Accelerating Universe in the Dark @ YITP

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Blue-tilted primordial gravitational waves from massive gravity

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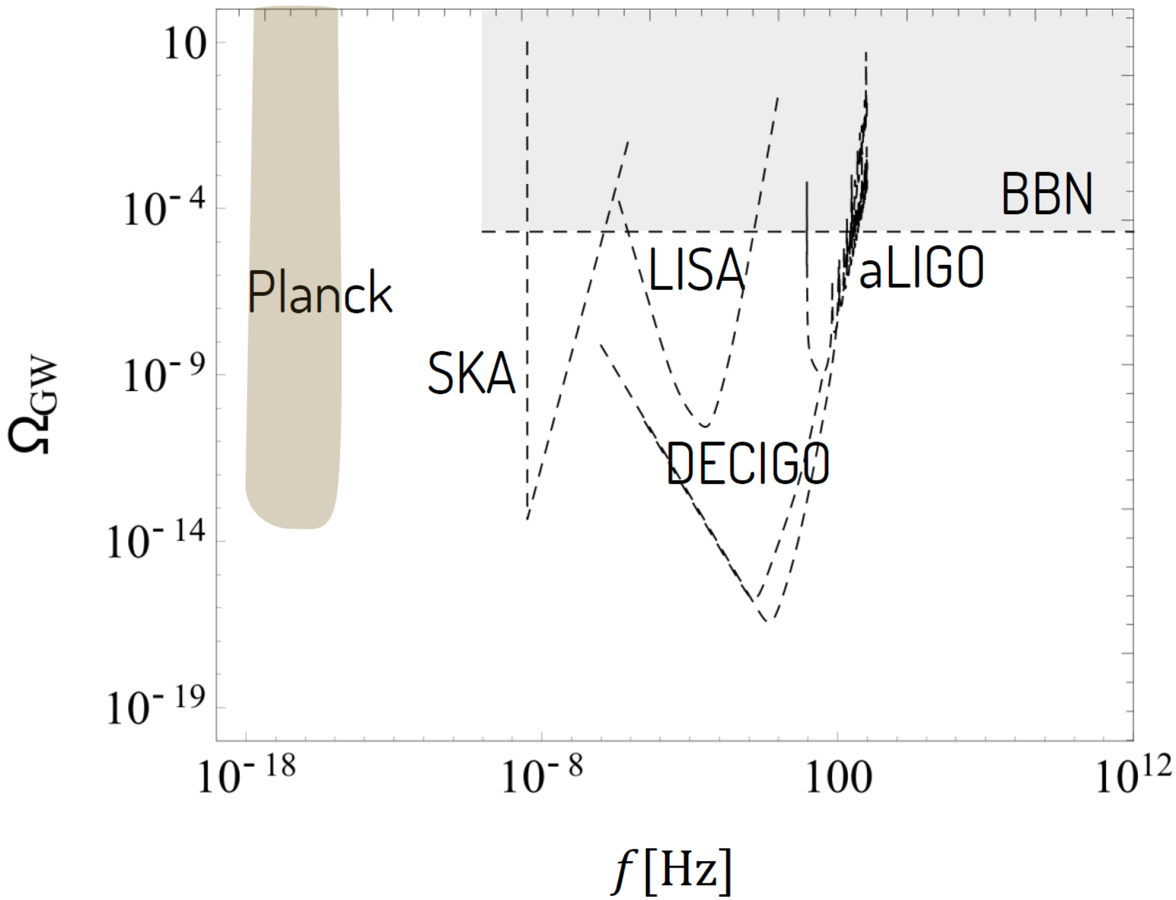
Fujita, Kuroyanagi, SM, Mukohyama Phys. Lett. B789, 215

Fujita, SM, Mukohyama in preparation

Primordial Gravitational Waves (PGWs)



Density parameter

$$\Omega_{\rm GW} \equiv \frac{1}{\rho_{\rm tot}} \frac{d\rho_{\rm GW}}{d\ln k}$$

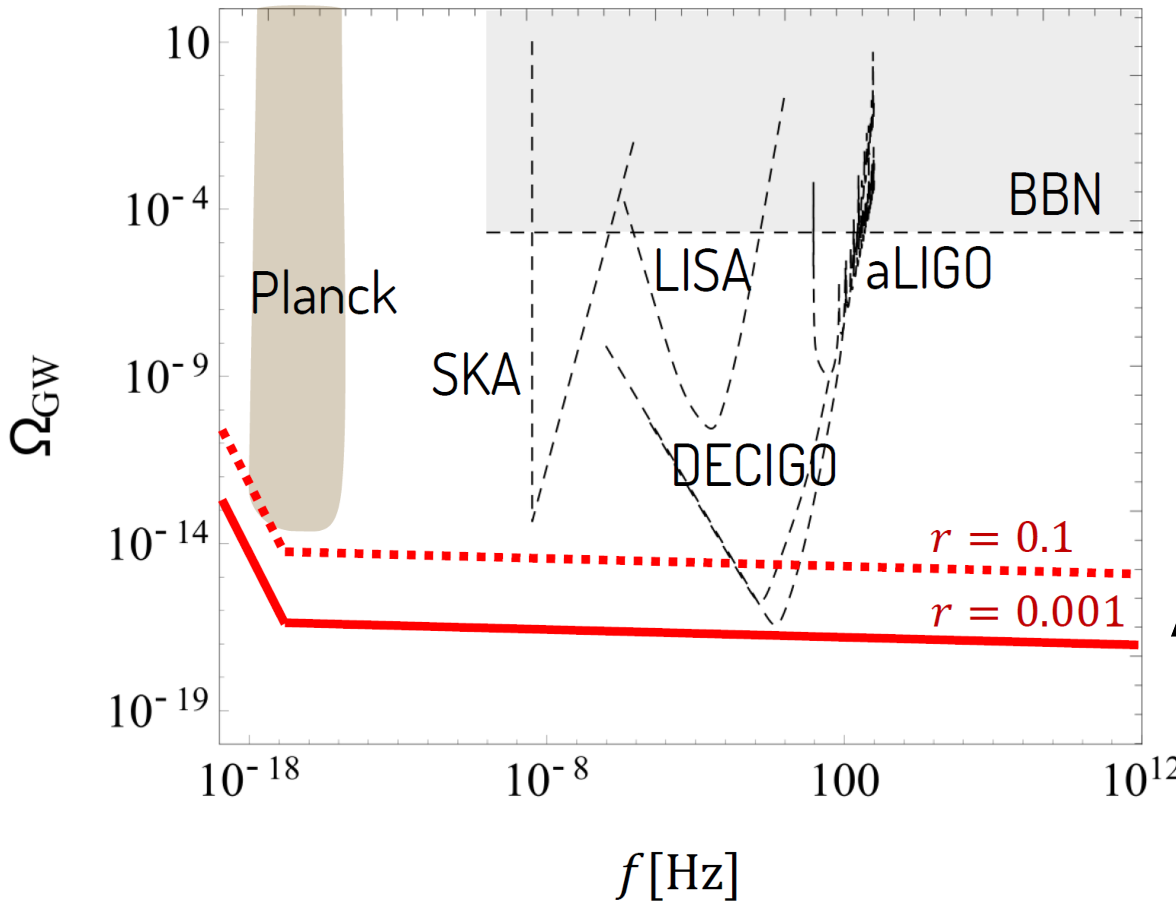
Frequency

$$f \equiv k/2\pi$$

We have already constraints on $\Omega_{\rm GW}$ from BBN and Planck

Interferometers can get information of PGWs on various scales !!

Interferometers and Standard PGWs



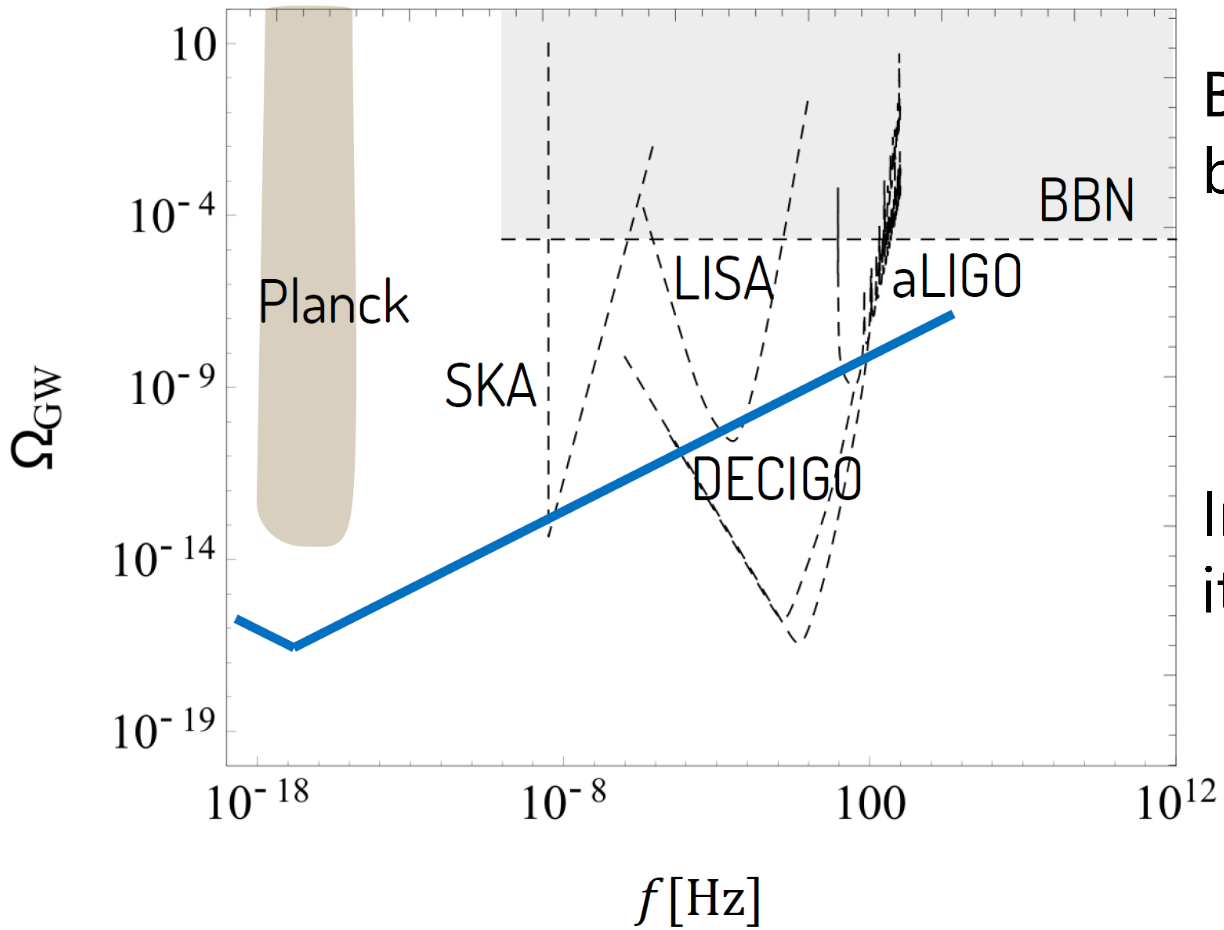
PGWs from inflation

$$\mathcal{P}_h^{\rm standard} = \frac{2H^2}{\pi^2 M_{\rm Pl}^2}$$

Almost flat (Red-tilted) PGWs spectrum

Planck constrains $\Omega_{GW} \lesssim 10^{-15}$ on interferometers' scales !!

Interferometers and Blue-tilted PGWs



Blue-tilted PGWs are better for detection

In standard inflation, it is difficult to realize

Can we obtain consistent and detectable blue-tilted PGWs ?

PGWs from supersolid inflation

- Solid inflation and supersolid inflation

Endlich, Nicolis, Wang `12, Nicolis, Penco, Rosen `14

Effective Field Theory of Inflation with spatial rotations

$$\phi^0 = t + \pi \,, \quad \underline{\phi^i = \alpha\, x^i + \alpha\, \sigma^i} \quad , \; i = 1, 2, 3$$

inv. under SO(3)

$$\Rightarrow \quad \mathcal{L} = F(X, Y^i, Z^{ij})$$

$$X = \partial_\mu \phi^0 \partial_\nu \phi^0 g^{\mu\nu} \,, \quad Y^i = \partial_\mu \phi^0 \partial_\nu \phi^i g^{\mu\nu}$$

$$Z^{ij} = \partial_\mu \phi^i \partial_\nu \phi^j g^{\mu\nu}$$

- Power spectrum of PGWs from supersolid inflation

Cannone, Tasinato, Wands `14

$$\mathcal{P}_h = \frac{2H^2}{\pi^2 M_{\rm Pl}^2} \left(\frac{k}{k_{\rm CMB}} \right)^{n_T} \qquad n_T = -2\epsilon + \frac{2}{3} \frac{m^2}{H^2} \qquad \frac{|m|}{H} \ll 1$$

Blue-tilted PGWs without violating the null energy condition !!

Supersolid inflation and Massive Gravity (MG)

- Second order action of $h_{\mu\nu} \equiv g_{\mu\nu} - g^{(0)}_{\mu\nu}$ w/o derivatives

$$S_h^{(2)} = \frac{1}{4} M_{\rm Pl}^2 \int {\rm d}^4x \sqrt{-g} \Big[m_0^2\, h_{00}^2 + 2\, m_1^2\, h_{0i}^2 - m_2^2\, h_{ij}^2 + m_3^2\, h_{ii}^2 - 2 m_4^2\, h_{00}\, h_{ii} \Big]$$

The structure is same as (Lorentz-violating) massive gravity !!

$m_i / H \sim \mathcal{O}(\epsilon)$ Dubovsky `04

Degree of ~~time diffs~~ and ~~spatial diffs~~ are comparable

- Higuchi ghost in massive gravity Higuchi `87

Massive graviton has more DOFs than massless graviton

In de Sitter spacetime, the extra DOF becomes ghost if the mass of graviton m satisfies $0 < m^2 < 2H^2$

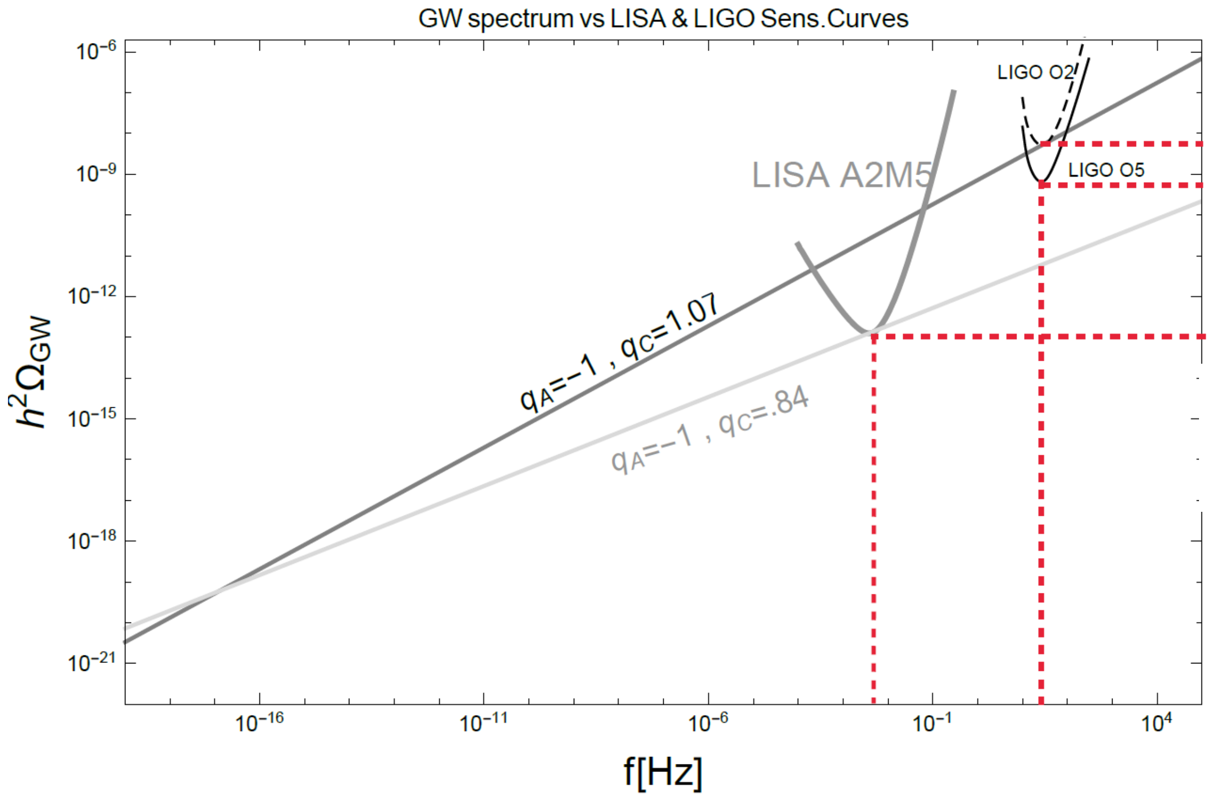
⇒ In supersolid inflation, n_T can be positive but still $\mathcal{O}(\epsilon)$...

PGWs from extended supersolid inflation

Ricciardone, Tasinato `17

Hierarchy between the degree of time diffs and spatial diffs

(Nonminimal couplings between fields and gravity)



$$n_T \simeq 3 - 2\nu$$

$$\nu \equiv \sqrt{\frac{9}{4} - \frac{m^2}{H^2}},$$

$\frac{m}{H} = \mathcal{O}(1)$ is possible

with small ϵ

Simpler way to realize blue-tilted spectrum?

Minimal theory of massive gravity (MTMG)

- Properties of MTMG   De Felice, Mukohyama `15

  Having only 2 propagating DOFs (No scalar & vector gravitons)

  Other points are same as dRGT de Rham, Gabadadze, Tolley `11

  ex.) FRW background, tensor perturbations around it,…

- Construction of MTMG

  Method to remove extra DOFs is based on ADM vielbein

  ⇒ Lorentz violating massive gravity

  Theoretical structure is similar to supersolid inflation

  But this is not EFT of inflation

Set-up

- Decomposition and quantization of h_{ij} with $g_{ij} = a^2 \left[e^h\right]_{ij}$

$$h_{ij}(\tau, \boldsymbol{x}) = \frac{2}{aM_{\rm Pl}} \sum_{\lambda=+,-} \int \frac{{\rm d}^3 k}{(2\pi)^3} e^{i\boldsymbol{k}\cdot\boldsymbol{x}} e_{ij}^{\lambda} \left[v_k^{\lambda}(\tau)\hat{a}_{\boldsymbol{k}}^{\lambda} + {\rm h.c.}\right]$$

- Equation of motion for the mode function

$$v_k'' + \left[k^2 + a^2\mu^2 - \frac{a''}{a}\right] v_k = 0,$$

	inflation	radiation dom.	radiation dom.
$a(\tau)$	$-1/(H_{\rm inf}\tau)$	$a_r\tau/\tau_r$	$a_r\tau/\tau_r$
	massive	massive	massless
$\mu(\tau)$	m	m	0

Inflation era

$$v_k'' + \left[k^2 - \frac{1}{\tau^2}\left(\nu^2 - \frac{1}{4}\right)\right] v_k = 0 \qquad \nu \equiv \sqrt{\frac{9}{4} - \frac{m^2}{H_{\rm inf}^2}}$$

- Power spectrum of PGWs

$$\mathcal{P}_h \equiv \frac{4k^3 |v_k(\tau)|^2}{\pi^2 M_{\rm Pl}^2 a(\tau)^2} \simeq \frac{2H_{\rm inf}^2}{\pi^2 M_{\rm Pl}^2}\left(\frac{k}{k_{\rm UV}}\right)^{3-2\nu} \quad \text{for} \quad k < k_{\rm UV}$$

(at the end of inflation) $k_{\rm UV} \equiv a_r H_{\rm inf}$

In MTMG, there is no Higuchi bound and $m/H_{\rm inf}$ can be $\mathcal{O}(1)$

(MTMG has only 2 propagating DOFs by construction)

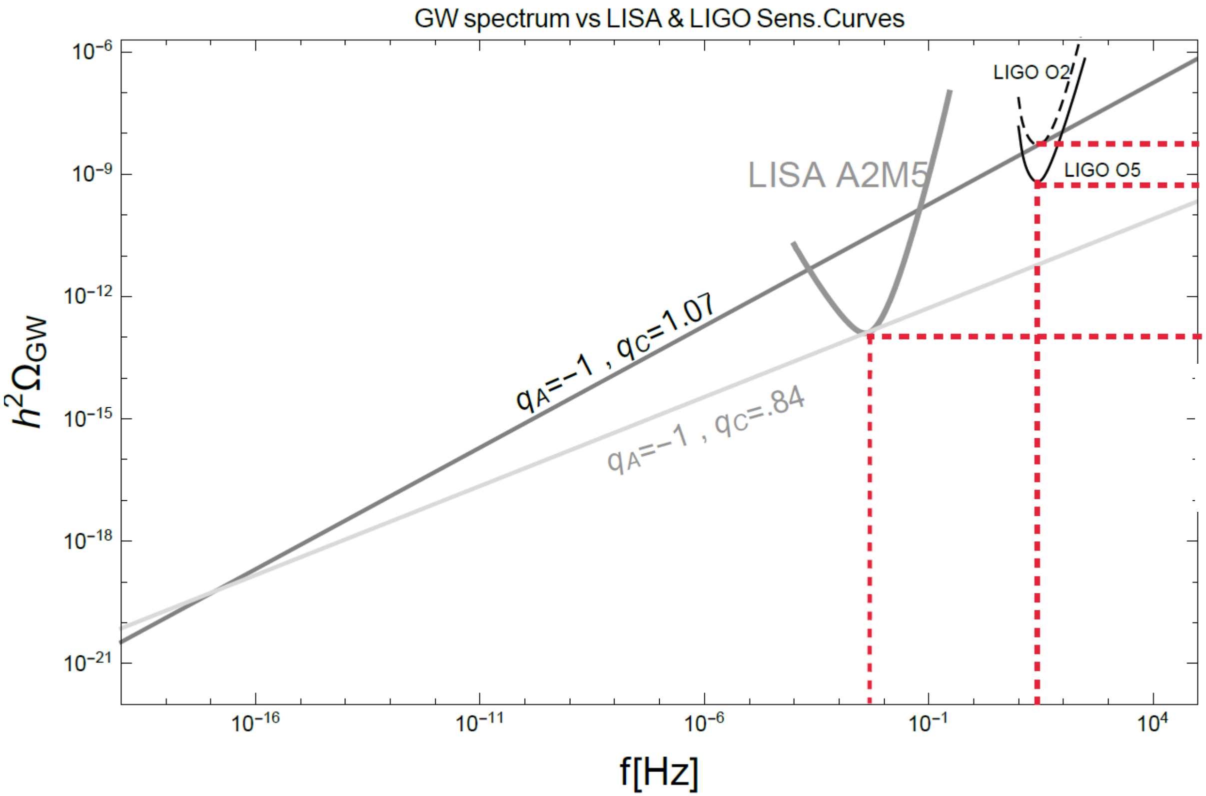
⇒ PGWs are highly blue-tilted (suppressed on large scales)

PGWs from extended supersolid inflation

Ricciardone, Tasinato `17

Hierarchy between the degree of time diffs and spatial diffs

(Nonminimal couplings between fields and gravity)



$$n_T \simeq 3 - 2\nu$$

$$\nu \equiv \sqrt{\frac{9}{4} - \frac{m^2}{H^2}},$$

$\frac{m}{H} = \mathcal{O}(1)$ is possible

with small ϵ

But we still need some enhancement mechanism for detection

Evolution of PGWs after inflation

- Graviton energy density

$$T_{\mu\nu}^{(\mathrm{GW})} = \frac{M_{\mathrm{Pl}}^2}{4}\langle \partial_\mu h_{ij} \partial_\nu h_{ij} \rangle \Longrightarrow \rho^{(\mathrm{GW})} \propto \frac{1}{2a^2}(h'_{ij})^2$$ (massless)

(analogy with scalar field) $$\Longrightarrow \rho^{(\mathrm{GW})} \propto \frac{1}{2a^2}(h'_{ij})^2 + \frac{1}{2} m^2 h_{ij}^2$$

- Massive phase

$$\rho_k^{\mathrm{GW}} \propto m^2 h_k^2 \propto a^{-2} v_k^2 \propto a^{-3}$$

decays like non-relativistic matter!!

- Massless phase

$$\rho_k^{\mathrm{GW}} \propto a^{-2} {h'_k}^2 \propto a^{-2}[(a^{-1} v_k)']^2 \propto a^{-4}$$

decays like relativistic matter (as usual)

Power spectrum of PGWs at late time

1. Inflation → **2. Mass-dominant** → **3. Massless**

1. Inflation

From BD-vacuum, GWs are produced and decay on super -horizon scales in same way as $\delta\phi_k$

Blue-tilt

$$\frac{\mathcal{P}_h}{\mathcal{P}_h^{\rm standard}} \sim \left(\frac{k}{k_{\rm UV}}\right)^{3-2\nu}$$

2. Mass-dominant

After instant re-heating, $k \ll am$ and gravitons behave as matter.

Slow decay

$$\rho_k^{\rm GW} \propto a^{-3}$$

3. Massless

At some point in RD era, gravitons lose the mass to avoid some obs. bounds.

Detection

$$\rho_k^{\rm GW} \propto a^{-4}$$

(at late time)

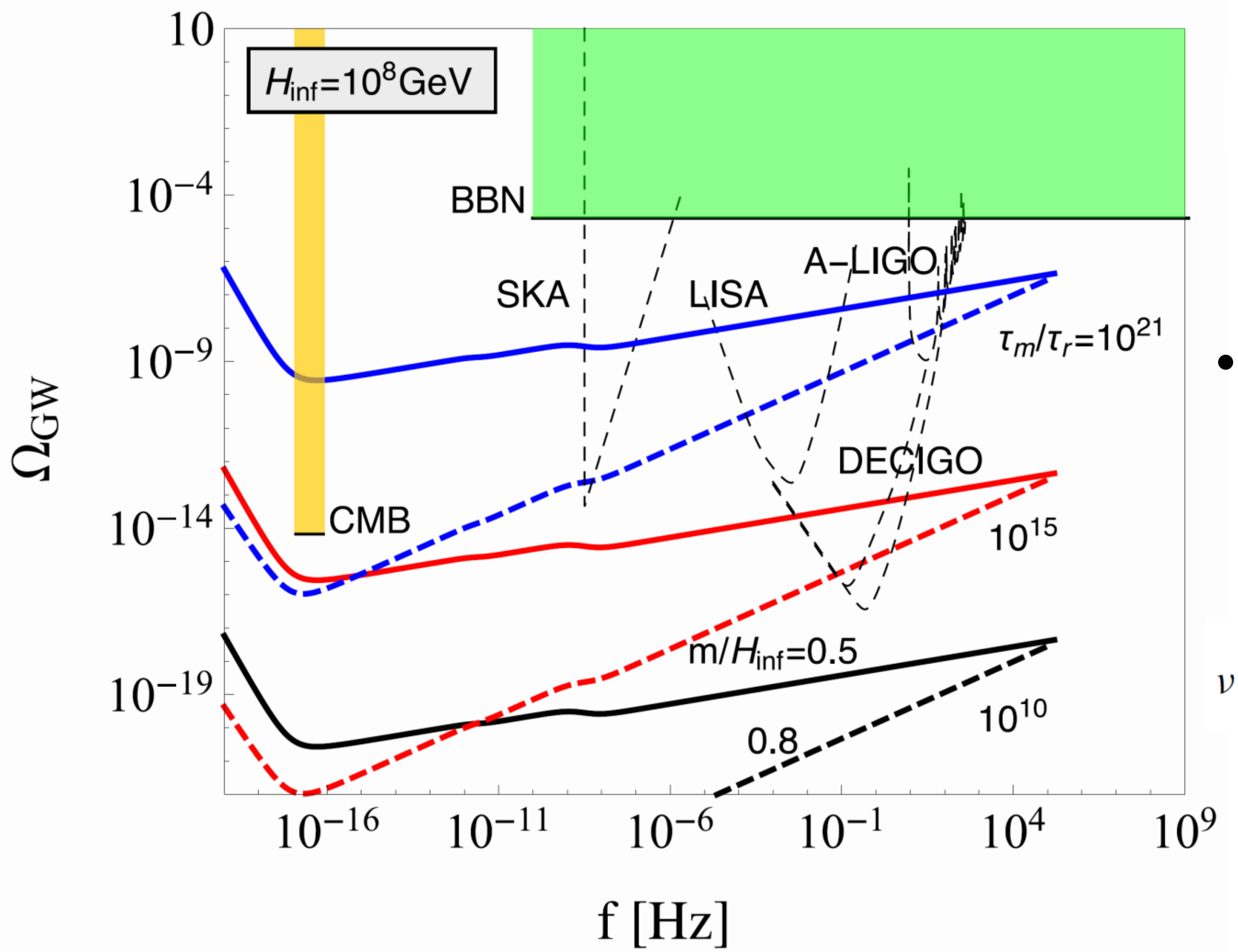
$$\mathcal{P}_h^{\rm massive} \sim \frac{\tau_m}{\tau_r}\left(\frac{k}{k_{\rm UV}}\right)^{3-2\nu} \mathcal{P}_h^{\rm standard}$$

$$\nu \equiv \sqrt{9/4 - m^2/H_{\rm inf}^2} \qquad k_{\rm UV} \equiv a_r H_{\rm inf}$$

Theoretical prediction for $\Omega_{\rm GW}$

$$\Omega_{\rm GW}(f) \simeq 10^{-15}\frac{\tau_m}{\tau_r}\left[\frac{H_{\rm inf}}{10^{14}{\rm GeV}}\right]^{\nu+\frac{1}{2}}\left[\frac{f}{2\times 10^8{\rm Hz}}\right]^{3-2\nu}$$

$f < f_{\rm UV}$



$$f_{\rm UV} = 2\times 10^8 H_{14}^{1/2}{\rm Hz}$$

$$H_{14} \equiv H_{\rm inf}/(10^{14}{\rm GeV})$$

- Constraints

$$\frac{\tau_m}{\tau_r} \lesssim 10^{10} H_{14}^{-2}, \quad ({\rm BBN})$$

$$\nu \lesssim \frac{75 - \log_{10}(H_{14}^{1/2}\tau_m/\tau_r)}{50 + \log_{10}(H_{14})}. \quad ({\rm CMB})$$

Primordial tensor non-Gaussianity

- How to distinguish scenarios with detectable PGWs ?

PGWs from vacuum fluctuations of metric are almost Gaussian

Maldacena `02

Stochastic GWs by uncorrelated astrophysical sources are also almost Gaussian (Central limit theorem)

⇒ Tensor non-Gaussianity is powerful discriminator

- Primordial tensor bispectrum

$$\langle h_{i_1j_1}(\tau,\boldsymbol{k}_1)h_{i_2j_2}(\tau,\boldsymbol{k}_2)h_{i_3j_3}(\tau,\boldsymbol{k}_3)\rangle \equiv (2\pi)^3\delta(\boldsymbol{k}_1+\boldsymbol{k}_2+\boldsymbol{k}_3)B^h_{i_1j_1i_2j_2i_3j_3}(k_1,k_2,k_3)$$

Depending on amplitude, shape of triangle, and chiralities

$$\langle h_{i_1j_1}(\tau,\boldsymbol{k}_1)h_{i_2j_2}(\tau,\boldsymbol{k}_2)h_{i_3j_3}(\tau,\boldsymbol{k}_3)\rangle = i\int_{-\infty}^{\tau} \mathrm{d}\eta \langle [H_{\rm int}(\eta), h_{i_1j_1}(\tau,\boldsymbol{k}_1)h_{i_2j_2}(\tau,\boldsymbol{k}_2)h_{i_3j_3}(\tau,\boldsymbol{k}_3)]\rangle$$

Shape of tensor bispectrum

- Interaction Hamiltonian at third order

$$H_{\rm int} = H_{\rm int}^{\rm (GR)} + H_{\rm int}^{\rm (mass)}$$

$$H_{\rm int}^{\rm (GR)} = -\frac{M_{\rm Pl}^2}{4} a^2 \int {\rm d}^3x\, h_{ij} h_{kl} \left(\partial_j \partial_l h_{ik} - \partial_i \partial_j \frac{1}{2} h_{kl} \right)$$

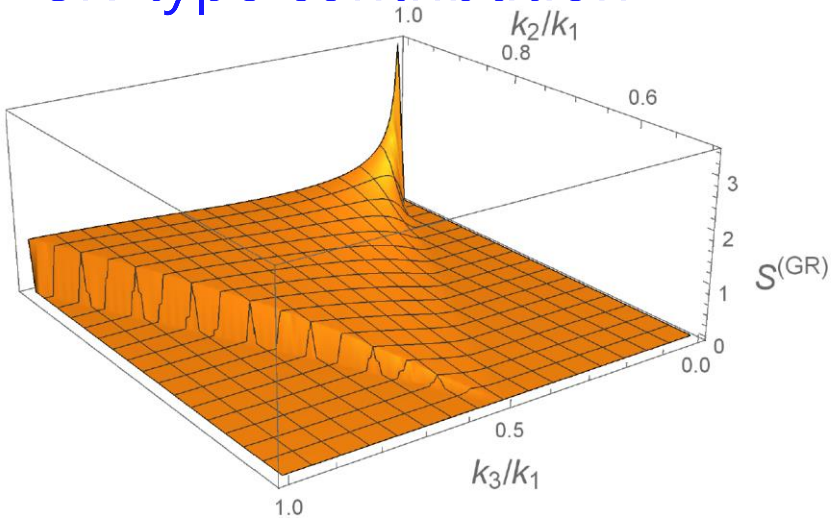
appears in GR

$$H_{\rm int}^{\rm (mass)} = -g \frac{M_{\rm Pl}^2}{4} a^4 \int {\rm d}^3x\, h_{ij} h_{jk} h_{ki}$$

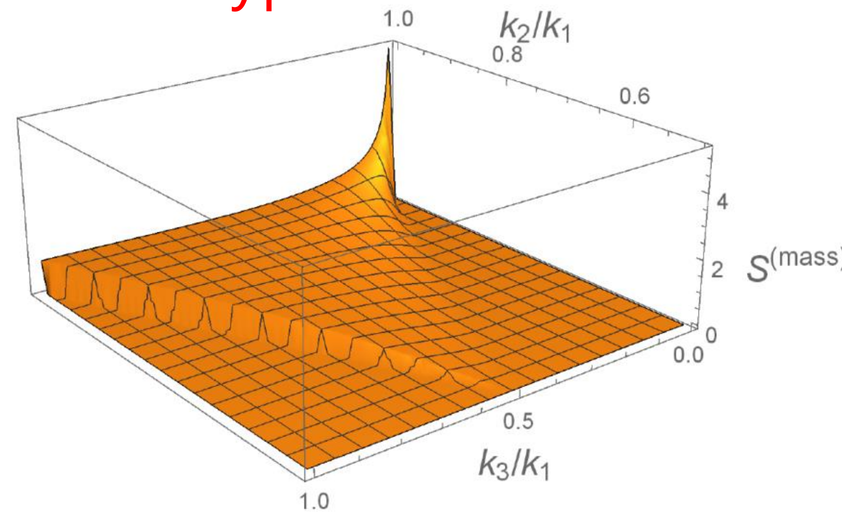
peculiar to MTMG

- ``Shape function" of tensor bispectrum ($m/H_{\rm inf} = 0.8$)

GR-type contribution



MTMG-type contribution



Both are maximized in the squeezed limit

Detectability of tensor bispectrum by LISA

- Amplitude of tensor bispectrum

$$\frac{(k_1 k_2 k_3)^2 B_h^{\rm equil}}{[\mathcal{P}_h(k_1)\mathcal{P}_h(k_2)\mathcal{P}_h(k_3)]^{1/2}} \simeq \left(\frac{g_{\rm inf}}{H_{\rm inf}^2}\right)\left(\frac{H_{\rm inf}}{10^{-3}M_{\rm Pl}}\right)$$

(``test of non-Gaussianity" for LISA) $B_h = \delta_{j_1 i_2}\delta_{j_2 i_3}\delta_{j_3 i_1} B^h_{i_1 j_1 i_2 j_2 i_3 j_3}$

Bartolo et al `18

⇨ chance for LISA to detect for $g_{\rm inf} \geq 10^{-3} H_{\rm inf} M_{\rm Pl}$

If curvature perturbation is generated by single-field inflation

$$\frac{H_{\rm inf}}{M_{\rm Pl}} = \sqrt{8\pi\epsilon\mathcal{P}_\zeta} \approx 10^{-4}\left(\frac{\epsilon}{0.1}\right)^{\frac{1}{2}}\left(\frac{\mathcal{P}_\zeta}{2\times 10^{-9}}\right)^{\frac{1}{2}} \qquad g_{\rm inf} \geq 10 H_{\rm inf}^2$$

But for models with suppressed curvature perturbation,

$g_{\rm inf}/H_{\rm inf}^2 \sim \mathcal{O}(1)$ is possible

Conclusions

- Highly blue-tilted PGWs can be detected by interferometers, even if their signal is not observed on the CMB scales

- We construct a consistent model producing highly blue-tilted and largely amplified PGWs based on MTMG

- We also calculate the non-Gaussianity of PGWs for the model and discuss the detectability by LISA

Discussions

- Squeezed limit of tensor bispectrum

Consistency Relation (CR) for adiabatic tensor perturbations

Maldacena `02

$$\lim_{\boldsymbol{q}\to 0}\langle h^{s_1}_{\boldsymbol{q}} h^{s_2}_{\boldsymbol{k}}\ h^{s_3}_{-\boldsymbol{k}}\ \rangle' = \frac{3}{2}\delta^{s_2 s_3}\mathcal{P}_h(q)\mathcal{P}_h(k)e^{s_1}_{ij}(\boldsymbol{q})\frac{k^i k^j}{k^2}$$

If CR holds, effect of superhorizon mode is unobservable

Pajer, Schmidt, Zaldarriaga `13

In solid inflation, CR breaks and there are observable effects

Bordin, Creminelli, Mirbabayi, Norea `16

$$\mathcal{P}_{h,\bar{h}}(k) = \mathcal{P}_{h,0}(k)\left(1 + \mathcal{Q}_{ij}\frac{k^i k^j}{k^2}\right) \qquad \mathcal{Q}_{ij} \propto f_{\rm NL}^{h,\rm squeezed}$$

- Relation between curvature perturbation and PGWs

Thank you very much !!