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TOKYO UNIVERSITY OF SCIENCE
理学部第二部

“Accelerating Universe in the Dark”, YITP

Primordial Black Holes from R^2 gravity

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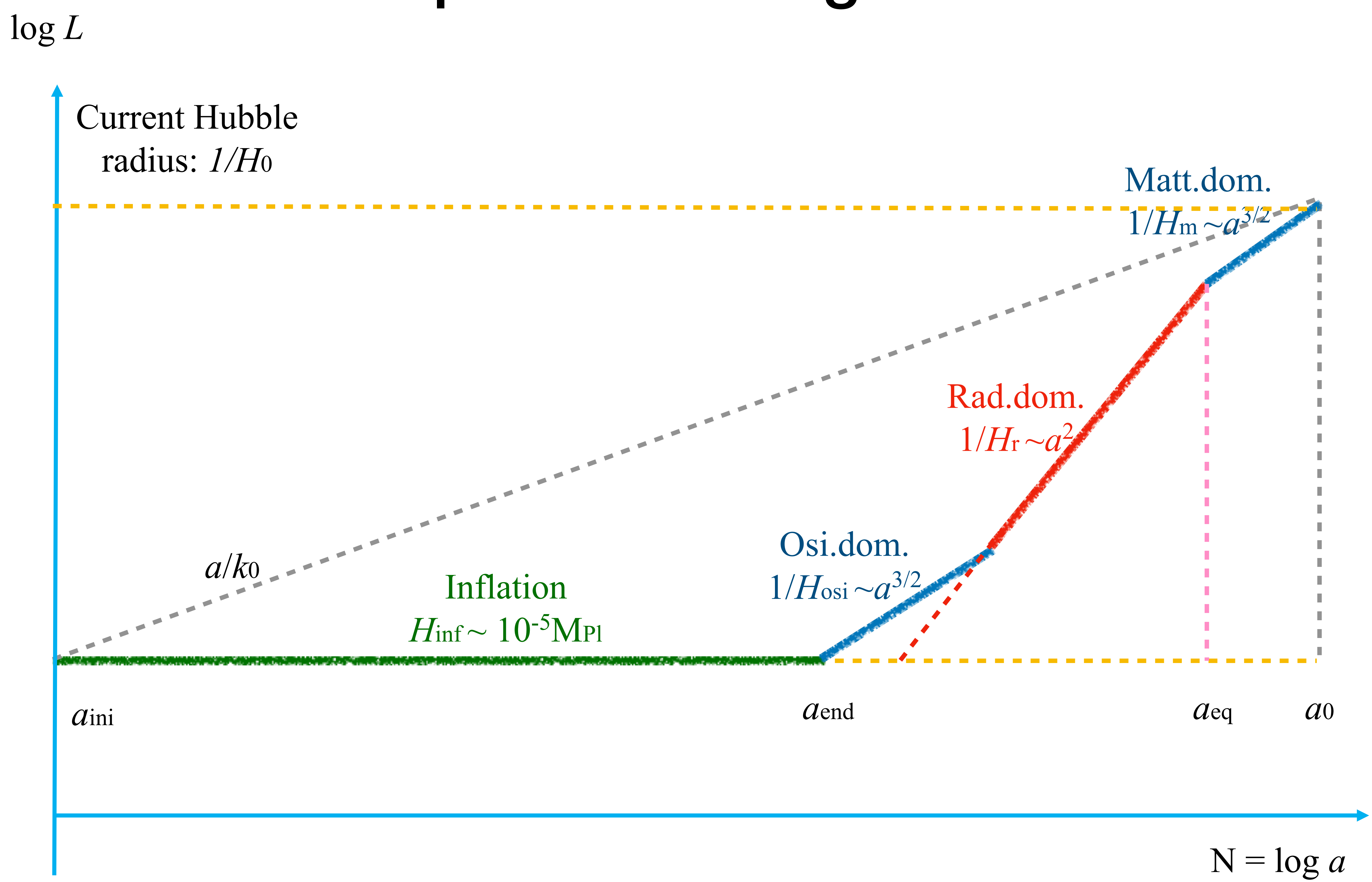
7, March, 2019

- Based on: Shi Pi, YZ, Qing-guo Huang, and Misao Sasaki, JCAP 1805 (2018) 042

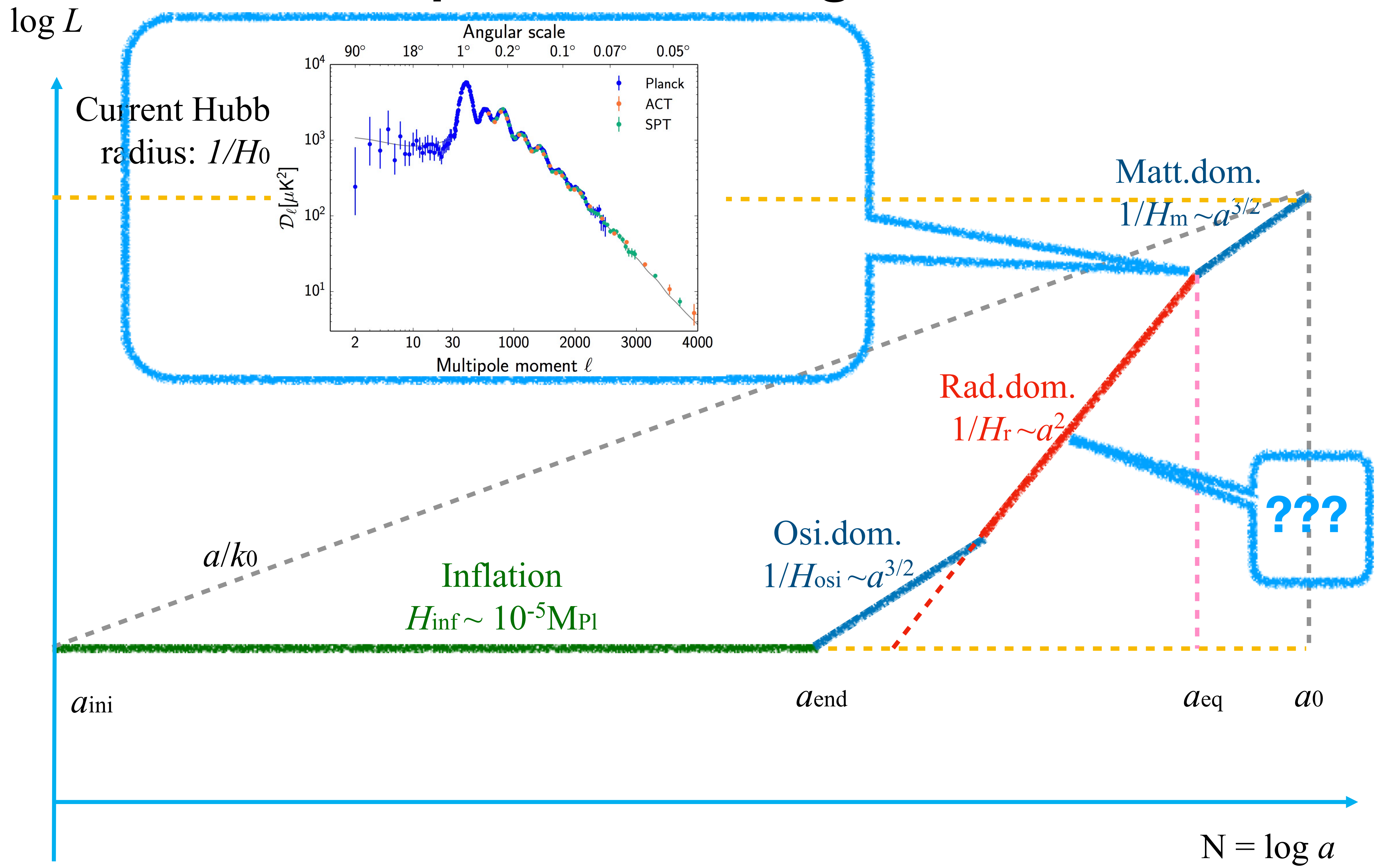
Contents

- Basics on PBH formation
- Possible inflation models for PBH formation
- Our model: Scalar field in R^2 gravity
- Discussions

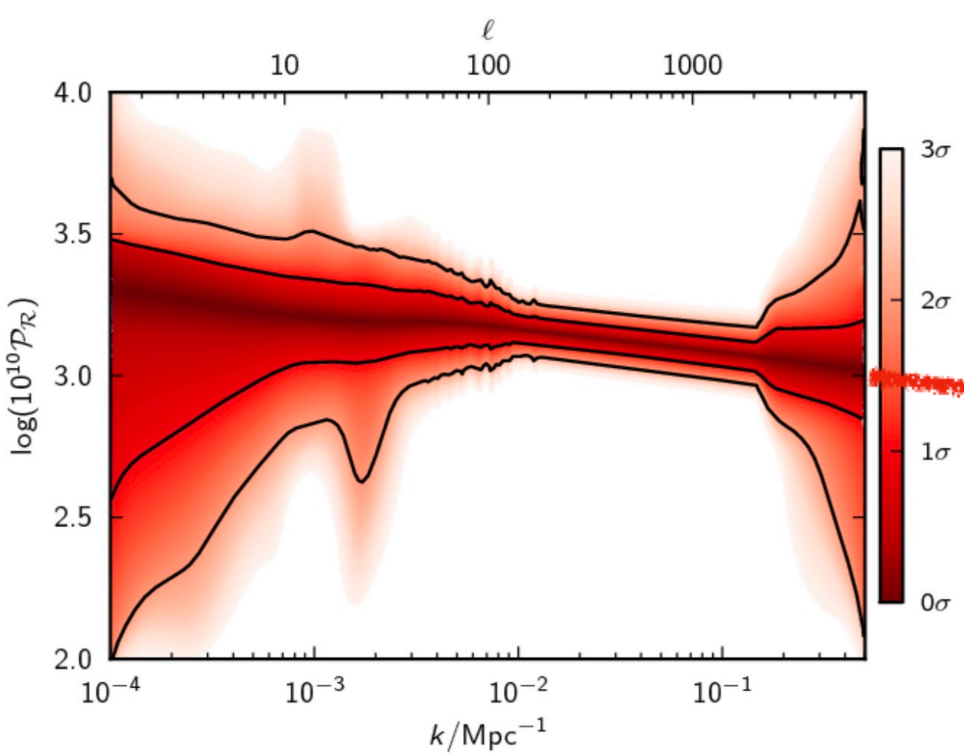
Spacetime diagram

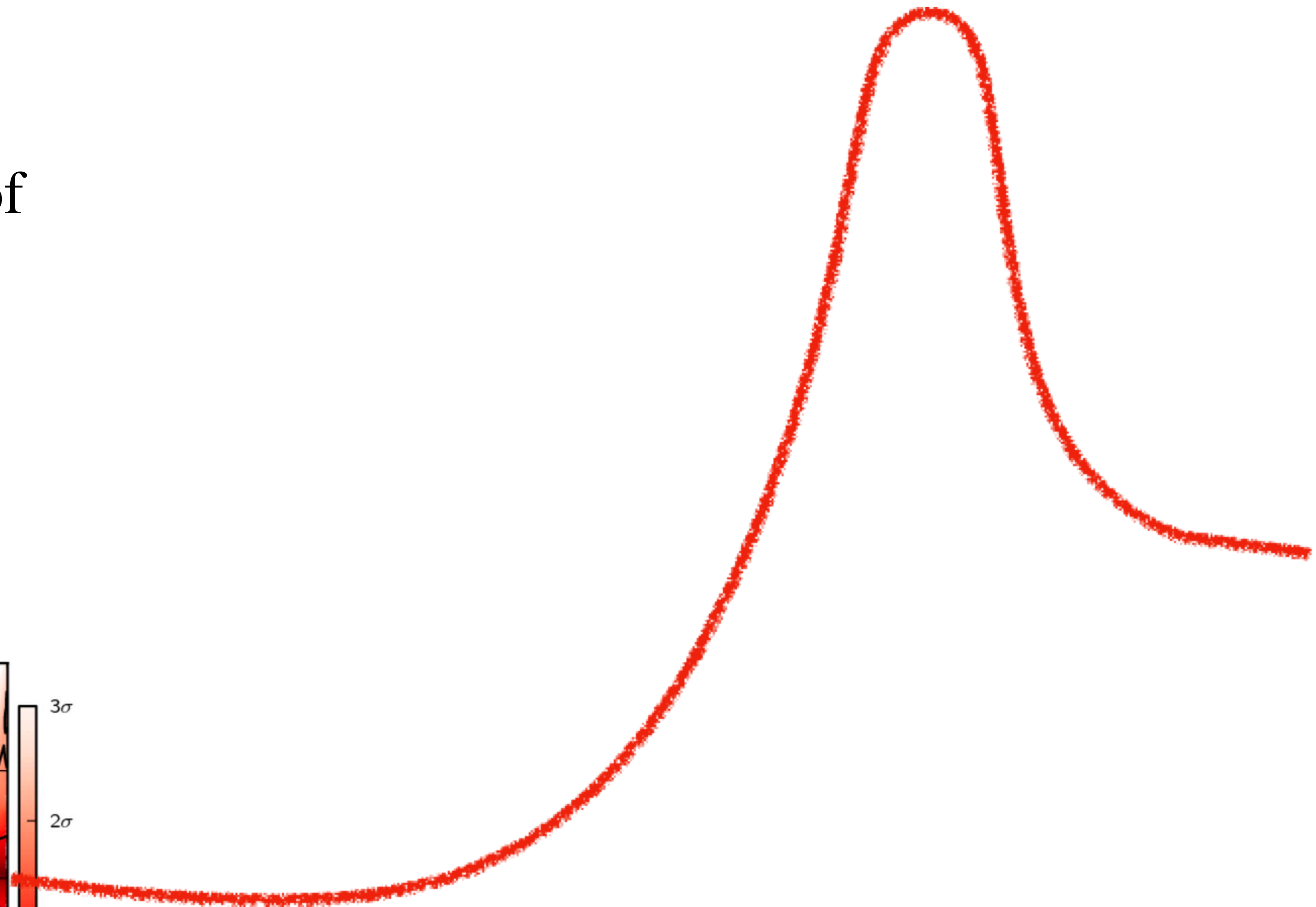


Spacetime diagram

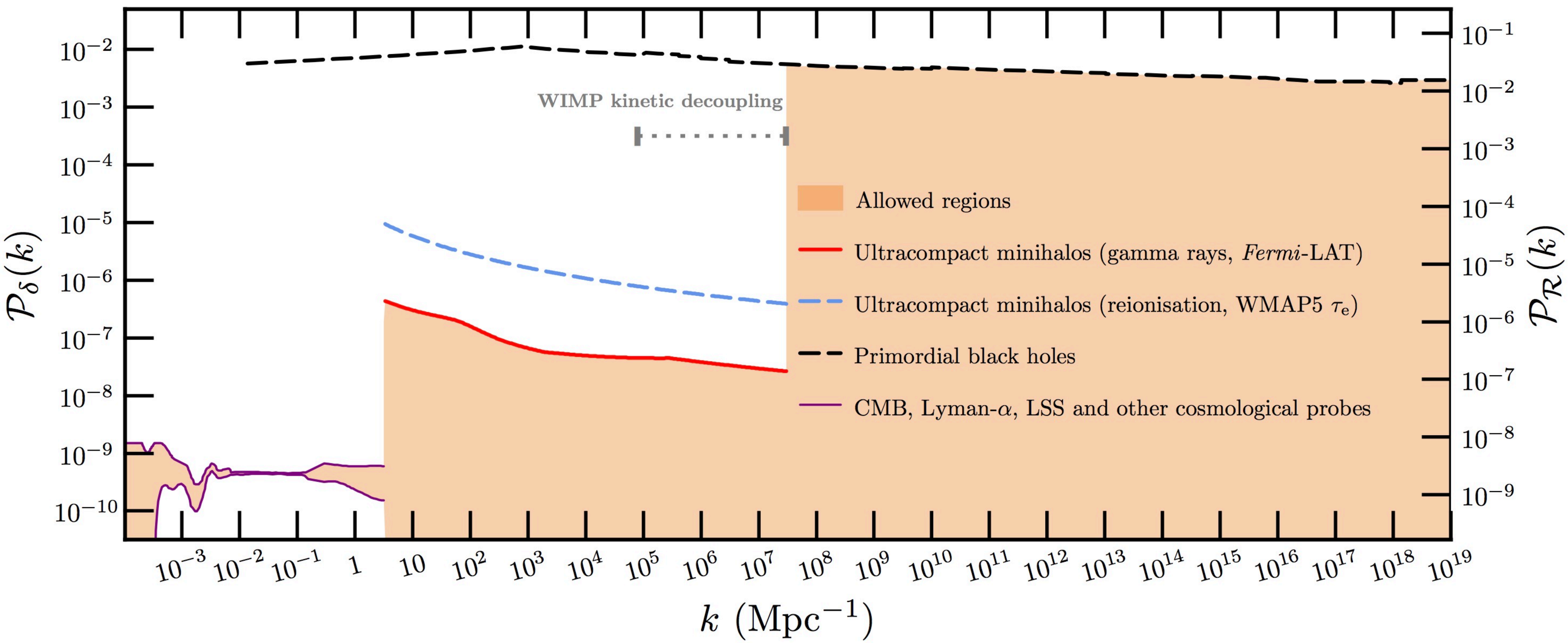


Bayesian reconstruction of the primordial power spectrum for l<2300. (Planck 2015)



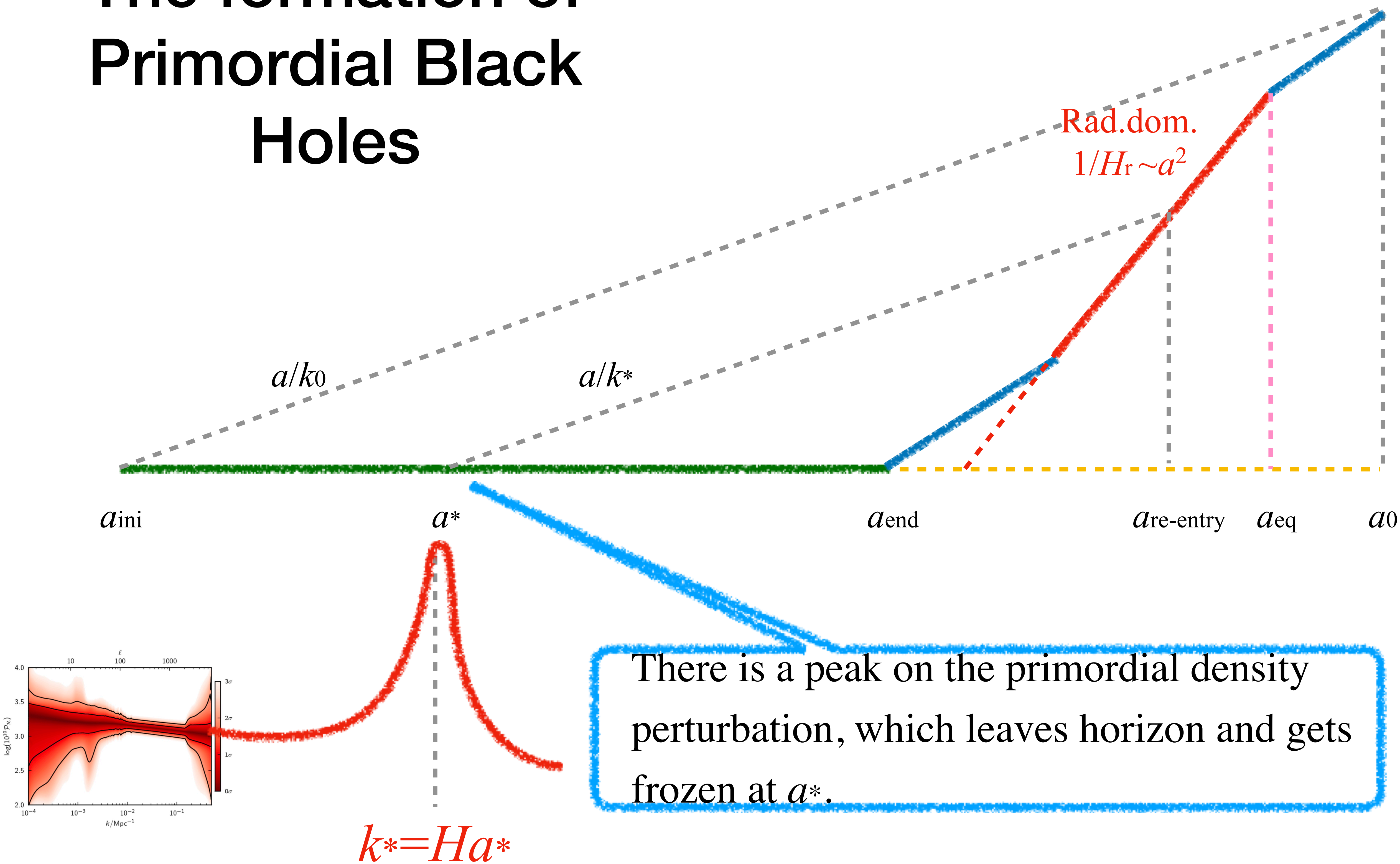


The resolution is lacking to say anything precise about higher l.



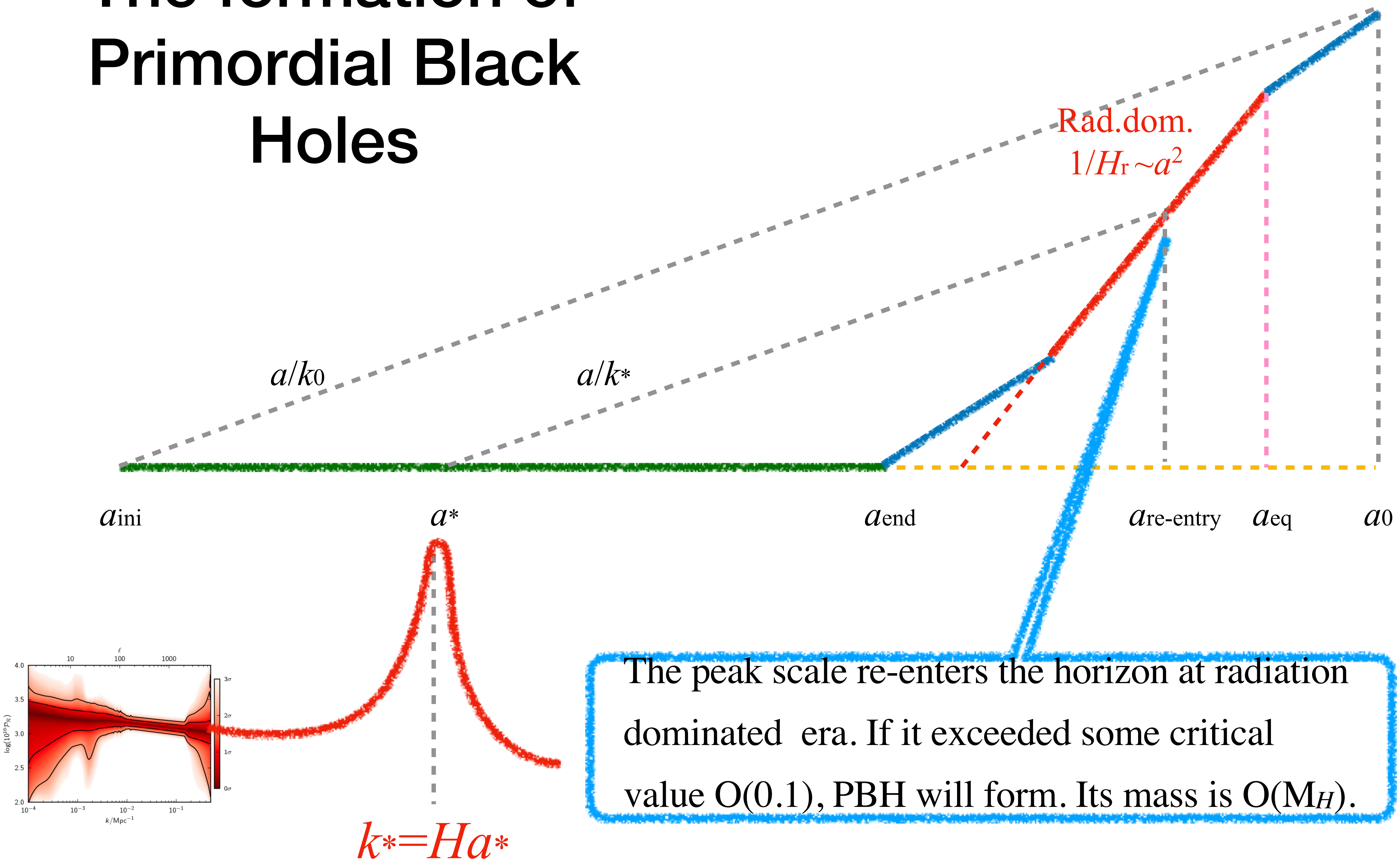
There are some constraints on small scales, but quite loose. (Bringmann et. al. 2011)

The formation of Primordial Black Holes



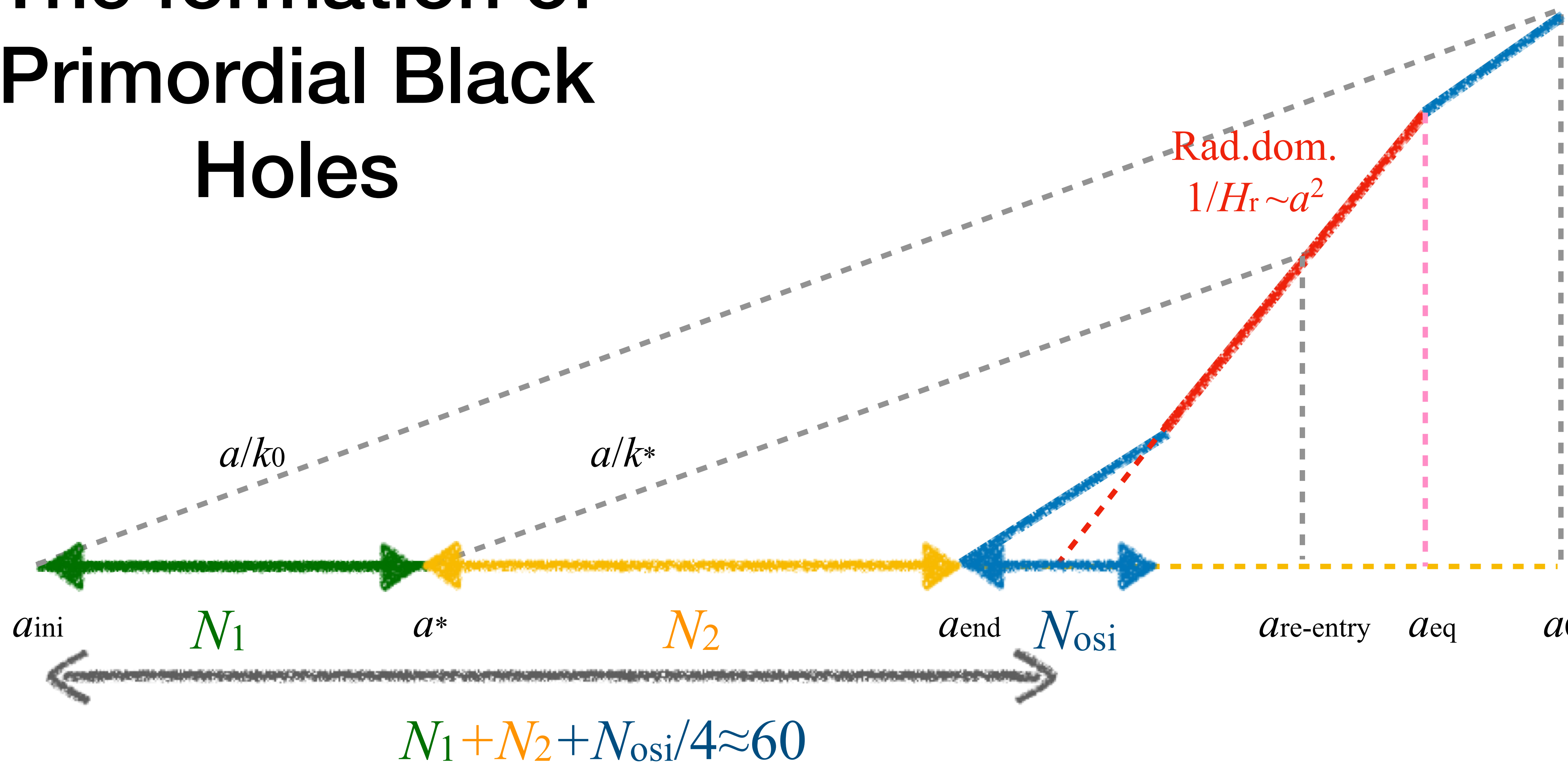
There is a peak on the primordial density perturbation, which leaves horizon and gets frozen at a_*.

The formation of Primordial Black Holes



The peak scale re-enters the horizon at radiation dominated era. If it exceeded some critical value O(0.1), PBH will form. Its mass is O(M_H).

The formation of Primordial Black Holes



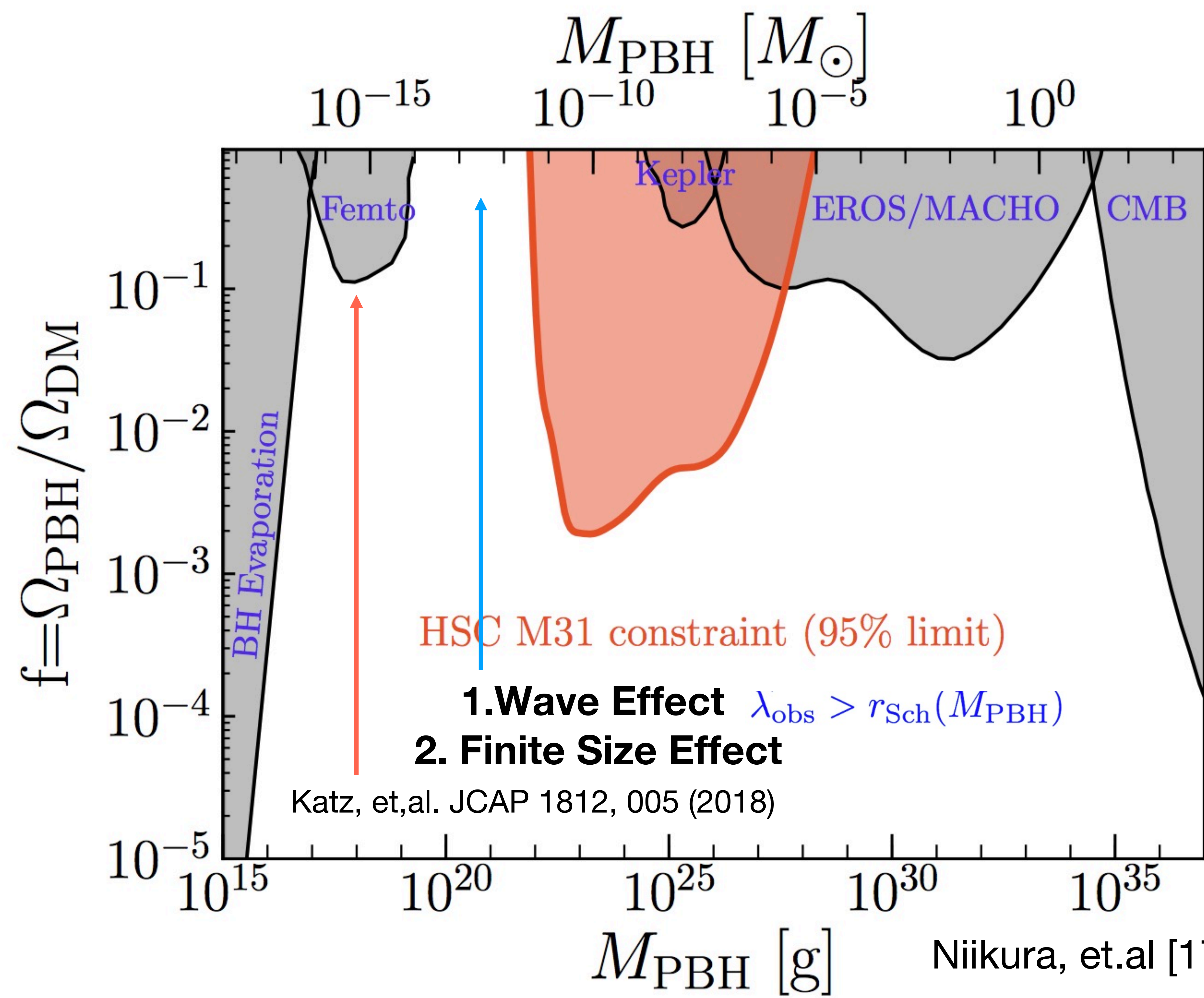
PBH mass: $M_{PBH} = \gamma M_H \sim \frac{M_{Pl}^2}{H} = 10^{58} M_{Pl} e^{-2N_1} = M_{Pl} 10^{58-0.87 N_1}$

Inverse relation: $N_1 = 44.4 - \frac{1}{2}\ln\left(\frac{M_{\rm PBH}}{10^{16}\ {\rm g}}\right).$

PBH mass scale does NOT depend on the reheating physics

What is Primordial Black Hole?

- PBH = BH formed before the recombination epoch (i.e. z>>2000) (conventionally during the radiation-dominated era)
- Hubble sized region with $\delta\rho/\rho = O(1)$ collapses to form BH
- Such a large perturbation could be produced by inflation
- PBHs may dominate Dark Matter
- …



Niikura, et.al [1701.02151]

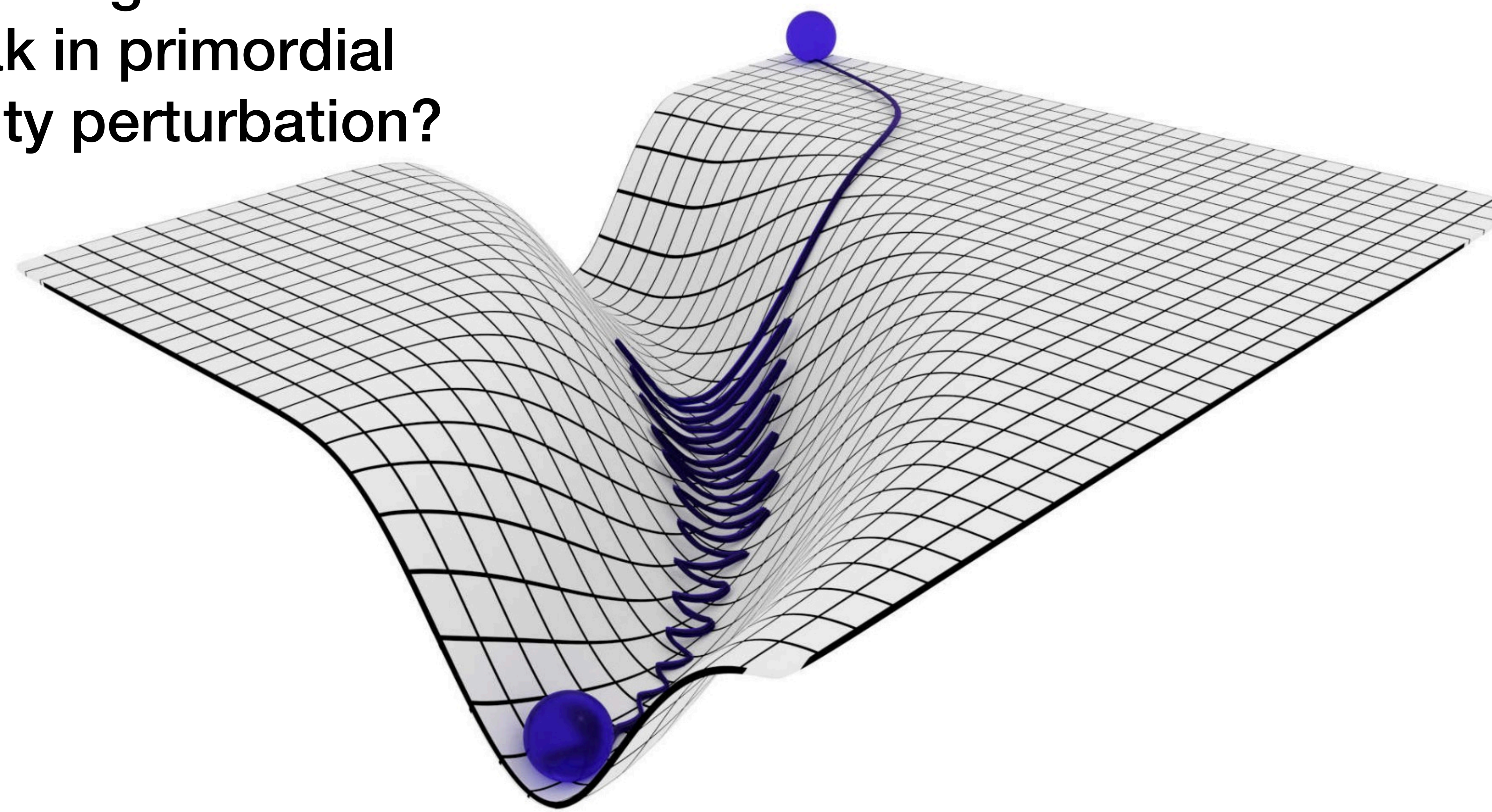
Inflation Models for PBH Formation

- PBH will form if there is a bump in the primordial power spectrum for the curvature perturbation.

- This means there will be a “ultra-slow-roll” stage of inflation.

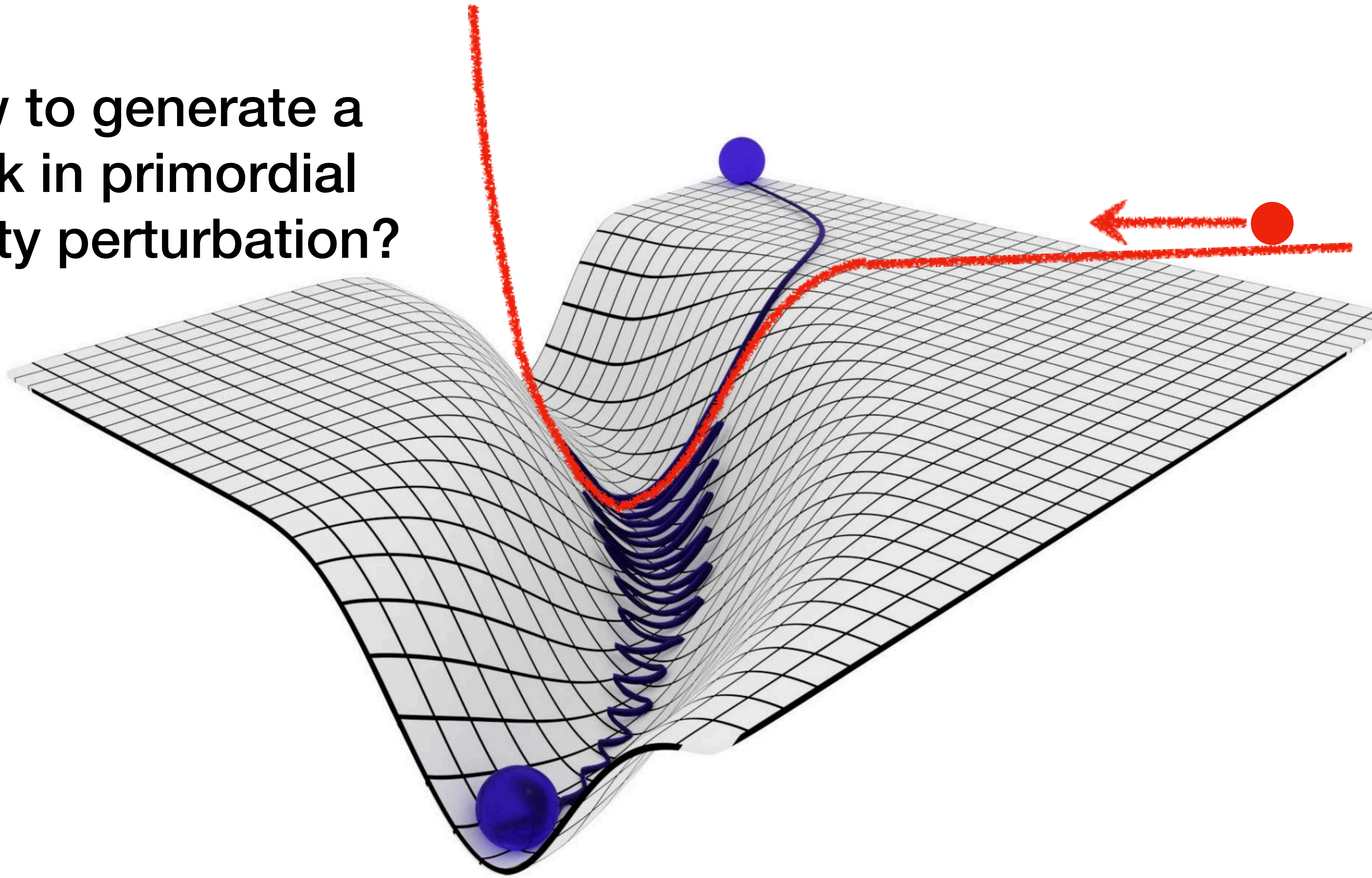
- Multifield: Scalar field in R^2 gravity (Shi Pi, YZ, Qing-guo Huang, and Misao Sasaki, JCAP 1805 (2018) 042)

How to generate a peak in primordial density perturbation?



- Typical potential for ‘inflation + massive field’, where oscillations will be generated.

How to generate a peak in primordial density perturbation?



- We impose a Starobinsky-like potential: $\sim \left(1 - e^{-\phi/M_{\mathrm{Pl}}}\right)^2$
- This is equivalent to: $R + \frac{R^2}{6M} - \frac{1}{2}(\partial\chi) - V(\chi)$

Motivation

- A natural way to realize it is just R^2 gravity plus inflaton.
- R^2 gravity itself can generate inflation. (Starobinsky 1980)
- Also, it is the best-fit inflation model. (Planck 2015)
- It is equivalent to study the scalar field(s) in Starobinsky model.

Setup

- We propose the Lagrangian as the Starobinsky R^2 gravity plus a scalar field χ, nonminimally coupled to gravity

$$S_J = \int d^4x\sqrt{-g}\left\{\frac{M_{\rm Pl}^2}{2}\left(R+\frac{R^2}{6M^2}\right)-\frac{1}{2}g^{\mu\nu}\partial_\mu\chi\partial_\nu\chi - V(\chi) - \frac{1}{2}\xi R\chi^2\right\}.$$

- $V(\chi)$ is potential for χ, which we pick for the small-field form:

$$V(\chi) = V_0 - \frac{1}{2}m^2\chi^2 + \cdots$$

- ξ-term is the non-minimally coupled term to solve the initial condition problem.

Setup

- It has been proved that the action with R^2 is equivalent to Einstein-Hilbert action plus one scalar field (scalaron). (Whitt 1984, Maeda 1988)

- After transferred to Einstein frame, our model becomes Hilbert Einstein action with two scalar fields: scalaron ϕ + light field χ, with nontrivial metric in field space (Starobinsky et. al. 2001)

$$S_E = \int d^4x\sqrt{-\tilde{g}} \cdot \left\{ \frac{M_{\rm Pl}^2}{2}\tilde{R} - \frac{1}{2}\tilde{g}^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - \frac{1}{2}e^{-\sqrt{\frac{2}{3}}\frac{\phi}{M_{\rm Pl}}}g^{\mu\nu}\partial_\mu\chi\partial_\nu\chi \right.$$
$$\left. - \frac{3}{4}M^2M_{\rm Pl}^2\left(1 - e^{-\sqrt{\frac{2}{3}}\frac{\phi}{M_{\rm Pl}}} + e^{-\sqrt{\frac{2}{3}}\frac{\phi}{M_{\rm Pl}}}\xi\frac{\chi^2}{M_{\rm Pl}^2}\right)^2 - e^{-2\sqrt{\frac{2}{3}}\frac{\phi}{M_{\rm Pl}}}V(\chi)\right\}.$$

EoM

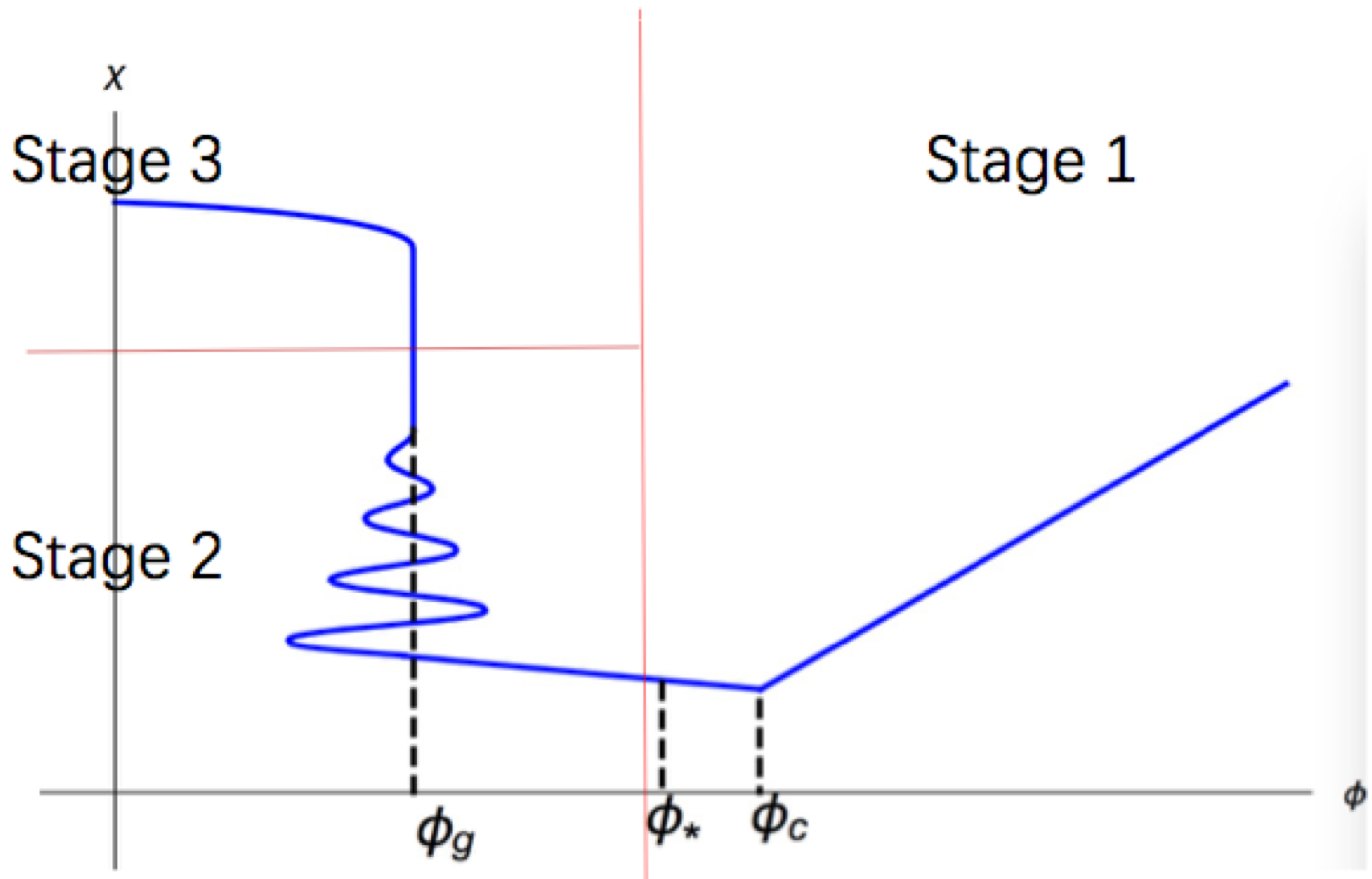
- The equations of motion:

$$3M_{\rm Pl}^2 H^2 = \frac{1}{2}\dot{\phi}^2 + F^{-1}\frac{1}{2}\dot{\chi}^2 + \frac{3}{4}M^2 M_{\rm Pl}^2 \left[1 - F^{-1}\left(1 - \xi\frac{\chi^2}{M_{\rm Pl}^2}\right)\right]^2 + F^{-2}V(\chi),$$

$$\ddot{\phi} + 3H\dot{\phi} + \sqrt{\frac{3}{2}}M^2 M_{\rm Pl}F^{-1}\left\{1 - \xi\frac{\chi^2}{M_{\rm Pl}^2} + \frac{\dot{\chi}^2}{3M^2M_{\rm Pl}^2} - F^{-1}\left[\left(1 - \xi\frac{\chi^2}{M_{\rm Pl}^2}\right)^2 + \frac{4V}{3M^2M_{\rm Pl}^2}\right]\right\} = 0,$$

$$\ddot{\chi} + \left(3H - \sqrt{\frac{2}{3}}\frac{\dot{\phi}}{M_{\rm Pl}}\right)\dot{\chi} + 3M^2\left[1 - F^{-1}\left(1 - \xi\frac{\chi^2}{M_{\rm Pl}^2}\right)\right]\xi\chi + F^{-1}V'(\chi) = 0,$$

- with $F = \exp\left(\sqrt{\frac{2}{3}}\frac{\phi}{M_{\rm Pl}}\right)$ and $V(\chi) = V_0 - \frac{1}{2}m^2\chi^2 + \cdots\ ,$



x
Stage 3
Stage 1
Stage 2
ϕ_g
ϕ_*
ϕ_c
ϕ

Background solutions

We derive the background solutions by dividing into two stages:

<u>Stage 1</u>: $\boxed{\kappa\phi \gg 1}$ $\Longrightarrow$ **slow rolling ϕ and χ**

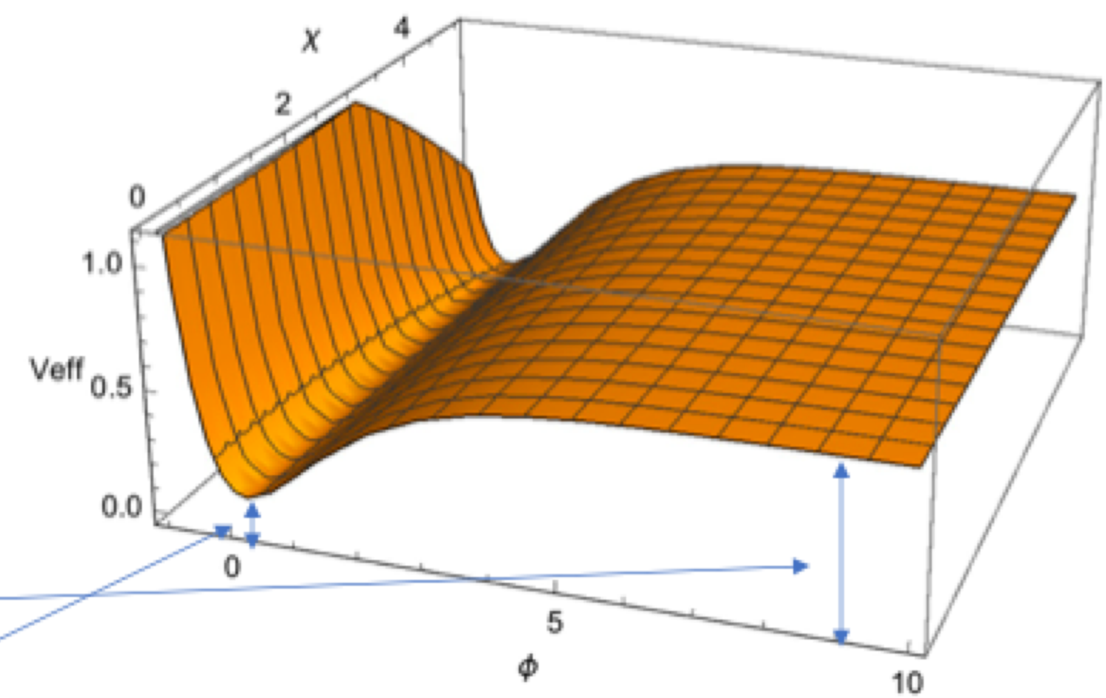
In this region, EOM are simplified as

$$H^2 = \frac{M^2}{4}\left[\left(1-e^{-\alpha\kappa\phi}\right)^2 + e^{-2\alpha\kappa\phi}\frac{4}{\mu^2}\right],$$

$$3H\dot{\phi} = -\frac{M^2}{\alpha\kappa}e^{-\alpha\kappa\phi}\left[1-e^{-\alpha\kappa\phi}\left(1+\frac{4}{\mu^2}\right)\right],$$

$$\left(3H-\alpha\kappa\dot{\phi}\right)\dot{\chi} = -3M^2\xi\left(1-e^{-\alpha\kappa\phi}\right)\chi + e^{-\alpha\kappa\phi}m^2\chi.$$

$$\boxed{\mu^2 \equiv \frac{3M^2M_{\rm Pl}^2}{V_0}}$$



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$$\left(3H-\alpha\kappa\dot{\phi}\right)\dot{\chi} = \boxed{-3M^2\xi}\left(1-e^{-\alpha\kappa\phi}\right)\chi + e^{-\alpha\kappa\phi}m^2\chi\,.$$

$dN \equiv -Hdt$ $\Longrightarrow$ $\boxed{\phi(N) = \frac{1}{\alpha\kappa}\ln\left[\frac{4}{3}\left(N-N_*\right)+e^{\alpha\kappa\phi_*}\right]}$

$$\left|\frac{\ddot{\phi}}{H\dot{\phi}}\right|_* = 1$$

Stablize the X~0 point for $\kappa\phi \gg 1$

Background solutions

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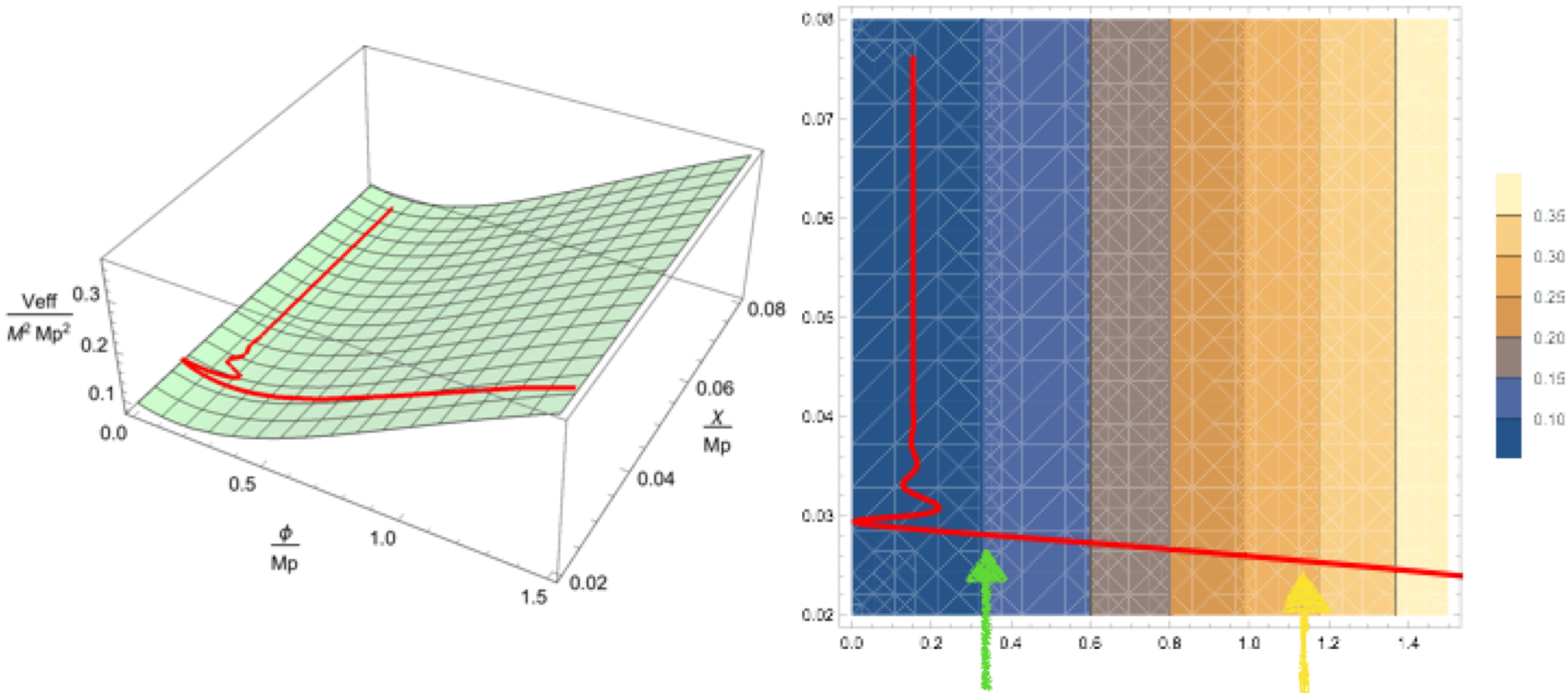
$$\left(3H-\alpha\kappa\dot{\phi}\right)\dot{\chi} = -3M^2\xi\left(1-e^{-\alpha\kappa\phi}\right)\chi + e^{-\alpha\kappa\phi}m^2\chi\,.$$

$$\Longrightarrow \quad \frac{d\ln\chi}{dN} = \frac{4\xi N+\lambda}{N+\beta} \quad \Longrightarrow \quad N_c \equiv -\frac{\lambda}{4\xi}$$

$$\xRightarrow{\ \phi_c > \phi_*\ } \quad |\xi| < \frac{4m^2}{7M^2}$$



End of the first stage of inflation,
marked as ϕ^*

End of Starobinsky inflation

Power Spectrum in the First Stage

- We use δN formalism to calculate the power spectrum in the first stage

$$P_\zeta = N_{,\phi}^2 \langle \delta\phi^2 \rangle = \frac{3H^2}{32\pi^2 M_{\rm Pl}^2} \left(\frac{1 - 2F^{-1} + F^{-2}\left(1 + 4/\mu^2\right)}{F^{-1} + F^{-2}\left(1 + 4/\mu^2\right)} \right)^2$$

$$= \frac{3M^2}{128\pi^2 M_{\rm Pl}^4} \frac{\left(1 - 2F^{-1} + F^{-2}\left(1 + 4/\mu^2\right)\right)^3}{\left(F^{-1} + F^{-2}\left(1 + 4/\mu^2\right)\right)^2},$$

$$= \frac{V_0}{24\pi^2 M_{\rm Pl}^4} \left(\frac{3}{16}\mu^2 \right) \frac{\left(1 - 2F^{-1} + F^{-2}\left(1 + 4/\mu^2\right)\right)^3}{\left(F^{-1} + F^{-2}\left(1 + 4/\mu^2\right)\right)^2},$$

- With the slow-roll parameters

$$\epsilon_H^{(1)} = \frac{8}{3} \frac{\left(F - \left(1 + 4/\mu^2\right)\right)^2}{\left(F^2 - 2F + \left(1 + 4/\mu^2\right)\right)^2},$$

$$\eta_H^{(1)} = \frac{8}{3} \frac{F\left(F^2 - 2F\left(1 + 4/\mu^2\right) + \left(1 + 4/\mu^2\right)\right)}{\left(F^2 - 2F + \left(1 + 4/\mu^2\right)\right)^2}$$

<u>Stage 2</u>: $\boxed{\kappa\phi \lesssim 1}$ $\Longrightarrow$ **slowing rolling** χ

In this region, the field ϕ quits the slow-roll region and trapped in the effective potential, with small oscillations. We use the method developed in

A. J. Tolley and M. Wyman, "The Gelaton Scenario: Equilateral non-Gaussianity from multi-field dynamics," Phys. Rev. D **81**, 043502 (2010) doi:10.1103/PhysRevD.81.043502 [arXiv:0910.1853 [hep-th]].

Defining the effective potential as

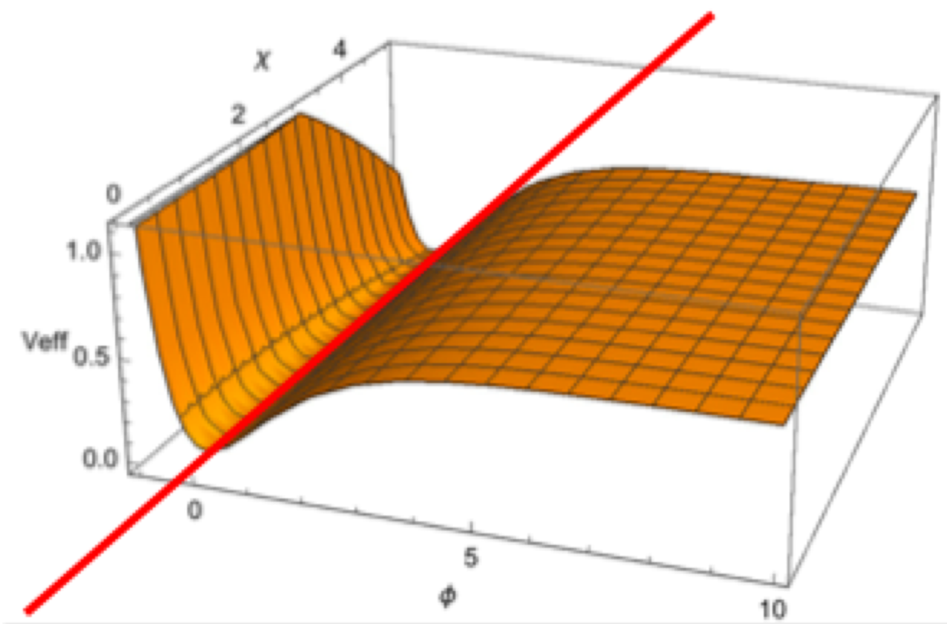
$$V_{\text{eff}} \equiv U(\phi) + \frac{1}{2} e^{-\alpha\kappa\phi} \tilde{g}^{\mu\nu} \partial_\mu \chi \partial_\nu \chi + e^{-2\alpha\kappa\phi} V(\chi)$$

The vacuum trajectory can be found as

$$\left.\frac{\partial V_{\rm eff}}{\partial \phi}\right|_{\phi=\phi_g} = 0 \quad \Rightarrow \quad e^{-\alpha\kappa\phi_g} = \frac{1+\sigma X}{1-\kappa^2\xi\chi^2+2\sigma V}$$

$\boxed{X = \dot{\chi}^2/2}$

$\boxed{\sigma \equiv 2\kappa^2/[3M^2(1-\kappa^2\xi\chi^2)]}$



$$S_g = \int d^4x\sqrt{-g}\left(\frac{M_p^2}{2}R - V_{\rm eff,g}\right)$$

$$\boxed{V_{\rm eff,g} = \frac{V - X\left(1-\kappa^2\xi\chi^2\right) - \frac{\kappa^2}{3M^2}X^2}{\left(1-\kappa^2\xi\chi^2\right)^2 + \frac{4\kappa^2}{3M^2}V}}$$

$$3H^2\frac{d\chi}{dN}\left[\left(1-\kappa^2\xi\chi^2\right)^2 + \frac{4\kappa^2}{3M^2}V\right] \approx \left(1-\kappa^2\xi\chi^2\right)V_{,\chi} + 4\kappa^2\xi V\chi$$

Under the assumption $V_0 \gg m^2\chi^2/2$, the solution can be found as

$$\chi \approx \chi_f \exp\left[\left(-\frac{1}{\kappa^2}\frac{m^2}{V_0}+4\xi\right)(N-N_f)\right]$$

Furthermore, the slow-roll parameters can be calculated as

$$\epsilon_H \equiv -\frac{\dot{H}}{H^2} = \frac{\kappa^2}{2}\frac{1}{1+\frac{4}{\mu^2}}\left(\frac{d\chi}{dN}\right)^2 = \frac{\kappa^2}{2}\left(4\xi-\frac{m^2}{\kappa^2 V_0}\right)^2\frac{\chi^2}{1+\frac{4}{\mu^2}},$$

$$\eta_H \equiv \frac{\dot{\epsilon}_H}{H\epsilon_H} = -\frac{d\ln\epsilon_H}{dN} = -\frac{2}{\chi}\frac{d\chi}{dN} = -2\left(-\frac{m^2}{\kappa^2 V_0}+4\xi\right) \lesssim 0.01,$$

$$\Rightarrow \quad \epsilon_H = \frac{\eta_H^2}{8}\frac{\chi^2\kappa^2}{1+\frac{4}{\mu^2}} \ll \eta_H$$

$$\Downarrow$$

$$n_s - 1 = -2\epsilon_H - \eta_H \approx -\eta_H \sim -0.04$$

$$\Big\Downarrow \; r = 16\epsilon_H < 0.1$$

$$\epsilon_H \lesssim 0.005 \ll \eta_H \sim 0.04$$

Oscillations of power spectrum

Now we study the oscillations of ϕ field around its classical vacuum trajectory by setting $\phi \to \phi_g + \Delta\phi$, so the second-order action can be expressed as

$$\Delta S_E^{(2)} = \int d^4x\sqrt{-g}\left[-\frac{1}{2}(\partial\Delta\phi)^2 - \frac{1}{2}\left(\frac{\partial^2 V_{\rm eff}}{\partial\phi^2}\right)\bigg|_{\phi=\phi_g}\Delta\phi^2\right]$$

$$\approx \int d^4x\sqrt{-g}\left\{-\frac{1}{2}(\partial\Delta\phi)^2 + \alpha^2 e^{-\alpha\kappa\phi_g}\left[\frac{3}{4}M^2 - e^{-\alpha\kappa\phi_g}\left(\frac{3}{2}M^2 + 2\kappa^2 V_0\right)\Delta\phi^2\right]\right\}$$

$$\Longrightarrow \quad \frac{d^2\Delta\phi}{dN^2} - 3\frac{d\Delta\phi}{dN} + \mu^2\Delta\phi = 0$$

$$\Longrightarrow \quad \Delta\phi = \beta e^{3(N-N_*)/2}\cos\left[\omega\left(N - N_*\right) + \theta\right], \qquad \omega \equiv \sqrt{\mu^2 - \frac{9}{4}}$$

$$\Delta\phi(N_*) \equiv \phi_* - \phi_g \qquad \dot{\Delta\phi}(N_*) = \dot{\phi}(N_*)$$

- There is oscillation only for $\mu \gtrsim 2.08$
- There is also an upper bound for not violate inflation during the transition: $\mu \lesssim 8.95$

Power Spectrum In the Second Stage

- Since ϕ is a heavy field, its perturbations are exponentially suppressed.

- We can use δN formalism to calculate the power spectrum in the second stage, mainly contributed by the quantum fluctuations of χ.

- The dependence of e-folding number can be calculated by its slow-roll EoM, which is dynamically coupled to $\Delta\phi$.

$$3H\left(1-\frac{1}{3}\sqrt{\frac{2}{3}}\frac{\dot{\phi}}{HM_{\rm Pl}}\right)\dot{\chi}+e^{-\sqrt{\frac{2}{3}}\frac{\phi}{M_{\rm Pl}}}V'(\chi)=0.$$

$$\left(3H - \alpha\kappa\dot{\phi}\right)\dot{\chi} = -3M^2\xi\left(1 - e^{-\alpha\kappa\phi}\right)\chi + e^{-\alpha\kappa\phi}m^2\chi$$

$$\Downarrow \quad \phi = \phi_g + \Delta\phi$$

$$\frac{d\ln\chi}{dN} \approx -\frac{\eta_H}{2}\left(1 - \frac{\alpha\kappa}{3}\frac{d\phi}{dN}\right) + \mu^2\left(\xi + \frac{m^2}{3M^2}\right)\alpha\kappa\Delta\phi$$

$$\Downarrow$$

$$\ln\frac{\chi}{\chi_0} = -\frac{\eta_H}{2}(N - N_0) + \frac{\alpha\kappa\eta_H}{6}(\phi - \phi_0) + \alpha\kappa\mu^2\left(\xi + \frac{m^2}{3M^2}\right)\int_{N_0}^{N}\Delta\phi dN$$

$$\Downarrow$$

$$\frac{1}{\chi} = -\frac{\eta_H}{2}\frac{dN}{d\chi} + \frac{\alpha\kappa\eta_H}{6}\frac{d\phi}{d\chi} + \alpha\kappa\mu^2\left(\xi + \frac{m^2}{3M^2}\right)\frac{dN}{d\chi}\Delta\phi$$

$$\Downarrow$$

$$\frac{\partial N}{\partial\chi} = \frac{2}{\eta_H\chi}\left(1 - \frac{8}{3}\frac{\epsilon_H}{\mu^2} + \sqrt{\frac{2}{3}}\frac{\Delta\phi}{M_{\rm Pl}} + \frac{1}{3}\sqrt{\frac{2}{3}}\frac{\partial}{\partial N}\frac{\Delta\phi}{M_{\rm Pl}}\right)$$

Define the canonical field

$$\hat{\chi} = F^{-1/2}\chi$$

$$\Downarrow$$

$$\langle\delta\chi\delta\chi\rangle = F\langle\delta\hat{\chi}\delta\hat{\chi}\rangle = e^{\sqrt{\frac{2}{3}}\frac{\phi}{M_{\rm Pl}}}\left(\frac{H}{2\pi}\right)^2$$

$$\approx \left(1 + \frac{4}{\mu^2}\right)\left(\frac{H}{2\pi}\right)^2$$

Power Spectrum In the Second Stage

$$\mathcal{P}_{\mathcal{R}}^{(0)} \equiv \frac{H^2}{8\pi\epsilon_H^{(2)} M_{\rm Pl}^2} \approx \frac{V_0}{24\pi^2 M_{\rm Pl}^2} \underbrace{\left(\frac{M_{\rm Pl}}{\chi_*}\right)^2 \frac{8}{\eta_H^{(2)2}}}_{\text{Large Enhancement}} e^{\frac{\eta_H^{(2)}}{2}(N-N_*)},$$

$$\begin{aligned}\mathcal{P}_{\mathcal{R}} &= \left(\frac{\partial N}{\partial \chi}\right)^2 \langle \delta\chi\delta\chi \rangle, \\ &= \mathcal{P}_{\mathcal{R}}^{(0)} \Bigg\{ 1 - \frac{2}{3} e^{\frac{3}{2}(N-N_*)} \ln \frac{F_*}{1+\frac{4}{\mu^2}} \sqrt{1+\Upsilon^2} \Big[\omega \sin\left(\omega(N-N_*) + \tan^{-1}\Upsilon\right) \\ &\quad - 6\cos\left(\omega(N-N_*) + \tan^{-1}\Upsilon\right) \Big] \Bigg\}^2,\end{aligned}$$

Power Spectrum
In the Second Stage

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Large Enhancement

$$\omega = \sqrt{\mu^2 - 9/4}$$

$$\mathcal{P}_{\mathcal{R}} = \left(\frac{\partial N}{\partial \chi}\right)^2 \langle \delta\chi\delta\chi \rangle,$$

$$= \mathcal{P}_{\mathcal{R}}^{(0)} \left\{ 1 - \frac{2}{3} e^{\frac{3}{2}(N-N_*)} \ln \frac{F_*}{1+\frac{4}{\mu^2}} \sqrt{1+\Upsilon^2} \left[\omega \sin\left(\omega(N-N_*) + \tan^{-1}\Upsilon\right) \right.\right.$$

$$\left.\left. -\ 6\cos\left(\omega(N-N_*) + \tan^{-1}\Upsilon\right)\right]\right\}^2,$$

Power Spectrum In the Second Stage

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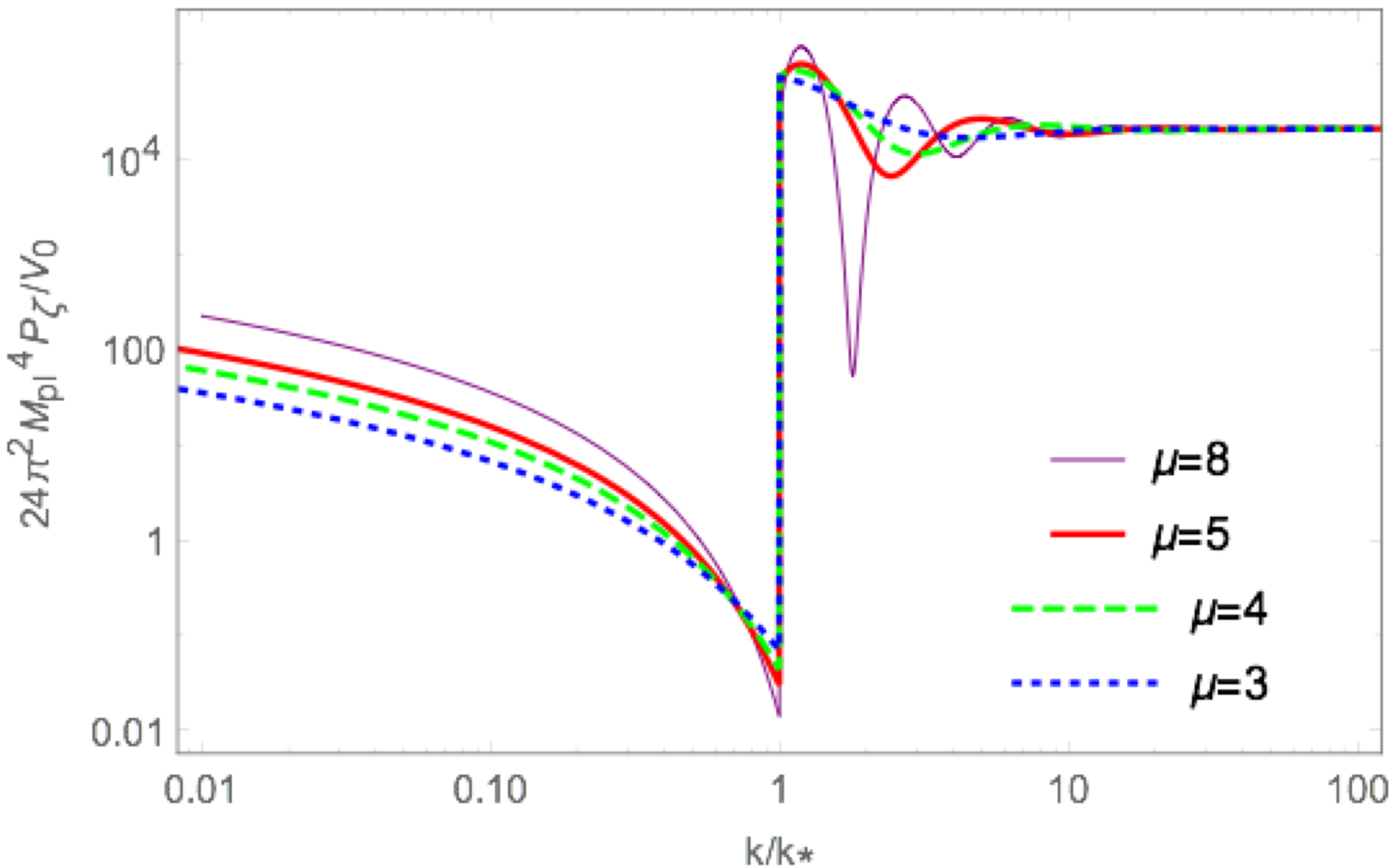
$$\left.\left. - \; 6\cos\left(\omega(N-N_*) + \tan^{-1}\Upsilon\right)\right]\right\}^2,$$



$\eta_H = 0.02, \ \ \chi_*/M_{\rm Pl} = 0.1,$

$\mu = 2, 3, 5, 8$

PBH formation

- Initial mass fraction:

$$\beta(M) \sim \int_{\delta_c}^{\infty} P(\delta(M_H))d\delta(M_H)$$

- For the Press-Schechter distribution:

$$\beta(M) = \mathrm{erfc}\left(\frac{\delta_c}{\sqrt{2}\sigma(M_H)}\right)$$

- And β can be transferred to the mass spectrum today by

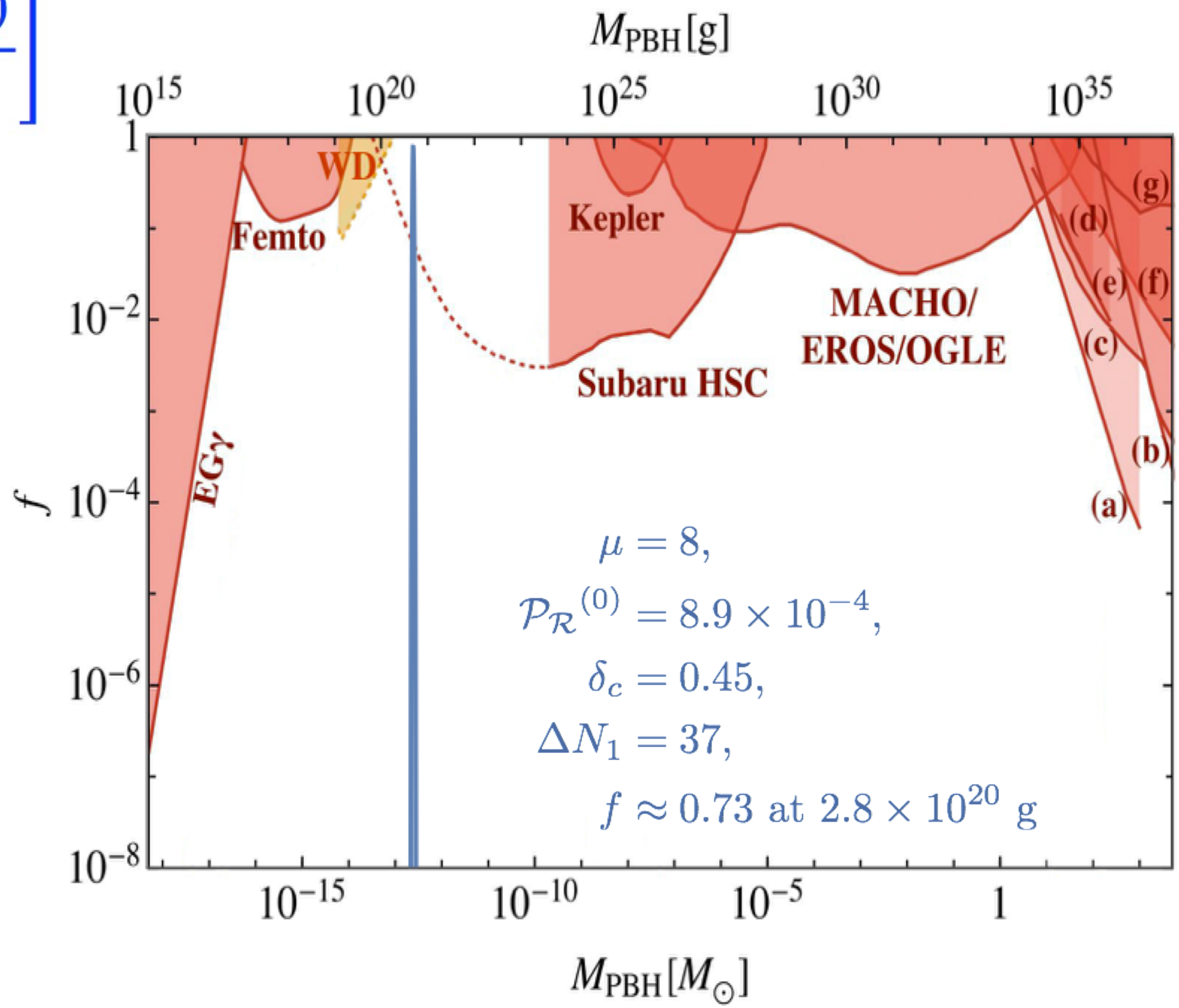
$$f \sim 10^9 \left(\frac{M}{M_\odot}\right)^{1/2} \beta(M)$$

$$f(M) \propto \exp\left[-\frac{O(0.1)}{\mathscr{P}(k)}\right]$$

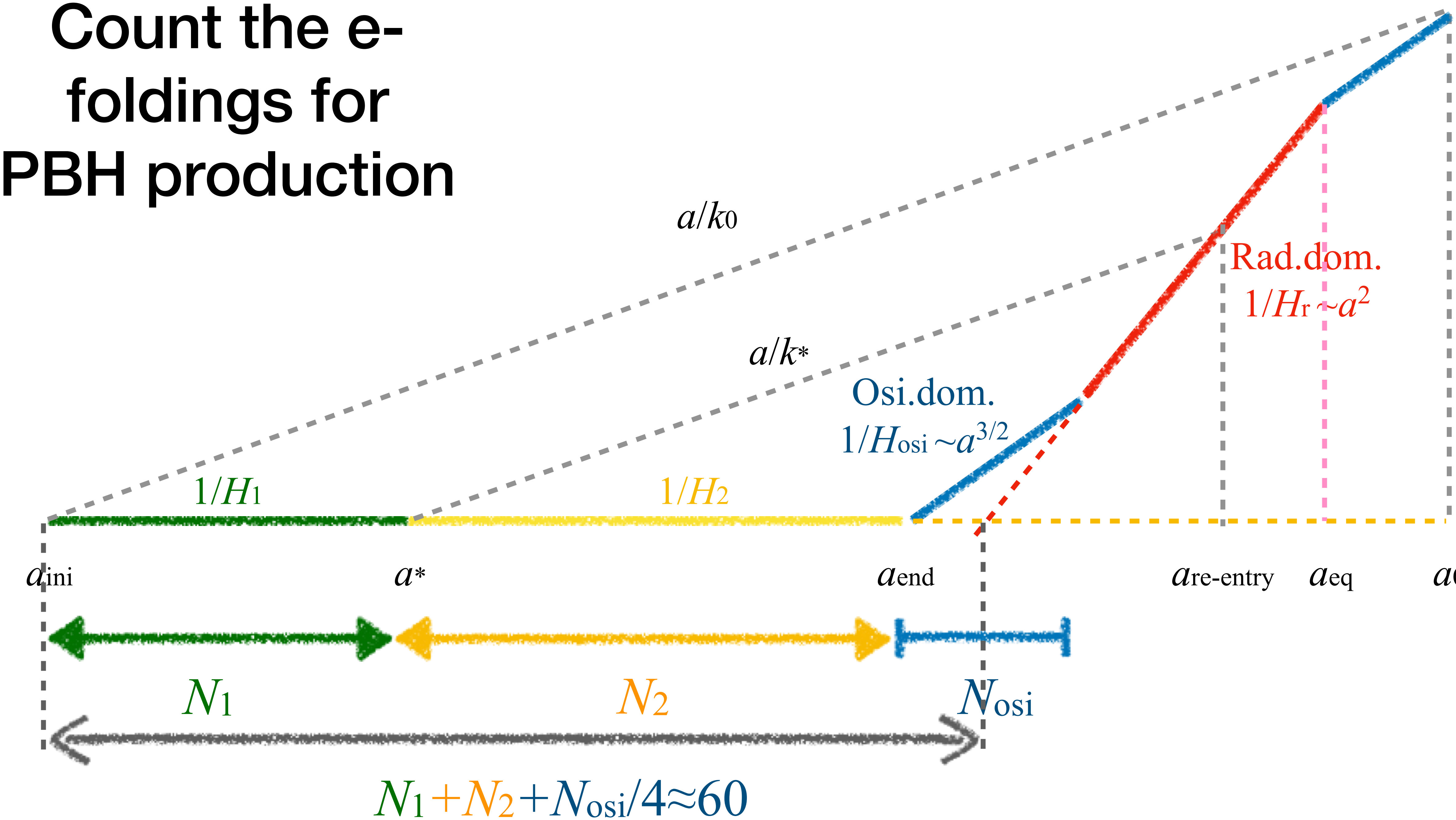
a sharp peak in $P(k)$

⇩

a spike in $f(M)$

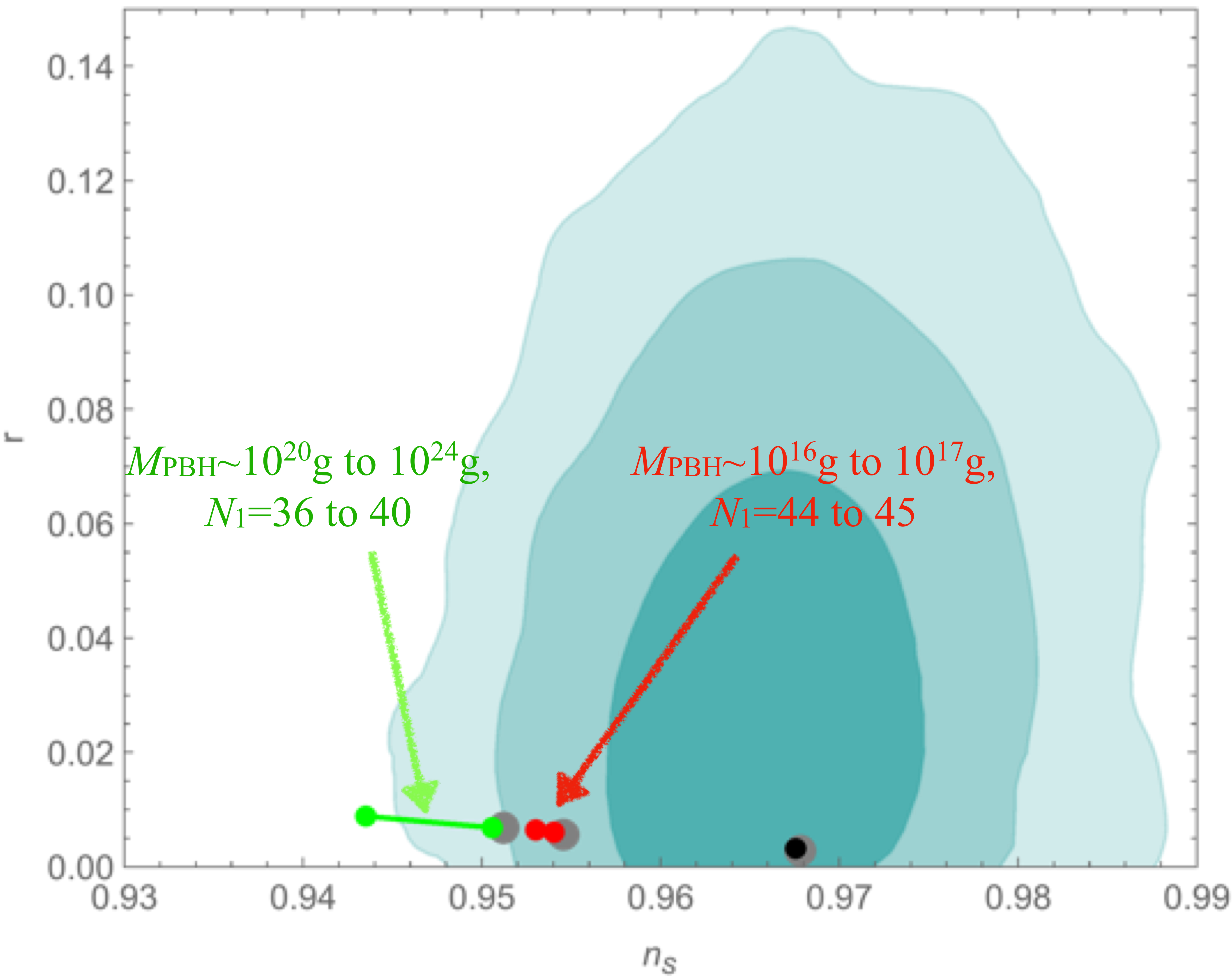


Count the e-foldings for PBH production



PBH mass: $M_{\rm PBH} \sim M_H \sim \dfrac{M_{\rm Pl}^2}{H_*} e^{2(N_2+N_{\rm osi}/4)} = \dfrac{M_{\rm Pl}^2}{H_*} e^{2(60-N_1)}$

Inverse relation: $N_1 = 44.4 - \dfrac{1}{2}\ln\left(\dfrac{M_{\rm PBH}}{10^{16}\ {\rm g}}\right).$



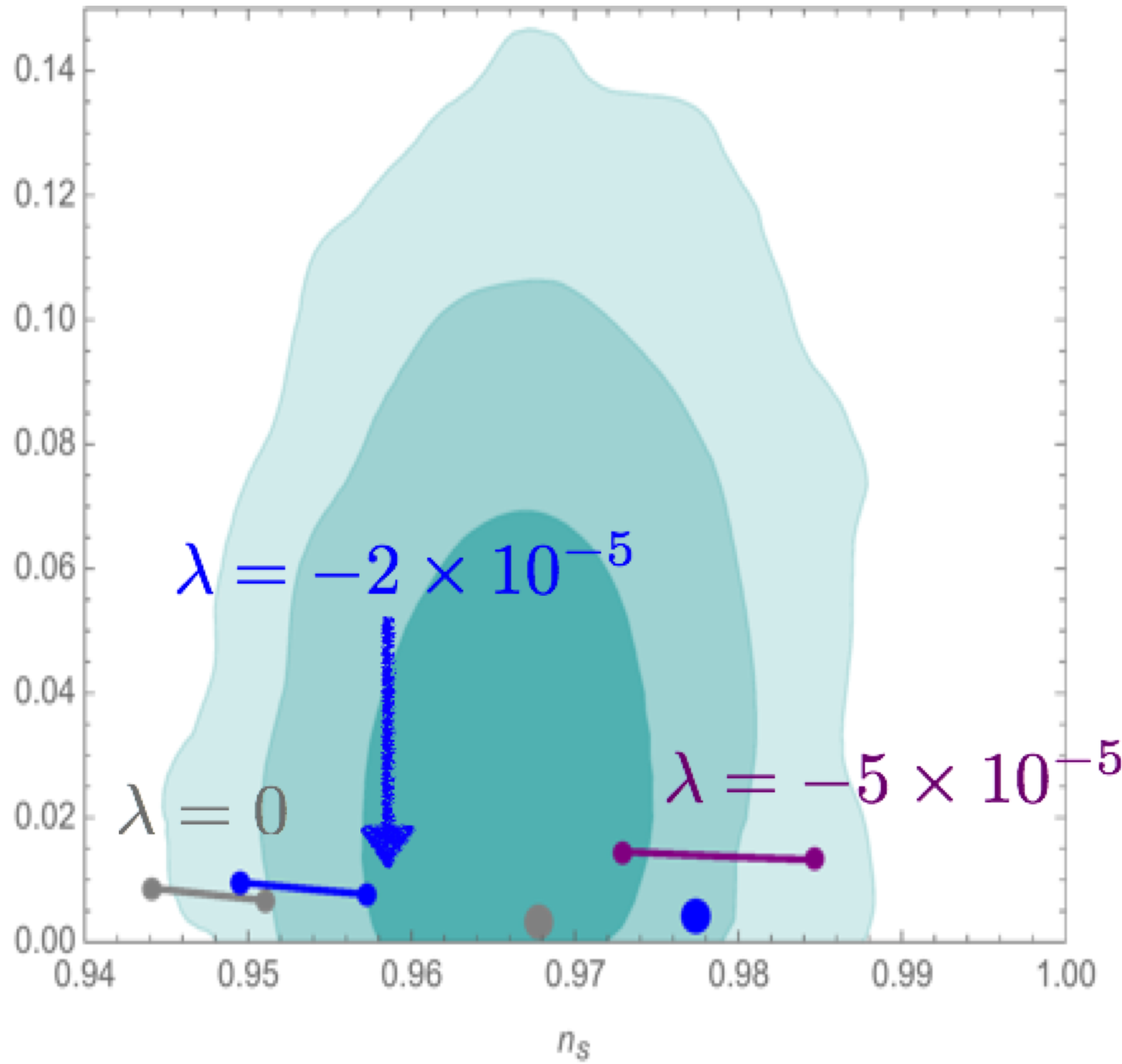
0.14
0.12
0.10
0.08
0.06
0.04
0.02
0.00
r
0.93
0.94
0.95
0.96
0.97
0.98
0.99
n_s
$M_{\rm PBH}$~10^{20}g to 10^{24}g,
N_1=36 to 40
$M_{\rm PBH}$~10^{16}g to 10^{17}g,
N_1=44 to 45

Extension

- Example: Higher Ricci scalars.

$$f(R) = R + \frac{R^2}{6M^2} + \lambda \frac{R^3}{3M^4}.$$

- We only consider the N_1=36 to 40 window.



Summary

- R^2+scalar field $\equiv$ two-field with non-trivial field metric.
- Scalar field may provide a second stage inflation after the end of Starobinsky-stage.
- The transition of two stages may give enhanced features on the power spectrum.
- This enhanced ``feature'' can be used to produce PBHs as dark matter.