
0.25
0.50
0.75
1.00
1.25
1.50
1.75
2.00
r/R_{200}



p=0
p=0
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1
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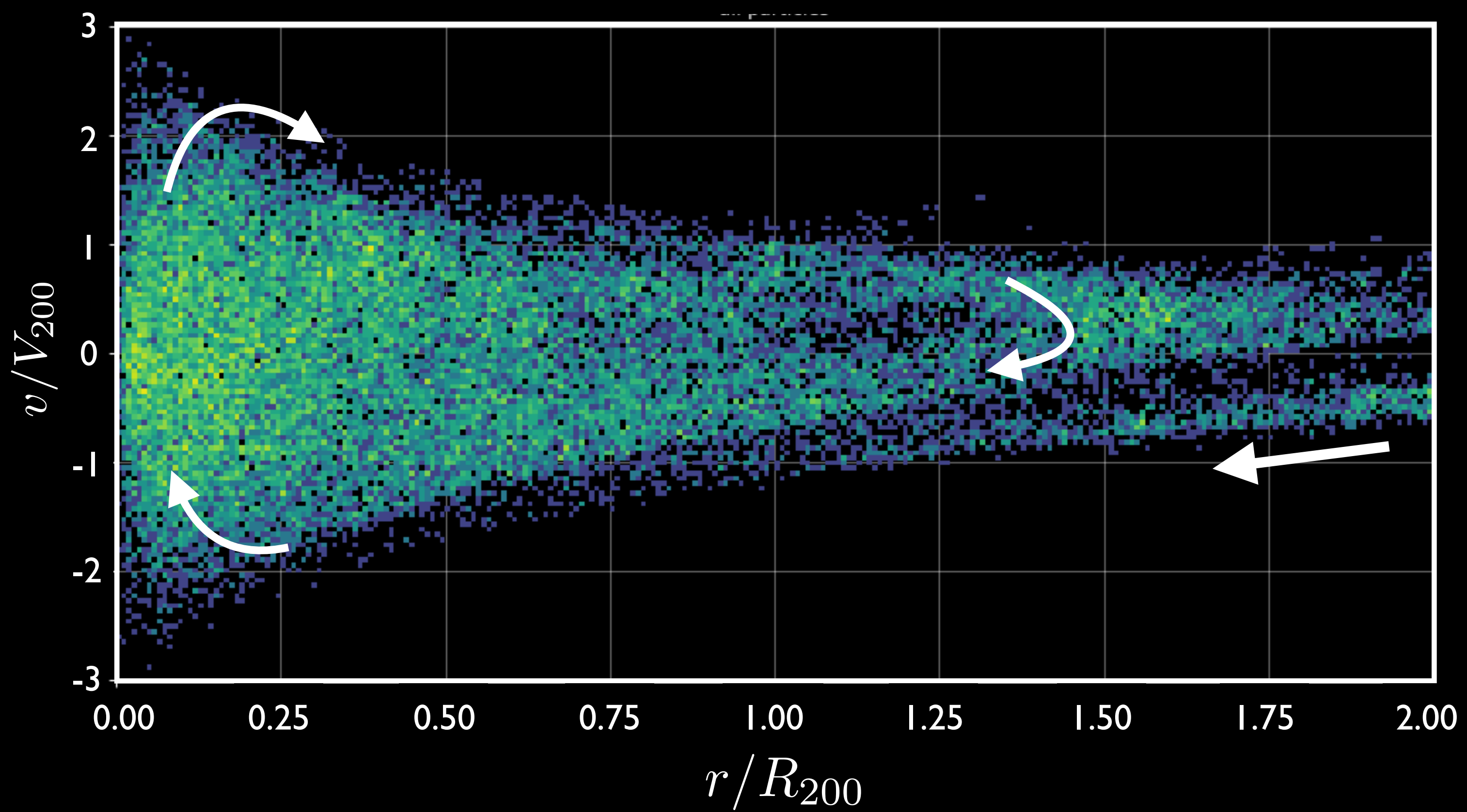
-1
-2
-3
v/V_{200}
0.00
0.25
0.50
0.75
1.00
1.25
1.50
1.75
2.00
r/R_{200}

Result



3
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-3
v/V_{200}
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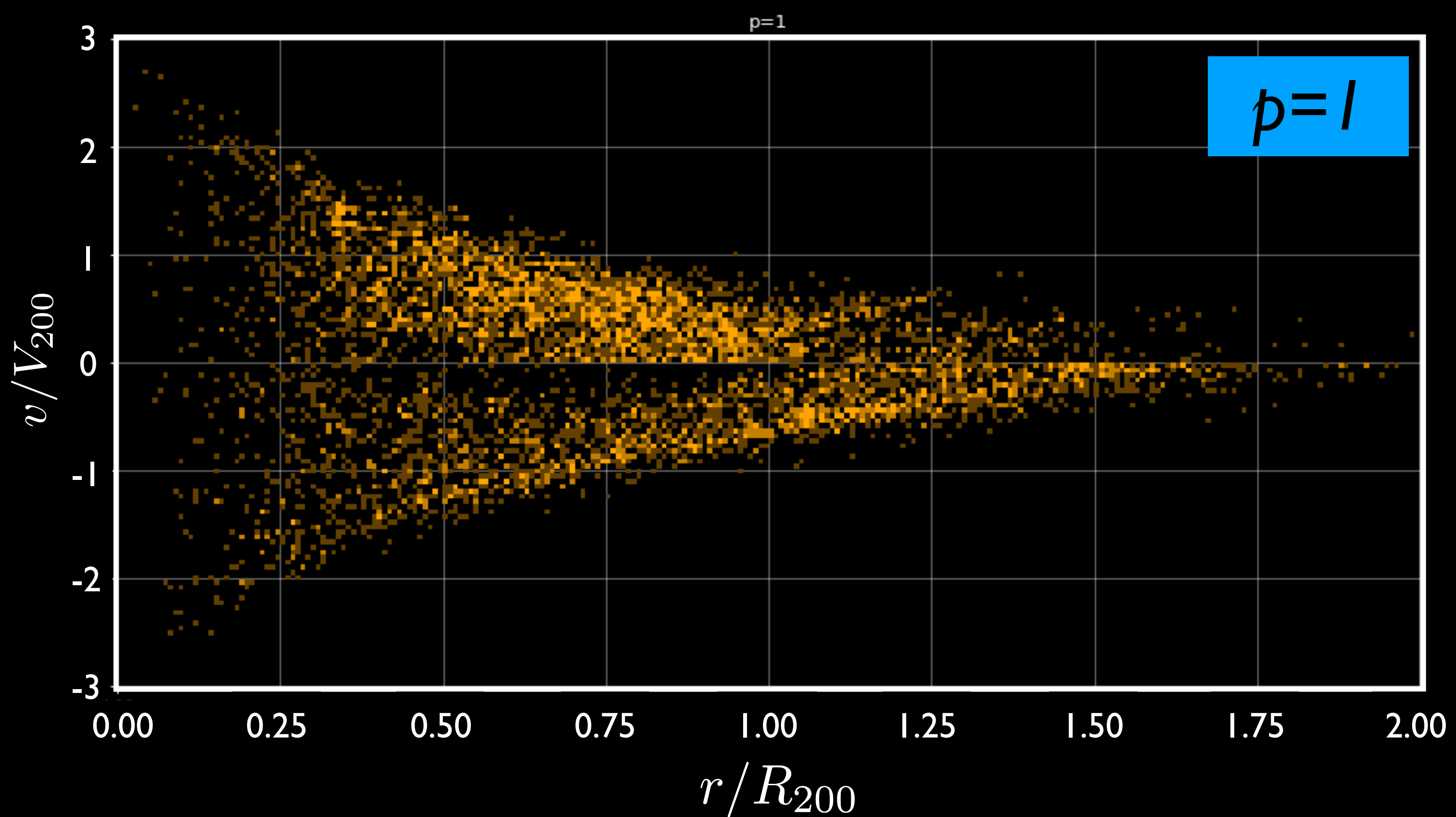
1.75
2.00
r/R_{200}



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-2
-3
v/V_{200}
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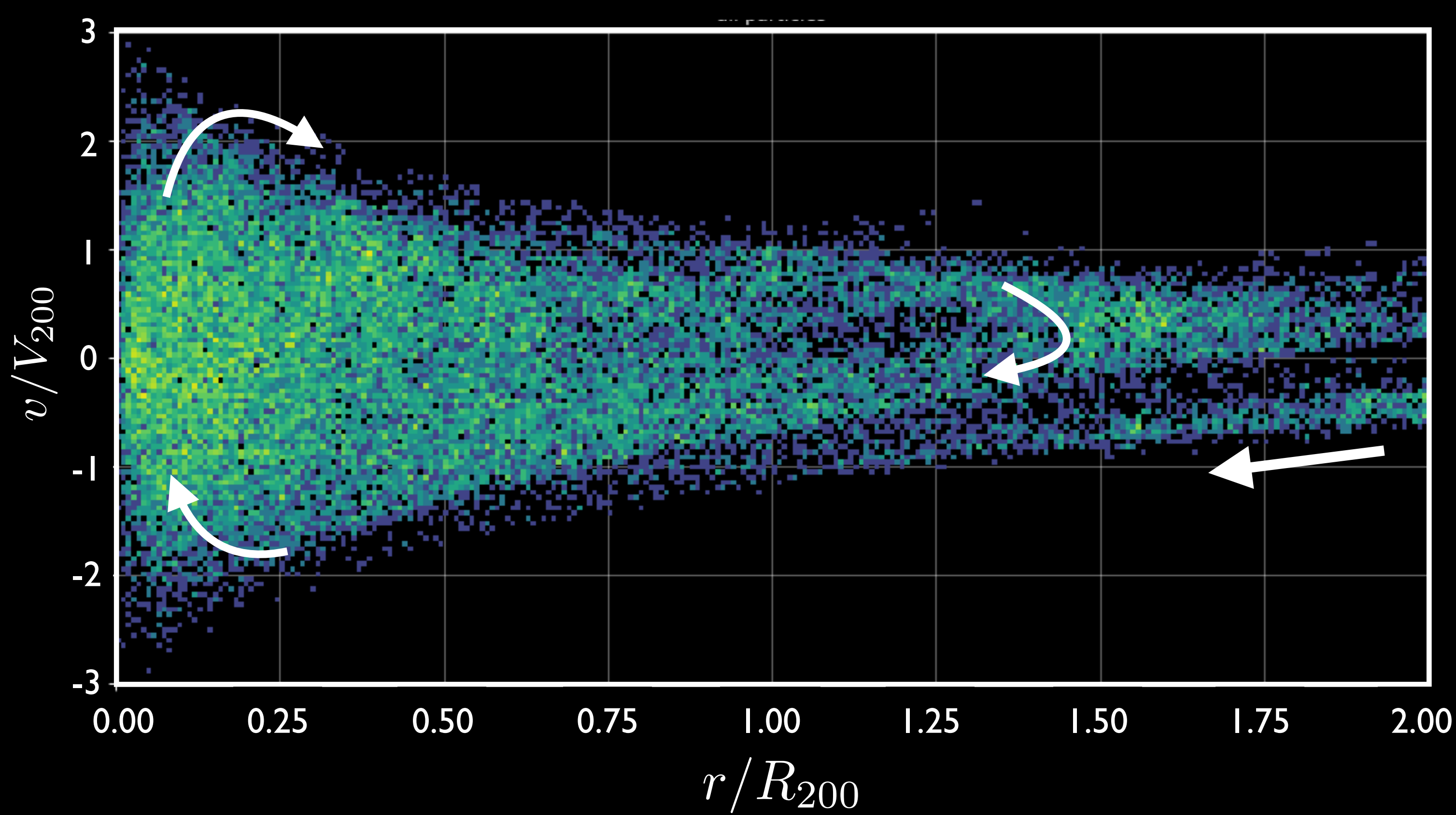
1.75
2.00
r/R_{200}

Result



3
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v/V_{200}
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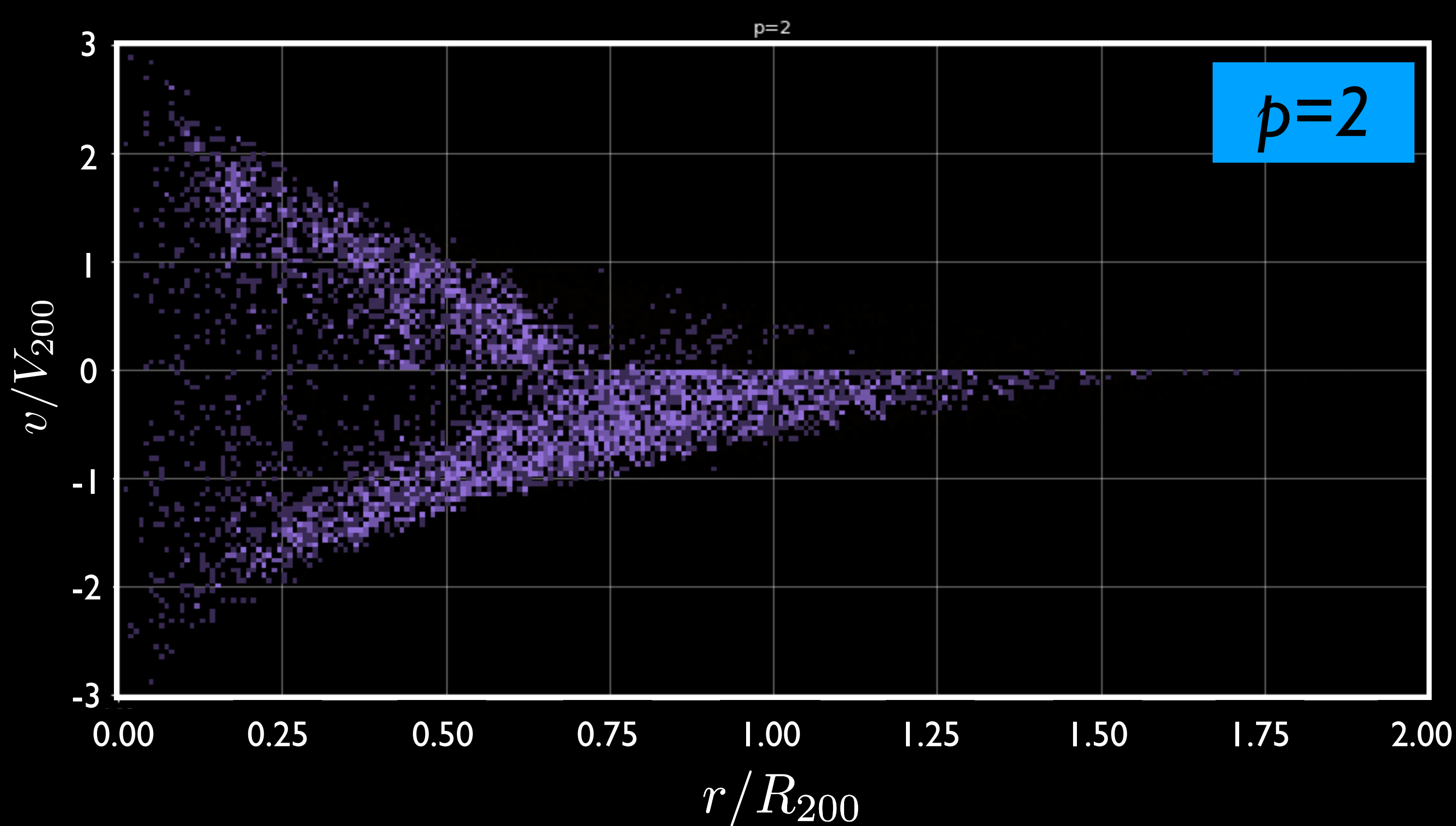
1.75
2.00
r/R_{200}



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p=2
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0
-1
-2
-3
v/V_{200}
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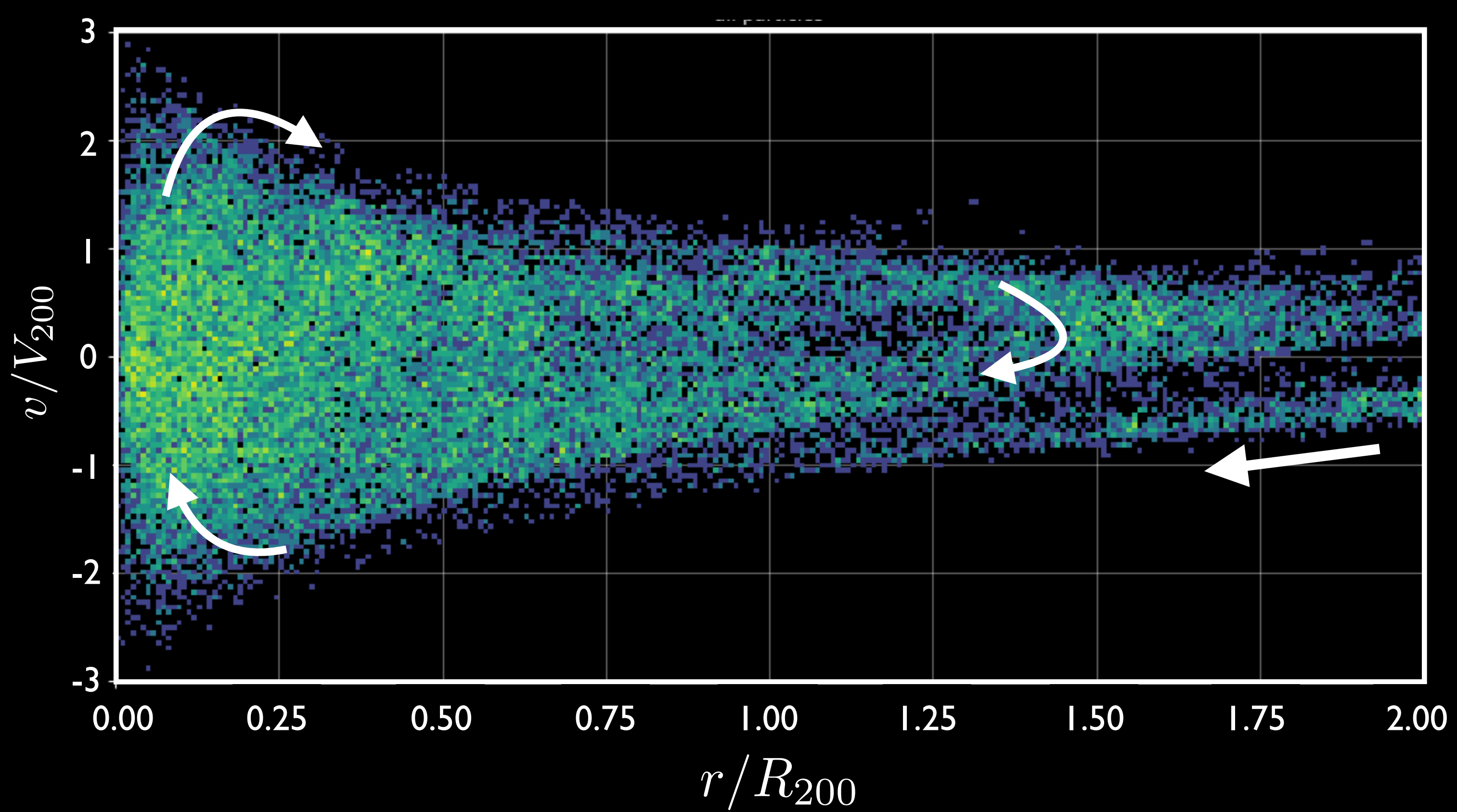
1.00
1.25
1.50
1.75
2.00
r/R_{200}

Result



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v/V_{200}
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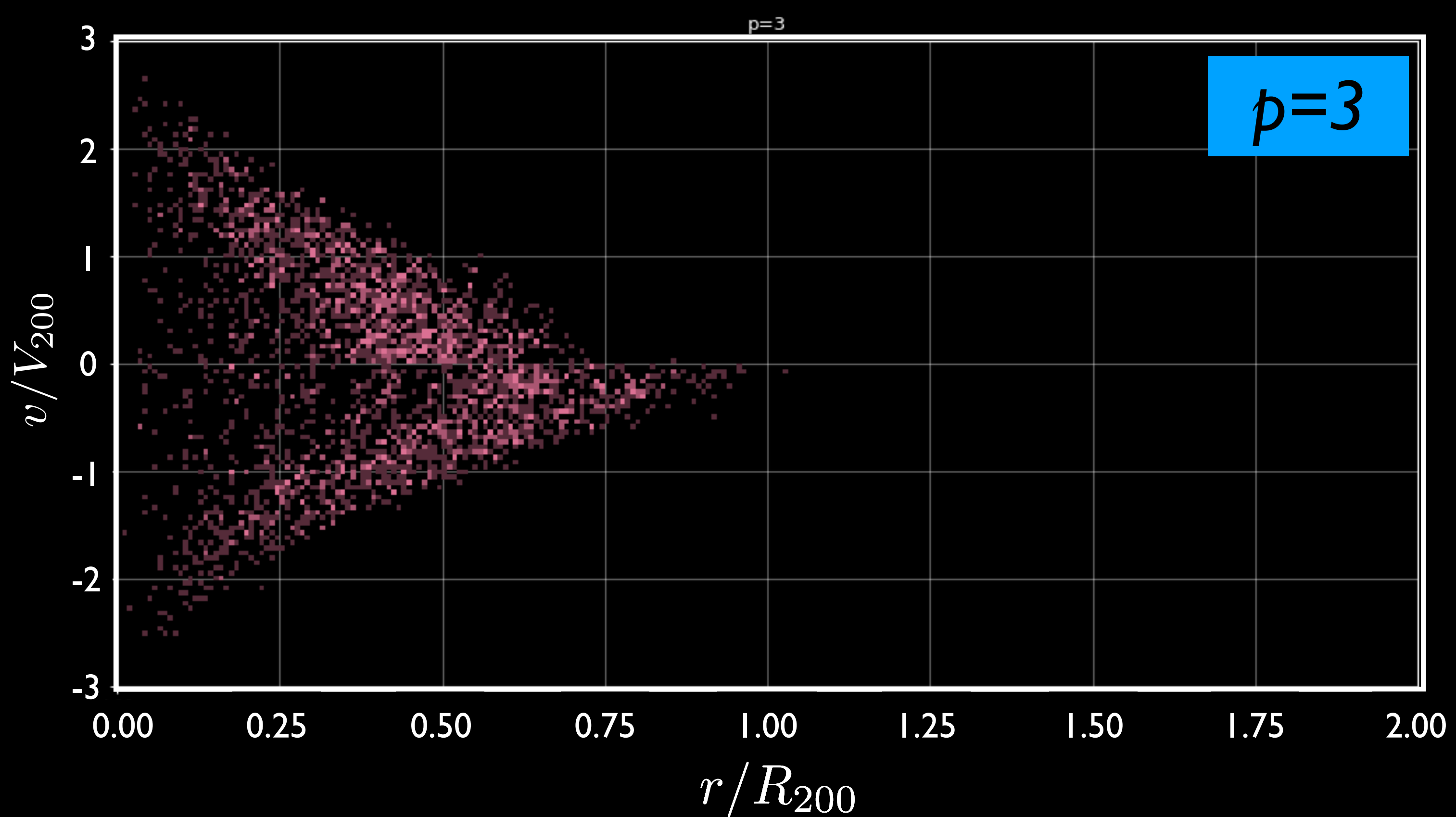
1.75
2.00
r/R_{200}



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-2
-3
v/V_{200}
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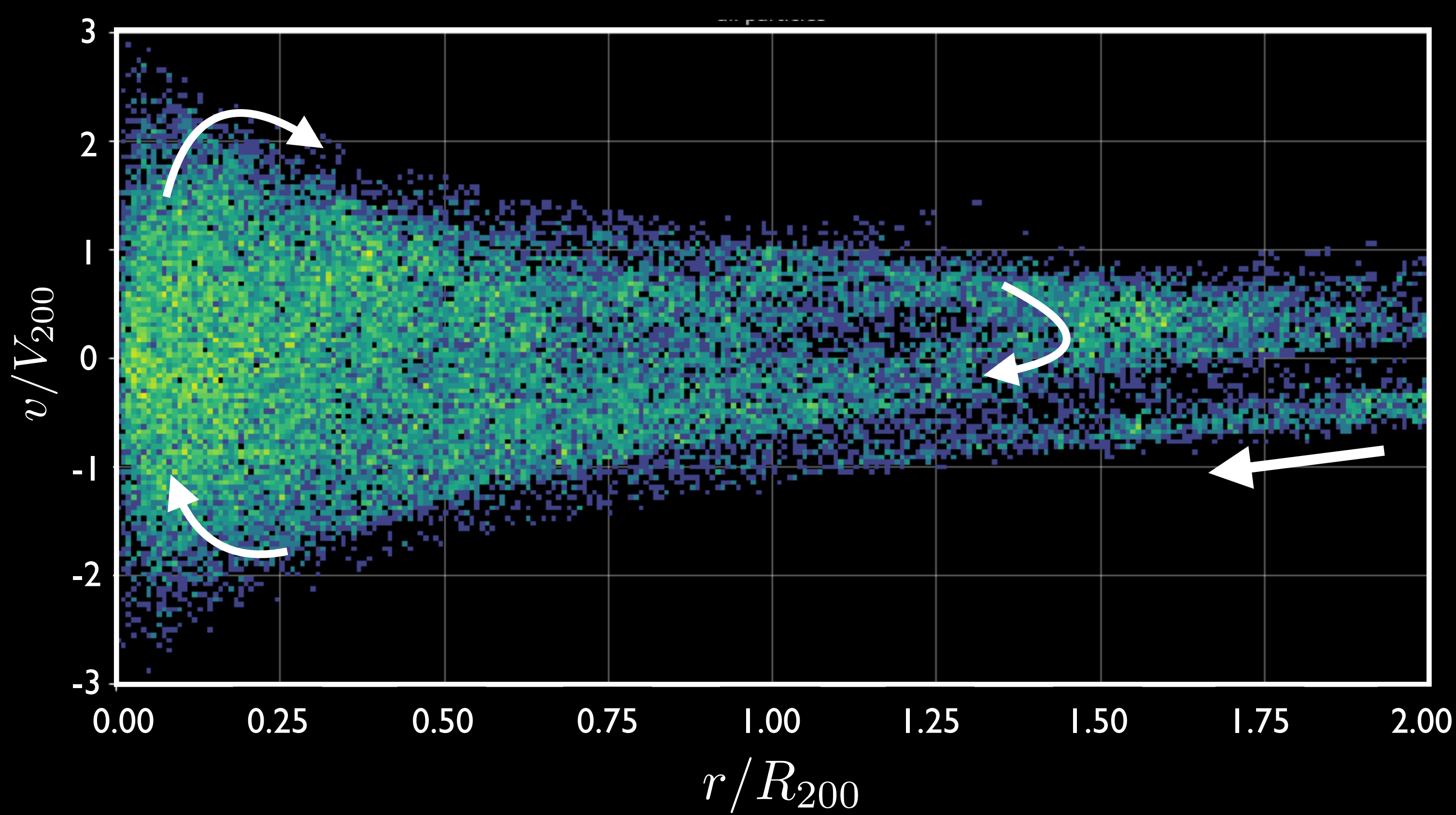
0.50
0.75
1.00
1.25
1.50
1.75
2.00
r/R_{200}

Result



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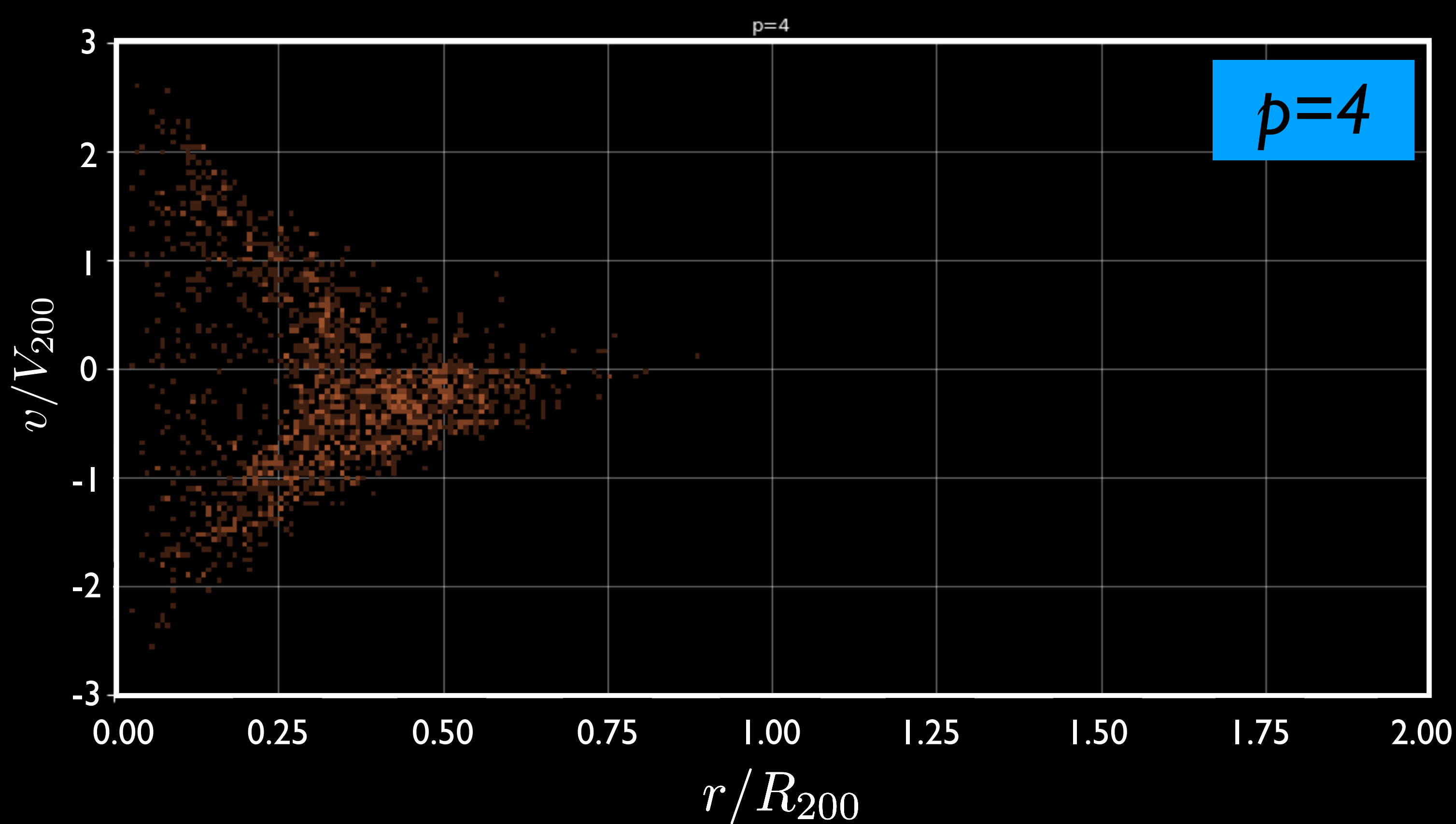
v/V_{200}
0.00
0.25
0.50
0.75
1.00
1.25
1.50
1.75
2.00
r/R_{200}



p=4
p=4
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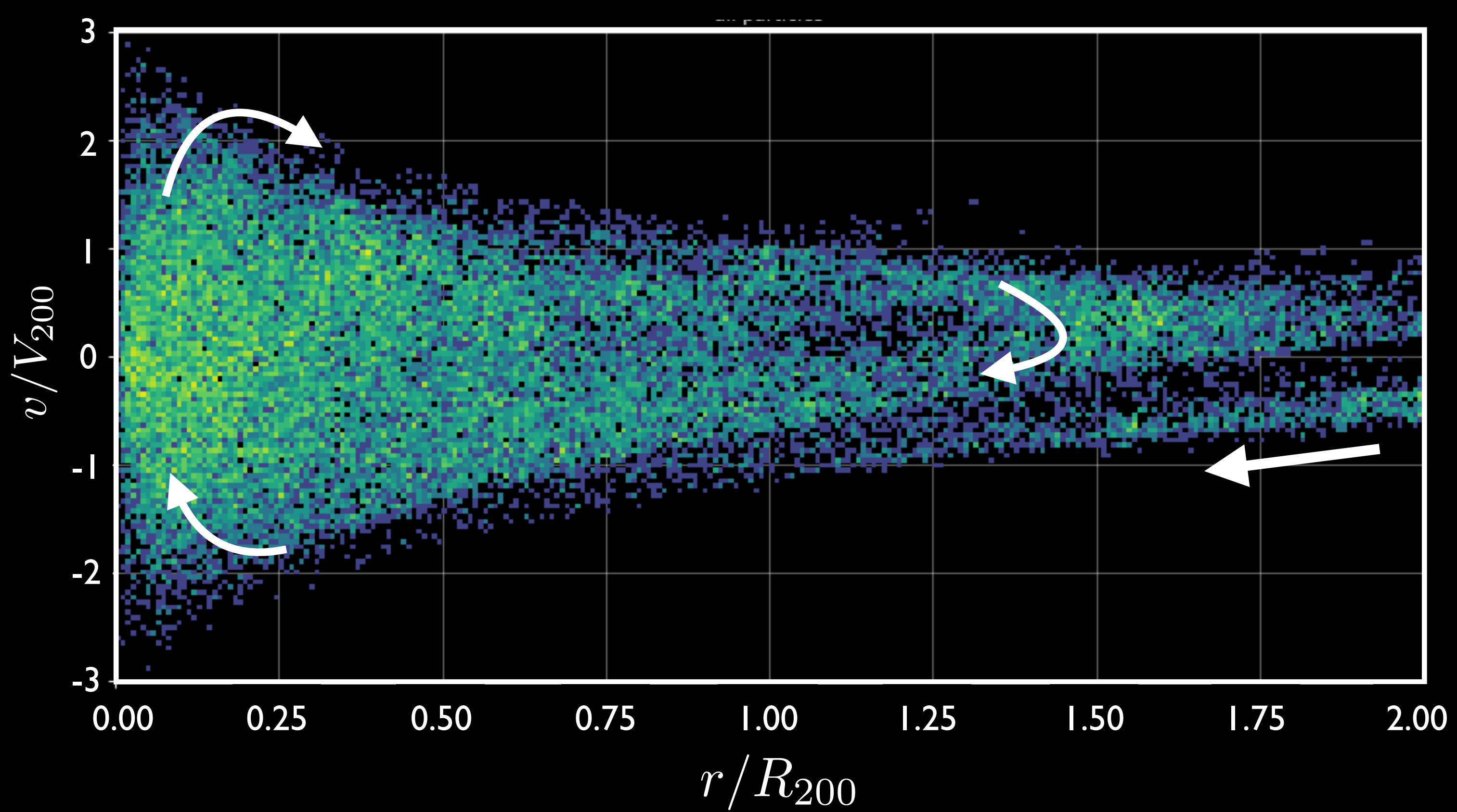
1
0
-1
-2
-3
v/V_{200}
0.00
0.25
0.50
0.75
1.00
1.25
1.50
1.75
2.00
r/R_{200}

Result



3
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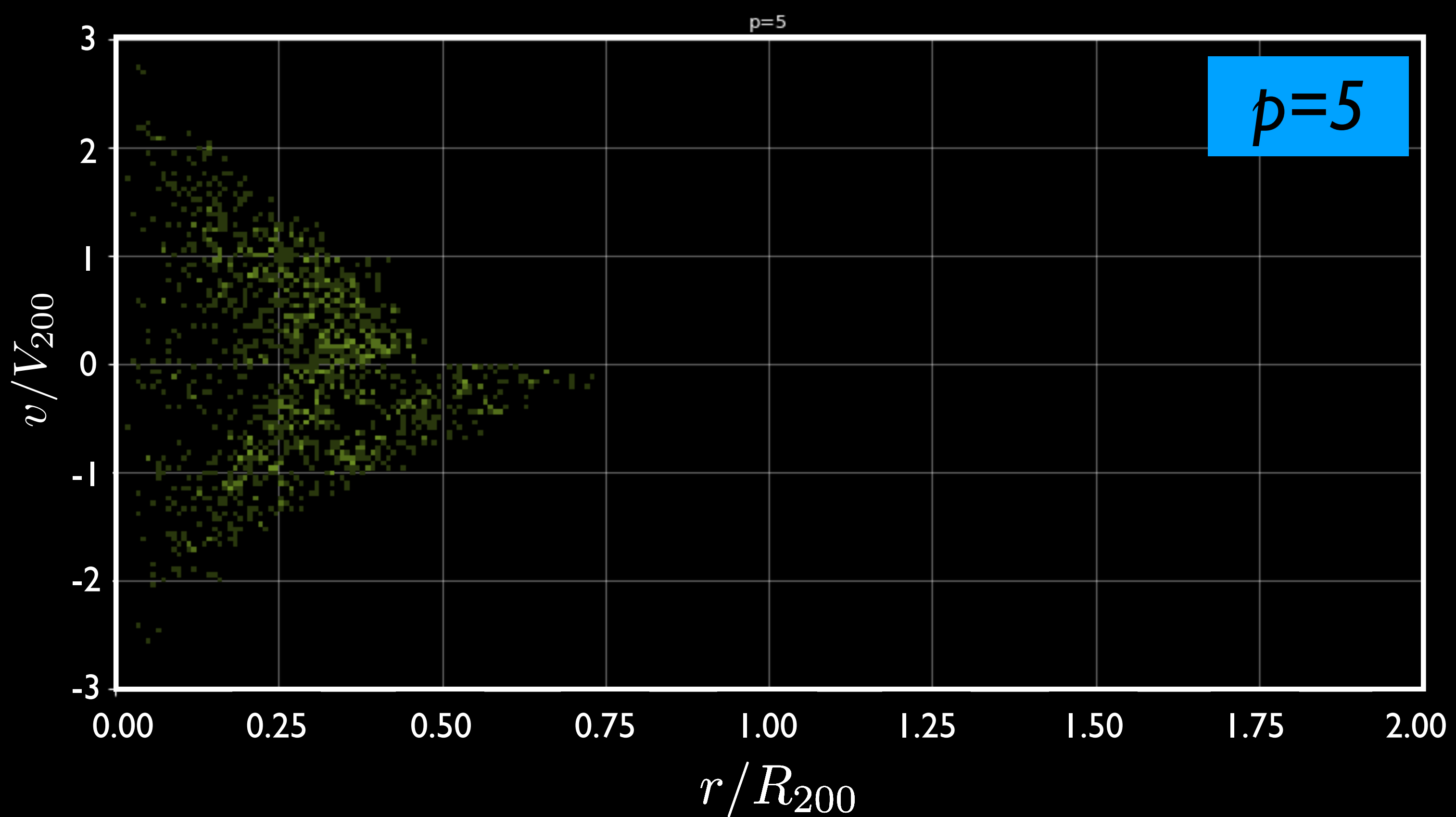
-3
v/V_{200}
0.00
0.25
0.50
0.75
1.00
1.25
1.50
1.75
2.00
r/R_{200}



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p=5
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0
-1
-2
-3
v/V_{200}
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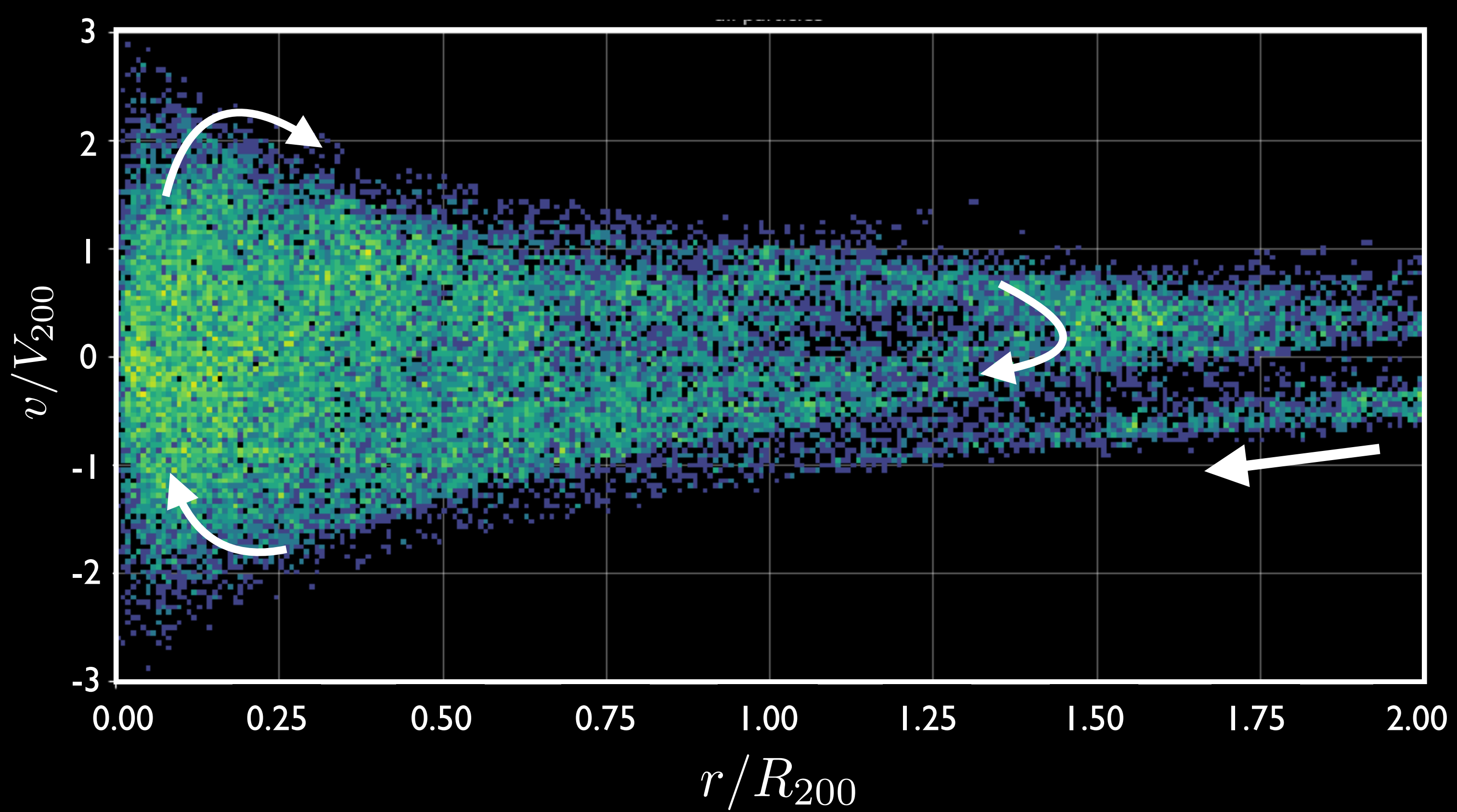
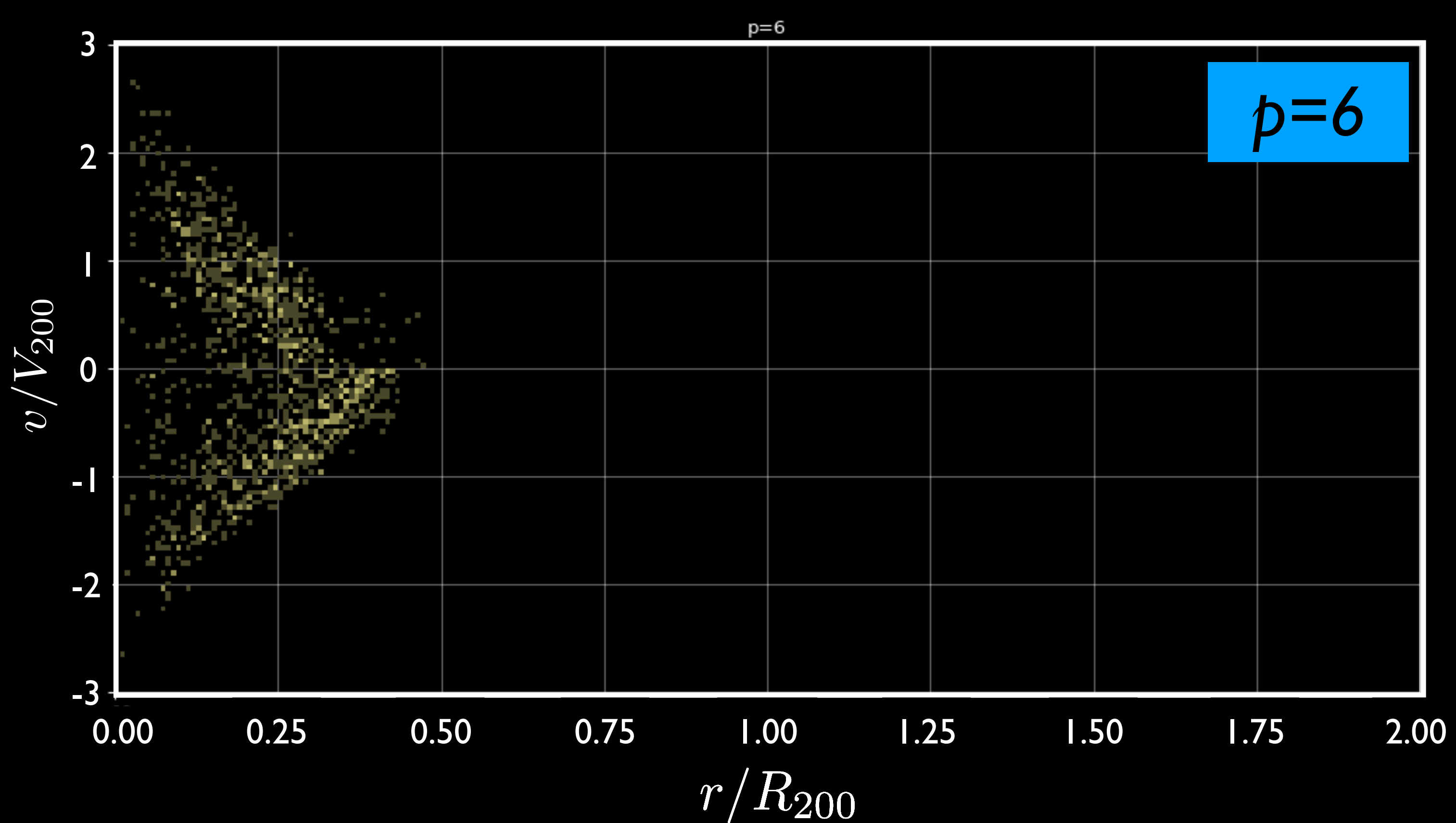
0.25
0.50
0.75
1.00
1.25
1.50
1.75
2.00
r/R_{200}

Result



3
2
1
0
-1
-2
-3
v/V_{200}
0.00
0.25
0.50
0.75
1.00
1.25
1.50
1.75
2.00
r/R_{200}



p=6
p=6
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2
1
0
-1
-2
-3
v/V_{200}
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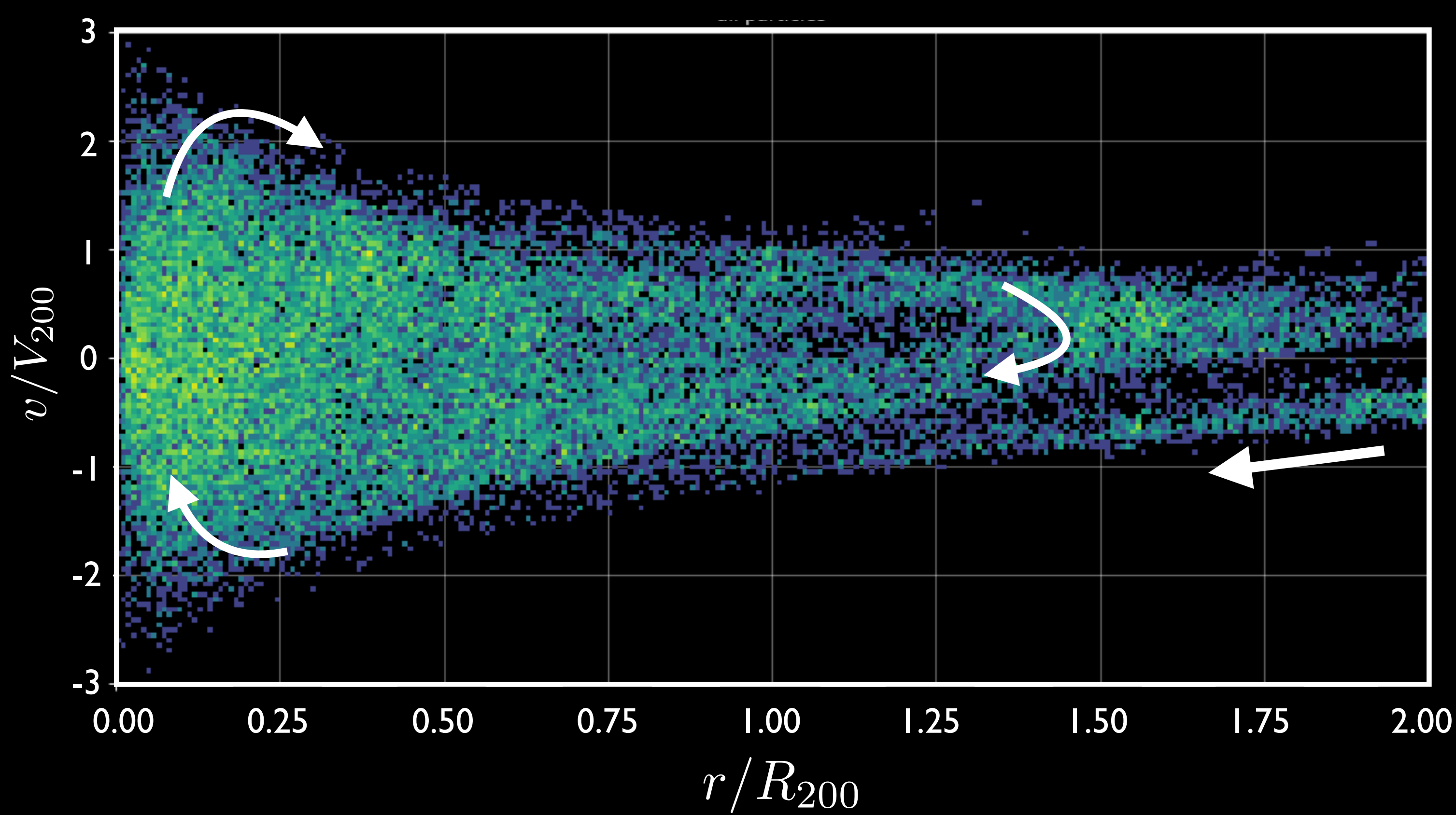
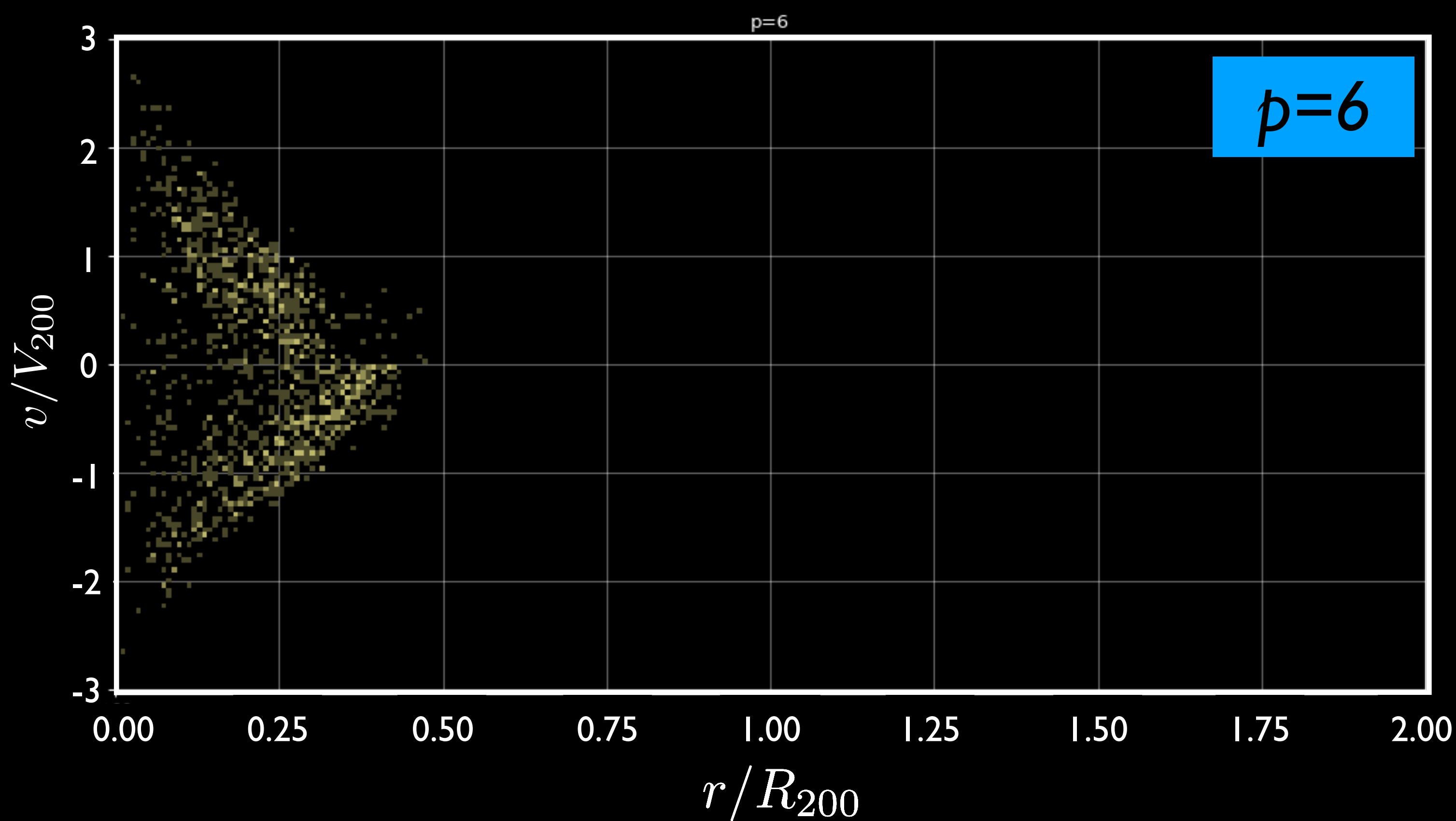
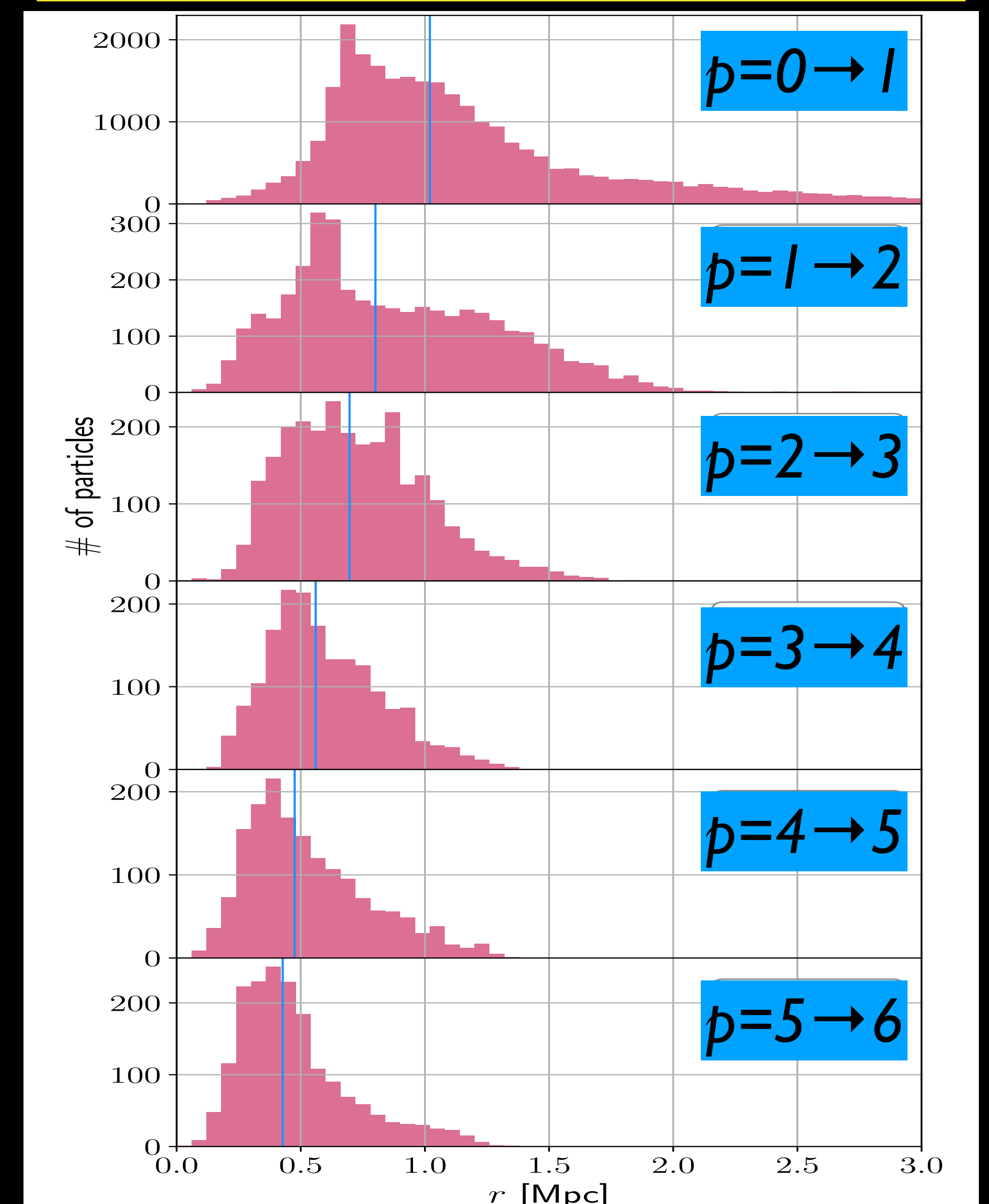
0.25
0.50
0.75
1.00
1.25
1.50
1.75
2.00
r/R_{200}

Result





Distribution of apocenter passage position



Fit to self-similar solution (Fillmore & Goldreich'84)

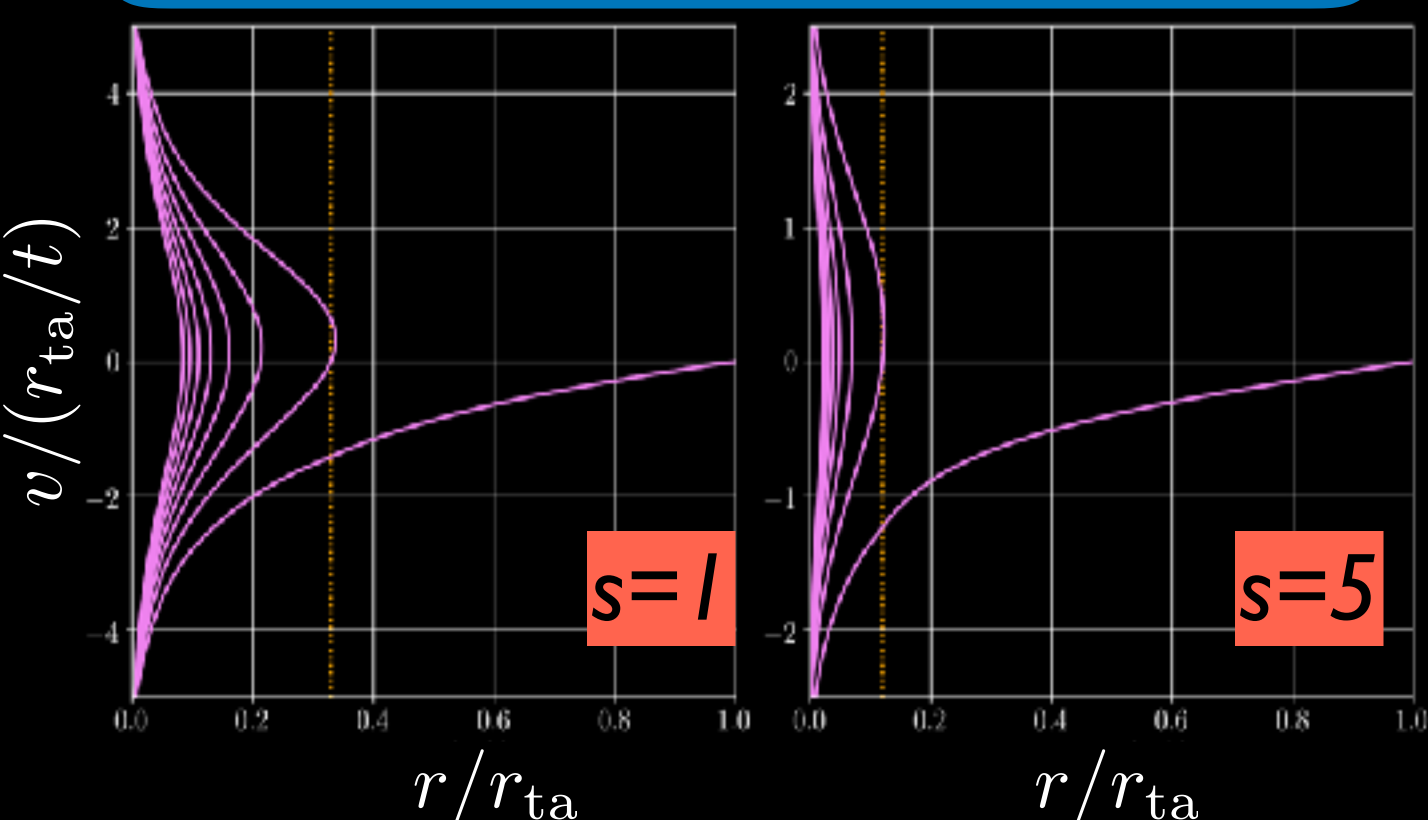
Comparison with self-similar solution

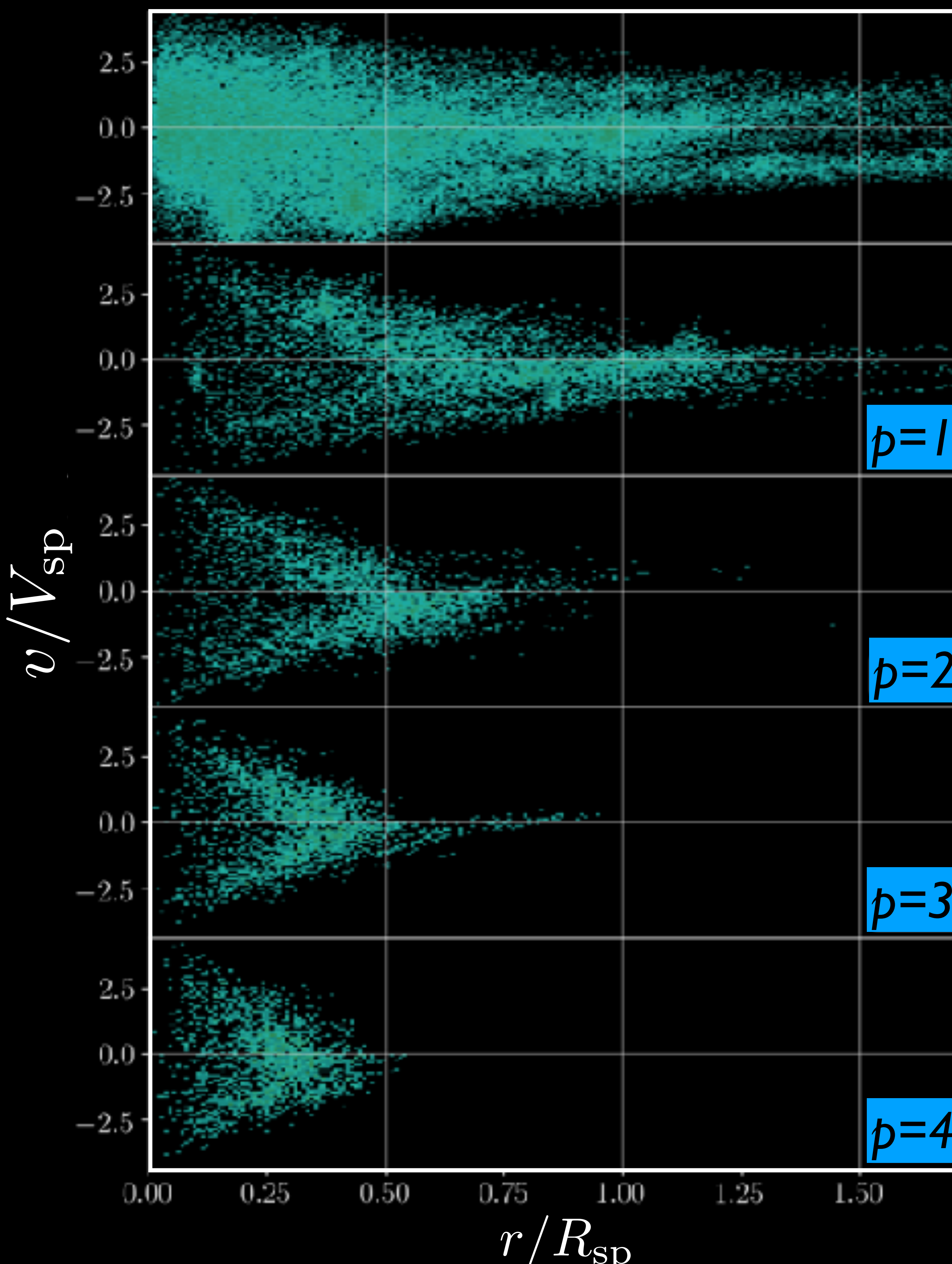
Use apocenter-passage positions
p=1~5 to fit to self-similar solution
by Fillmore & Goldreich ('84)

Fitting parameters:

$$s = (1/\epsilon), \quad r_{\rm ta}(t_{\rm ta})$$

accretion rate $(M \propto a^s)$ scale radius





(Master thesis by H. Sugiura)

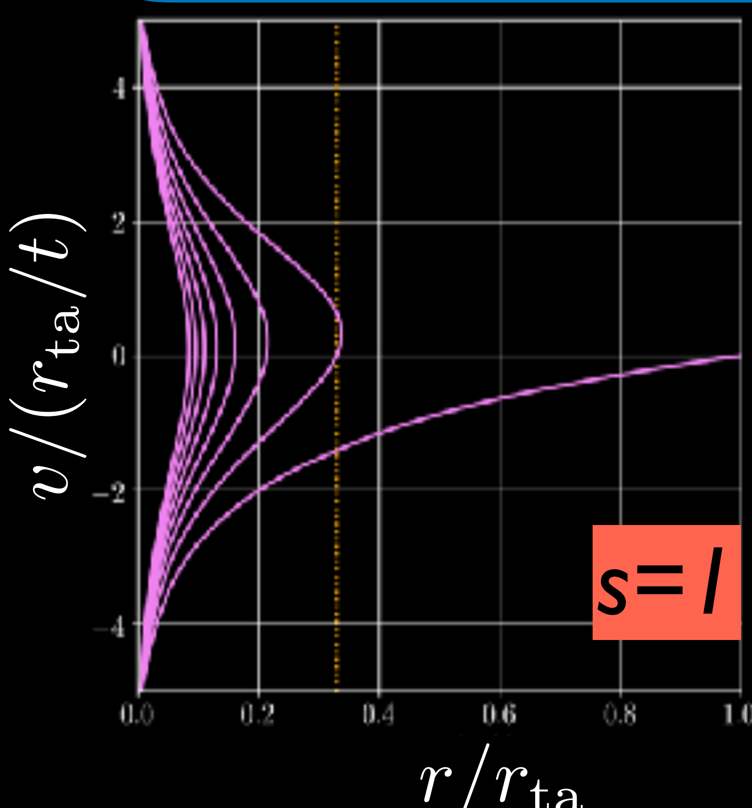
Comparison with self-similar solution

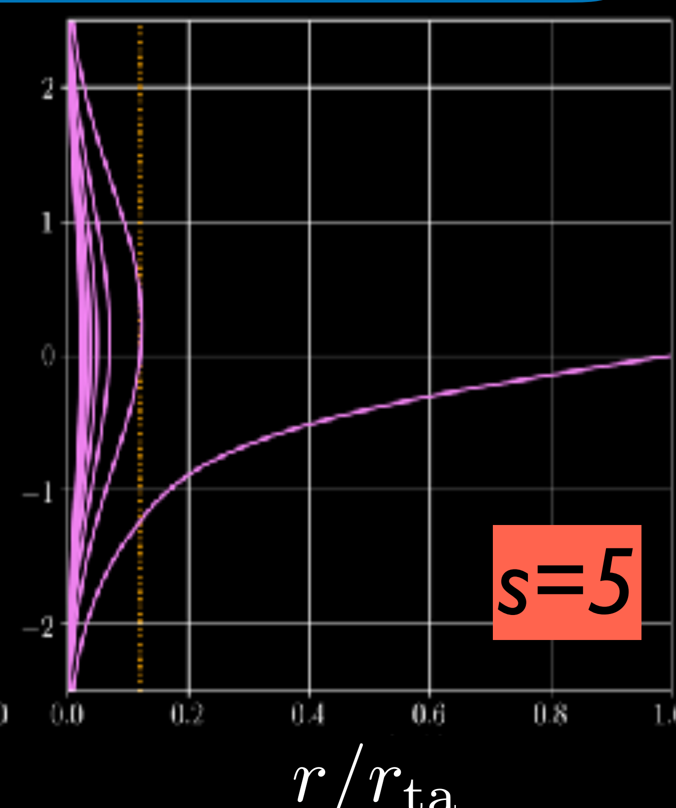
Use apocenter-passage positions
p=1~5 to fit to self-similar solution
by Fillmore & Goldreich ('84)

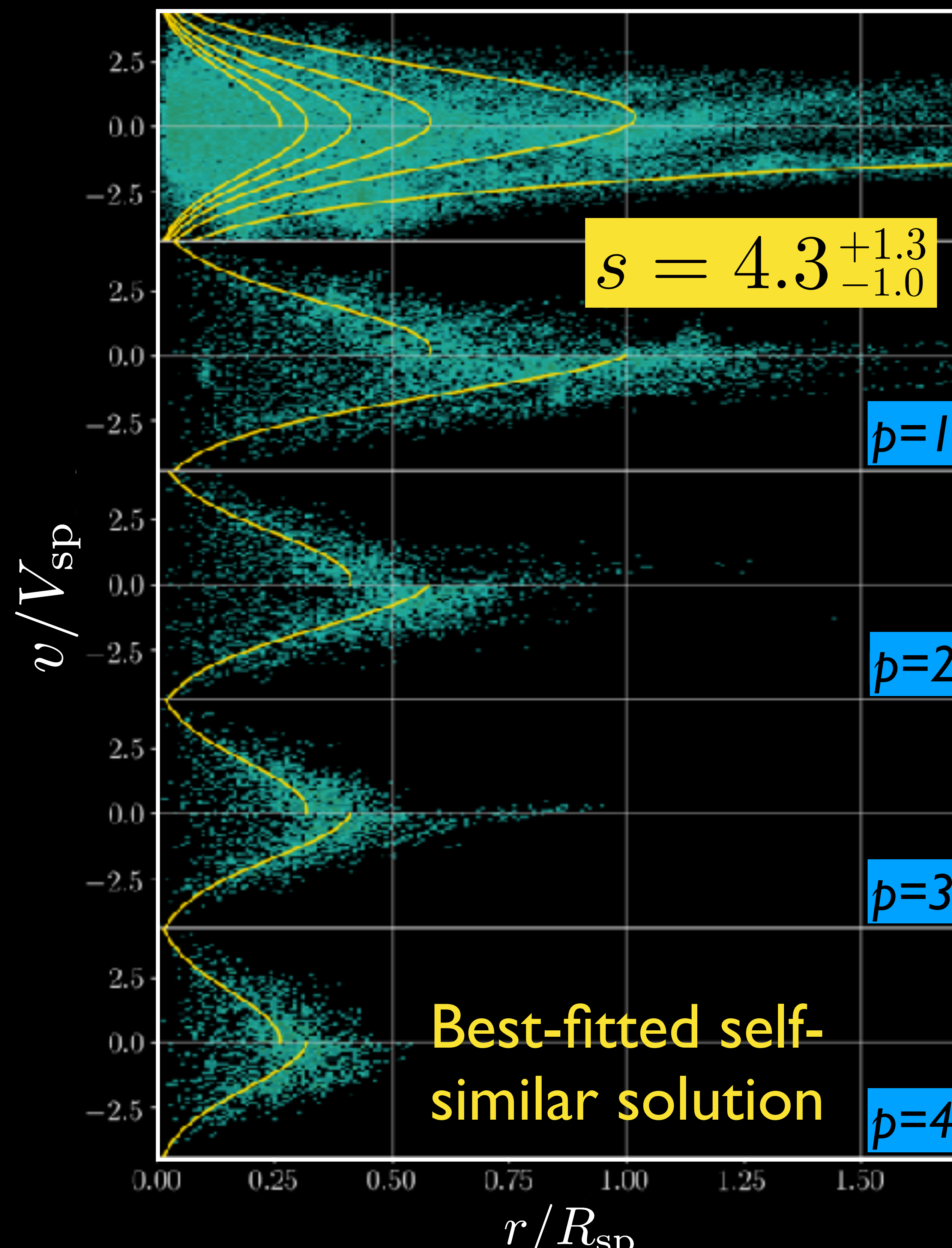
Fitting parameters:

$$s = (1/\epsilon), \quad r_{\rm ta}(t_{\rm ta})$$

accretion rate $(M \propto a^s)$ — scale radius





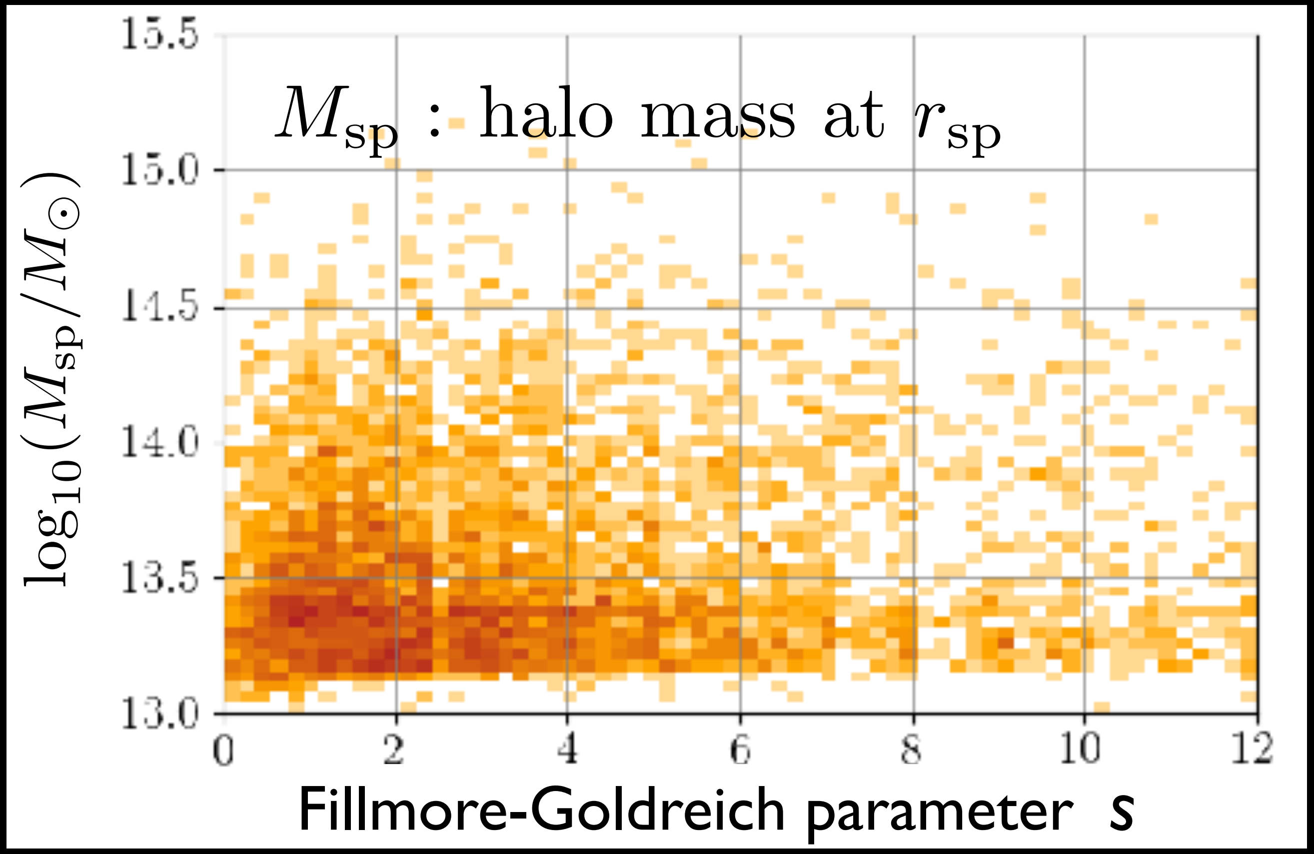


(Master thesis by H. Sugiura)

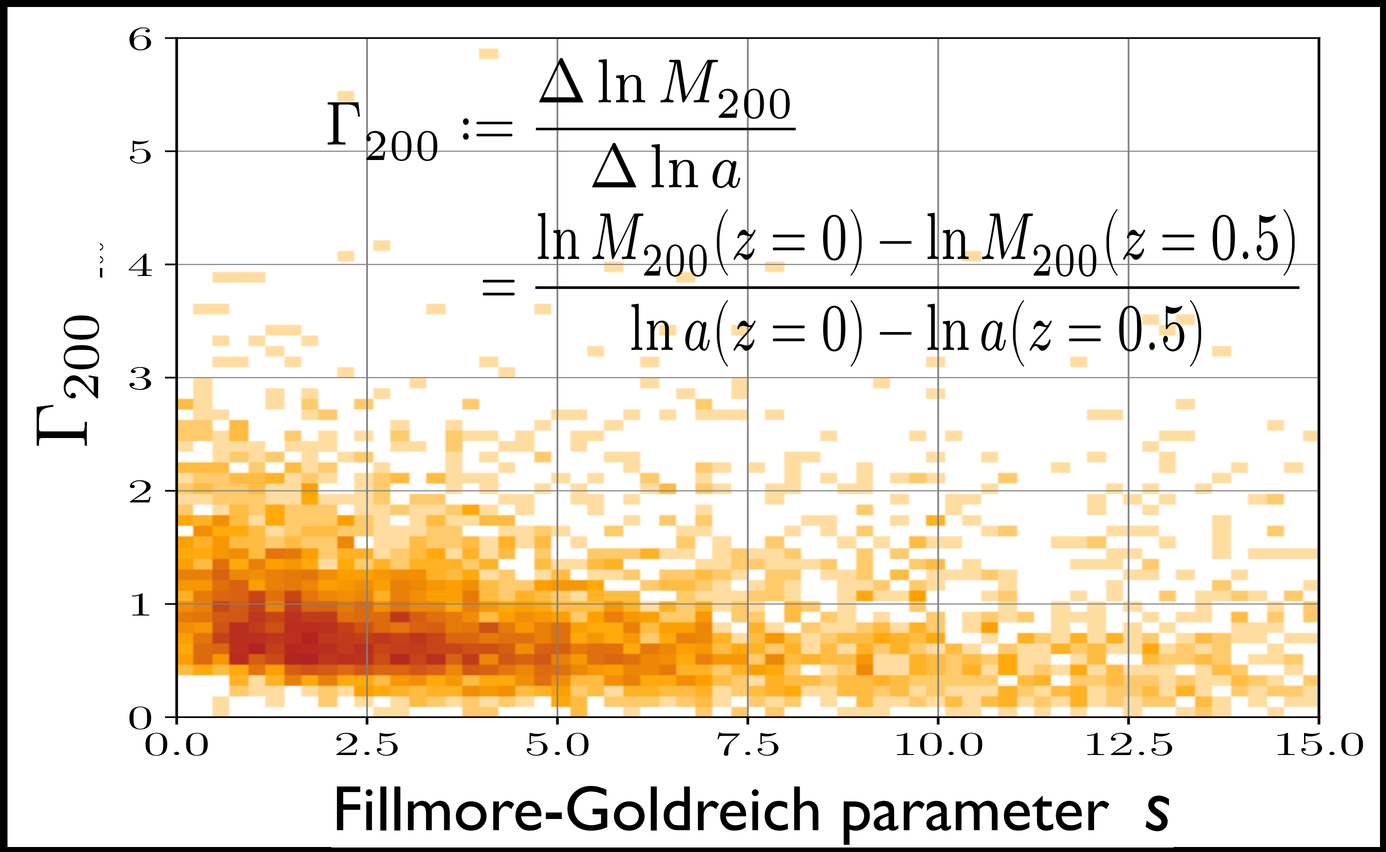
Results

Preliminary

- ~50% of halos are successfully fitted to the self-similar solution
  (Bad fitting is partly due to miss-identification of apocenter passage)
- Good fit is obtained even for non-spherical halos
- No tight correlation between fitting parameter *s* and $M_{\rm sp}$ & Γ_{200}



$M_{\rm halo}(t) \propto a(t)^s$



(Master thesis by H. Sugiura)

Summary

Shell-crossing & multi-stream flows as distinctive features of nonlinear structure formation in CDM cosmology

✓ A first detailed comparison between Lagrangian PT & Vlasov-Poisson simulation

Generic convergence behavior of Lagrangian PT is found
⟶ used to predict shell-crossing structures

✓ Confrontation of self-similar solution (SSS) against dark halos from *N*-body simulations

A technique to trace multi-stream flows with particle trajectories
⟶ SSS is found to describe outer halo structure remarkably well

New test of CDM paradigm and clue to clarify nature of CDM