中山大學
SUN YAT-SEN UNIVERSITY

Spatially covariant gravity and the extension of scalar-tensor theories

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School of Physics and Astronomy, Sun Yat-sen University

2019-3-8

YITP, Kyoto University

Based on papers with:

C. Deffayet, T. Fujita, C. Kang, D. Steer, M. Yamaguchi,
Z. Yao, J. Yokoyama, D. Yoshida G. Zahariade

Why modified gravity

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Phenomenological:

To explain the early and late accelerated expansion of our universe.

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To explain the early and late accelerated expansion of our universe.

Theoretical:

To understand why GR is unique.

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To explain the early and late accelerated expansion of our universe.

Theoretical:

To understand why GR is unique.

"The best way to understand something is to modify it."

How to modify gravity

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Einstein equation is quite unique.

$$\alpha G_{\mu\nu} + \lambda g_{\mu\nu} = 0$$

[Lovelock, 1971]

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Any metric theory of gravity alternative to GR must satisfy (at least):

- extra degrees of freedom (with exotic couplings with gravity),
- extra dimensions (e.g., brane world),
- higher derivative terms (e.g., $f(R)$),
- extension of Riemannian geometry (e.g., $f(T)$),
- giving up locality.

Covariant scalar-tensor theories

From *k*-essence to Horndeski and beyond

1915 • GR

$$\mathcal{L} = \frac{1}{16\pi G} R$$

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k-essence:
the most general scalar-tensor theory,
the Lagrangian involves up to the first derivative of the scalar field.

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Beyond *k*-essence?

Introducing second derivatives?

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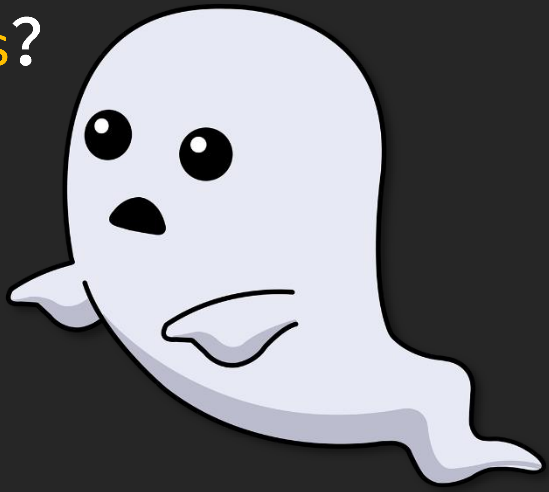
Introducing second derivatives?

⟶ ghost



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***Phys.Rev. D** 84 (2011) 064039]*

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Generalized galileon/Horndeski theory: the most general scalar-tensor theory,

- Lagrangian/EoMs involve up to the second derivatives,
- Propagates 1 scalar + 2 tensor dofs.

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International Journal of Theoretical Physics, Vol. 10, No. 6 (1974), pp. 363–384

Second-Order Scalar-Tensor Field Equations in a Four-Dimensional Space

GREGORY WALTER HORNDESKI

Department of Applied Mathematics, University of Waterloo, Waterloo, Ontario, Canada

Received: 10 July 1973

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Beyond galileon/Horndeski ***theory***?

- Higher derivatives of the scalar field and the metric.
- Propagates 1 scalar + 2 tensor dofs.

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2015 DHOST

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DHOST theory:

- Higher derivatives in EoMs (2nd derivatives in the action),
- Propagates 1 scalar + 2 tensor dofs.

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Even beyond?

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We may need some alternative approach.

Spatially covariant gravity

Spatially covariant gravity

“不忘初心，方得始终。”

“Never forget why you started,
and your mission can be accomplished.”

「初心忘るべからず、終始心得るべし。」

Spatially covariant gravity

初心 (beginner's mind) = a scalar degree of freedom

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Reduction of symmetries → Extra mode(s)

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Breaking U(1) → Massive vector (2 vector + 1 scalar)

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Reduction of symmetries	→	Extra mode(s)
Breaking U(1)	→	Massive vector (2 vector + 1 scalar)
Breaking whole spacetime diff	→	Massive gravity (2 tensor + 2 vector + 1 scalar)

Spatially covariant gravity

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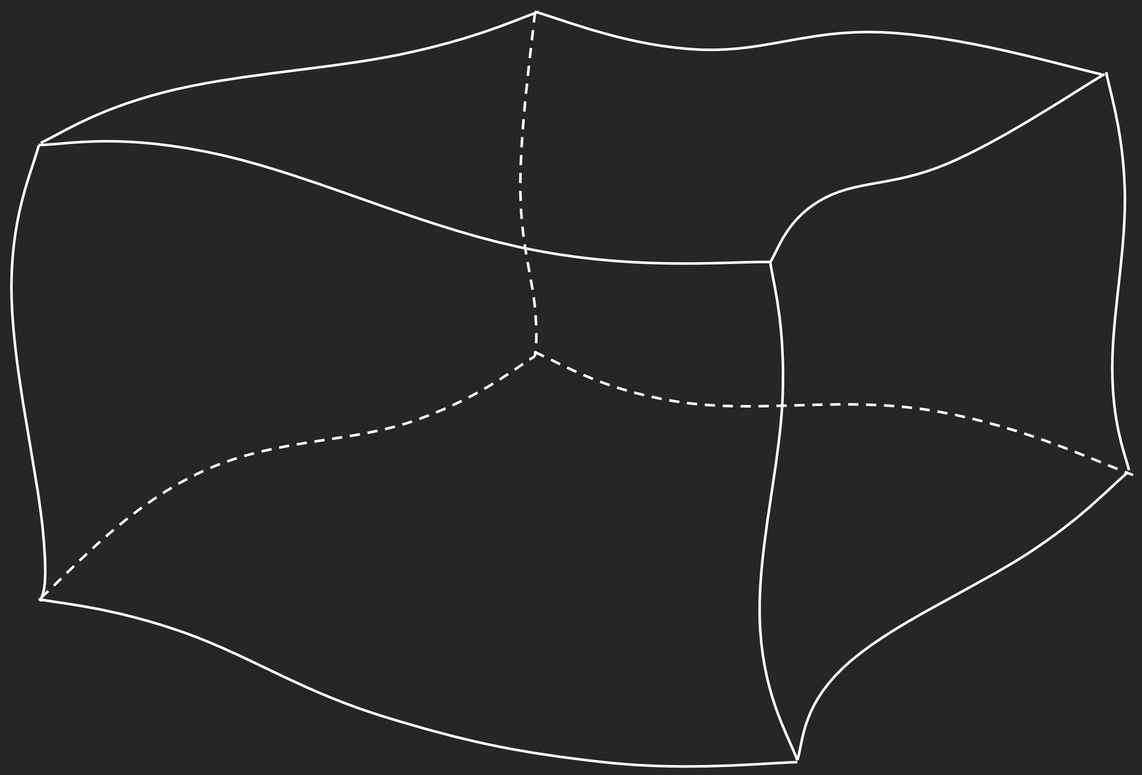
Reduction of symmetries → Extra mode(s)

Breaking U(1) → Massive vector (2 vector + 1 scalar)

Breaking whole spacetime diff → Massive gravity (2 tensor + 2 vector + 1 scalar)

Breaking time diff, respecting only spatial symmetries → 2 tensor + 1 scalar

Foliation of spacetime



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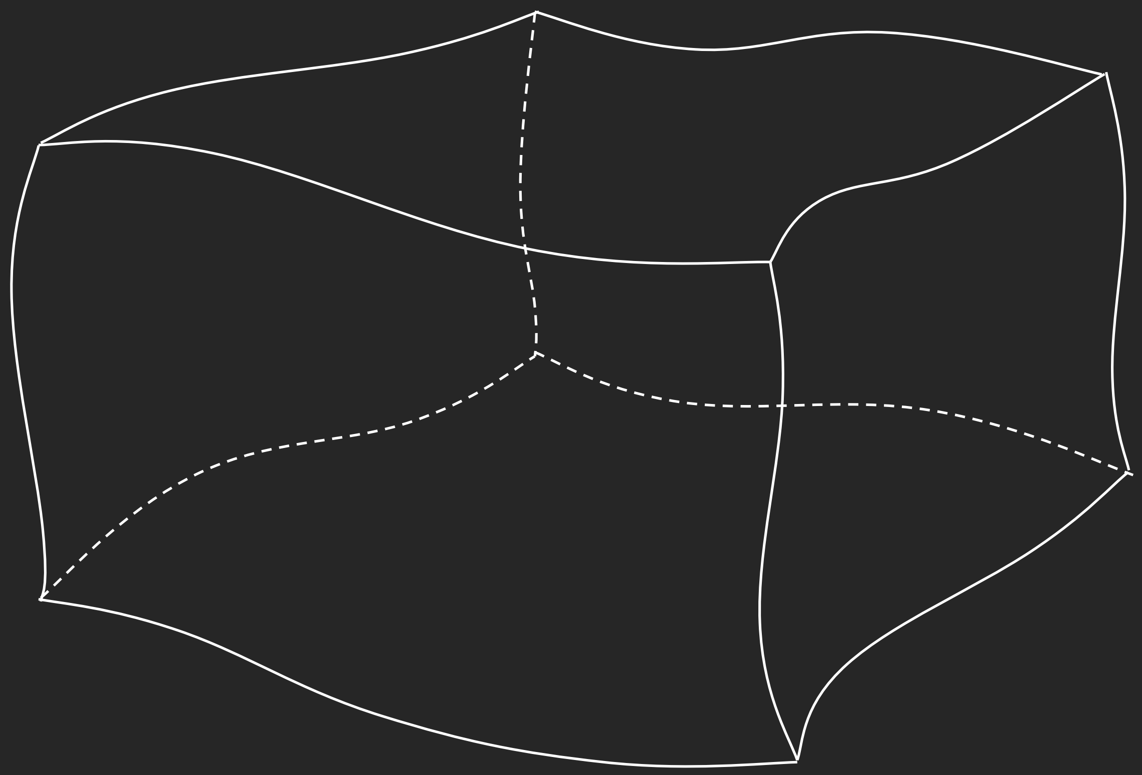
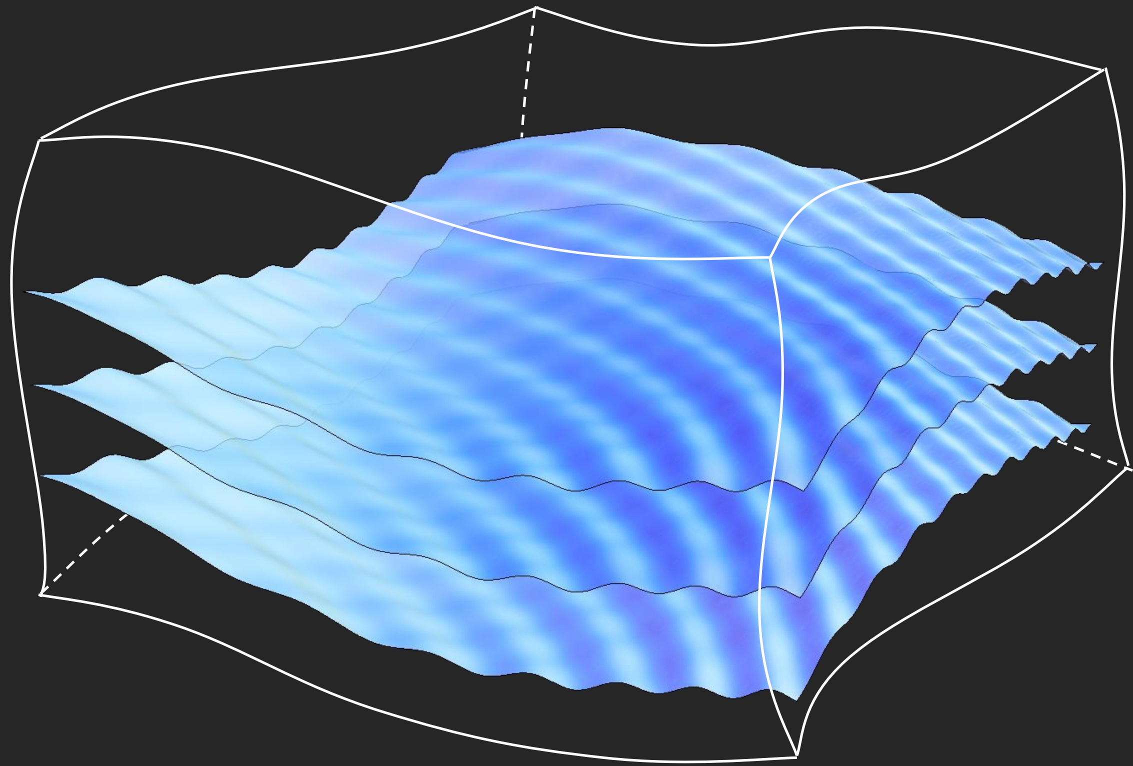


Spacetime covariant

4-D quantities

$\phi, g_{\mu\nu}, R_{\mu\nu\rho\sigma}, \nabla_\mu$

Foliation of spacetime



Spacelike hypersurfaces

Spacetime covariant

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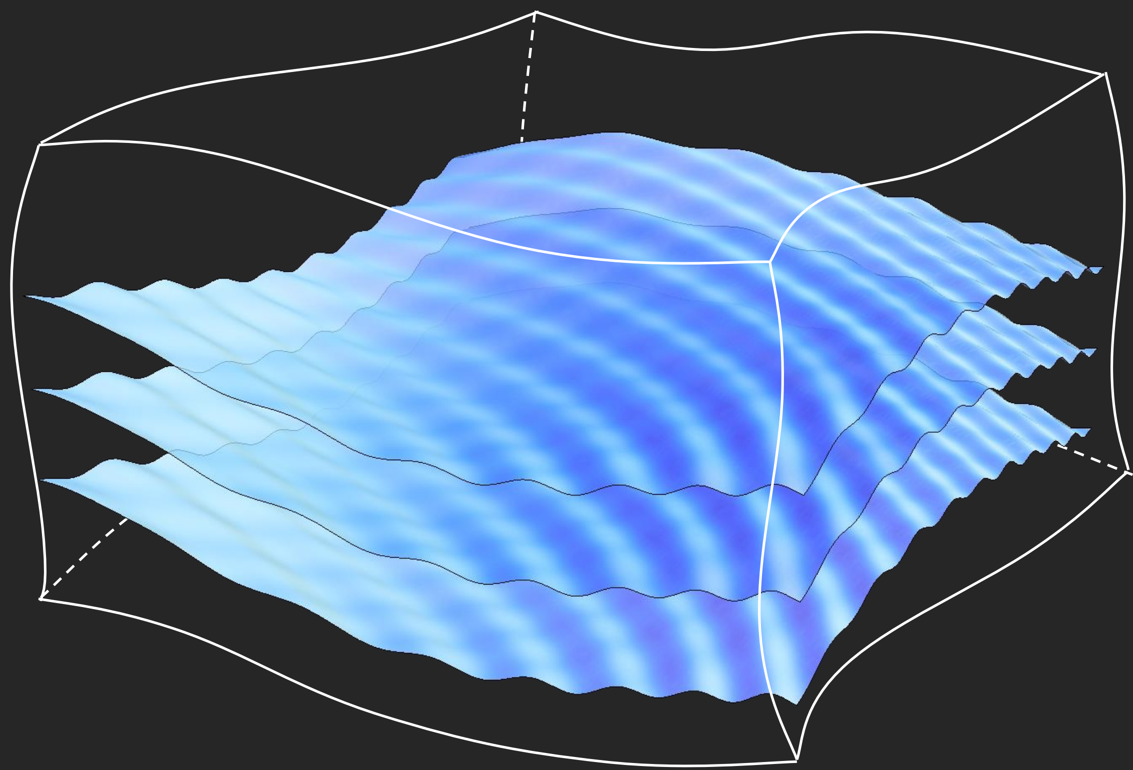

Spacetime covariant

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➡

Spatially covariant

3-D quantities

$t, N, h_{ij}, R_{ij}, \nabla_i, K_{ij}$

Early examples

2004 **Ghost condensation**

[Arkani-Hamed, Cheng, Luty & Mukohyama]

2007 **Effective field theory of inflation**

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Cosmological background naturally breaks time diff, which has a preferred time direction or set of spatial sclices, on which:

$$\phi(t,\vec{x}) = \bar{\phi}(t)$$ ⟹ unitary gauge
(uniform scalar field gauge)

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$$S = \int d^4x\sqrt{-g}\Big[\frac{1}{2}R + \Lambda(t) + f_1(t)\,\delta N + f_2(t)\,\delta N^2 + \cdots$$
$$+ g_1(t)\,\delta K^i_i + g_2(t)\left(\delta K^i_i\right)^2 + g_3(t)\,\delta K_{ij}\delta K^{ij} + \cdots\Big]$$

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- The Lagrangians are built of spatial invariants;
- The theories propagates one scalar mode (besides the two tensor modes).

Two faces of scalar-tensor theories

Spacetime covariant
Scalar-tensor theories

$$\mathcal{L}(\phi, g_{\mu\nu}, R_{\mu\nu\rho\sigma}, \nabla_\mu)$$

Two faces of scalar-tensor theories

Spacetime covariant
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Spatially covariant
gravity theories

$$\mathcal{L}(t, N, h_{ij}, R_{ij}, K_{ij}, \nabla_i)$$

Two faces of scalar-tensor theories

Gauge fixing: $\phi(t, \vec{x}) \to t$
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Spacetime covariant Scalar-tensor theories

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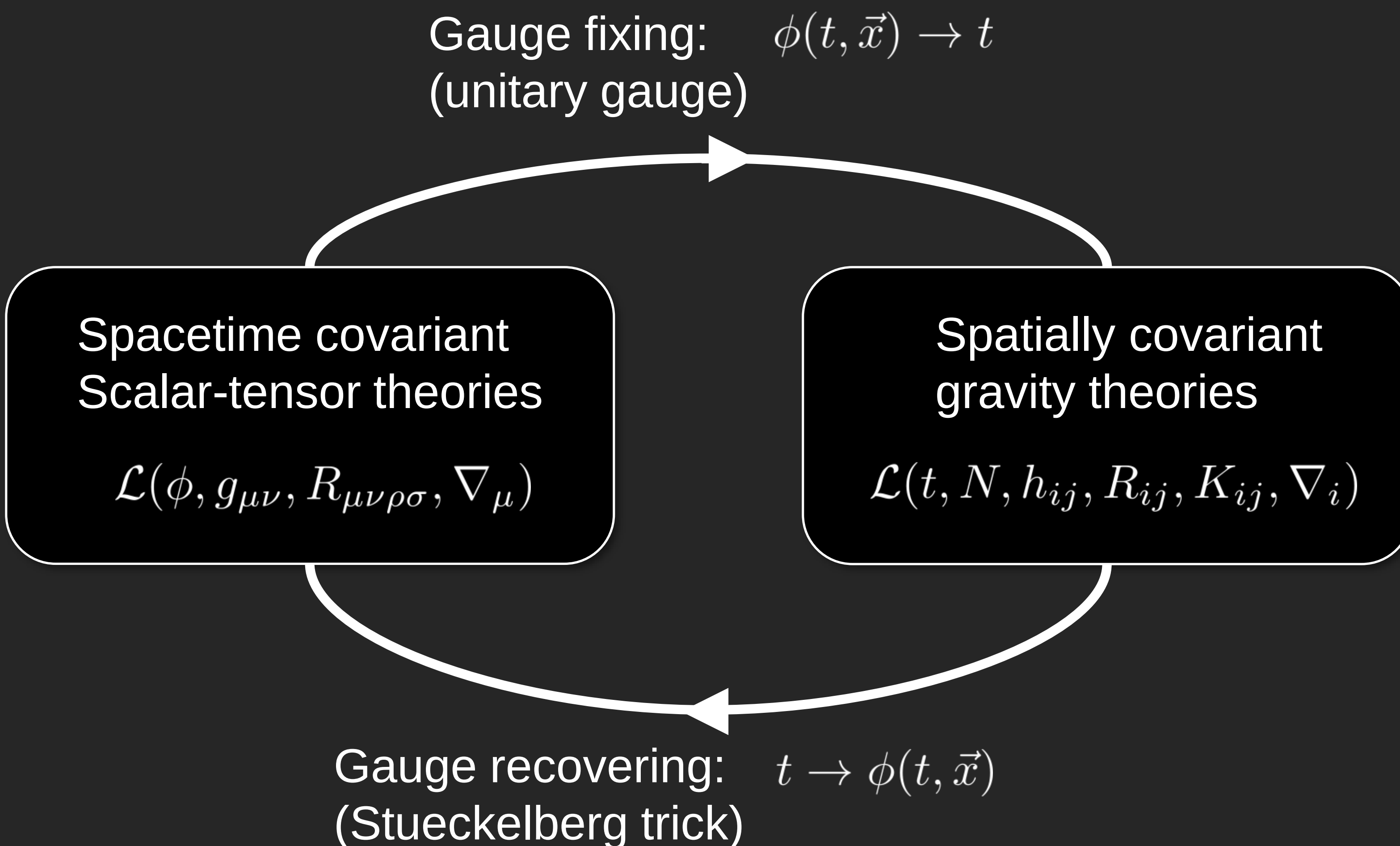

[H. Motohashi, T. Suyama, K. Takahashi, 2016]

[A. De Felice, D. Langlois, S. Mukohyama, K. Noui & A. Wang, 2018]

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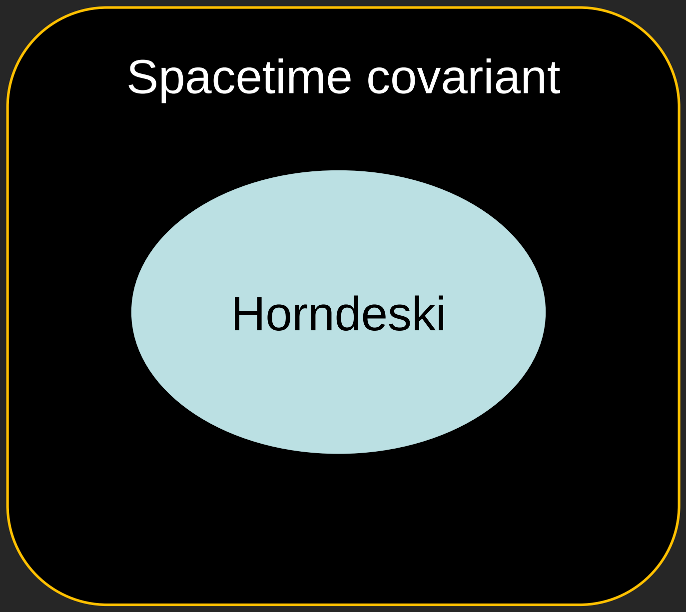

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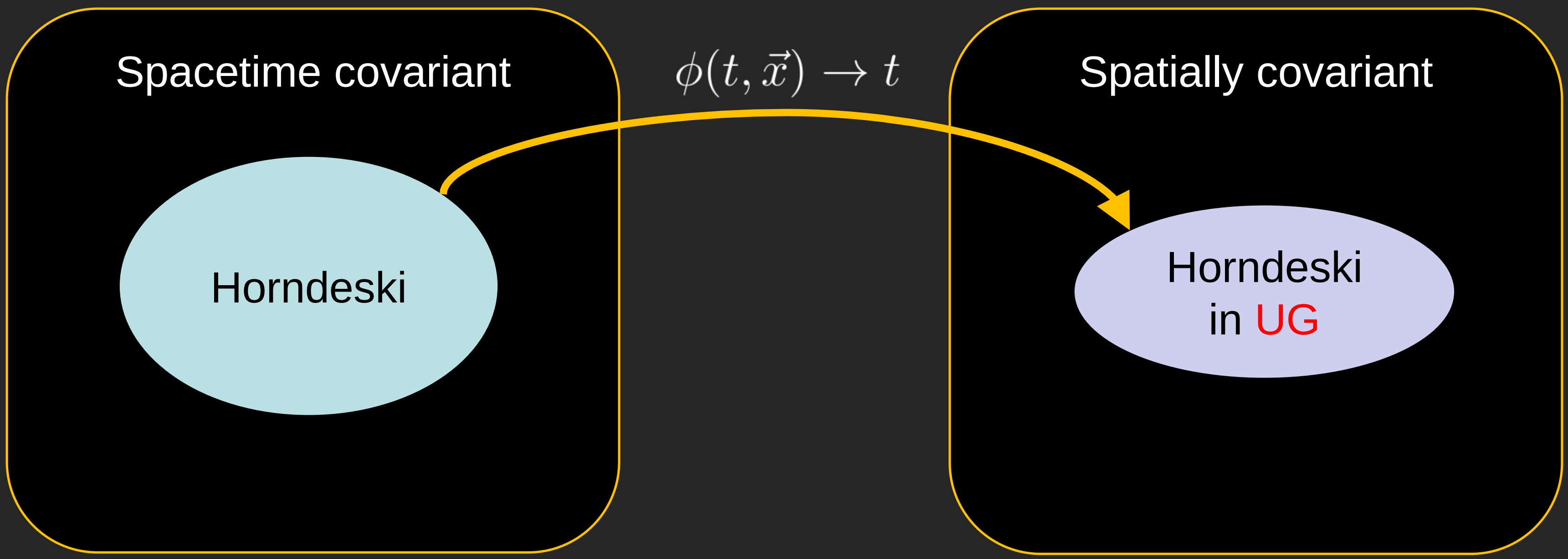

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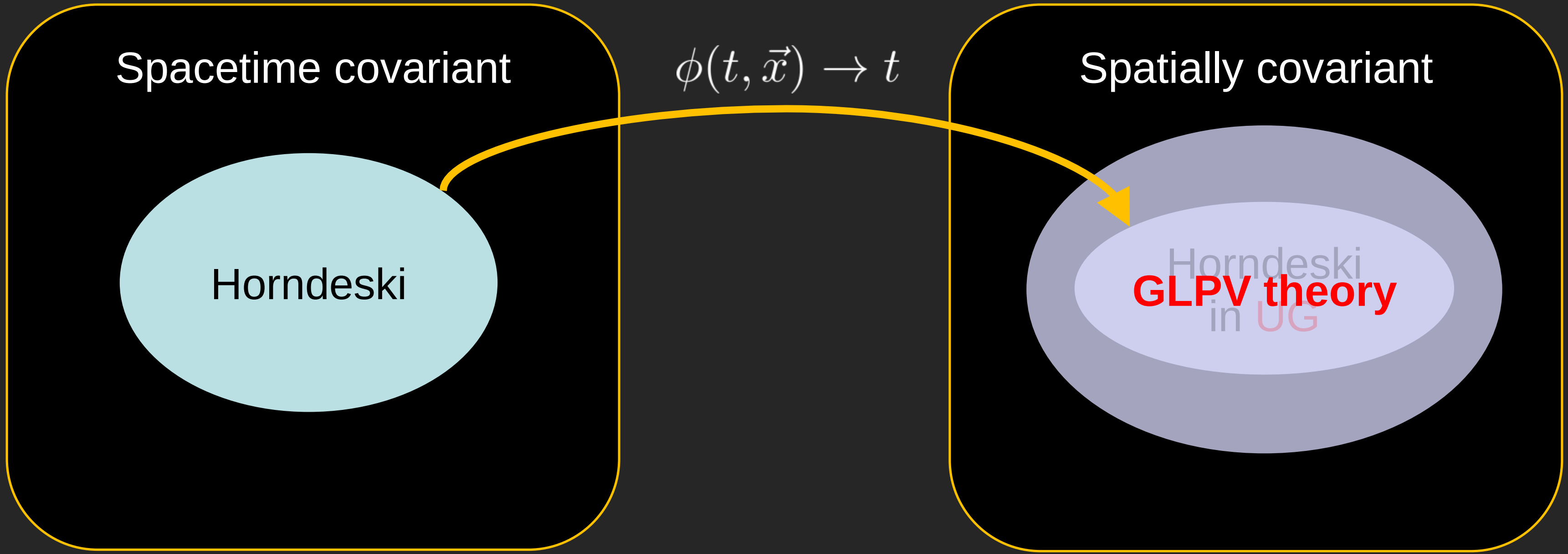

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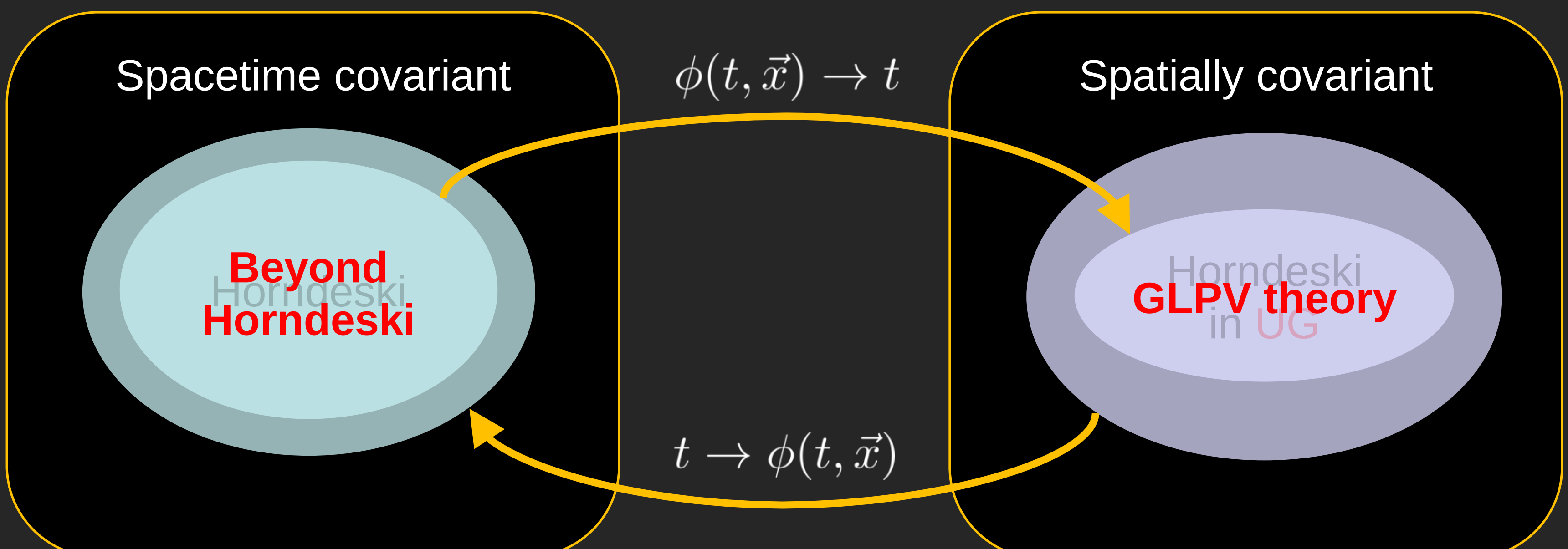

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2 tensor + 1 scalar DoFs with higher derivative EoMs.
[XG, Phys.Rev. D90 (2014) 104033]

With Velocity of the lapse function

Geometric motivation

The basic picture:

4d spacetime + foliation of spacelike hypersurfaces

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$\phi,\ N,\ h_{\mu\nu}$ with derivatives in terms of

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Also, field transformations typically introduces $\dot{N}$.

[G. Domènech, S. Mukohyama, R. Namba, A. Naruko, R. Saitou, Y. Watanabe, 2015]

The action

[XG & Zhi-Bang Yao, arXiv:1806.02811]

General action (in the unitary gauge):

$$S = \int \mathrm{d}t\mathrm{d}^3x\, N\sqrt{h}\,\mathcal{L}\left(t, N, h_{ij}, F, K_{ij}, \nabla_i\right)$$

with $$F = \frac{1}{N}\left(\dot{N} - \pounds_{\vec{N}} N\right), \qquad K_{ij} = \frac{1}{2N}\left(\dot{h}_{ij} - \pounds_{\vec{N}} h_{ij}\right)$$

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Generally, such kind of theories have 2 scalar dof's, one of which is an Ostrogradsky ghost.

Conditions for 3 dofs

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Two conditions must be satisfied:

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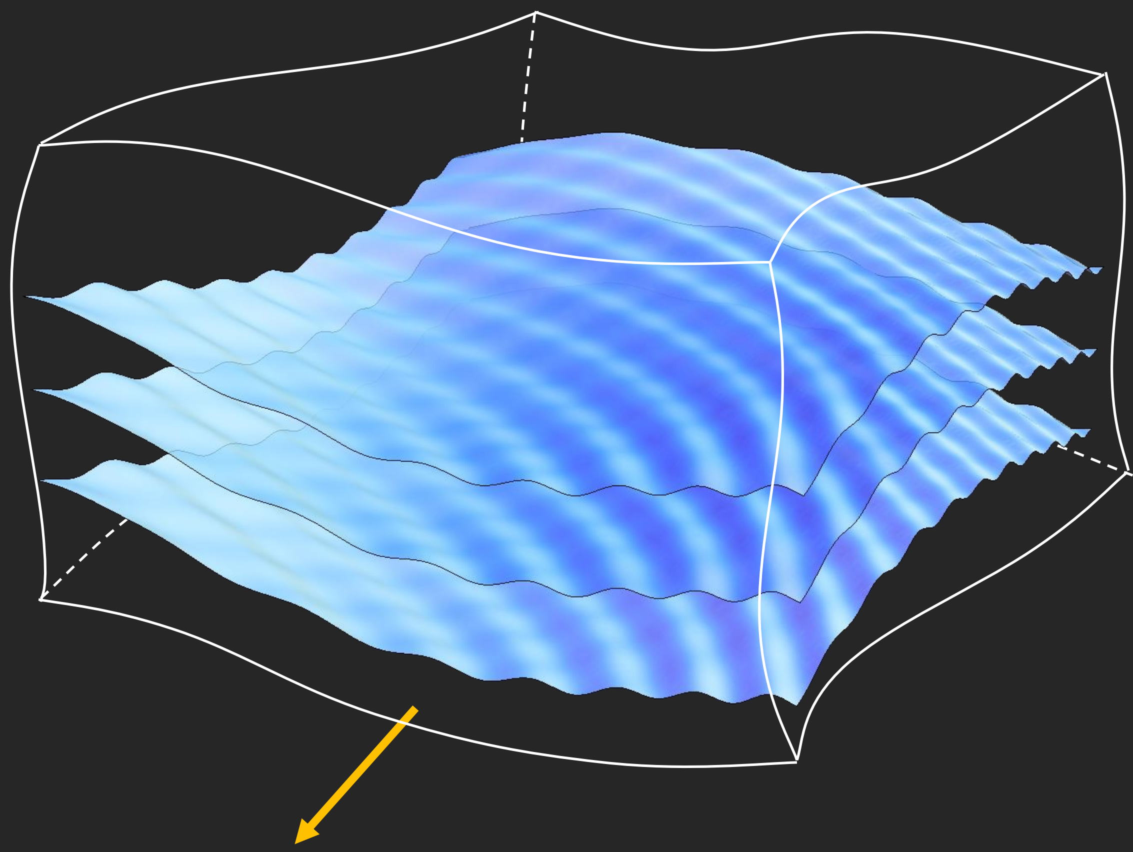
A perturbative analysis shows that:

- The degeneracy condition guarantees no ghost at linear order around FRW;
- Ghost reappears at nonlinear order around FRW, or linear order around inhomogeneous background. (consistency condition needed)

With a non-dynamical scalar field

How if spacelike?

How if the scalar field acquires a spacelike gradient?



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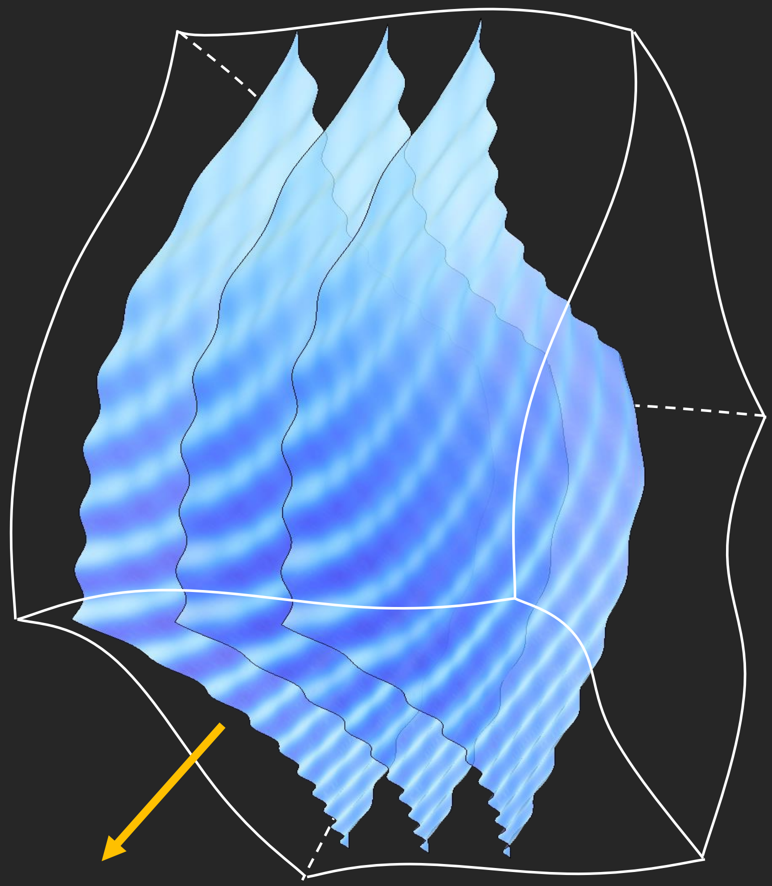

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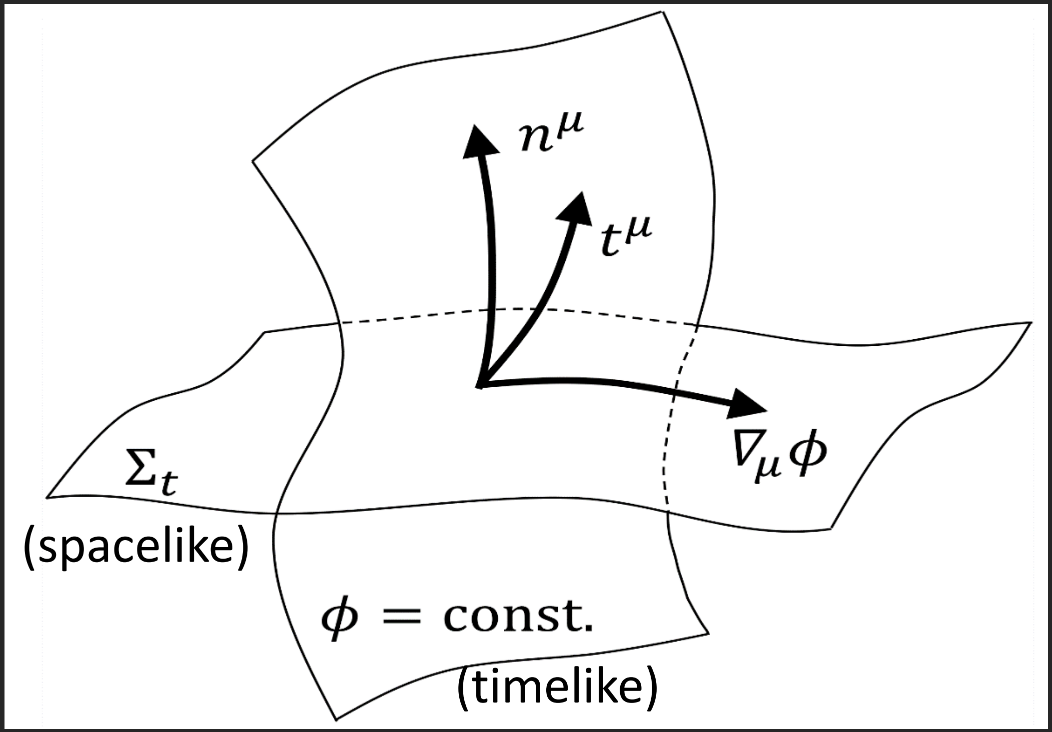
Spatial gauge

Assuming $\nabla_\mu \phi \neq 0$ everywhere.



n^μ : tangent to ϕ=const. hypersurface, ⟶ $\pounds_n \phi = n^\mu \nabla_\mu \phi = 0$

hypersurface orthogonal ⟶ usual 3+1 decomposition

Spatial gauge

timelike spacelike

$$\nabla_\mu \phi = -n_\mu \pounds_n \phi + \mathrm{D}_\mu \phi$$

Spatial gauge

timelike | spacelike

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$\mathrm{D}_\mu \phi \rightarrow 0$

unitary gauge

Spatial gauge

timelike — spacelike

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$-n_\mu \pounds_n \phi \to 0$

spatial gauge

$$\to \mathrm{D}_\mu \phi = \delta^0_\mu \mathrm{D}_i \phi$$

Spatial gauge

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	↓		
	0		

spatial gauge

$$\rightarrow \mathrm{D}_\mu \phi = \delta^0_\mu \mathrm{D}_i \phi$$

$$\nabla_\mu \nabla_\nu \phi = n_\mu n_\nu (\pounds^2_n \phi - a^\rho \, \mathrm{D}_\rho \phi) - 2 n_{(\mu} \big(\mathrm{D}_{\nu)} \pounds_n \phi - K_{\nu)}{}^{\rho} \, \mathrm{D}_\rho \phi \big) - \pounds_n \phi \, K_{\mu\nu} + \mathrm{D}_\mu \mathrm{D}_\nu \phi$$

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spatial gauge

$$\to -N^2 \delta^0_\mu \delta^0_\nu a^i \mathrm{D}_i \phi - 2N \delta^0_{(\mu} \delta^i_{\nu)} K_i{}^j \, \mathrm{D}_j \phi + \delta^i_\mu \delta^j_\nu \mathrm{D}_i \mathrm{D}_j \phi$$

Only spatial derivatives survive!

Horndeski in the spatial gauge

$$\mathcal{L}_2^{\mathrm{H},(\mathrm{s.g.})} = \mathcal{L}_2^{\mathrm{H},(\mathrm{s.g.})}(\phi, X)\,, \qquad \text{with} \qquad X \equiv \frac{1}{2}\mathrm{D}_i\phi\mathrm{D}^i\phi$$

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$$\begin{aligned}\mathcal{L}_4^{\mathrm{H},(\mathrm{s.g.})} = \;& G_4\left(R + K_{ij}K^{ij} - K^2\right) + 2\frac{\partial G_4}{\partial X}\left(R^{ij} + K^{ik}K_k{}^j - KK^{ij}\right)\mathrm{D}_i\phi\mathrm{D}_j\phi \\ & -\frac{\partial G_4}{\partial X}\left[\left(\mathrm{D}^2\phi\right)^2 - \mathrm{D}_i\mathrm{D}_j\phi\mathrm{D}^i\mathrm{D}^j\phi\right] \\ & +2\frac{\partial^2 G_4}{\partial X^2}\left(\mathrm{D}_i\mathrm{D}_j\phi\mathrm{D}^i\phi\mathrm{D}^j\phi\mathrm{D}^2\phi - \mathrm{D}^i\mathrm{D}_j\phi\mathrm{D}^j\phi\mathrm{D}_i\mathrm{D}_k\phi\mathrm{D}^k\phi\right) \\ & +2\frac{\partial^2 G_4}{\partial X\partial\phi}\left(2\mathrm{D}_i\mathrm{D}_j\phi\mathrm{D}^i\phi\mathrm{D}^j\phi - \mathrm{D}^i\phi\mathrm{D}_i\phi\mathrm{D}^2\phi\right) \\ & -2\frac{\partial^2 G_4}{\partial\phi^2}\mathrm{D}_i\phi\mathrm{D}^i\phi - 2\frac{\partial G_4}{\partial\phi}\mathrm{D}^2\phi\end{aligned}$$

Horndeski in the spatial gauge

$$\begin{aligned}
\mathcal{L}_5^{\mathrm{H},(\mathrm{s.g.})} \quad = \quad & \frac{\partial G_5}{\partial X} G_{ij} \mathrm{D}^i \mathrm{D}^j \phi \mathrm{D}^k \phi \mathrm{D}_k \phi + \frac{1}{2} \frac{\partial G_5}{\partial X} \mathrm{D}^i \mathrm{D}^j \phi \mathrm{D}_i \phi \mathrm{D}_j \phi \left(K^2 - K_{kl} K^{kl} \right) \\
& - \frac{\partial G_5}{\partial X} \left(K_k{}^i K^{kj} \mathrm{D}^2 \phi - K^{ki} K^{lj} \mathrm{D}_k \mathrm{D}_l \phi \right) \mathrm{D}_i \phi \mathrm{D}_j \phi \\
& + 2 \frac{\partial G_5}{\partial X} \left(K^{ki} K^j_k - K K^{ij} \right) \mathrm{D}_i \mathrm{D}_l \phi \mathrm{D}^l \phi \mathrm{D}_j \phi \\
& - \frac{\partial G_5}{\partial X} \left(K^{kl} K^{ij} - K K^{kl} h^{ij} \right) \mathrm{D}_i \mathrm{D}_j \phi \mathrm{D}_k \phi \mathrm{D}_l \phi \\
& + \frac{1}{3} \frac{\partial G_5}{\partial X} \left[\left(\mathrm{D}^2 \phi \right)^3 - 3 \mathrm{D}_i \mathrm{D}_j \phi \mathrm{D}^i \mathrm{D}^j \phi \mathrm{D}^2 \phi + 2 \mathrm{D}_i \mathrm{D}_j \phi \mathrm{D}^j \mathrm{D}^k \phi \mathrm{D}_k \mathrm{D}^i \phi \right] \\
& + \frac{1}{2} \frac{\partial G_5}{\partial \phi} R \mathrm{D}^i \phi \mathrm{D}_i \phi - 2 \frac{\partial G_5}{\partial \phi} R_{ij} \mathrm{D}^i \phi \mathrm{D}^j \phi - \frac{1}{2} \frac{\partial G_5}{\partial \phi} \left(K^2 - K_{kl} K^{kl} \right) \mathrm{D}_i \phi \mathrm{D}^i \phi \\
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General framework

*[**XG**, M. Yamaguchi, D. Yoshida, JCAP 1903 (2019) 006]*

General action:

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↓

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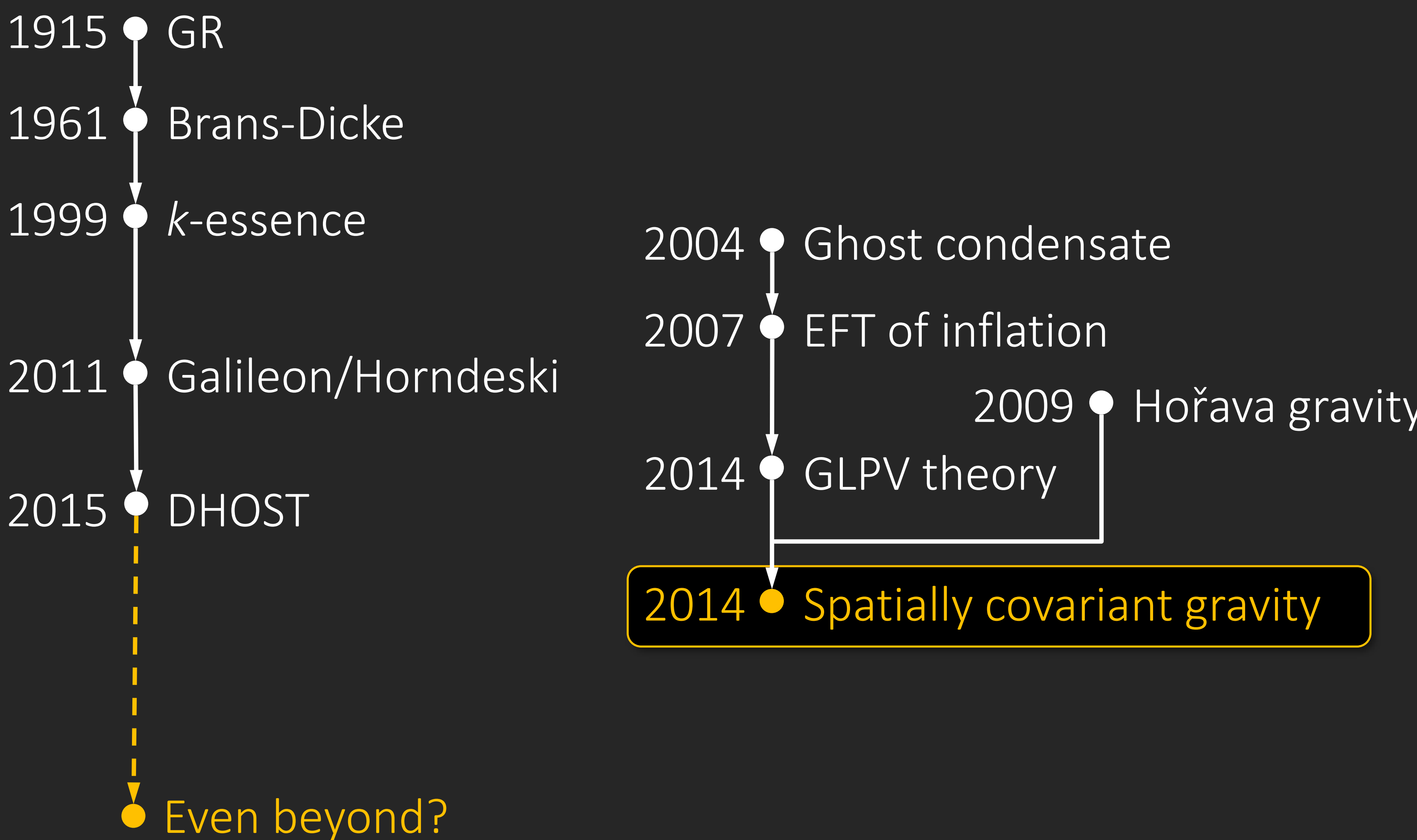

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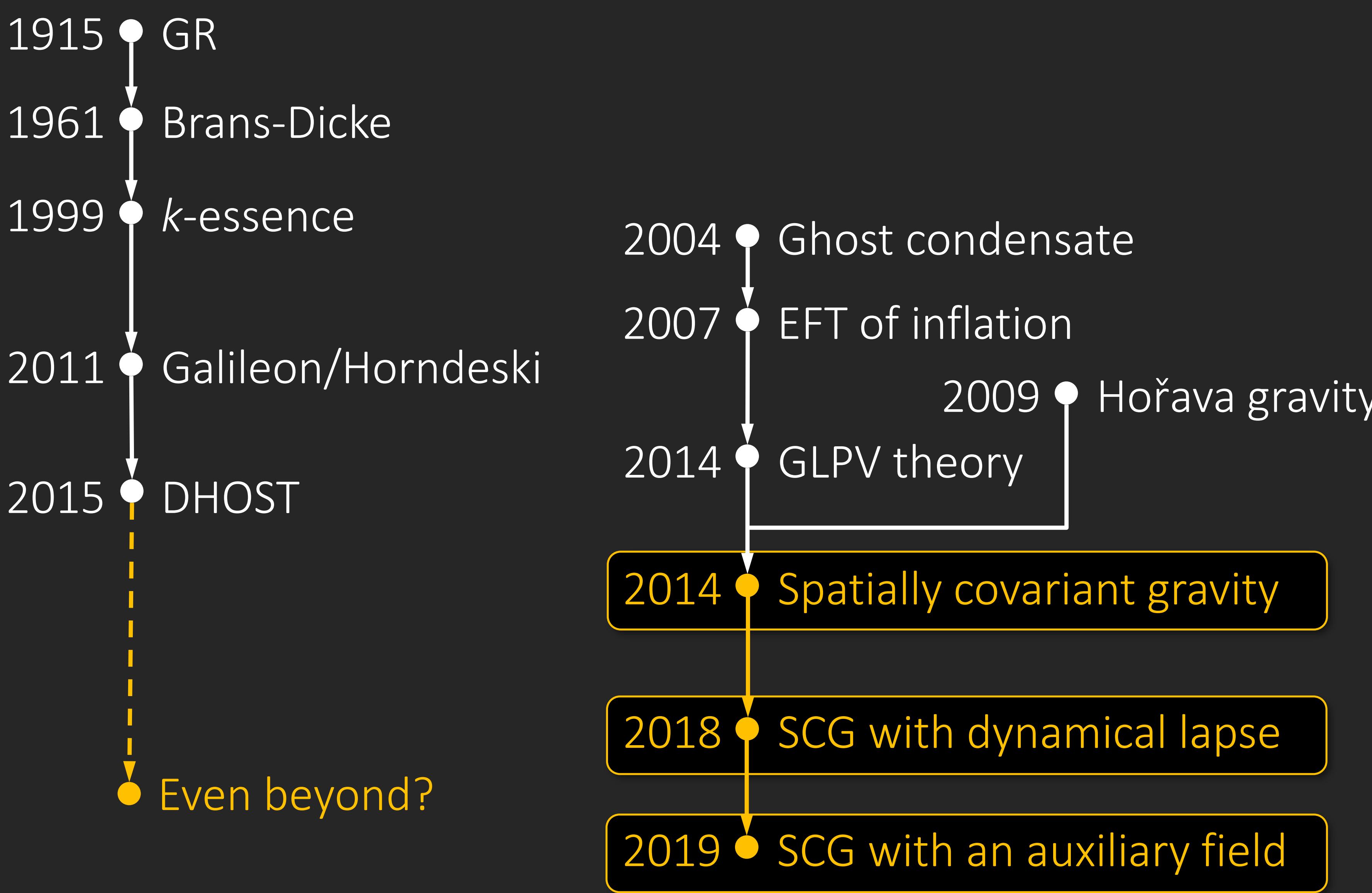

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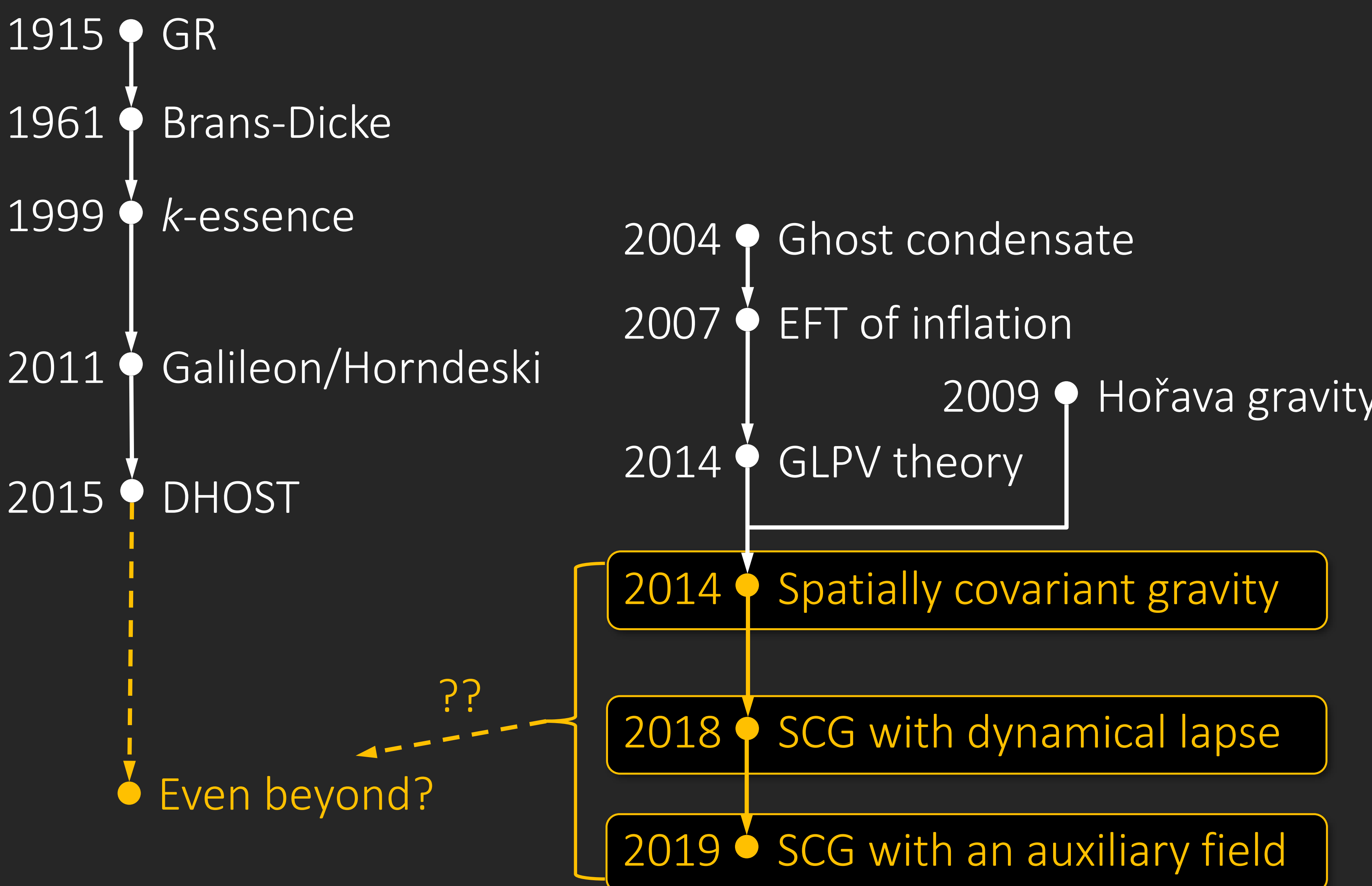

Thank you for your attention!