A model of dark energy spatially varying on very large scales

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Based on collaboration with

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Outline of talk

1. Introduction
2. Super-curvature mode dark energy model
3. Spatially varying property of the dark energy model
4. Observational effect on CMB anisotropies
5. Summary and conclusions

1. Introduction

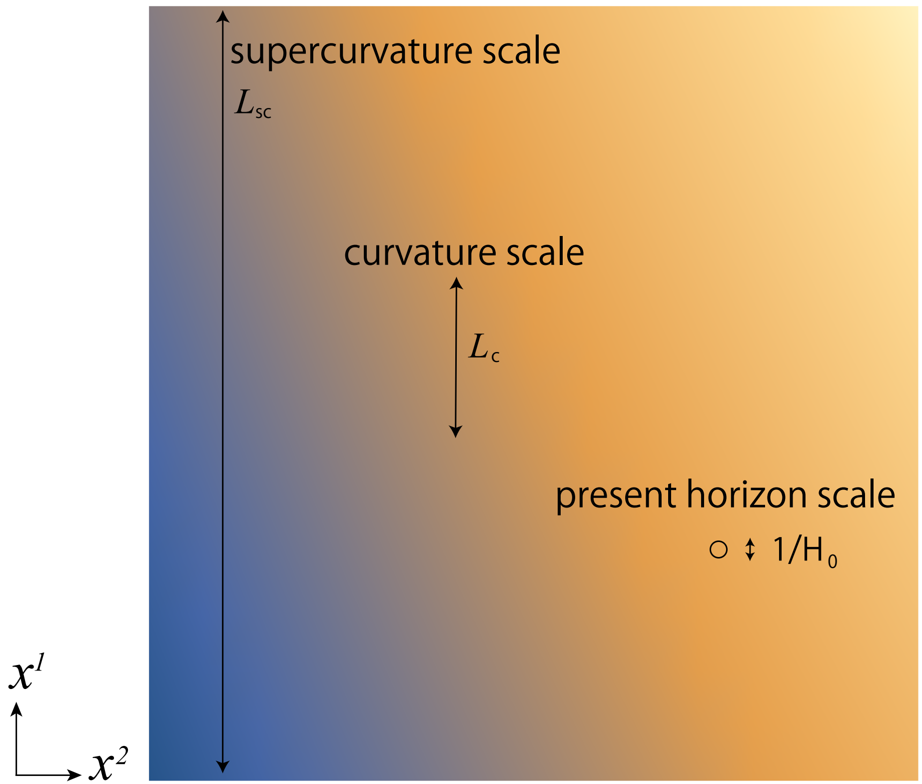
Isotropy and homogeneity of the background universe are fundamental assumptions of the standard cosmological model.

This assumption is tested with the cosmological observations, the cosmic microwave background (CMB) radiation and the large scale structure, ….

The isotropy and the homogeneity are widely accepted, but there are arguments on the large-scale anomalies in CMB, e.g., the power asymmetry, the low CMB quadrupole anomaly, and so on.

We note that the tests are limited by the fact that we only observe down the past light-cone, and limited by the redshift-evolution and finiteness of the observable region of the present horizon.

We propose a cosmological-constant-like dark energy model with inhomogeneity on very large scale.

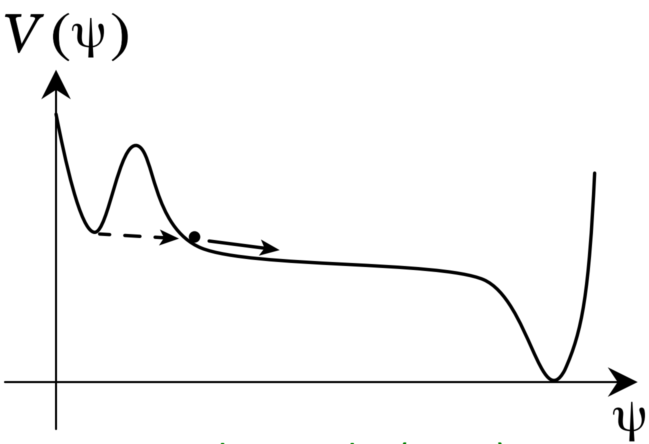


This figure shows a schematic spatial dependence of the dark energy density of the model by the color.
The dark energy has the density contrast of $\mathcal{O}(1)$ on the large scale, the super-curvature scale.
The scale of the inhomogeneity is much larger than the horizon size of our universe.

2. Super-curvature mode dark energy model

H. Aoki, S. Iso, D-S. Lee, Y. Sekino, C-P. Yeh, PRD (2018)

Assume a scalar field ϕ coupled to another scalar field ψ, where ψ induces a bubble nucleation and an open inflation.



Yamamoto, Sasaki, Tanaka (1995)
Bucher, Turok (1995)
Cf. de Sitter conjecture, Ooguri-san's talk

mass of ϕ → Hubble parameter of false vacuum inflation

If $m_A \ll H_A$

↓

super-curvature mode of ϕ is produced, remains until present without decaying.

↓

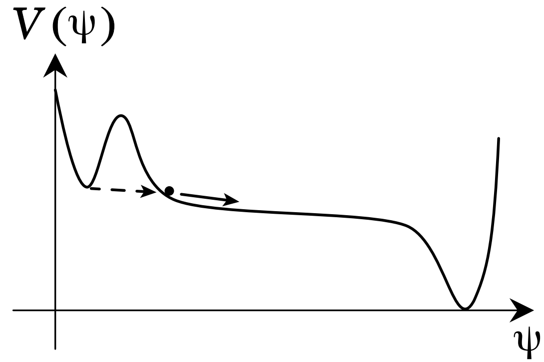
A role of dark energy, if the mass is sufficiently light in the present universe though we need a tuning of parameters.

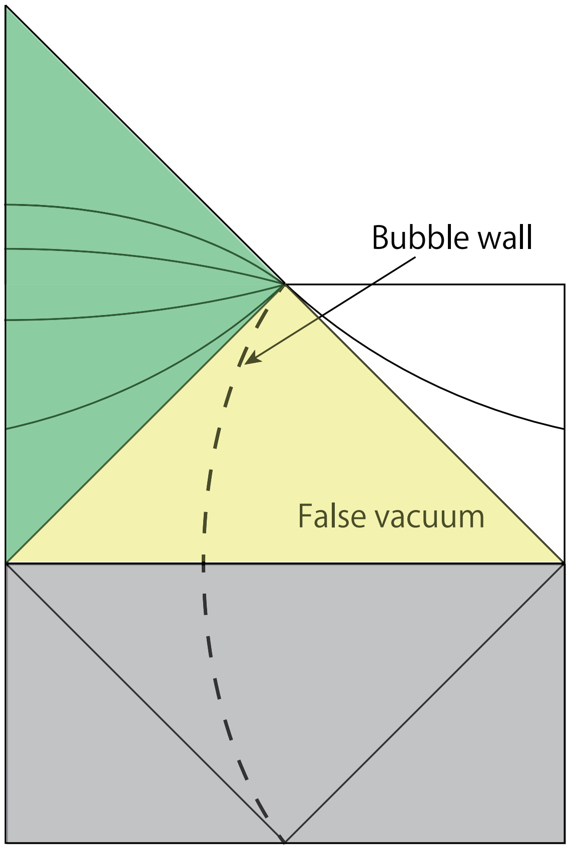
Open inflation scenario is induced by a bubble nucleation in a false vacuum

Nomura-san's talk

Tunneling process of a false vacuum in de Sitter space

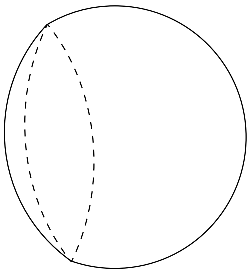
Then a slow-roll inflation follows inside the bubble





The tunneling process of the bubble nucleation is described by the Coleman De Luccia bounce solution, assuming the Euclidean metric

$$ds^2 = a_E^2(\tau)(d\tau^2 + \underbrace{d\rho^2 + \sin^2\rho d\Omega_{(2)}^2}_{\text{3-shpere line element}})$$



The bounce solution has the O(4) symmetry.

Analytic continuation of the bounce solution to Lorentzian region describes the solution of an expanding bubble.

The green region describes an **open inflation universe,** whose symmetry comes from the symmetry of the bounce solution

$$ds^2 = a^2(\eta)(-d\eta^2 + dr^2 + \sinh r d\Omega_{(2)}^2)$$

$$\tau \to \eta + \frac{\pi}{2} i$$
$$\rho \to ir$$

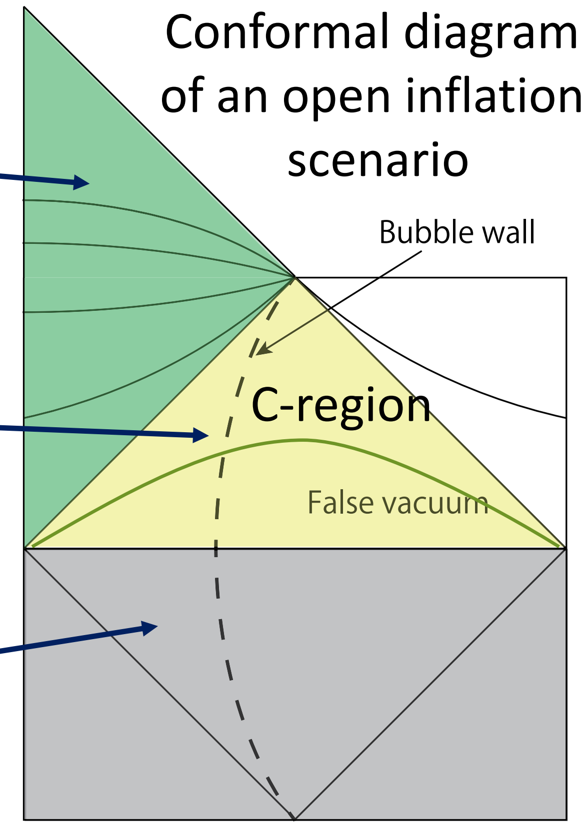
$$ds^2 = a^2(\xi)(-dr_C^2 + d\xi^2 + \cosh^2 r_C d\Omega_{(2)}^2)$$

$$\tau \to \xi$$
$$\rho \to ir_C + \frac{\pi}{2}$$

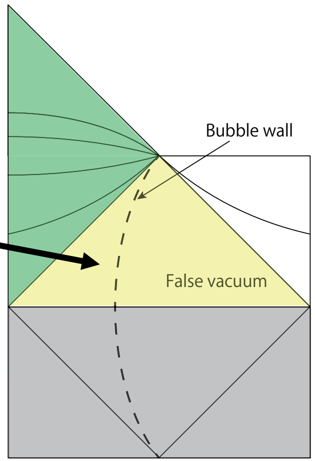
Bounce solution

$$ds^2 = a_E^2(\tau)(\underbrace{d\tau^2 + d\rho^2 + \sin^2 \rho d\Omega_{(2)}^2}_{\text{3-sphere metric}})$$

Conformal diagram of an open inflation scenario



We consider quantum field theory of a massive scalar field on the background of a bubble.



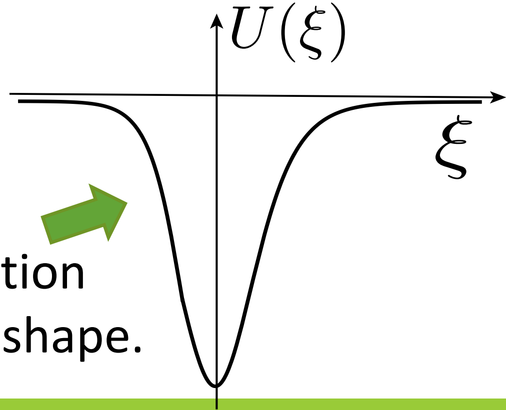
Mode expansion in the C-region

$$\phi(r_C,\xi,\Omega_{(2)}) = \underbrace{f_{p\ell}(r_C)}\frac{\chi_p(\xi)}{a(\xi)}Y_{\ell m}(\Omega_{(2)})$$

$$f_{p\ell}(r_C) = N_{p\ell}\frac{P^{-\ell-1/2}_{ip-1/2}(\cosh r_C)}{\sqrt{\sinh r_C}}$$

$$\left[-\frac{d^2}{d\xi^2} + \underbrace{\frac{a''(\xi)}{a(\xi)} + \mathcal{M}^2(\xi)a^2(\xi) - 1}\right]\chi_p(\xi) = p^2\chi_p(\xi)$$

Eigenvalue problem → modes functions



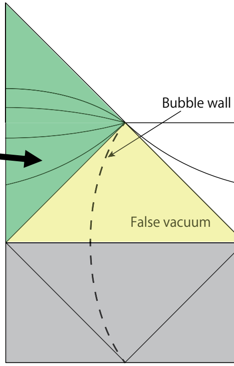
The bounce solution determines the shape.

- Continuous mode $p^2 > 0 \ (-\infty < p < \infty)$
- Discrete mode $p^2 < 0 \ (p : \text{pure imaginary})$
  $p = p_* = i|p_*|$

One discrete mode appears when $m \lesssim H$

Sasaki, Tanaka, Yamamoto (1995)

Continuation of the mode functions yields the quantum field in an open universe inside a bubble



$$\hat{\phi}(\eta, \mathbf{x}) = \underbrace{\sum_{\sigma=L,R} \sum_{\ell m} \int_0^\infty \frac{dp}{2\pi} \left[\varphi_p(\eta) f^{\sigma}_{k\ell m}(r,\Omega) \hat{a}^{\sigma}_{p\ell m} \right]}_{\text{Continuous mode}} + \underbrace{\sum_{\ell m} \varphi_{p_*}(\eta) f_{p_* \ell m}(r,\Omega) \hat{a}_{p_* \ell m}}_{\text{discrete mode}} + \text{h.c.}$$

Length scale of the discrete mode

$$L_{\rm sc} = \frac{1}{\epsilon\sqrt{-K}} \gg L_{\rm c} = \frac{1}{\sqrt{-K}} \gg 1/H_0$$

$\epsilon \ll 1$

curvature scale

The discrete mode is called super-curvature mode

$$p_* = i(1-\epsilon)$$

$$\epsilon = \mathcal{O}(1) \times \left(\frac{m_A}{H_A}\right)^2 \ll 1$$

Mass and Hubble parameter of the false vacuum inflation (Ancestor vacuum)

Evolution of the super-curvature mode is different from those of the continuous modes.

$$\varphi_p(\eta) \propto \frac{e^{-ip\eta}}{a(\eta)}$$ → The continuous modes decay

$$\varphi_{p_*}(\eta) \propto e^{-\epsilon\eta} \sim \text{constant}$$ → The super-curvature mode freezes

$$\epsilon \ll 1$$

Amplitude of the super-curvature mode fluctuations is $\langle \phi^2 \rangle \sim \mathcal{O}(1)\frac{H_A^4}{m_A^2}$

If the mass is sufficiently light in the present universe $m_A \to m_0 \sim H_0$

Potential energy of the super-curvature mode can explain the dark energy

$$\rho_{V(\phi)} \simeq \frac{m_0^2}{2}\langle \phi^2 \rangle \sim M_{\rm Pl}^2 H_0^2$$ ← $$\frac{M_{\rm Pl}}{H_A} \sim \frac{H_A}{m_A}$$

H. Aoki, S. Iso, D-S. Lee, Y. Sekino, C-P. Yeh, (2018)

3. Spatially varying property of super-curvature model

Energy density

$$\rho_{DE}(\eta, \mathbf{x}) \simeq \frac{m_0^2}{2}\phi^2(\eta, \mathbf{x})$$

Density contrast

$$\delta_{DE}(\eta, \mathbf{x}) = \frac{\rho_{\rm DE}(\eta, \mathbf{x}) - \langle \rho_{\rm DE}(\eta, \mathbf{x})\rangle}{\langle \rho_{\rm DE}(\eta, \mathbf{x})\rangle}$$

Correlation function

$$\xi_{DE}(R) \equiv \langle \delta_{DE}(\eta, \mathbf{x})\delta_{DE}(\eta, \mathbf{y})\rangle$$

$$= \frac{2\langle \phi(\eta, \mathbf{x})\phi(\eta, \mathbf{y})\rangle^2}{\langle \phi^2(\eta, \mathbf{x})\rangle^2}$$

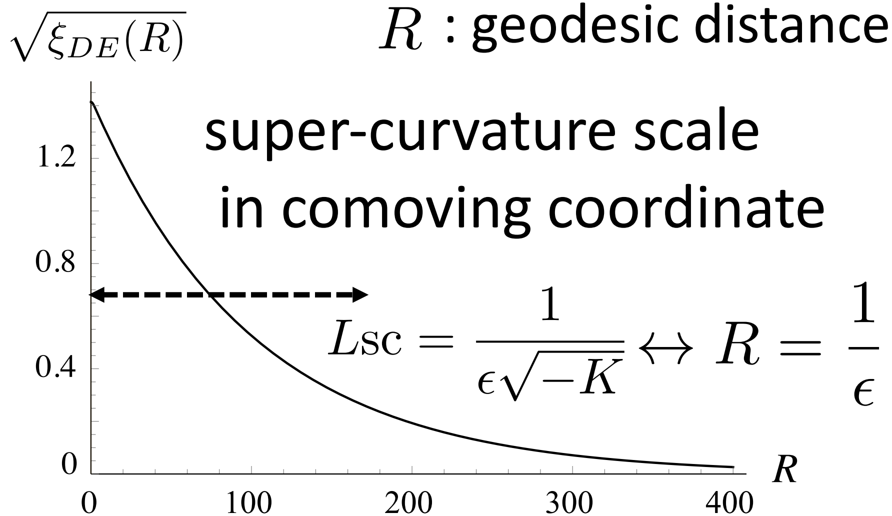
$$= 2\left(\frac{\sinh(1-\epsilon)R}{(1-\epsilon)\sinh R}\right)^2$$

$$\simeq 2 \times \begin{cases} 1 & \text{for} \quad R \ll 1 \\ e^{-2\epsilon R} & \text{for} \quad R \gg 1 \end{cases}$$

Two point function of the super-curvature mode

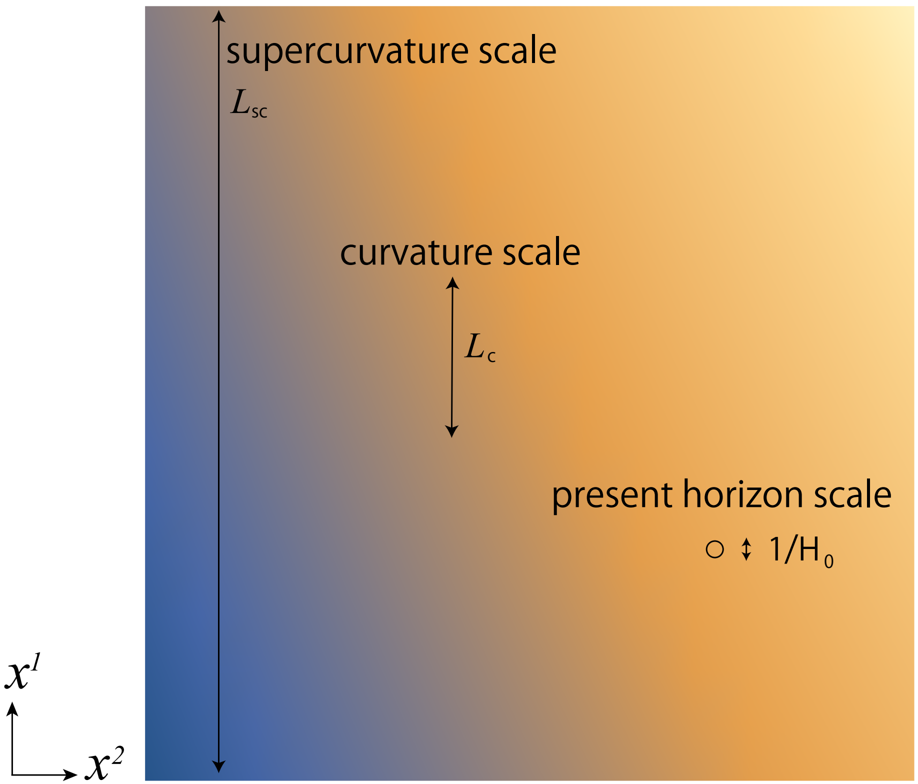
$$\langle \phi(\eta, \mathbf{x})\phi(\eta', \mathbf{x}')\rangle = \varphi(\eta)\varphi(\eta')\frac{\sinh(1-\epsilon)R}{(1-\epsilon)\sinh R}$$

R : geodesic distance



Density contrast of dark energy has inhomogeneities of the O(1) on the scale of the super-curvature.

Schematic of dark energy density: the density contrast has an inhomogeneity of the O(1) on the scale of the super-curvature.



Dark energy density is almost constant within the horizon of our universe,

→ Probability distribution function of dark energy density

The large scale inhomogeneities of dark energy is an interesting feature of the model.

Observational signature of the inhomogeneity of dark energy on very large scales ?

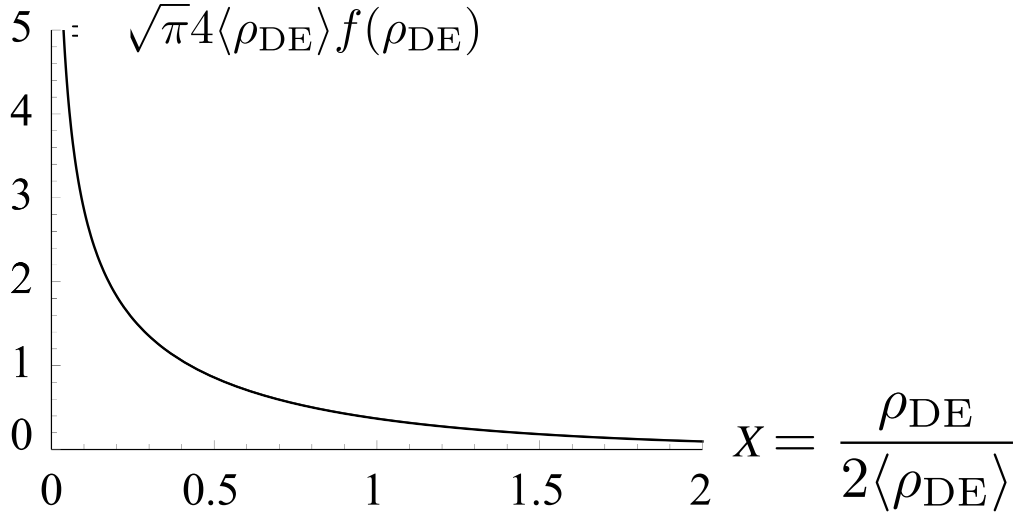
(cf. Kanno, Sasaki, Tanaka 2014)

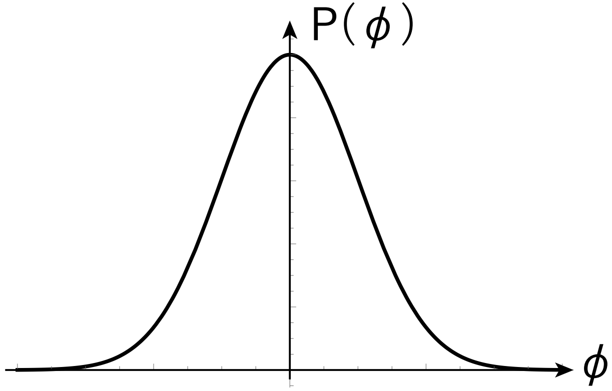
One point probability distribution function for the dark energy density

$$\rho_{DE}(\eta, \mathbf{x}) \simeq \frac{m_0^2}{2}\phi^2(\eta, \mathbf{x})$$

$$f(\rho_{\rm DE}) = \int_{-\infty}^{\infty} d\phi\, \delta\left(\rho_{\rm DE} - \frac{m_0^2}{2}\phi^2\right) P(\phi)$$

ϕ follows the Gaussian probability distribution





Wide distribution of the dark energy density

4. Observational effect on CMB anisotropy

Inhomogeneities of dark energy may imprint observational signature.

Newtonian gauge

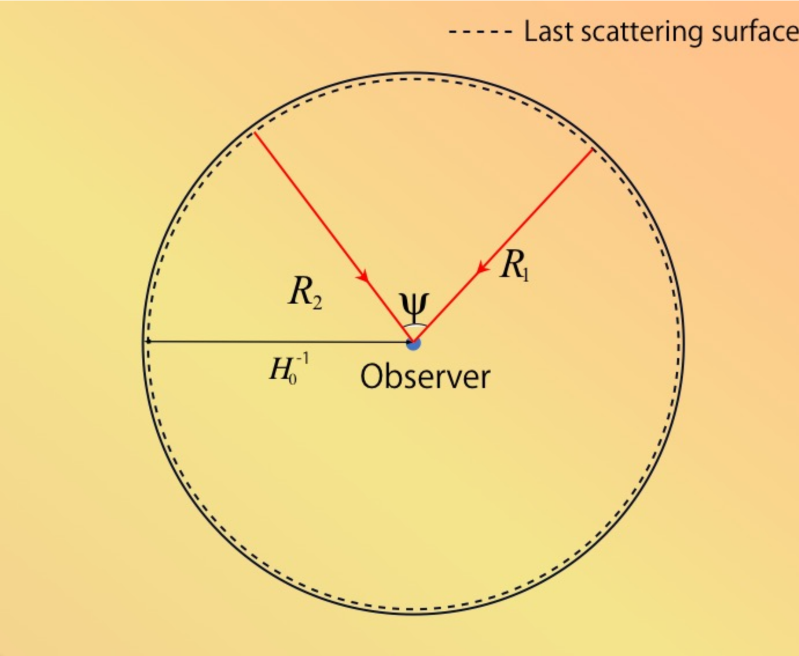
$$ds^2 = a^2(\eta)\left[-(1+2\Psi)d\eta^2 + (1+2\Phi)\gamma_{ij}dx^i dx^j\right]$$

$$\gamma_{ij}dx^i dx^j = d\chi^2 + \left(\frac{\sinh\sqrt{-K}\chi}{\sqrt{-K}}\right)^2 (d\theta^2 + \sin^2\theta d\varphi^2)$$

Boltzmann equation for photons

→ Integrated Sachs-Wolfe effect

$$\frac{\Delta T}{T}(\gamma) = \int_{\eta_*}^{\eta_0} d\eta \left(\frac{\partial\Psi(\eta,\chi,\gamma)}{\partial\eta} - \frac{\partial\Phi(\eta,\chi,\gamma)}{\partial\eta}\right)_{\chi=\eta_0-\eta}$$



We assume no initial curvature perturbation of super-curavature mode,

→isocurvature dark energy density perturbations of super-curvature scale.

Evolution of the gravitational potentials for the super-curvature mode

$$k_*^2 \simeq -K\epsilon \ll -K \ll \mathcal{H}^2$$

super-curvature mode has long-length scale than curvature and horizon $\mathcal{H} = \frac{\dot{a}}{a}$

Einstein equation

$$\delta G^0{}_0 \simeq \frac{2}{a^2}\left[3\mathcal{H}^2\Psi - 3\mathcal{H}\dot{\Phi}\right] = 8\pi G\left(\delta T^0{}_{0(\phi)} + \delta T^0{}_{0(\mathrm{m})}\right)$$

$$\Psi + \Phi \simeq 0$$

Dark matter perturbations

$$\delta_{\mathrm{m}}(\eta) + 3\Phi(\eta) \simeq 0$$

$$6\frac{\mathcal{H}}{a^2}\dot{\Psi} + \left(6\frac{\mathcal{H}^2}{a^2} + 24\pi G\rho_m\right)\Psi = 8\pi G\delta T^0{}_{0(\phi)}$$

Solution →

$$\Psi(\eta,\chi,\vec{\gamma}) = \frac{1}{F(\eta)}\int_{\eta_*}^{\eta} d\eta_1 \frac{8\pi G F(\eta_1)}{B(\eta_1)}\delta T^0{}_{0(\phi)}(\eta_1,\chi,\vec{\gamma})$$

$$F(a) = \frac{a^{5/2}}{\sqrt{\Omega_m + (1-\Omega_m)a^3}} \qquad B(\eta) = \frac{6H_0}{a}\sqrt{\frac{\Omega_m}{a^3} + 1 - \Omega_m}$$

$$\delta T^0{}_0(\eta,\chi\vec{\gamma}) = -\frac{m_0^2}{2}\left(\phi^2(\eta,\chi\vec{\gamma}) - \phi^2(\eta,\chi\vec{\gamma})\big|_{\chi=0}\right)$$

Two point function of the CMB temperature anisotropy due to ISW effect

$$\left\langle \frac{\Delta T}{T}(\vec{\gamma})\frac{\Delta T}{T}(\vec{\gamma}')\right\rangle = \int_0^{\eta_0} d\eta_1 \int_0^{\eta_0} d\eta_2 \left[\frac{\partial}{\partial \eta_1}\frac{1}{F(\eta_1)}\int_0^{\eta_1} d\eta_3 \frac{8\pi G m_0^2 F(\eta_3)}{B(\eta_3)}\right]\left[\frac{\partial}{\partial \eta_2}\frac{1}{F(\eta_2)}\int_0^{\eta_2} d\eta_4 \frac{8\pi G m_0^2 F(\eta_4)}{B(\eta_4)}\right]$$

$$\times\left\langle \left(\phi(\eta_3,\chi_3,\vec{\gamma})^2-\phi(\eta_3,0)^2\right)\left(\phi(\eta_4,\chi_4,\vec{\gamma}')^2-\phi(\eta_4,0)^2\right)\right\rangle\Big|_{\chi_3=\eta_0-\eta_3,\chi_4=\eta_0-\eta_4}.$$

$$\langle \phi(\eta,\mathbf{x})\phi(\eta',\mathbf{x}')\rangle = \varphi(\eta)\varphi(\eta')\frac{\sinh(1-\epsilon)R}{(1-\epsilon)\sinh R}$$

$$= \frac{1}{4\pi}\sum_\ell (2\ell+1)C_\ell P_\ell(\cos\psi)$$

$$C_\ell = \alpha_\ell S_\ell^2$$

$$S_\ell = \int_0^1 da\left(\sqrt{-K}(\eta_0-\eta(a))\right)^\ell \frac{\partial}{\partial a}\left(\frac{1}{F(a)}\int_0^a da' \frac{8\pi G\rho_{\rm DE}(a')F(a')}{3a'H^2(a')}\right)$$

$$\alpha_1 = \frac{32\pi}{9} \qquad \alpha_2 = \frac{32\pi}{75} \quad \cdots$$

$$\begin{cases} C_1 \simeq 0.14\times\epsilon\Omega_K \\ C_2 \simeq 0.01\times\epsilon\Omega_K^2 \\ C_\ell \simeq \mathcal{O}(\epsilon\Omega_K^\ell)\ (\ell\geq 3)\end{cases}$$

For $\epsilon \ll 1,\ \Omega_K \ll 1$, dipole and quadrupole are the components of possible observational signature.

Comparison with observations puts a constraint on the model parameters

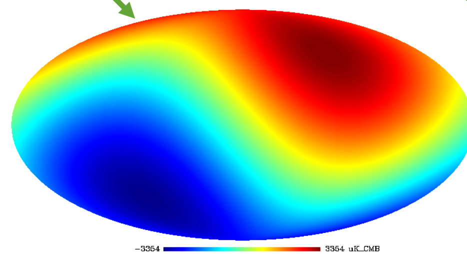
CMB temperature anisotropies

Dipole and quadrupole

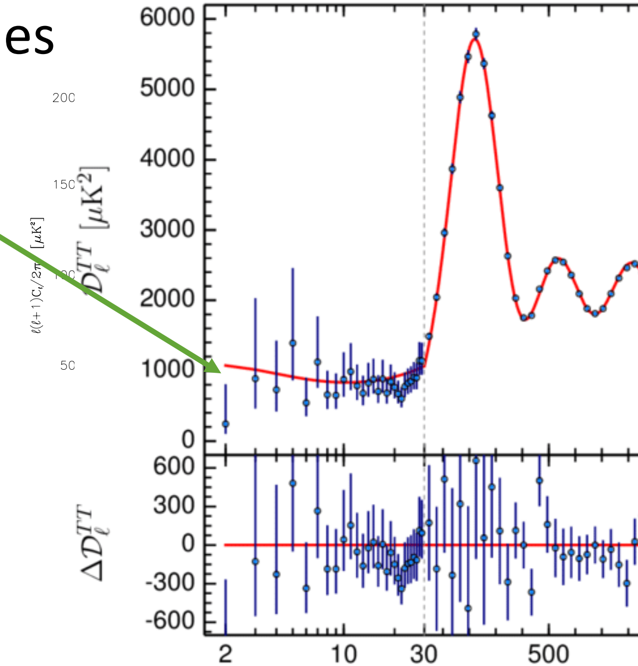
$$C_1^{\rm obs} \simeq 6.3 \times 10^{-6}$$

$$C_2^{\rm obs} \lesssim 1.2 \times 10^{-10}$$

$$\begin{cases} \epsilon \Omega_K \lesssim 4.9 \times 10^{-5} \\ \epsilon \Omega_K^2 \lesssim 1.2 \times 10^{-8} \end{cases}$$



$$\frac{\Delta T_{\rm Dipole}}{T} \sim \frac{v}{c}$$



Constraint on the spatial curvature

$$\Omega_K \lesssim 10^{-2} \sim 10^{-3}$$ (Planck collaboration)

$$\Omega_K = 10^{-3}$$

$$\epsilon \lesssim 10^{-2} \qquad \epsilon = \mathcal{O}(1) \times \left(\frac{m_A}{H_A}\right)^2 \ll 1$$

Future Discussion necessary to check predictions of the model

e.g., Yamauchi, Aoki, SIso, Lee, Sekino, Yeh (arXiv:1807.07904)

Other possible deviation from a cosmological constant model

$$\rho_{DE}(\eta, \mathbf{x}) = \frac{1}{2a^2}(\underbrace{\dot{\phi}^2}_{\mathcal{O}(\epsilon^2)} + \underbrace{\nabla\phi\nabla\phi}_{\mathcal{O}(\epsilon)}) + \frac{m_0^2}{2}\phi^2(\eta, \mathbf{x})$$

$$P_{DE}(\eta, \mathbf{x}) = \frac{1}{2a^2}(\overbrace{\dot{\phi}^2}^{\mathcal{O}(\epsilon^2)} - \overbrace{\nabla\phi\nabla\phi}^{\mathcal{O}(\epsilon)}) - \frac{m_0^2}{2}\phi^2(\eta, \mathbf{x})$$

→ Equation of state can be $w \neq -1$

We need check further constraint, e.g., from the tensor perturbations and the large scale structure.

Tanaka, Sasaki (1997), Yamauchi et al. (2011)

5. Summary and Conclusions

Dark energy model of the super-curvature mode of a scalar field, induced in an open inflation scenario.

Spatially varying properties on the very large (super-curvature) scales

Possible signature in CMB large-angle anisotoropies through the ISW effect

Observational signature may appear in the dipole and quadrupole in CMB

Future investigation is necessary

Equation of state parameter can deviate from that of a cosmological constant

Cosmological consequences of the large scale inhomogeneity of dark energy on the structure formation

Constraint on the model from the tensor modes