Constraining the stochastic gravitational wave background using cosmic shear B-mode

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+possibly HSC collaborators

The Stochastic GW Background

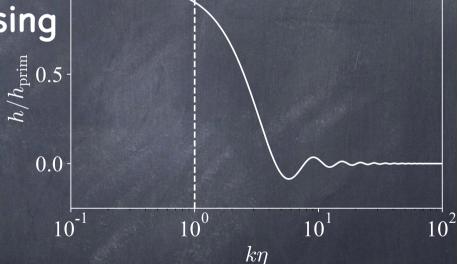
- GWs in the expanding universe
 - tensor perturbation $ds^2 = a^2(\eta)[-d\eta^2 + (\delta_{ij} + h_{ij})dx^idx^j]$

$$h_{ij}^{"} + 2\mathcal{H}h_{ij}^{"} + k^2h_{ij} = 0$$

decay after the horizon crossing

$$h_{ij}(\eta) \propto T_T(k,\eta) = 3\frac{j_1(k\eta)}{k\eta}$$

oscillation in subhorizon



- Sources of the stochastic GW background (SGWB)
 - Astrophysics: unresolved compact binary (BH, NS...)
 - Cosmology: inflation, phase transition, cosmic string...

Properties of the SGWB

- Characterized by its statistical properties
 - ▶ Isotropic, stationary...
 - Frequency spectrum
- Energy density parameter of the SGWB
 - Energy density of the SGWB

$$\rho_{\rm GW} \equiv \frac{1}{64\pi G a^2} \langle (h'_{ij})^2 + (\nabla h_{ij})^2 \rangle$$

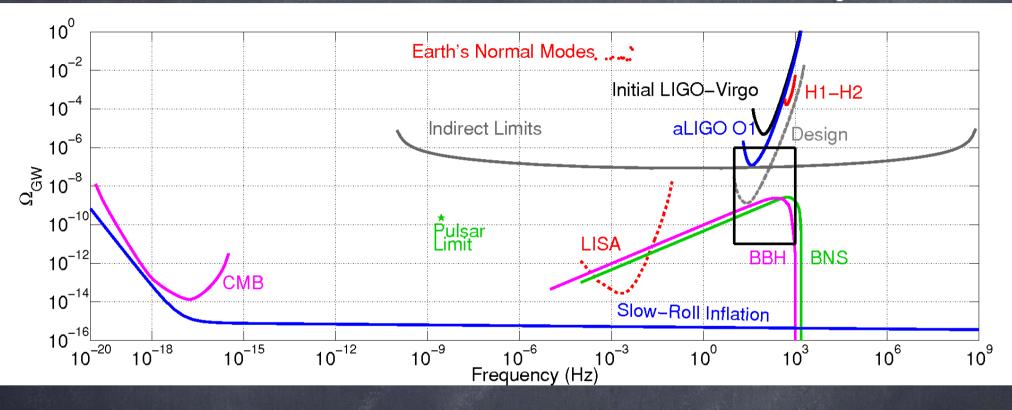
Energy density per unit log frequency or wavenumber

$$\Omega_{\rm GW}(f) \equiv \frac{1}{\rho_{\rm crit}} \frac{d\rho_{\rm GW}(f)}{d\ln f} = \frac{1}{12} \left(\frac{k}{H_0}\right)^2 \Delta_h^2(k) \qquad \Delta_h^2(k) = \frac{k^3 P_h(k)}{2\pi^2}$$

ho $\Omega_{
m GW} h^2$ is independent of the H_0

Current constraints on SGWB

LIGO&Virgo collaboration 2017

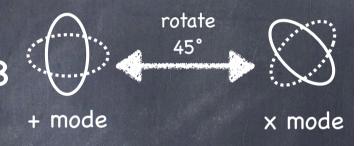


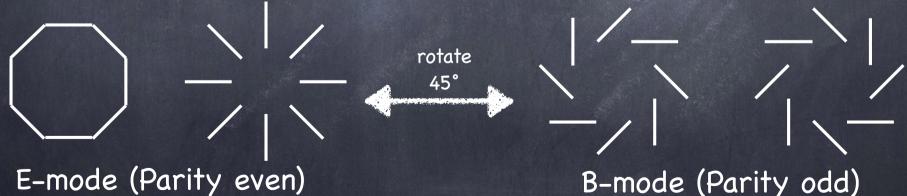
- Detection methods
 - CMB temperature/polarization anisotropy (Gpc)
 - Pulsar Timing Array (pc)
 - Laser interferometer experiments (km)

Cosmic shear as a probe of GW

Kaiser&Jaffe 1996, Dodelson+ 2003, Schmidt&Jeong 2012

- Galaxy shape(shear): spin-2 -> E/B decomposition
- Weak lensing: Bending light by metric perturbations
 - Scaler perturbation -> only EE
 - Tensor perturbation -> both EE&BB





B-mode shear is a unique signature of GWs

B-mode power spectrum from GWs

$$C_{\ell}^{BB}(\chi_1,\chi_2) = \frac{1}{\pi} \int d \ln k \; \Delta_h^2(k) F_{\ell}^B(k,\chi_1) F_{\ell}^B(k,\chi_2) \qquad \quad \Delta_h^2(k) = \frac{k^3}{2\pi^2} P_h(k)$$
 GW power

$$F_{\ell}^{B}(k,\chi) = -\frac{1}{4} \left[T_{T}(k,\eta_{0}) \left(\operatorname{Im} \hat{Q}_{1}(x) \frac{j_{\ell}(x)}{x^{2}} \right)_{x=0} + T_{T}(k,\eta_{0} - \chi) \left(\operatorname{Im} \hat{Q}_{1}(x) \frac{j_{\ell}(x)}{x^{2}} \right)_{x=k\chi} \right]$$

observer term

source term

Schmidt&Jeong 2012

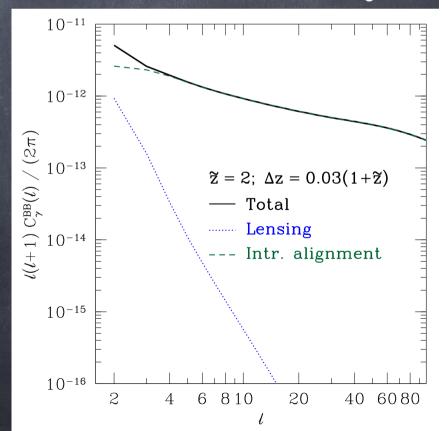
$$+ \int_0^{\chi} \frac{d\tilde{\chi}}{\tilde{\chi}} T_T(k, \eta_0 - \tilde{\chi}) \left(\operatorname{Im} \hat{Q}_2(x) \frac{j_{\ell}(x)}{x^2} \right)_{x = k\tilde{\chi}}$$

lensing term

- For GWs from the inflation
 - ▶ flat spectrum

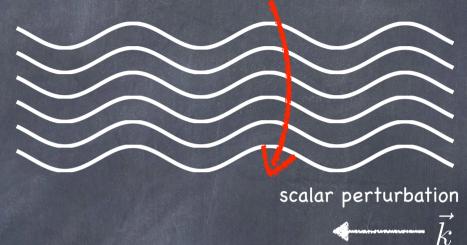
$$\Delta_h^2(k) = \frac{k^3}{2\pi^2} P_h(k) = \Delta_{T,\text{pivot}}^2 \left(\frac{k}{k_{\text{pivot}}}\right)^{n_T}$$
$$n_T = -r/8, \quad r = 0.2$$

very steeply falling with l

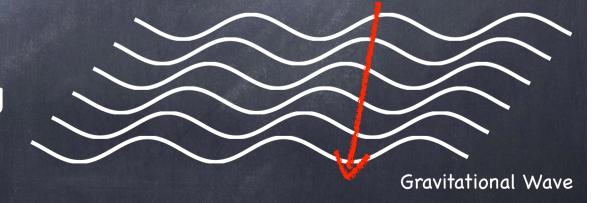


Weak lensing by density/GW

- Lensing by scalar perturbation
 - static
 - deflect light coherently along the line of sight(LOS)



- Lensing by GW
 - propagating!
 - cancellation of lensing effect along the LOS



B-mode signal from GWs

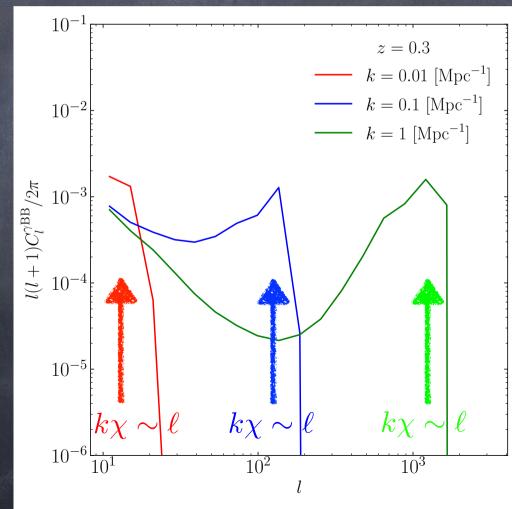
 $m{\mathsf{X}}$ Limber approximation: contribution from only $k\chi\sim\ell$

$$\int k^2 dk \; P_h(k;\chi,\chi') j_\ell(k\chi) j_\ell(k\chi') \; \simeq \; \frac{\pi}{2\chi^2} \delta_D(\chi-\chi') P_h\left(k = \frac{\ell+1/2}{\chi};\chi,\chi'\right)$$
 highly oscillatory function

$$P_h(k;\chi,\chi')=T_T(k\chi)T_T(k\chi')P_{T0}(k)$$
 $T_T(k\chi)\propto rac{j_1(k\chi)}{k\chi}$ in M.D.

- different from scalar case
- due to the high oscillation of tensor transfer func.
- require careful calculation
- low-l can probe short-mode





HSC first-year data

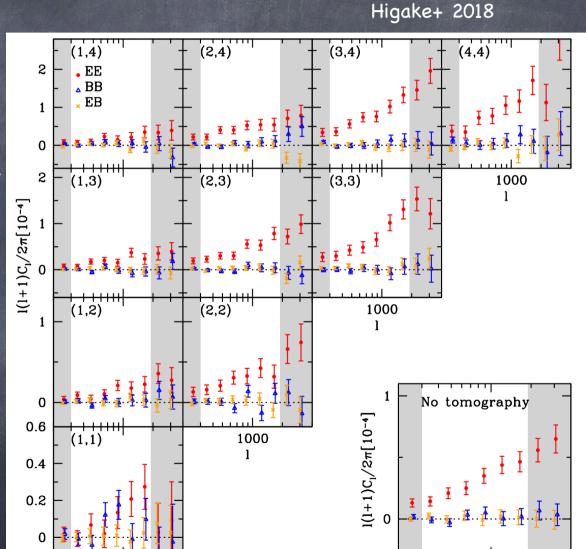
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- E-mode
 - from scalar mode
 - constrain cosmological parameters

(Hikage-san's talk)

- No B-mode

 - ightharpoonup can put upper limit on $\Omega_{
 m GW}$



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Method

Cosmic shear B-mode signal in the HSC survey (model)

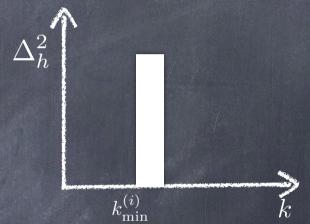
$$C_{\ell}^{(ij)} = \frac{1}{\pi} \int d\ln k \ \Delta_h^2(k) \ \bar{F}_{\ell}^{(i)}(k) \bar{F}_{\ell}^{(j)}(k)$$

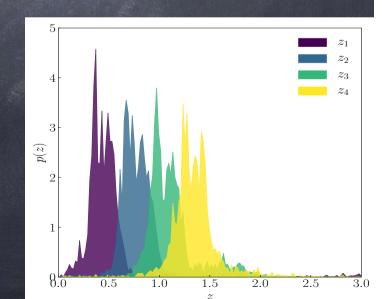
Tensor power spectrum: step-type

$$\Delta_h^2(k) = \begin{cases} \Delta_h^{2\ (i)} & (k_{\min}^{(i)} \le k < k_{\min}^{(i+1)}) \\ 0 & (\text{otherwise}) \end{cases}$$

- independent of GW sources
- Redshift distribution: $p_i(\chi)$
 - four redshift(photo-z) bins
 - averaged kernel

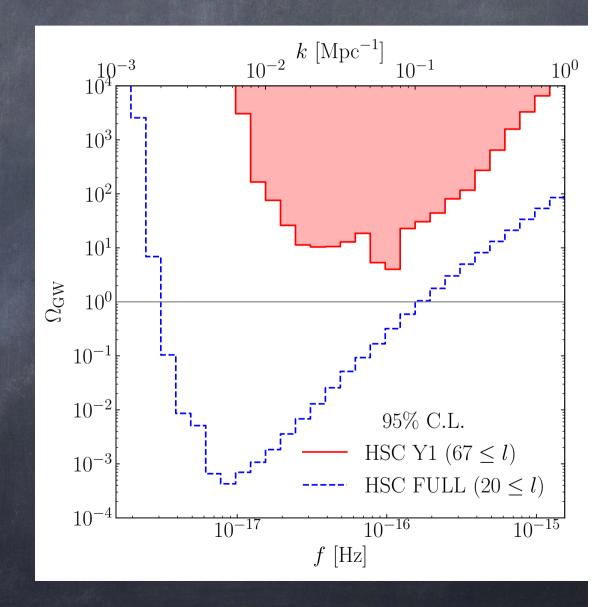
$$\bar{F}_{\ell}^{(i)}(k) = \int d\chi \ p_i(\chi) F_{\ell}(k, \chi)$$





Constraints from the HSC

- constraints are aroundGpc~Mpc scale
 - galaxy separation
- \odot HSC Y1: $\Omega_{\rm GW} \lesssim 1$
 - 140 deg², 1 > 67
- \odot HSC full: $\Omega_{\rm GW} \lesssim 10^{-3}$
 - ≥ 1400 deg², l > 20

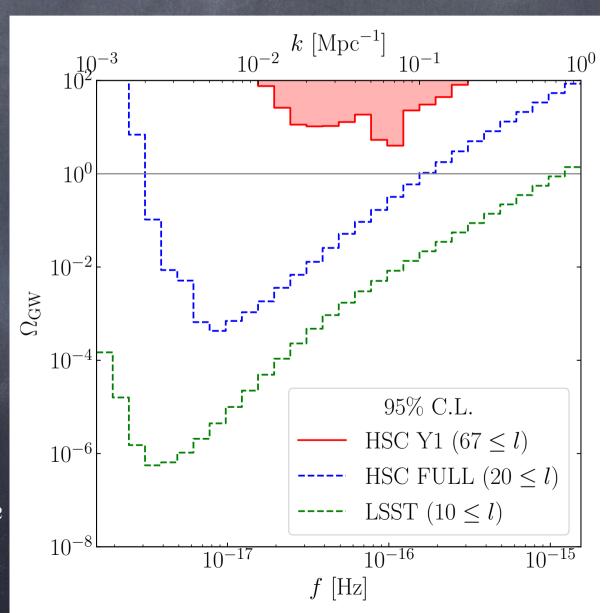


Future prospect (e.g. LSST)

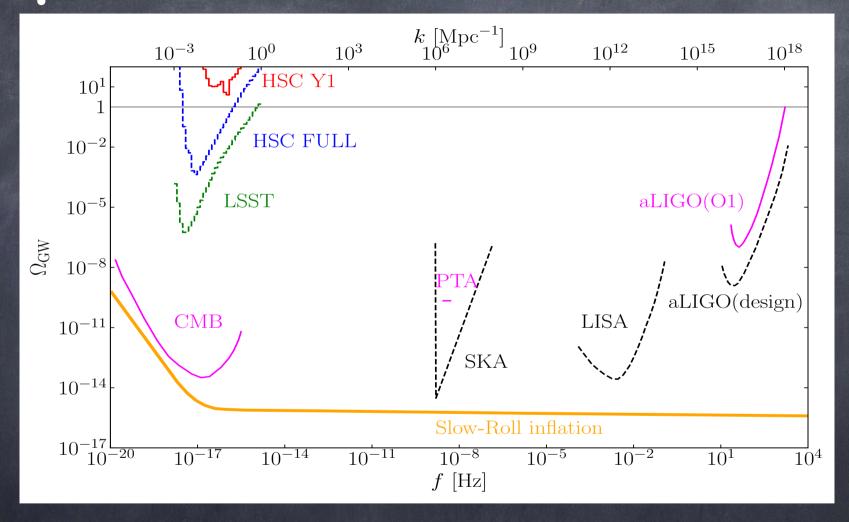
- To reduce errors
 - sky coverage
 - number density

$$\text{Cov} \propto (f_{\text{sky}}\bar{n}_g)^{-1}$$

- effect of decaying
 - high redshift
 - low multipole
- \odot LSST: $\Omega_{\rm GW} \lesssim 10^{-6}$
 - $f_{\rm sky} \sim 0.5, \ \bar{n}_{g,{\rm total}} \sim 40 \ {\rm arcmin}^{-2}$ $z < 3, \ \ell \ge 10$



Comparison with other results



Weak lensing can constrain GWs generated after recombination/reionization

Summary & Future works

- Weak lensing by GWs produces B-mode cosmic shear
- $\ \odot$ With the HSC full data, we can constrain $\Omega_{\rm GW} \lesssim 10^{-3} @ 10^{-17} \ \rm Hz$
- Tuture survey(e.g. LSST) will constrain $\Omega_{\rm GW} \lesssim 10^{-6}$
- Future works:
 - Removing higher-order contributions (cf. delensing)
 - Intrinsic Alignments by GWs

 Schmidt&Jeong 2012, Pajer+ 2013b
 - Cross correlation with CMB

Backup