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YUKAWA INSTITUTE FOR THEORETICAL PHYSICS

Rotating quantum black holes and ring-down gravitational waves

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Quantum BHs

Thermal excitation of quantum fields around a BH

→ Hawking radiation

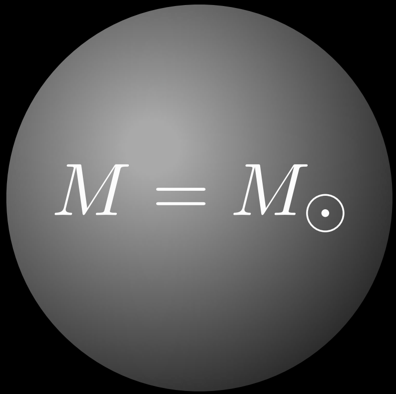
→ BH information loss paradox

→ stretched horizons, quantized BH area etc…

Quantum BHs has been discussed mainly in the theoretical side.
There are several predictions & conjectures in the literature.

How we can observationally probe the quantum properties of BH ?

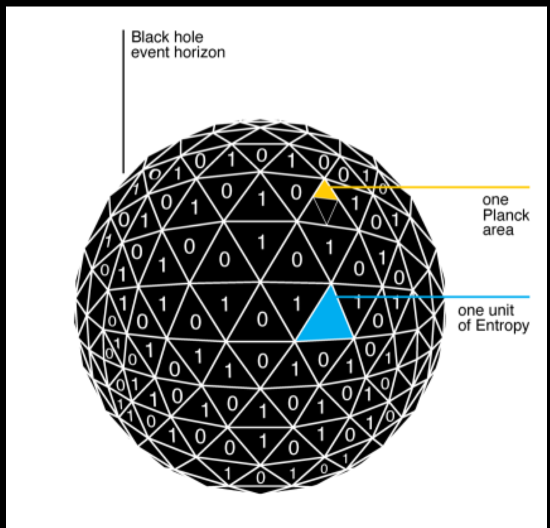
How about Hawking radiation ?



$$T_H \sim 10^{-6}\ \mathrm{K} \ll T_{\mathrm{CMB}} \simeq 2.7\ \mathrm{K}$$

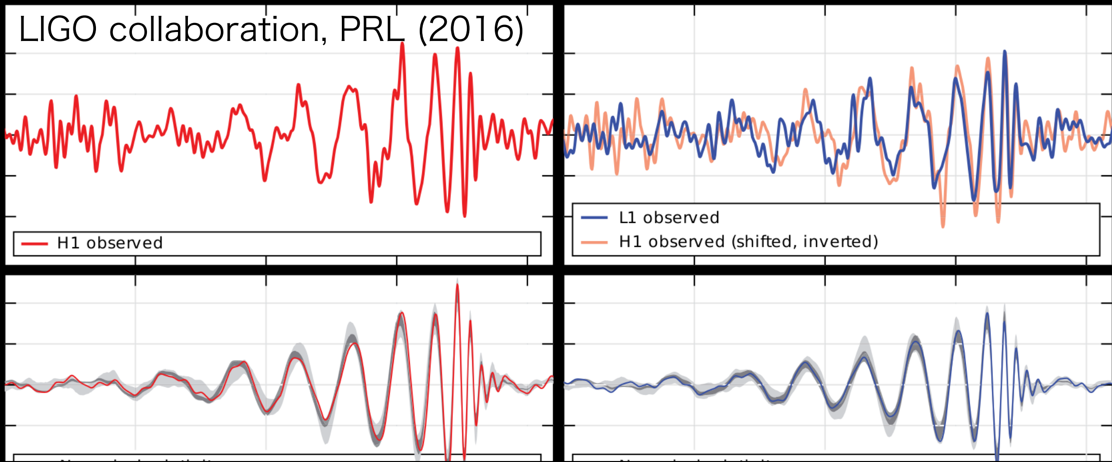
How about the Planck size structure of space?

To reach the Planck scale with a particle accelerator,
its size should be comparable to the radius of the Galaxy.



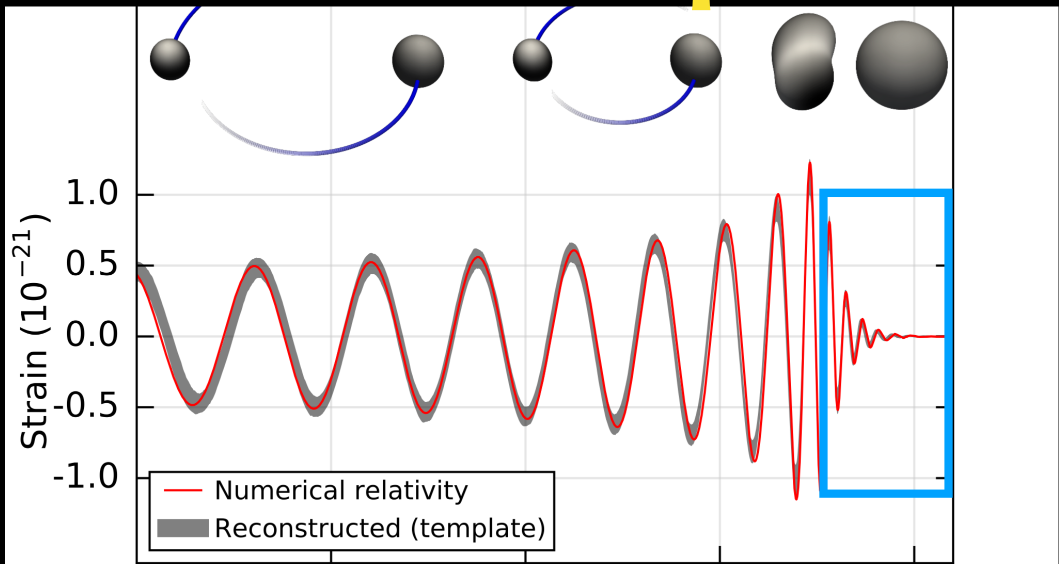
Ringdown GWs tell us about the horizon structure.

GWs from a BH binary



GW150914
(the first detection of GWs by LIGO)

ringdown GWs may be useful to test quantum gravity

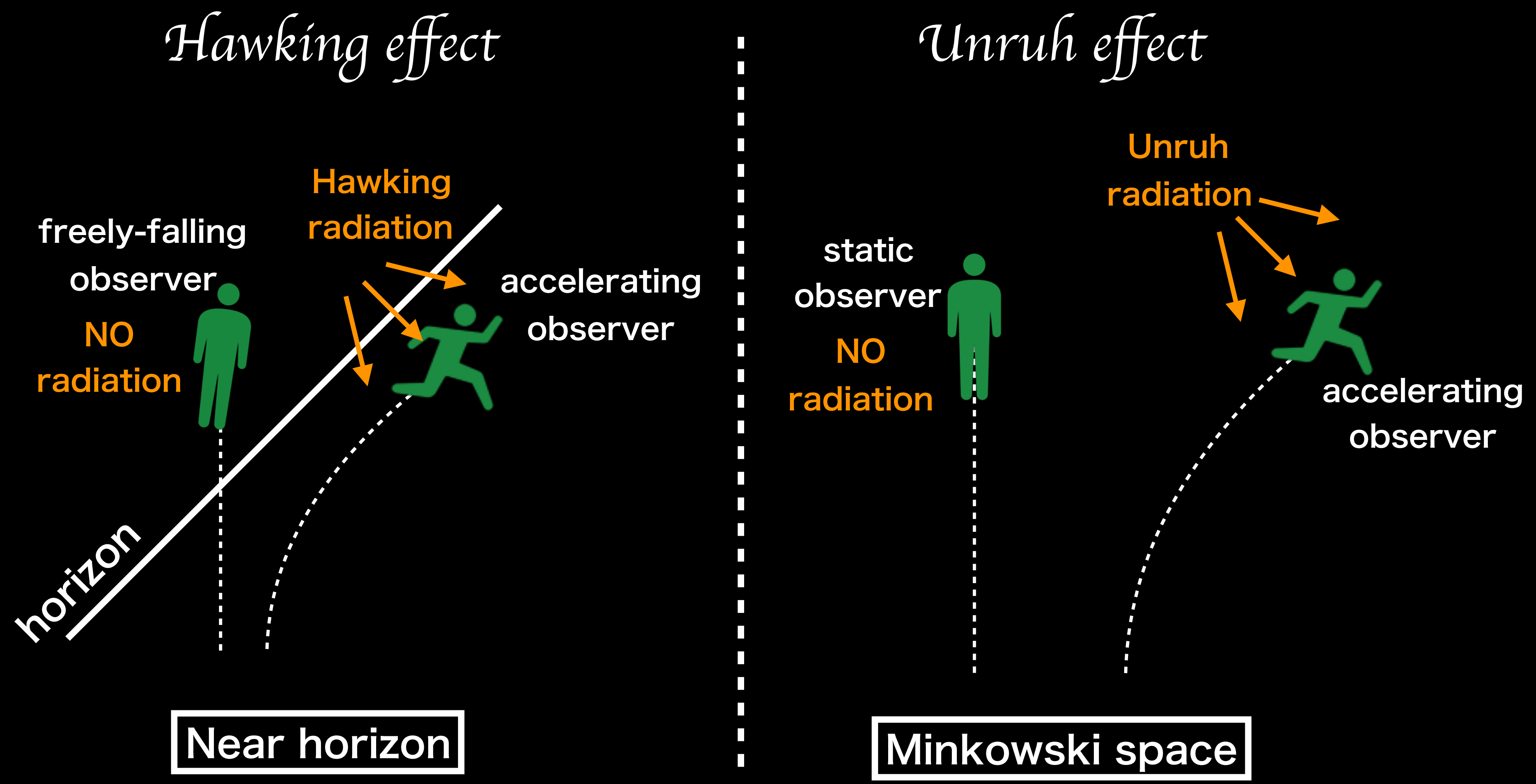


- QNMs depends only on its mass and angular momentum

↓

observation of ringdown GWs is useful to test gravity theories.

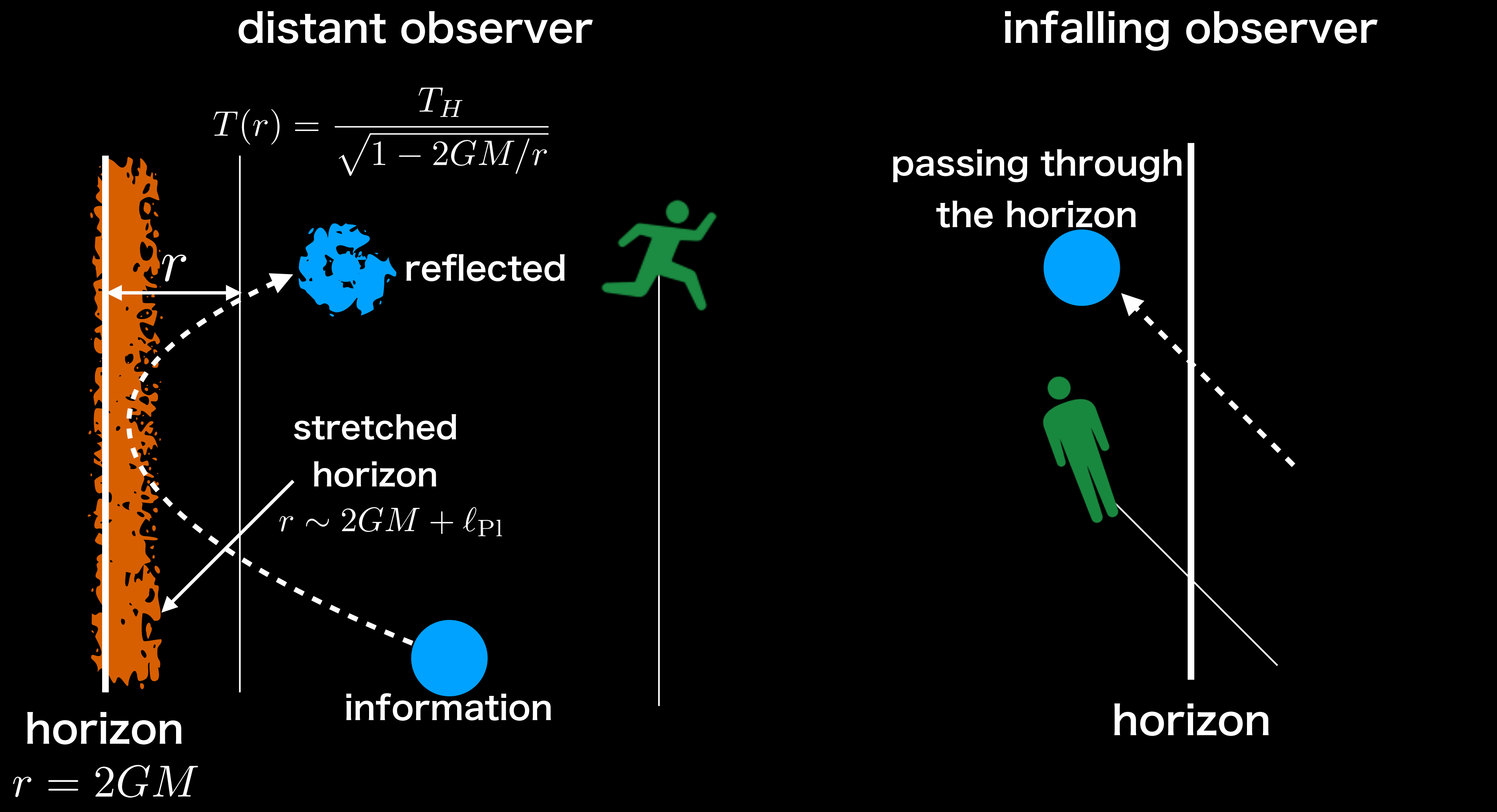
Hawking radiation is observer-dependent.



Black hole complementarity

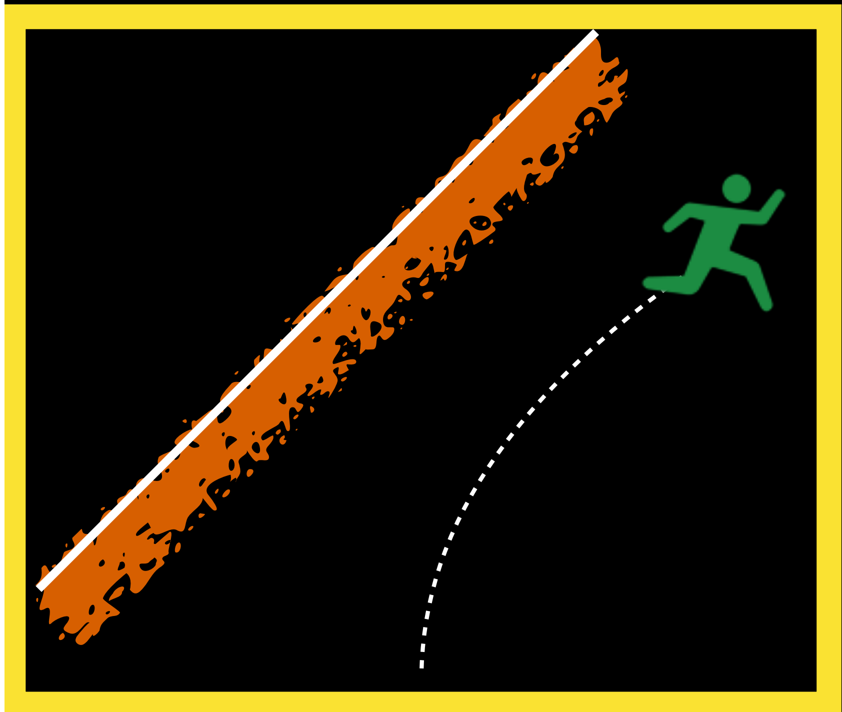
$$T_H = \frac{1}{8\pi GM}$$

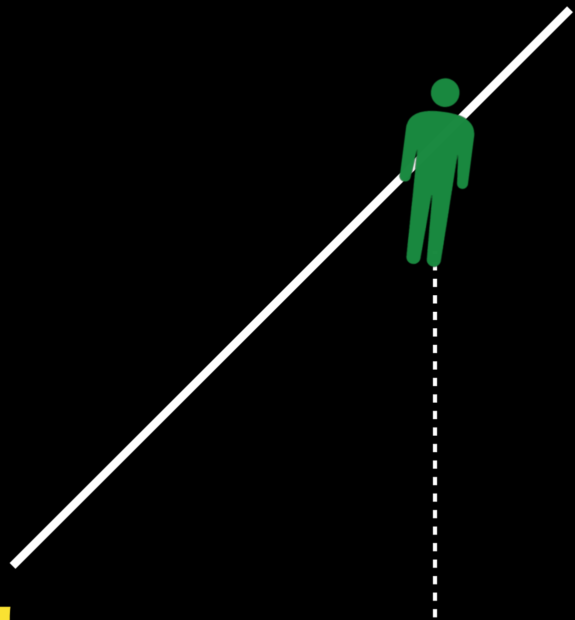
Susskind+ (1993)



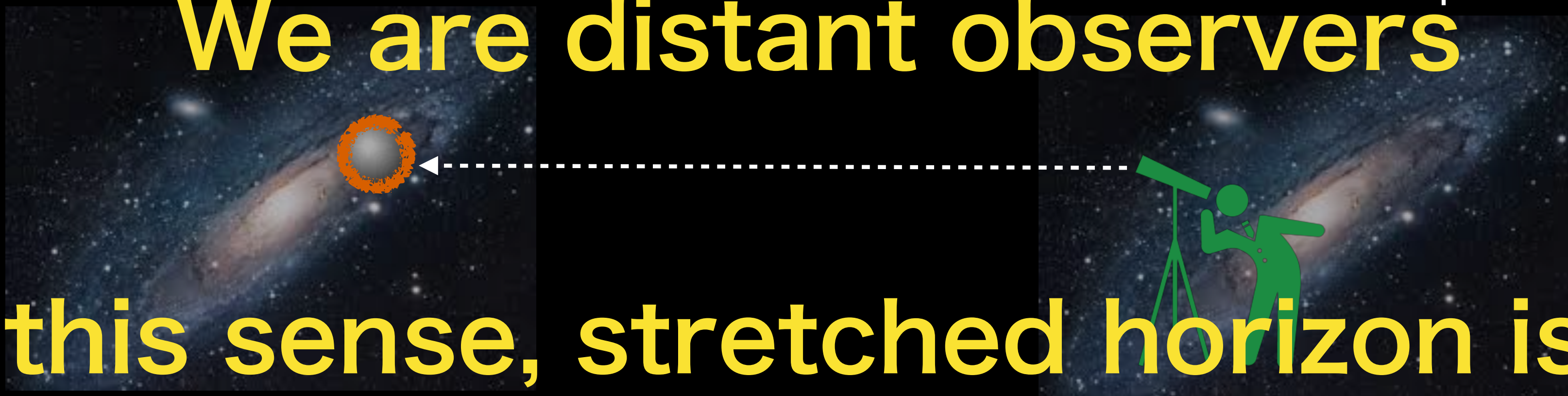
stretched horizon is observer-dependent

Susskind+ (1993)





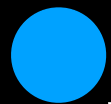
We are distant observers



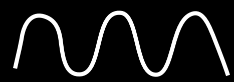
In this sense, stretched horizon is real !!

Fate of the ingoing GWs

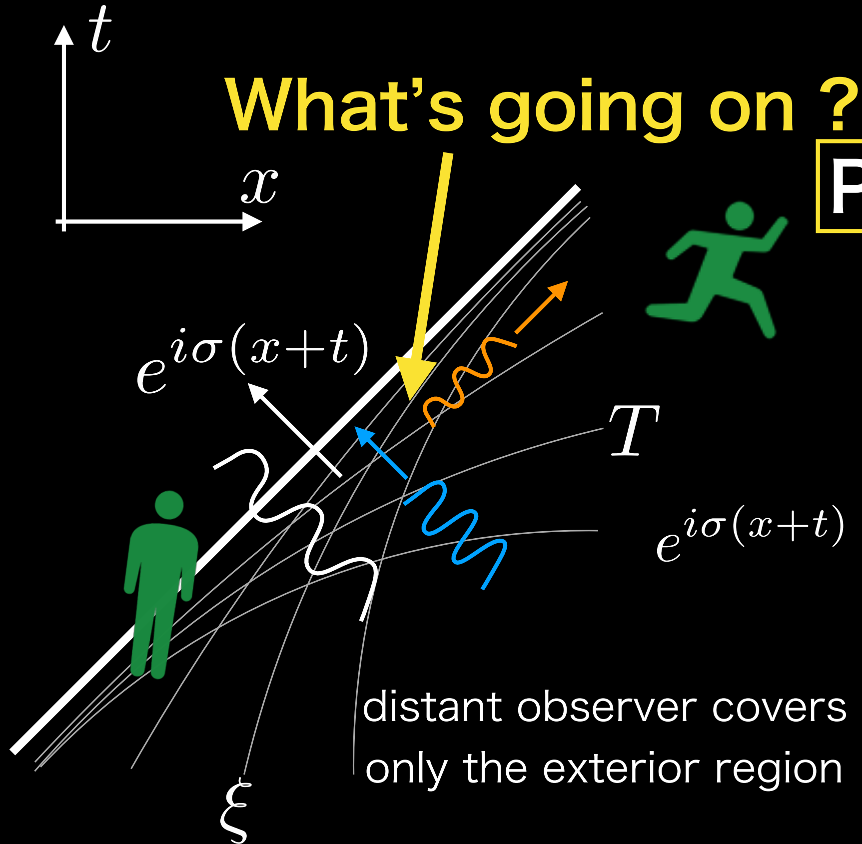
Let us suppose



information =



gravitational waves (GWs)



Near horizon

Picture for a distant observer

Rindler coordinates

$$x = \frac{1}{\kappa} e^{\kappa\xi} \cosh \kappa T \qquad t = \frac{1}{\kappa} e^{\kappa\xi} \sinh \kappa T$$

$$e^{i\sigma(x+t)} = \underline{e^{-\pi\omega/(2\kappa)} A e^{i\omega(\xi - T)}} + \underline{e^{\pi\omega/(2\kappa)} A^* e^{-i\omega(\xi+T)}}$$

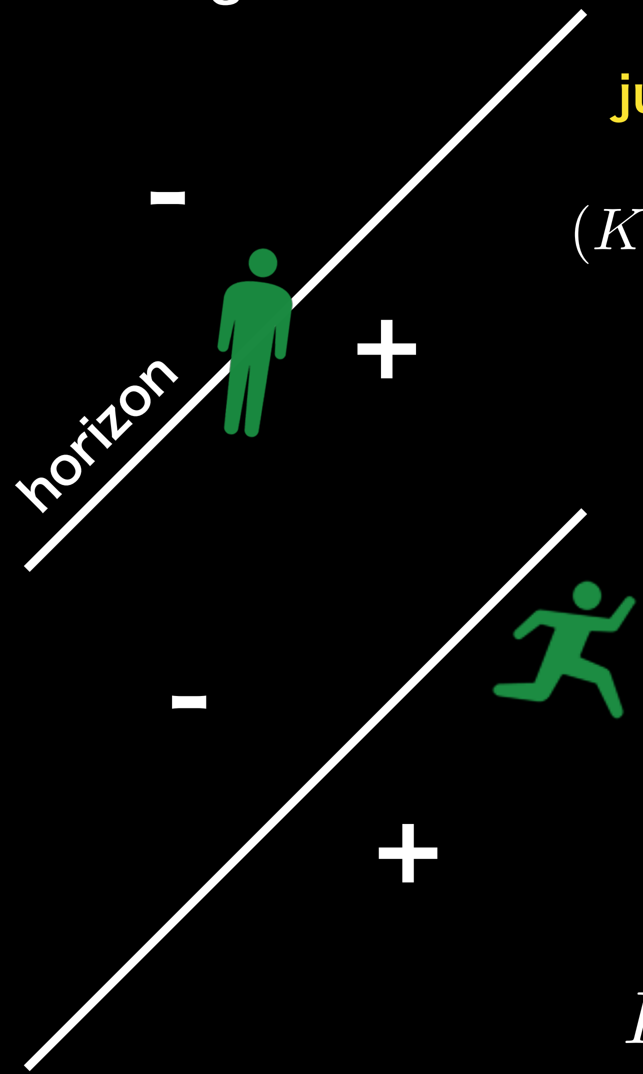
reflection rate = $e^{-\omega/T_H}$

Boltzmann reflection rate!!!

membrane paradigm

K. Thorne+ (1986)

According to a distant observer, BH horizons behave like **viscous fluid**.



junction between the interior and exterior

$$(K_+^{AB} - K_+ h^{AB}) - (K_-^{AB} - K_- h^{AB}) = 8\pi T^{AB} = 0$$

gravity (near horizon)

$$T^A_B = \frac{1}{8\pi}(K_+{}^A{}_B - K_+ h^A_B) = \frac{1}{8\pi}\left(-\sigma^A_B + \delta^A_B\left(\frac{\theta}{2} + g\right)\right)$$

shear, expansion, surface gravity

viscous fluid

$$T^A_B = P\delta^A_B - 2\eta\sigma^A_B - \zeta\theta\delta^A_B$$

$$P \leftrightarrow \frac{g}{8\pi} \qquad \eta \leftrightarrow \frac{1}{16\pi} \qquad \zeta \leftrightarrow -\frac{1}{16\pi}$$

pressure, shear viscosity, volume viscosity

membrane paradigm

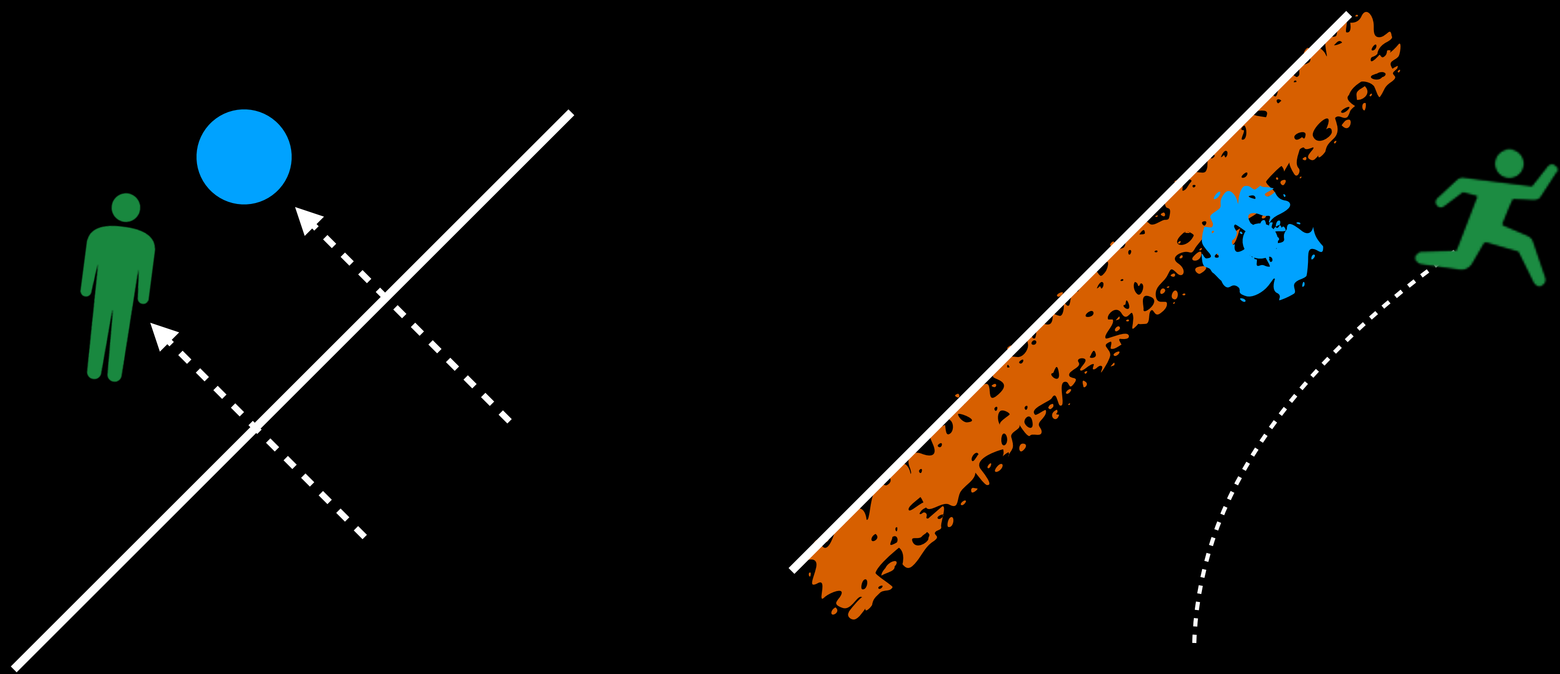
K. Thorne+ (1986)

According to an infalling observer,

information causally disappears.

According to a distant observer,

information is dissipated due to the viscosity.



Modeling viscous membrane

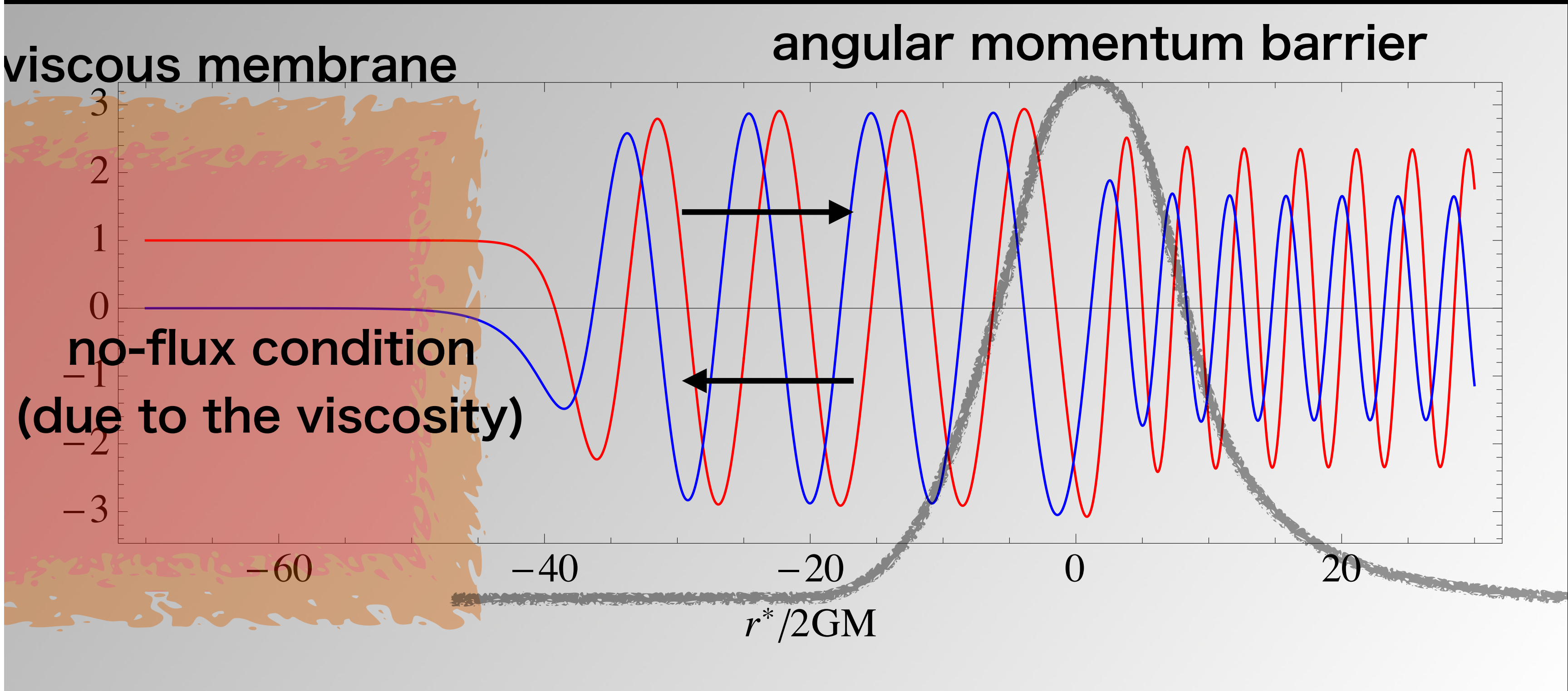
viscosity

blue-shifted frequency

Planck energy

classical general relativity

$$\left[\frac{d^2}{dr^{*2}} + \omega^2 - V_\ell(r^*)\right]\psi_\omega = 0$$



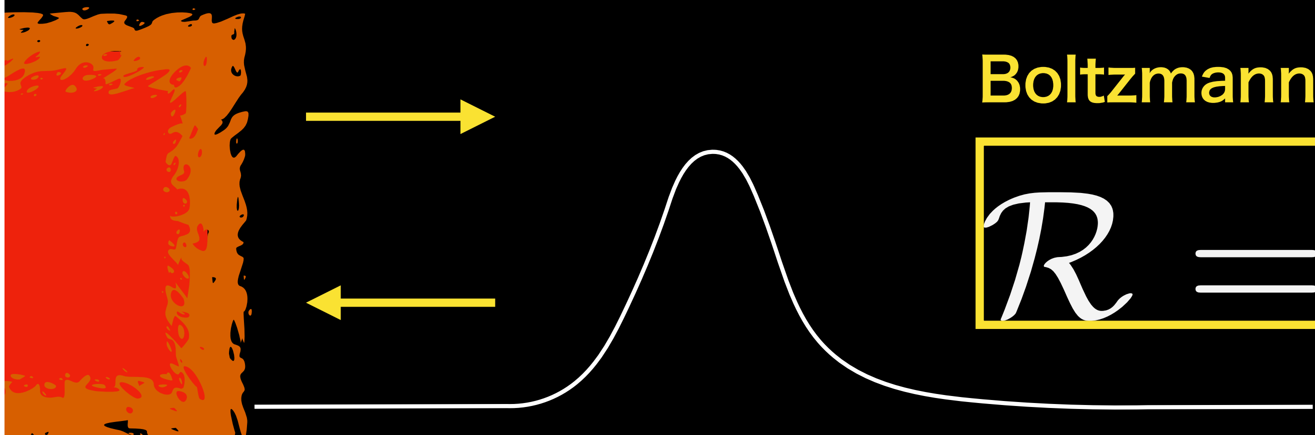
Boltzmann reflection from the membrane

$$\left[-i\frac{\gamma\Omega}{E_{\rm Pl}}\frac{d^2}{dr^{*2}}+\frac{d^2}{dr^{*2}}+\omega^2-V_\ell(r^*)\right]\psi_\omega=0$$

$$\lim_{r^*\to-\infty}\psi_\omega(r^*)=\text{const.}$$

$$\psi_\omega={}_2F_1\left[-i\frac{\omega}{\kappa},i\frac{\omega}{\kappa},1,-i\frac{E_{\rm Pl}e^{\kappa r^*}}{\gamma\omega}\right]$$

$$\psi_\omega=e^{-\pi\omega/(2\kappa)}Ae^{i\omega r^*}+e^{\pi\omega/(2\kappa)}A^*e^{-i\omega r^*}$$



Boltzmann reflection rate!!

$$\mathcal{R}=e^{-\omega/T_H}$$

Quantized BH area

$$S = \frac{A}{4\ell_{\mathrm{Pl}}^2}$$

BH area $A = 4\pi(2GM)^2$

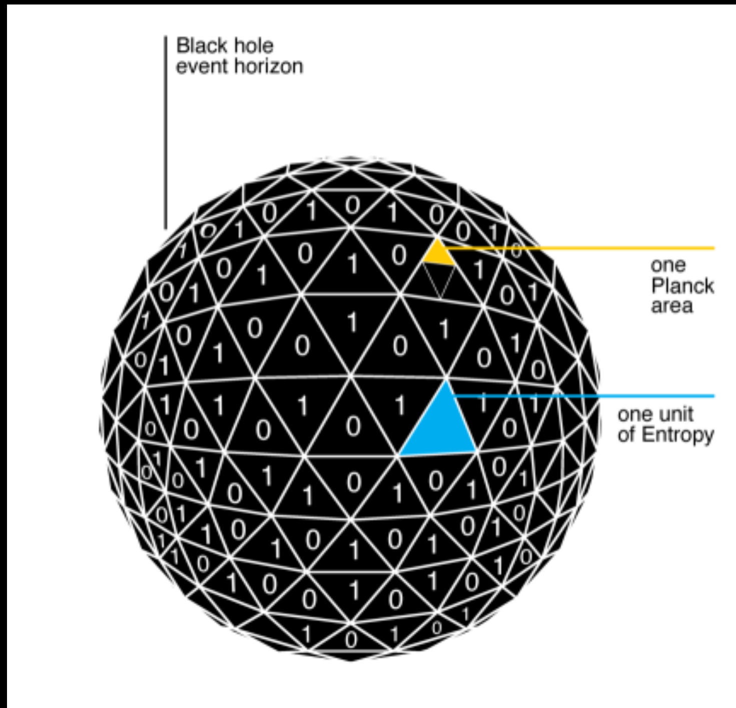
Planck area

The quantization of a BH area
has been discussed in several ways.

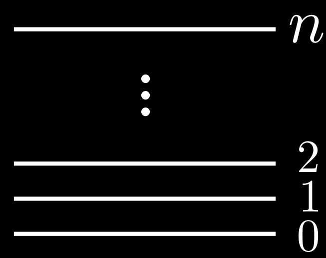
$$A = \alpha \ell_{\mathrm{Pl}}^2 N \qquad \alpha = \mathcal{O}(1)$$

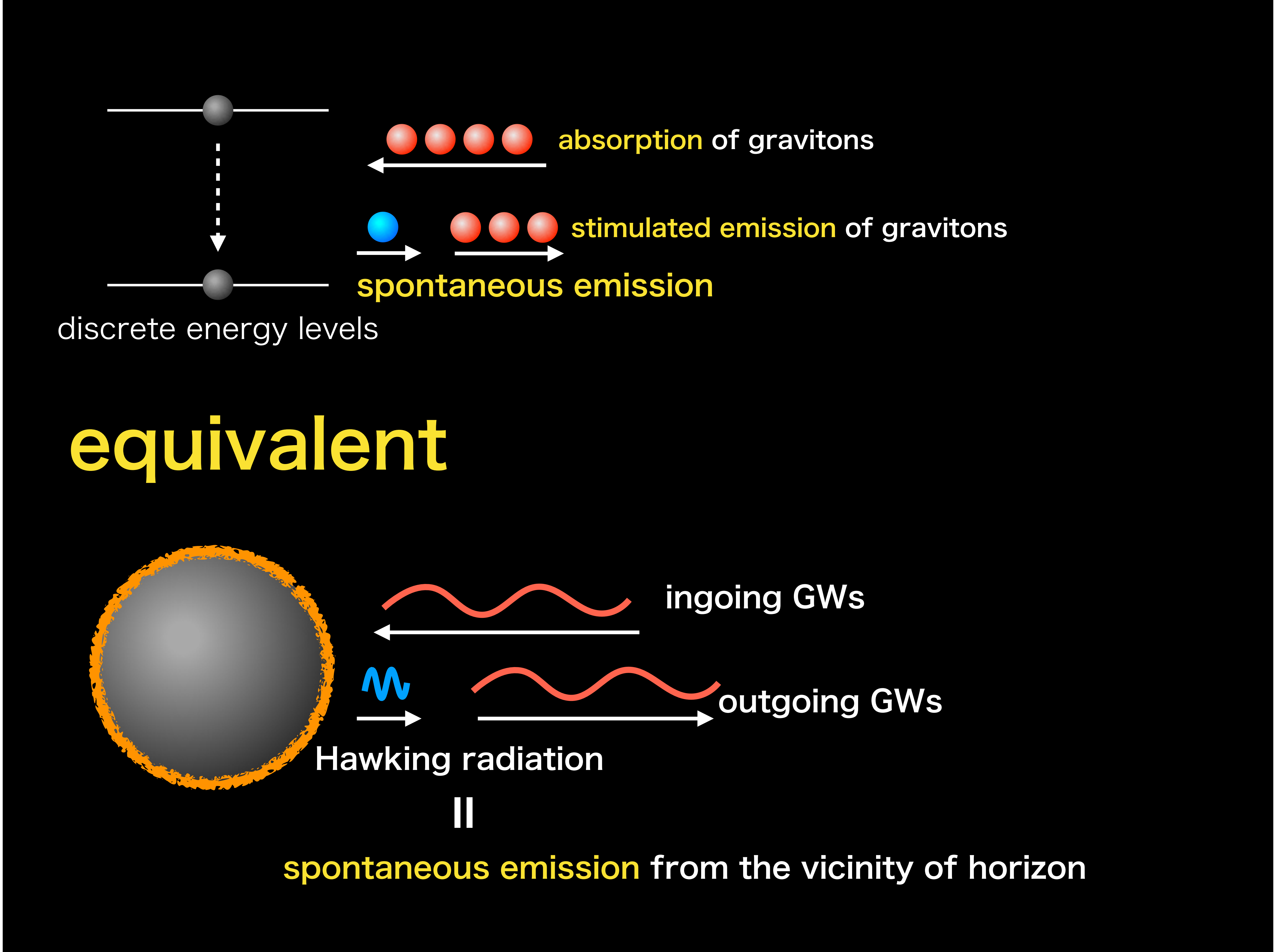
$$32\pi G^2 M \Delta M = \Delta A = \alpha G \Delta N$$

$$\omega_n = \Delta M = \frac{n\alpha}{32\pi}\frac{1}{GM} = \frac{\alpha}{4} T_H \times n$$

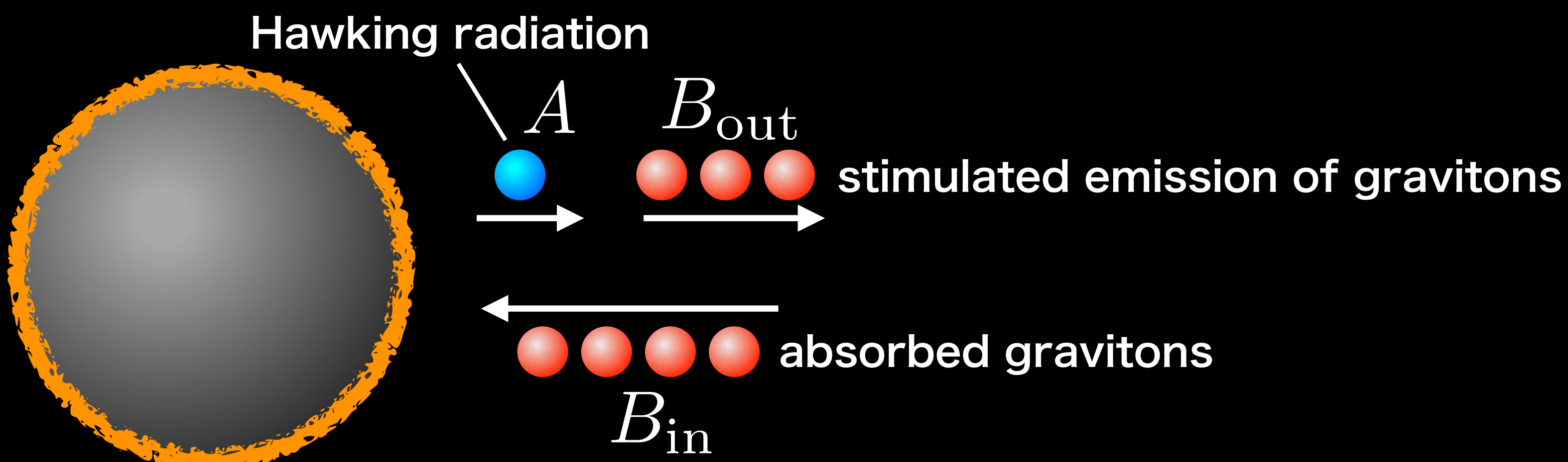


A BH can be regarded as
a quantum system with
discrete energy levels.




absorption of gravitons
stimulated emission of gravitons
spontaneous emission
discrete energy levels
equivalent
ingoing GWs
outgoing GWs
Hawking radiation
=
spontaneous emission from the vicinity of horizon

Detailed balance in QBHs



$$An_2 + B_{\rm out}\rho(\omega)n_2 - B_{\rm in}\rho(\omega)n_1 = 0$$

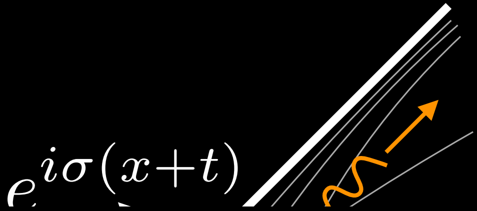
$\rho(\omega)\delta\omega$: energy density of the injected GWs of frequency ω

Assuming the gravitons are thermalized near horizon,

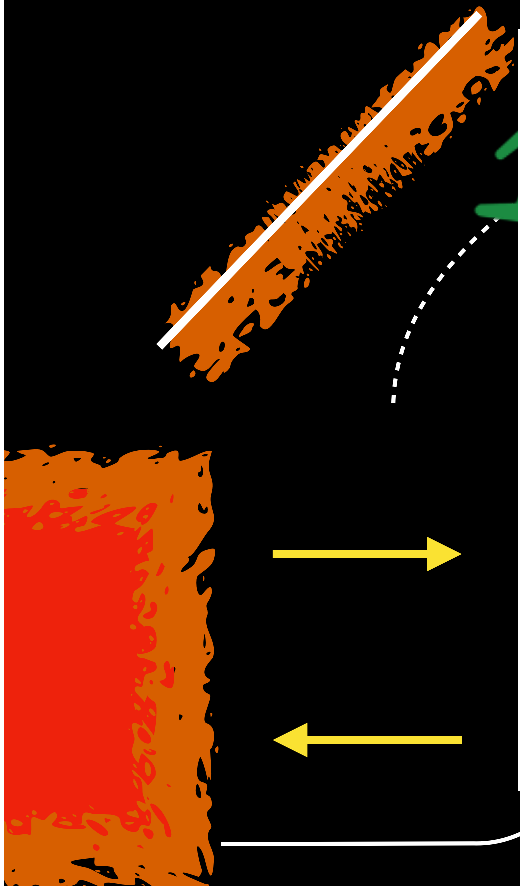
$$\rho(\omega) = \frac{\omega^3}{e^{\omega/T_H} - 1}$$

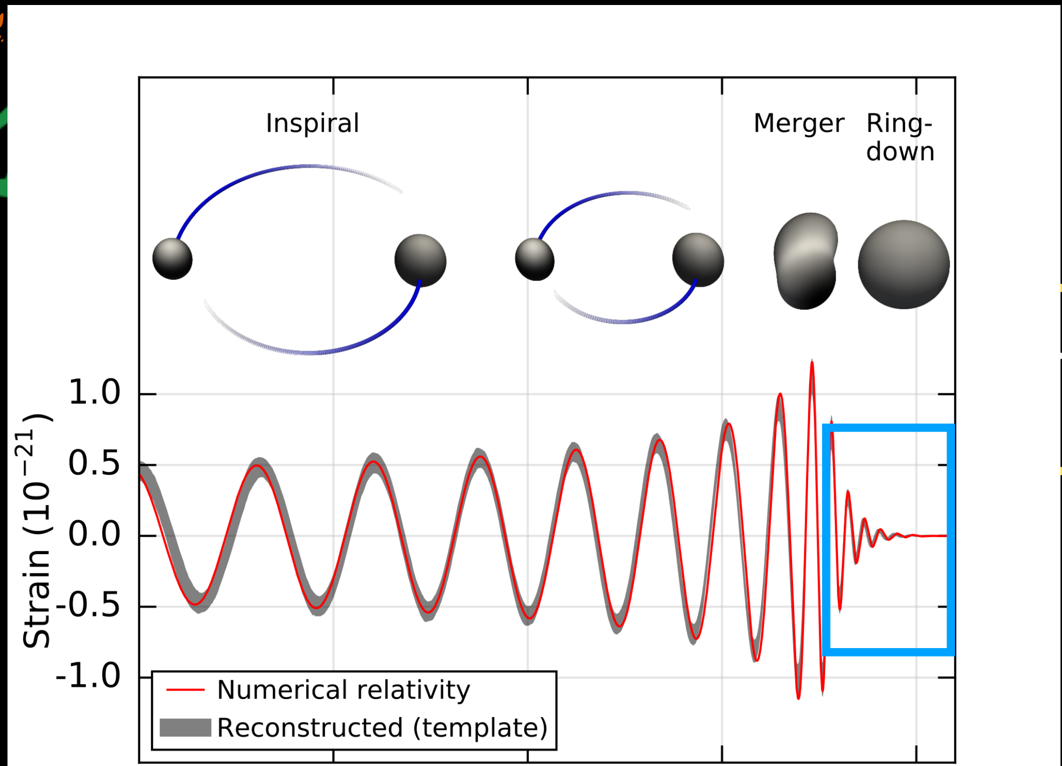
Boltzmann reflection rate $$\frac{B_{\rm in}\rho(\omega)n_1}{B_{\rm out}\rho(\omega)n_2} = e^{-\omega/T_H}$$

infalling observer



How is the ringdown GWs modified??





$-\omega/T_H$

Superradiance

Superradiance may cause the instability of ringdown GWs.

H. Nakano+ (2017)

PTEP

Prog. Theor. Exp. Phys. **2017**, 071E01 (10 pages)
DOI: 10.1093/ptep/ptx093

Letter

Black hole ringdown echoes and howls

Hiroyuki Nakano[1,2,*], Norichika Sago[3], Hideyuki Tagoshi[4], and Takahiro Tanaka[2,5]

The super-radiant amplification looks dangerous. There are extensive works on this problem (see, e.g., Refs. [24–26], and Ref. [27] for a review). The latest analysis [28] shows that the time scale can be larger than the age of the Universe if the location of the reflection boundary is sufficiently far from the horizon. The above means that if BHs have a complete reflecting boundary at a distance of the order of the Planck length from the horizon, all astrophysical BHs become non-rotating, i.e., Schwarzschild BHs. If we observe GW howls due to the super-radiant amplification, it means that only Schwarzschild BHs can exist in our universe.

We should confirm NO instability !

Rotating quantum BHs

$$\left(\frac{-i\gamma\omega}{\sqrt{\delta(r)}E_{\rm Pl}}\frac{d^2}{dr^{*2}}+\underbrace{\frac{d^2}{dr^{*2}}-\mathcal{F}\frac{d}{dr^*}-\mathcal{U}}_{\text{Sasaki Nakamura equation}}\right)\psi_\omega=0,$$

viscosity

Sasaki Nakamura equation

$$\lim_{r^*\to-\infty}\delta(r^*)=C^2\exp\left[2\kappa_+r^*\right],\qquad \lim_{r^*\to-\infty}\mathcal{F}(r^*)=0,$$

$$\lim_{r^*\to-\infty}\mathcal{U}(r^*)=-\tilde{\omega}^2(a)\equiv-\left(\omega-\frac{ma}{2r_+}\right)^2,$$

$$\boxed{\mathcal{R}=\exp\left[-\frac{|\tilde{\omega}(a)|}{T_H(a)}\right]}$$

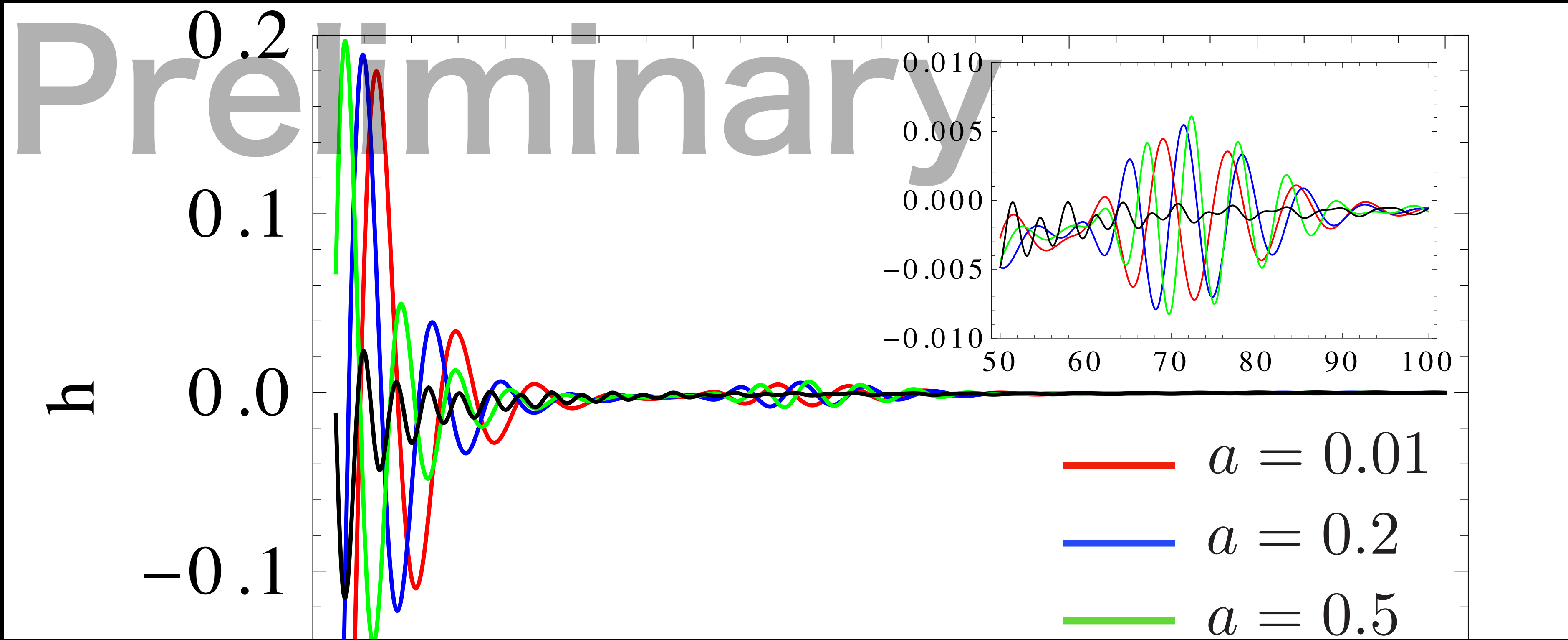
Hawking temperature

$$T_H(a)\equiv\frac{1}{\pi r_g}\frac{\sqrt{1-a^2}}{(2r_+/r_g)^2+a^2}$$

In the extreme limit (a -> 1), the temperature becomes zero.

$\mathcal{R}\to 0$ except for $\omega=\dfrac{ma}{2r_+}$

Calculation of ringdown GWs



If the echo is observed
and it is highly suppressed for highly spinning BHs,
it may be a supporting evidence for
the thermal nature of quantum BHs!!!

Conclusions

- Hawking radiation is observer-dependent.
- According to a distant observer, there would be stretched horizons due to gravitational blue-shift effect.
- Boltzmann reflection rate may be consistent from the BH complementarity and membrane paradigm.
- Slowly spinning BHs emanate the echo GWs while the echo is highly suppressed for rapidly spinning BHs.
- If the echo GWs are observed from a BH and the echo is highly suppressed for highly spinning BH, it may be a supporting evidence of thermal viscous membrane!!
- Quantum BHs with the Boltzmann reflection rate do not suffer from the superradiance.