(collaboration in C01)

Gravitational wave forest from string axiverse

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J.Soda & Y.U. Euro. Phys. J. C 78, 9, 779 (2018)

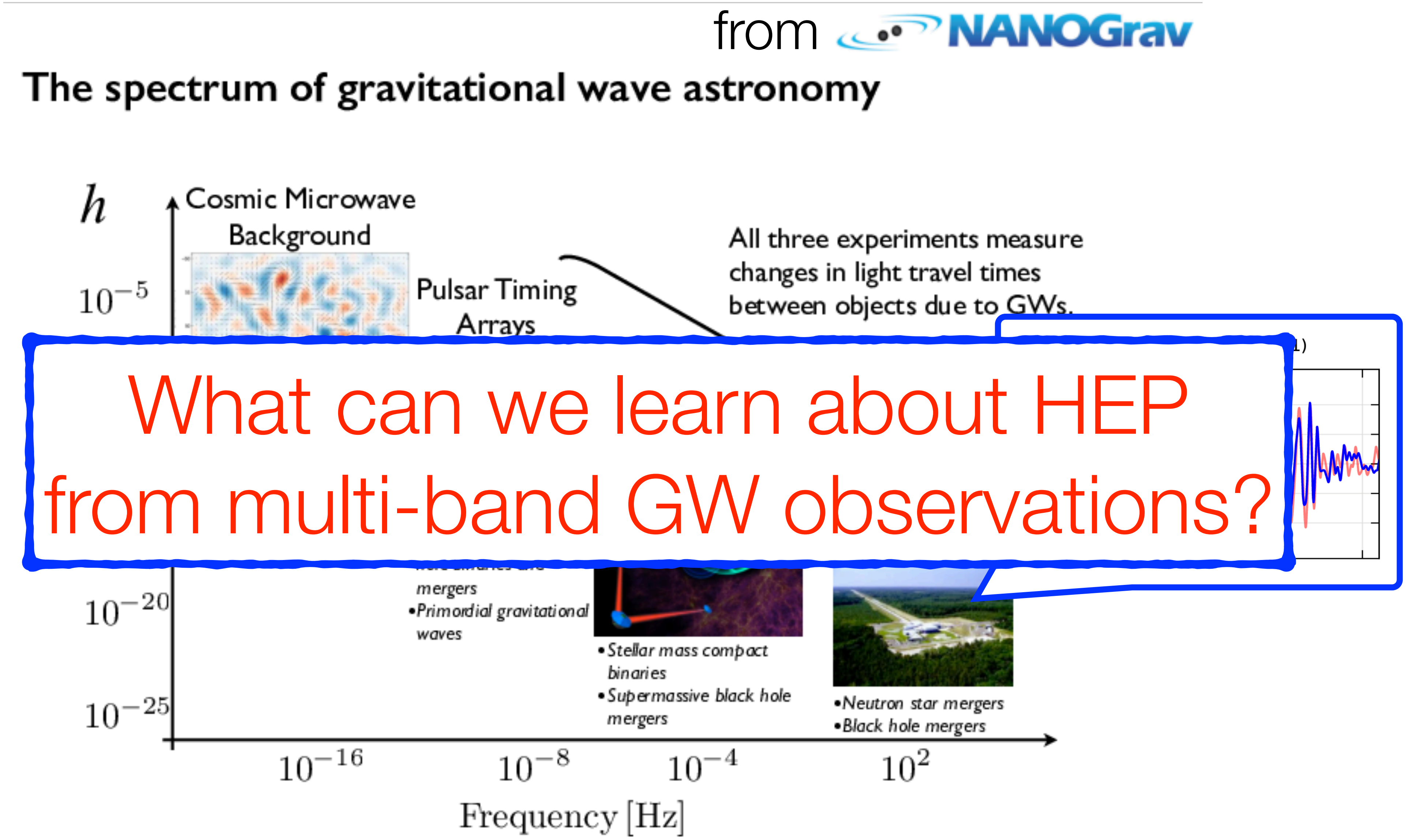
Kitajima, Soda & Y.U. JCAP 10, 008 (2018)

Kitajima, Soda & Y.U. in progress

Fukunaga, Kitajima & Y.U. 1903.*****

w/ Hayato Fukunaga, Naoya Kitajima (Nagoya U.), Jiro Soda (Kobe U.)

Multi-wavelength GW era



String axiverse

Arvanitaki et al. (10), $\cdots$

Superstring theory in compact 6D

↓

4D low energy EFT + Axions + Moduli ....

Wide mass ranges → Probe of exDim

$$m_a^2 \sim \frac{\mu^4}{f_a^2} e^{-\#\sigma_i}$$

↓

Inflaton, DM candidate (Fuzzy DM)

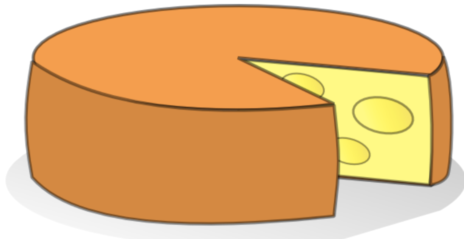
Hu et al.(00), $\cdots$

ex. Large Volume Scenario

Thraxions

Conlon et al. (05)

Hebecker + (18)



→ Predicts extremely light axions

Axion as dark matter

Marsh (15)

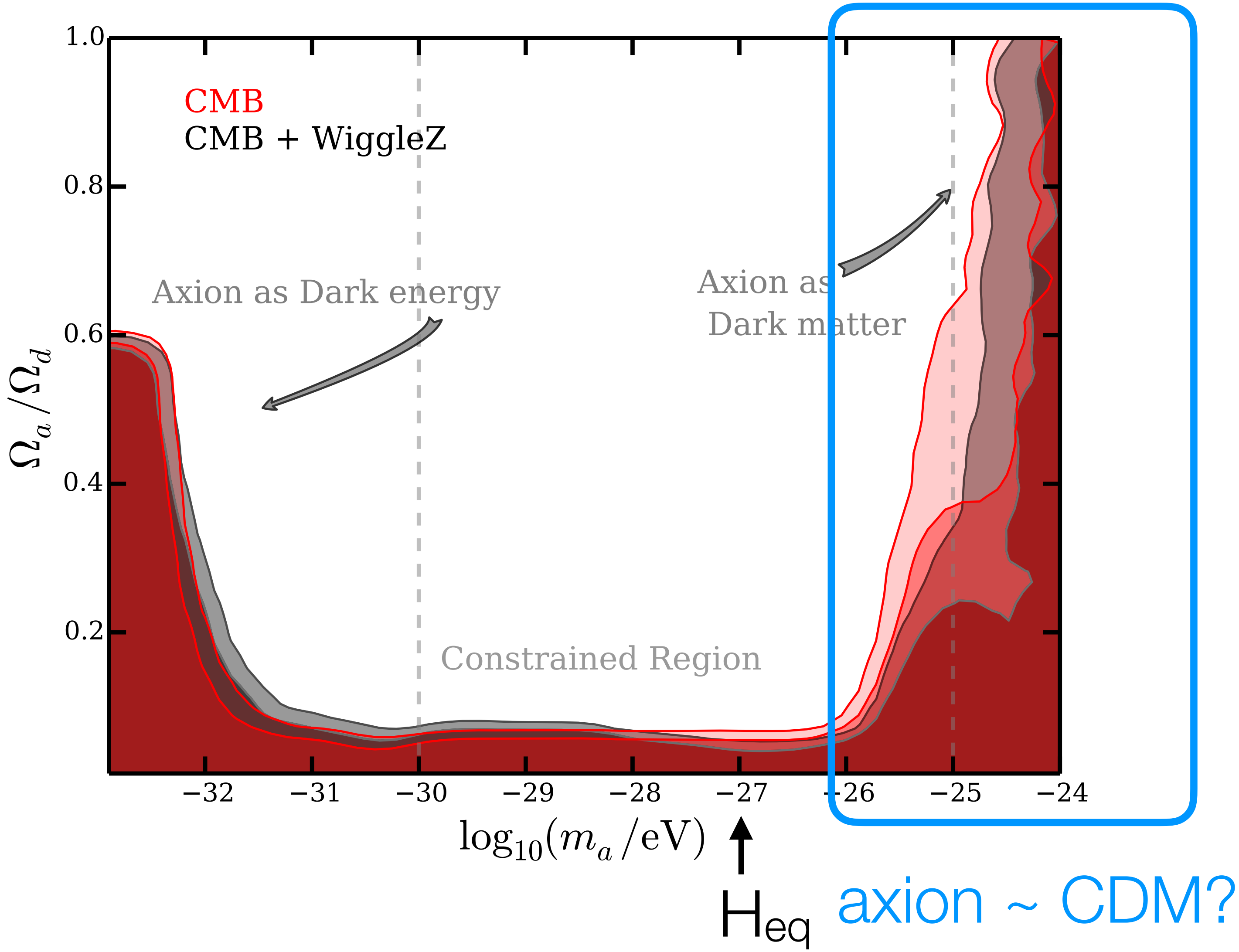
$m_a << H$

axion → Λ

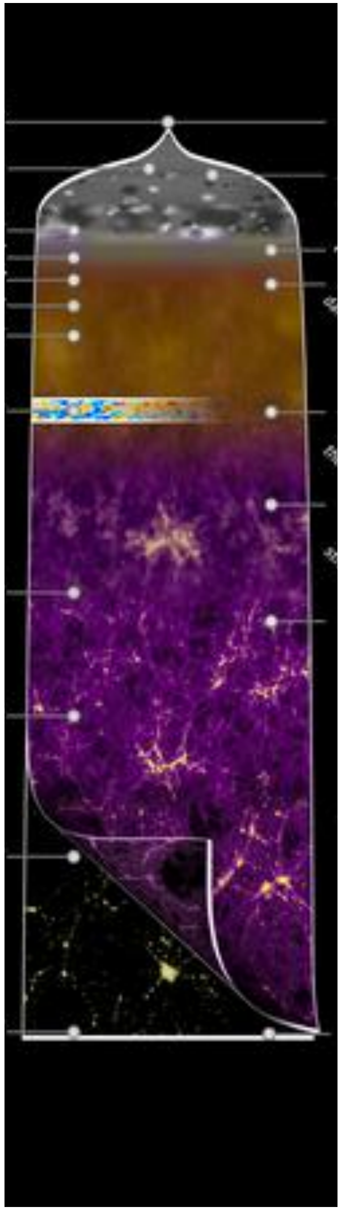
$m_a >> H$

axion → DM

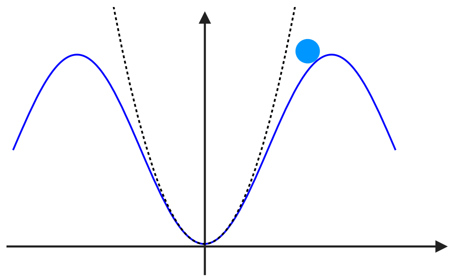
$$\Omega_d = \Omega_a + \Omega_c$$



Axion search from GWs



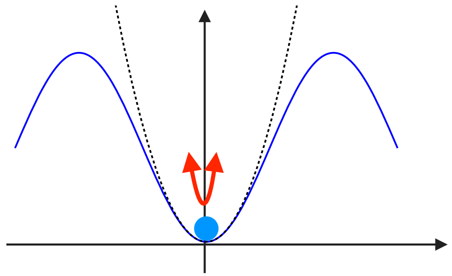
← Onset of oscillation



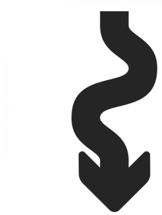
~ Λ (w=-1)

Transition

Resonance inst.

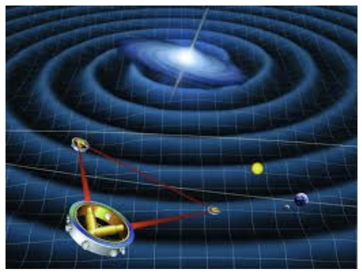


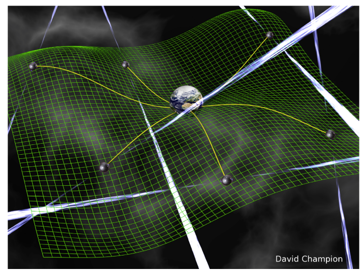
~ dust (w=0)



(b)GW

if onset is RD or MD





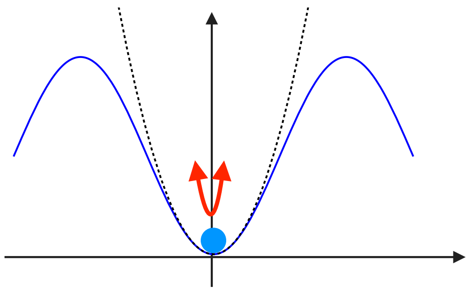
Bottom-line story

$$\ddot{h}_{ij} + 3H\dot{h}_{ij} - \frac{1}{a^2}\nabla^2 h_{ij} = \frac{2}{M_{\rm Pl}^2}\Pi_{ij}^{\rm TT}$$

$$\partial_i h_{ij} = 0 \text{ and } h_{ii} = 0$$

h: +, -

homogeneous axion



$\phi F\tilde{F}$

trigger : resonance instability

$\delta\phi$ → $h_{ij,\,h}$

$$\Pi_{ij}^{\rm TT} = \frac{1}{a^2}P_{ij}^{lm}\partial_l\phi\partial_m\phi$$

A_+, A_- → $h_{ij,\,+} \neq h_{ij,\,-}$

$$\Pi_{ij}^{\rm TT} = \frac{1}{a^2}P_{ij}^{lm}F_{l\mu}F_m^{\ \mu}$$

$$\epsilon_{ij}^{(\pm)} \sim \epsilon_i^{(\pm)}\epsilon_j^{(\pm)}$$

Normalization

Neglect back-reaction on geometry

⟶ Axion's dynamics is independent of (*m*, *f*)

$$\partial_t^2 \phi + 3H\partial_t \phi - \frac{\partial^2}{a^2}\phi + V_{,\phi} = 0$$

↓ $\tilde{x}^\mu \equiv m x^\mu \qquad \tilde{\phi} \equiv \frac{\phi}{f}$

$$V(\phi) = (mf)^2\, \tilde{V}(\tilde{\phi})\,, \qquad \tilde{\phi} \equiv \frac{\phi}{f}$$

e.g. $\tilde{V}(\tilde{\phi}) = 1 - \cos\tilde{\phi}$

$$\partial_{\tilde{t}}^2 \tilde{\phi} + 3\frac{H}{m}\partial_{\tilde{t}}\tilde{\phi} - \frac{\partial_{\tilde{\mathbf{x}}}^2}{a^2}\tilde{\phi} + \tilde{V}_{,\tilde{\phi}} = 0$$

m = m(ϕ=0, potential minimum)

Parametric resonance in cosmology

- Reheating: inflaton → SM
- Axion cosmology

Traschen & Brandenberger (90), ⋯

Kofman, Linde, Starobinsky (97)

e.g. periodic mass $m(t+T) = m(t)$

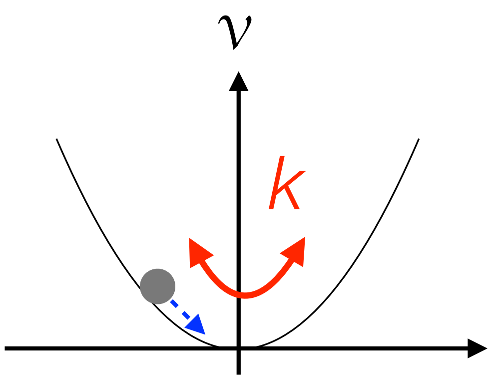
$$\frac{d^2 y_k(t)}{dt^2} + \omega_k^2 y_k(t) = 0$$

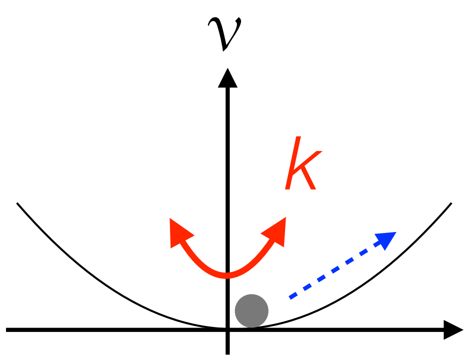
$$\omega_k^2(t) = k^2 + m^2(t)$$

→

$$\omega_k^2(t) = \left(\frac{k}{a(t)}\right)^2 + m^2(t)$$

Redshift disturbs.





potential opens

only k whose phases match to $m(t)$ grow

(parametric resonance)

Phenomenology 😀

Delayed onset of oscillation → Efficient PR

if initially $\left|\frac{\tilde{V}_{,\tilde{\phi}}}{\tilde{\phi}}\right|$ <<1, $\frac{H_{\rm osc}}{m} \sim \sqrt{\left|\frac{\tilde{V}_{\tilde{\phi}}}{\tilde{\phi}}\right|} \ll 1$ delayed oscillation

e.g. $\tilde{V}(\tilde{\phi}) = 1 - \cos\tilde{\phi}$ with $\tilde{\phi}_i \sim \pi$

(Time scale of cosmic exp.) (Time scale of V driven motion)

1/H $\sqrt{|V_\phi/\phi|} = \sqrt{|\tilde{V}_{\tilde{\phi}}/\tilde{\phi}|}/m$

Slow-roll << recall $\partial_t^2\phi + V_{,\phi} = 0$

Onset ~

Oscillation >>

Scalar potential of axion

continuous shift sym.

$\phi \to \phi + c$

NP effects
e.g. instanton effects

$\phi \to \phi + 2\pi n/f$

$n \in \mathbf{Z}$

$V(\phi) \sim \Lambda^4 \cos\phi/f$

Potential can be more flatten than $\cos\phi/f$

i) ~~Dilute instanton gas approximation~~

see. implications for axion=inflaton, *Nomura + (17, 18)* $V(\phi) = M^4\left[1 - \frac{1}{(1+(\phi/F)^2)^p}\right]$

ii) Non-min. coupling w/gravity, Non-canonical kinetic term

Recall α attractor model for Re[T] *Kallosh & Linde + (13, 14, ⋯)*

etc…

Linear perturbation of KG eq. : Hill's equation

$$\frac{d^2 y_k(\tilde t\,)}{d\tilde t^2} + \omega_k^2(\tilde t\,) y_k(\tilde t\,) = 0 \qquad y_k(\tilde t) \equiv a^{3/2}\delta\phi_k(\tilde t)$$

$$\omega_k^2(t) = \left(\frac{k}{a(t)m}\right)^2 + \tilde V_{\tilde\phi\tilde\phi}(\tilde t) = A_k - 2q\psi(\tilde t\,) \qquad \psi(\tilde t\,) = \psi(\tilde t + T)$$

Floquet th. $\quad y_k(t) = P_1(\tilde t\,)\, e^{\mu_k \tilde t} + P_2(\tilde t\,)\, e^{-\mu_k \tilde t} \qquad P_i(\tilde t\,) = P_i(\tilde t + T)$

e.g. Mathieu eq. $\quad \psi(\tilde t\,) = \sin(2\tilde t\,)$

resonance band is characterized by

$A_k \sim (k/am)^2$: position $\qquad$ q : band width

Yet, for $\frac{H_{\rm osc}}{m} \sim \sqrt{\left|\frac{\tilde V_{\tilde\phi}}{\tilde\phi}\right|} \ll 1 \qquad \psi(\tilde t\,) \neq \sin(2\tilde t\,)$

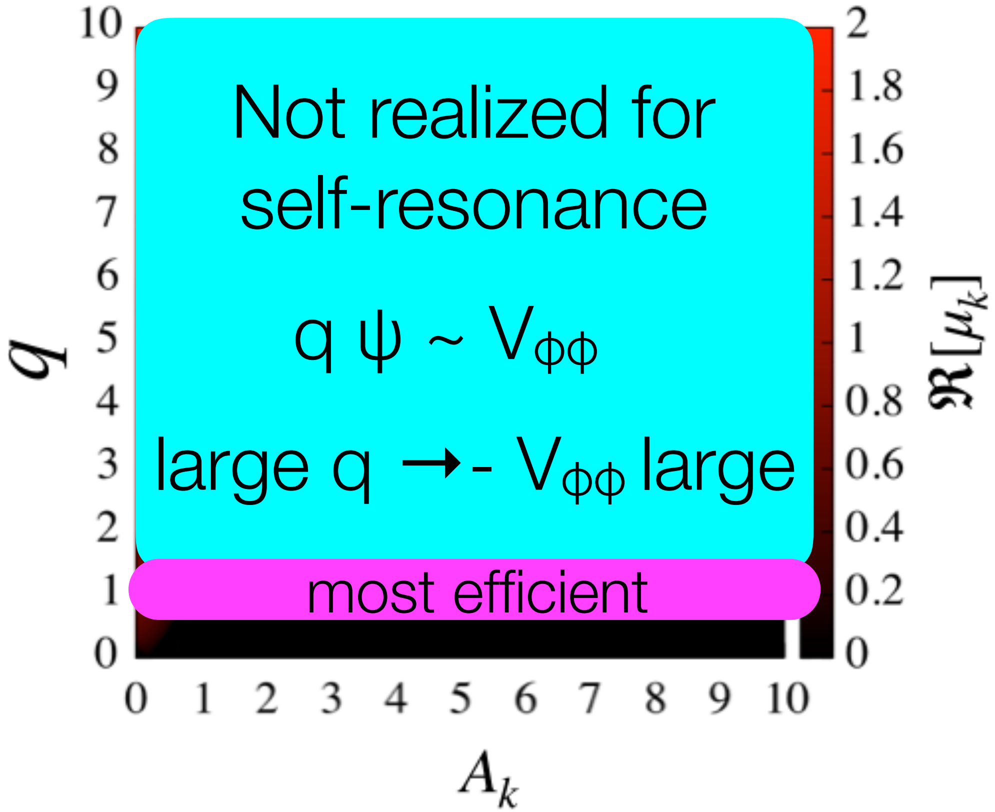
PR for general Hill's equation

Fukunaga, Kitajima, Y.U. (19)

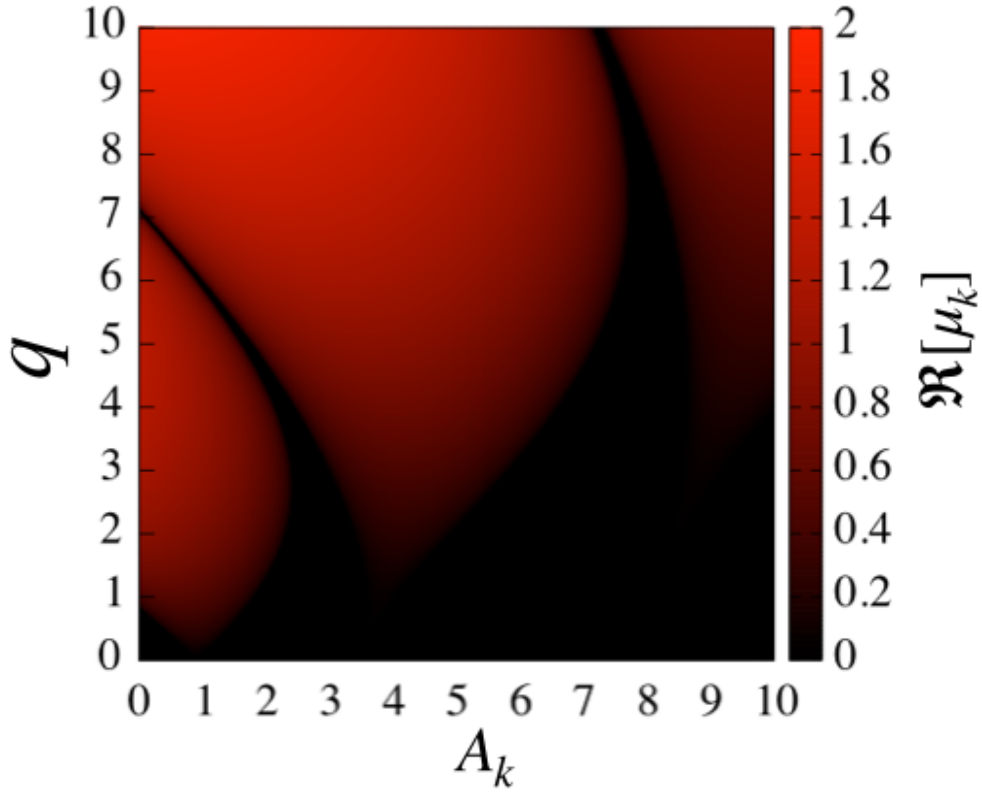
stability/instability chart is "generically" characterized by (A_k, q)

$$\omega_k^2 = A_k - 2q\psi(\tilde{t}\,) \qquad q = \sqrt{\frac{\langle(\omega_k^2 - \langle\omega_k^2\rangle)^2\rangle}{2}} \qquad \langle F(\tilde{t})\rangle \equiv \frac{1}{T}\int_{\tilde{t}-\frac{T}{2}}^{\tilde{t}+\frac{T}{2}} d\tilde{t}' F(\tilde{t}')$$

$\psi(\tilde{t}\,) = \sin(2\tilde{t}\,)$ (delay type)



$\psi(\tilde{t}\,) = \sin(2\tilde{t}\,)$



General Hill's equation

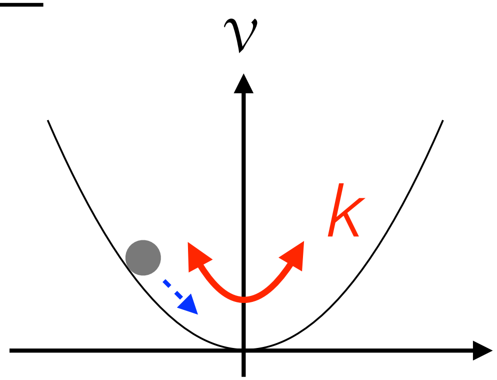
Fukunaga, Kitajima, Y.U. (19)

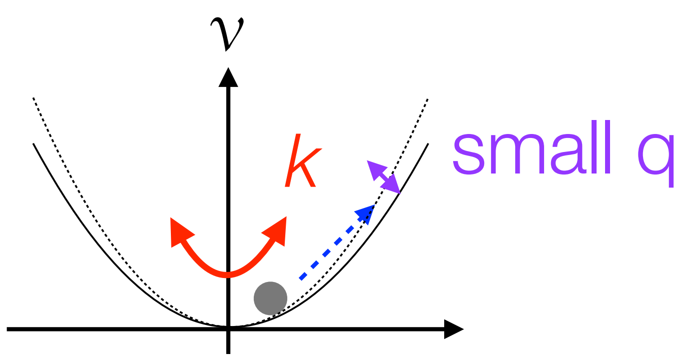
$$\omega_k^2(t) = \left(\frac{k}{a(t)m}\right)^2 + \tilde{V}_{\tilde{\phi}\tilde{\phi}}(\tilde{t})$$

large q → rapid growth, wide band

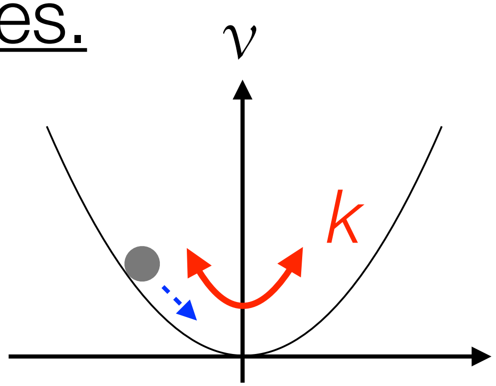
potential opens

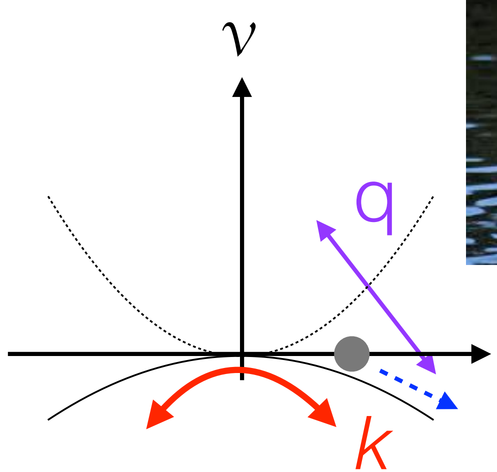
Narrow res.





Flapping res.





q = O(1)

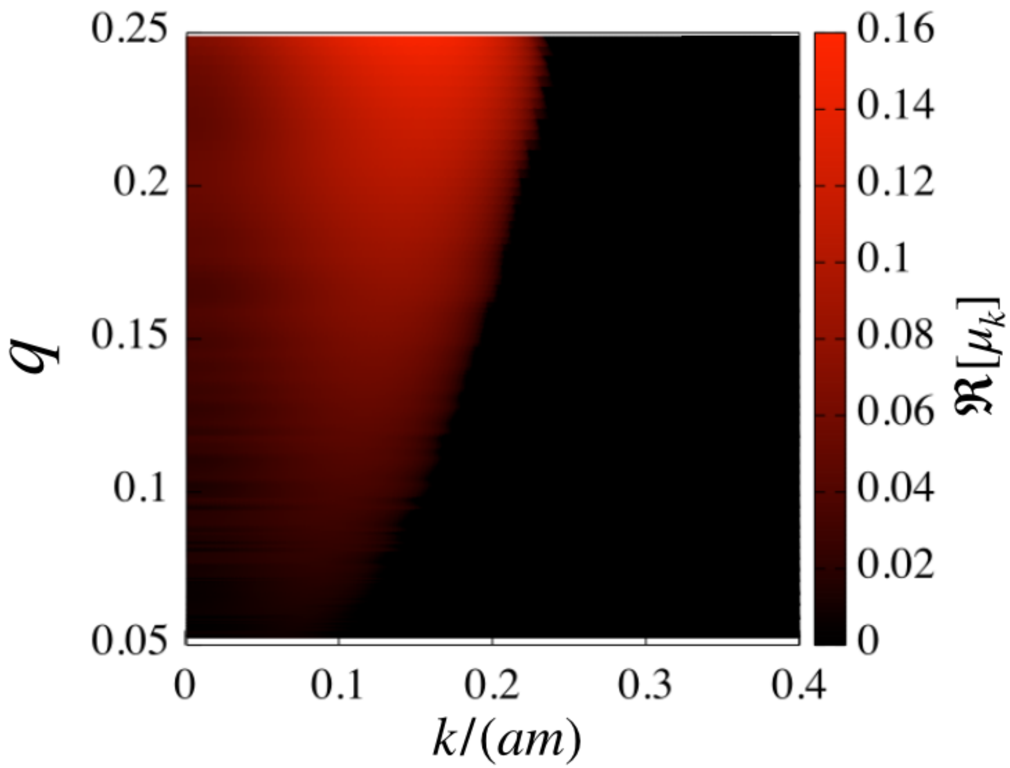
Kitajima, Soda, Y.U. (18)

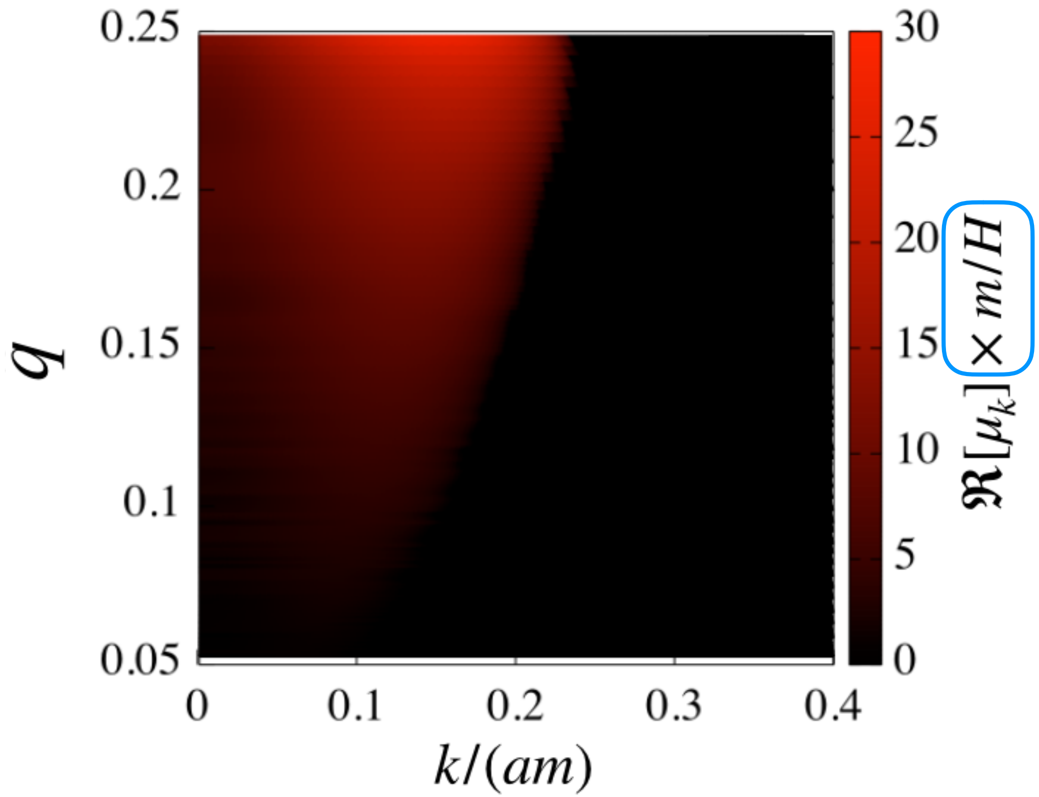
Resonance band for pure natural potential

$$\tilde{V}(\tilde{\phi}) = \frac{1}{2}\left[1 - \frac{1}{(1+\tilde{\phi}^2/c)^c}\right] \qquad (c>0)$$

Nomura + (17, 18)

Growth rate Re[μ_k] for linear perturbation (in RD) eg. axion DM

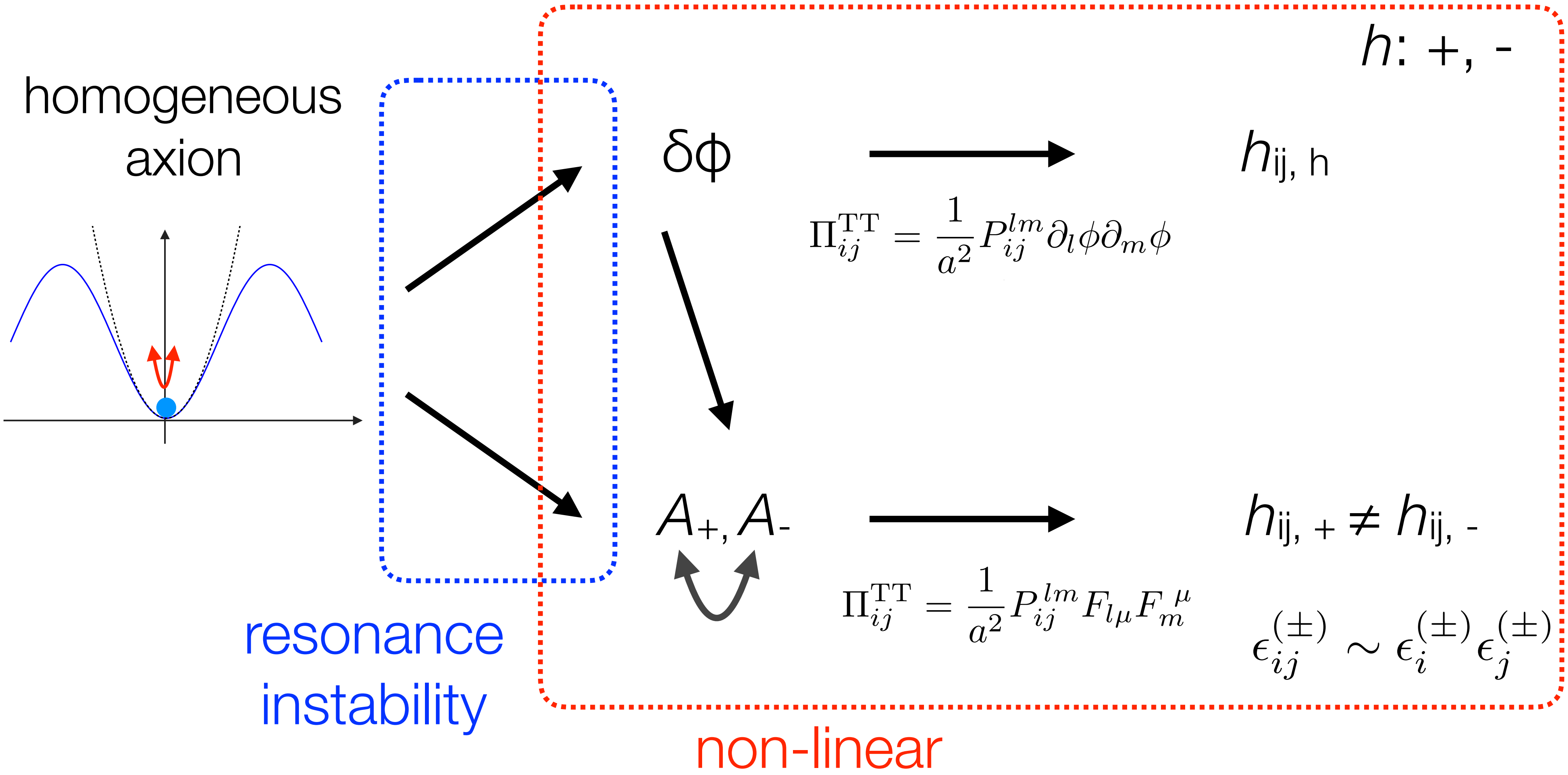




Expo. growth much faster than cosmic exp.

Resonant GW production from axions

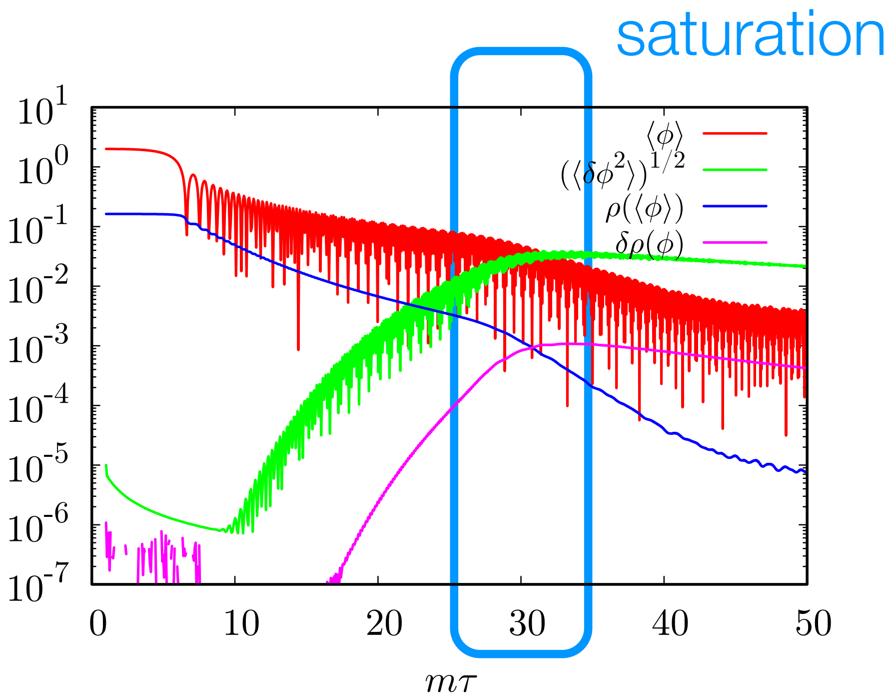
$$\ddot{h}_{ij} + 3H\dot{h}_{ij} - \frac{1}{a^2}\nabla^2 h_{ij} = \frac{2}{M_{\rm Pl}^2}\Pi_{ij}^{\rm TT} \qquad \partial_i h_{ij} = 0 \text{ and } h_{ii} = 0$$



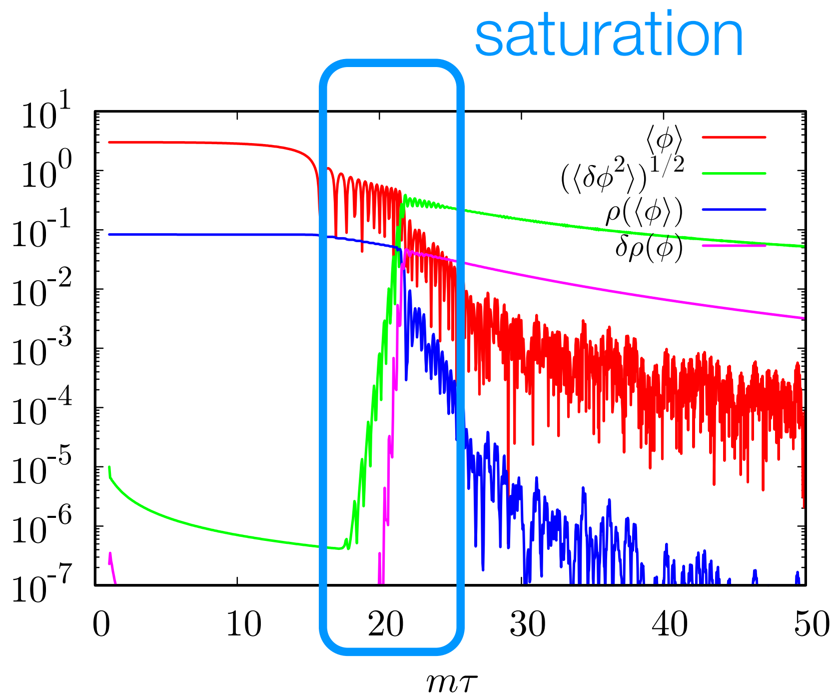
no $F\tilde{F}$

Lattice simulation

Kitajima, Soda & Y.U. (18)



(b) $c = 2, \phi_i = 2f$



(a) $c = 5, \phi_i = 3f$

Narrow res. dominant

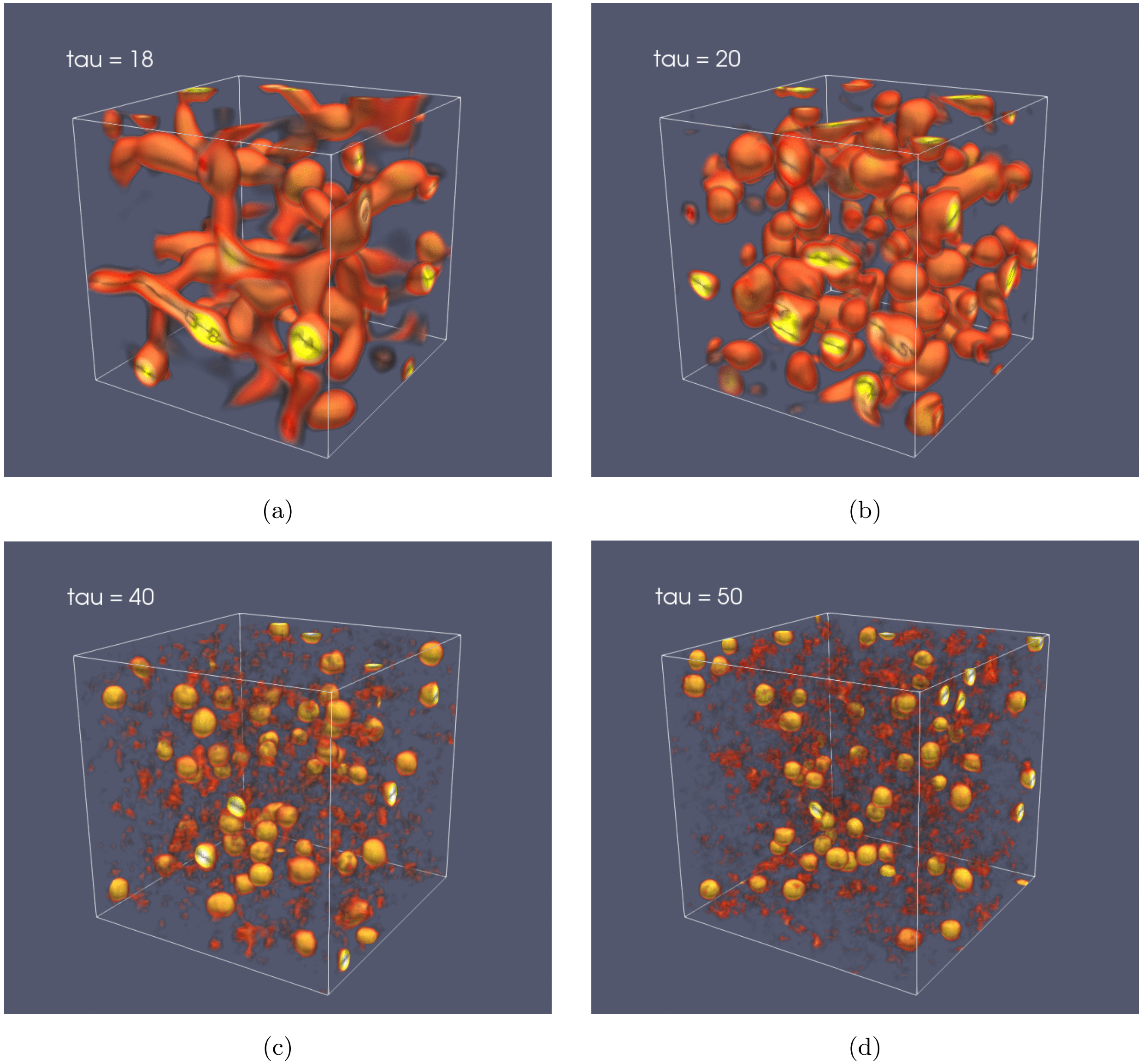
Flapping res. dominant

Cosmic exp. does not stop growth, but backreaction does.

Lattice simulation $N_{grid}=(256)^3$

Oscillon formation

Kitajima, Soda & Y.U. (18)



Kasuya et al.(03)
Amin & Shirokoff(10)
Amin et al.(2010)
Amin et al.(2014),···

$N_{grid}=(256)^3$

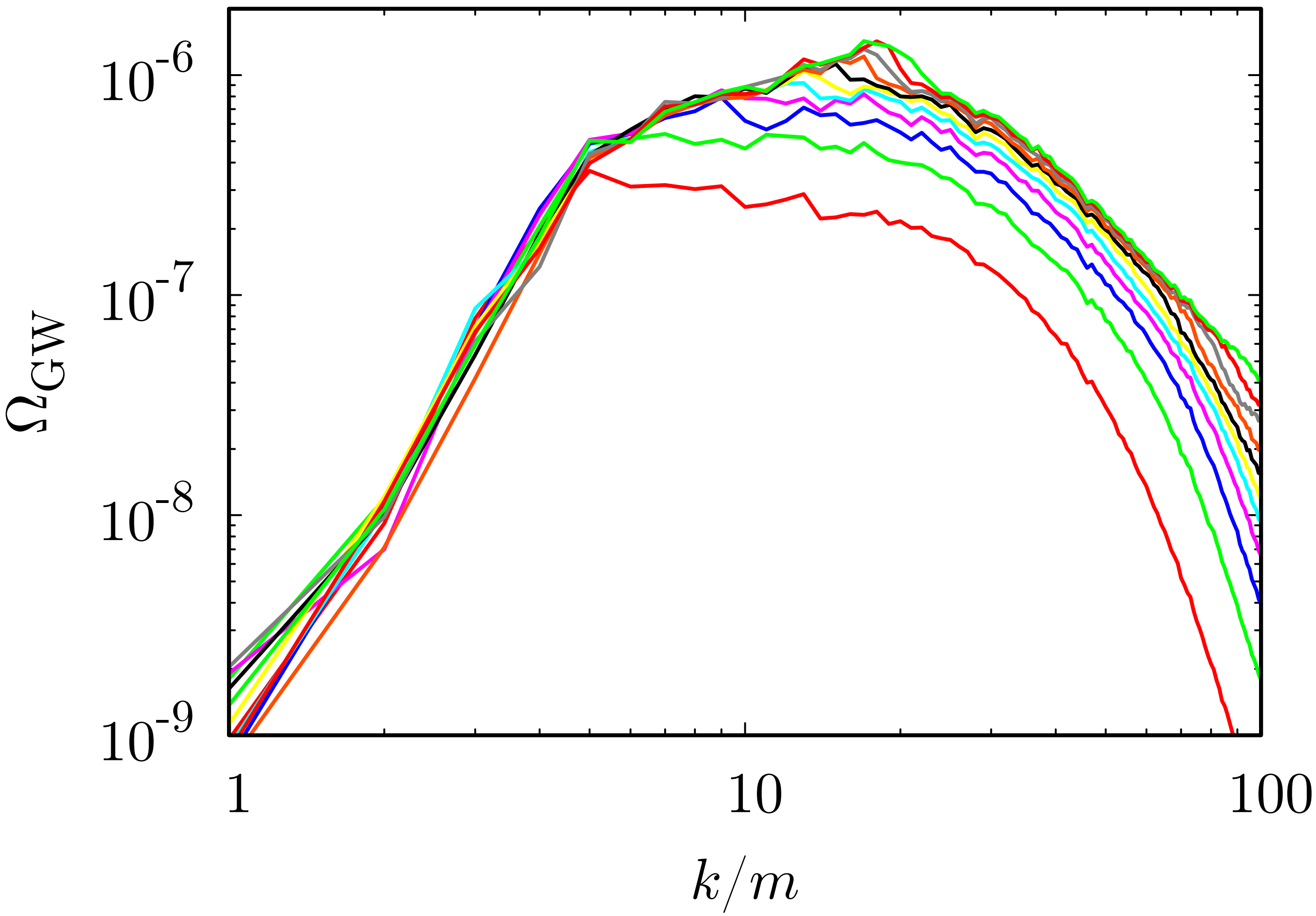
$c = 5$ and $\phi_i = 2f$.

The red, yellow and white region correspond to $\rho/\bar{\rho} > 2$, 4 and 10

no $F\tilde{F}$

GW spectrum

Kitajima, Soda & Y.U. (18)



$f = 10^{16}$ GeV, $c = 5$ and $\phi_i = 3f$

to evaluate the present value, x Ω_r

GW's frequency

Kitajima, Soda & Y.U. (18)

Redshifted frequency

$$\nu_0 = \frac{\kappa m}{2\pi}\left(\frac{a_{\rm em}}{a_0}\right)$$

$$\kappa \equiv \frac{\omega_{\rm phys}}{m} = \frac{k^{\rm em}_{\rm peak}}{m\, a_{\rm em}} = \frac{k^{(res)}_{\rm peak}}{m\, a_{\rm res}} \times \frac{k^{(em)}_{\rm peak}/a_{\rm em}}{k^{(res)}_{\rm peak}/a_{\rm res}}$$

momentum flow due to turbulence

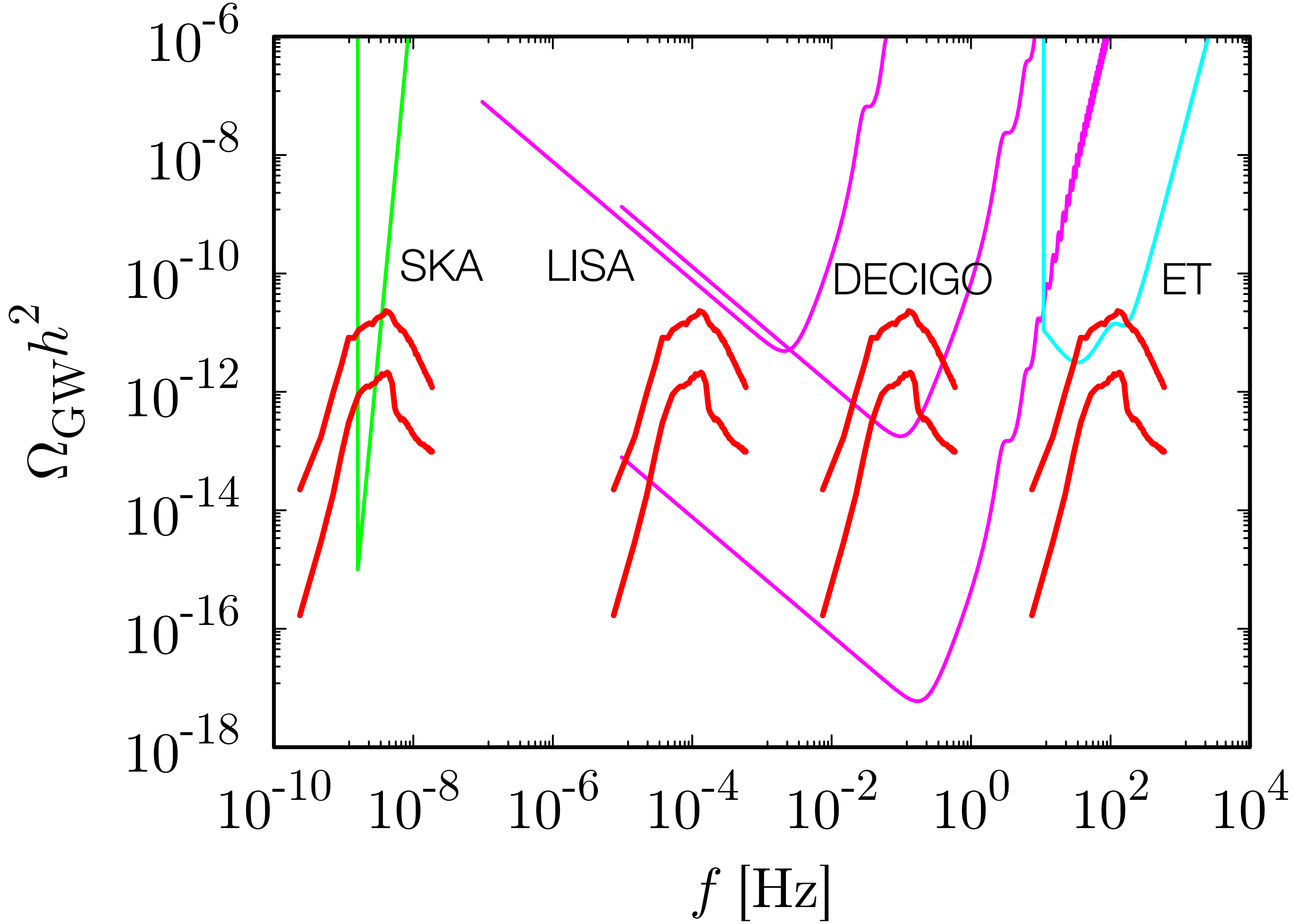
e.g. GWs emitted during radiation domination

$$\nu_0 = \frac{\kappa m}{2\pi} \times \left(\frac{\rho_{\rm r,0}}{\rho_{\rm r,\,em}}\right)^{1/4} \simeq 0.78 {\rm nHz}\, \kappa \left(\frac{m}{H_{\rm em}}\right)^{1/2} \left(\frac{m}{10^{-12}{\rm eV}}\right)^{1/2}$$

no $F\tilde{F}$

GW forest

Kitajima, Soda & Y.U. (18)

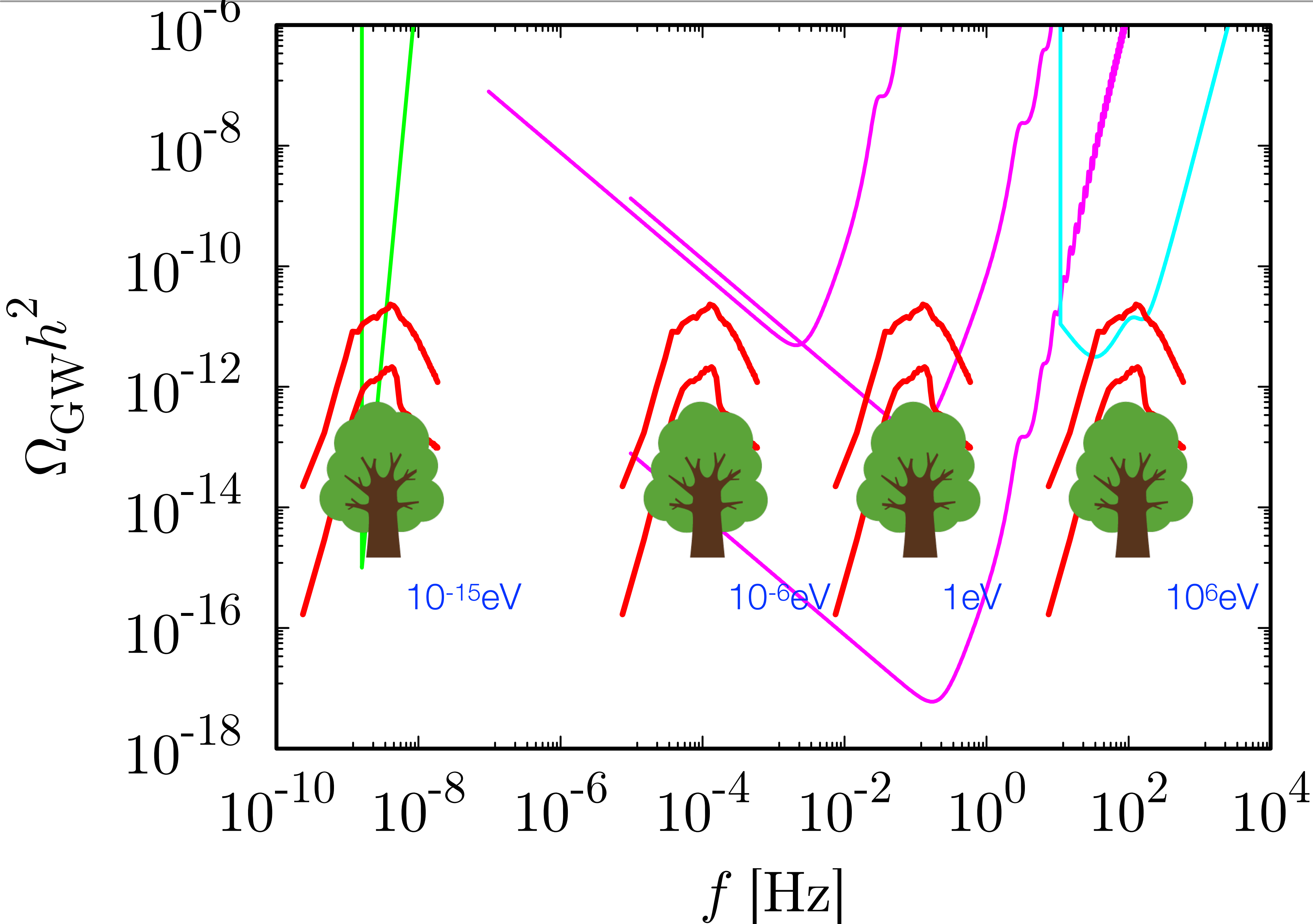


Axions from string theory $\quad$ ***f*** ~ 10^{16} GeV $\quad$ *e.g.,Svrcek & Witten (06)*

no $F\tilde{F}$

GW forest

Kitajima, Soda & Y.U. (18)



no $F\tilde{F}$

GWs from axion DM

Kitajima, Soda, Y.U. (18)

freq. of GW ν_0 ⟶ mass m

+

abundance of axion ⟶ decay const. f x mass m

Crude Order estimation

using $\varphi(t, x) \sim f\,(a_{\rm osc}/a)^{3/2}$

Δ : Sym. suppression (< 1)

$$\Omega_{\rm GW} h^2 \simeq 0.8 \times 10^{-18} \kappa^4 \Delta^2 \left(\frac{\rm nHz}{\nu_0}\right)^2 (\Omega_\phi h^2)^2$$

for κ=10 $\quad \Omega_{\rm GW} h^2 \simeq 10^{-16}$ at $\nu_0 = $nHz

or lower frequency btwn CMB & PTAs?

w/ $F\tilde{F}$

Prospects on polarized GW forest

Kitajima, Soda & Y.U. (in prep.)

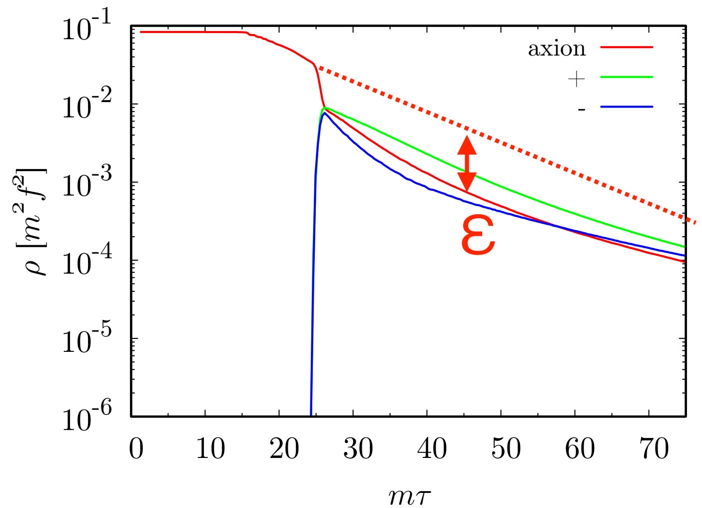
What about $\alpha \neq 0$?

- GW circularly polarized

  see also *Adshead* +(18)

- More prominent GW emission

  - Larger Δ (Less symmetric)
  - Weaker abundance restriction



GW from axion DM

$$\Omega_{\rm GW} h^2 \simeq 0.8 \times 10^{-18} \kappa^4 \Delta^2 \left(\frac{\rm nHz}{\nu_0}\right)^2 (\Omega_\phi h^2)^2 \quad \times \frac{1}{\varepsilon^2}$$

Summary

New window of axions in plateau

Theory side

Resonant instability leads to copious emission of GWs

Keys: Delayed oscillation

Phenomenology side

Predicts bGWs at various frequencies, multi-band obs.?

- Peaky spectrum
- Circularly polarized GWs

GWs from axion DM: sweet spot is btwn CMB till PTAs.