Constant-roll inflation in scalar-tensor theory

Hayato Motohashi
Center for Gravitational Physics
Yukawa Institute for Theoretical Physics

2019.03.08 Accelerating Universe in the Dark, YITP

Canonical single field Inflation

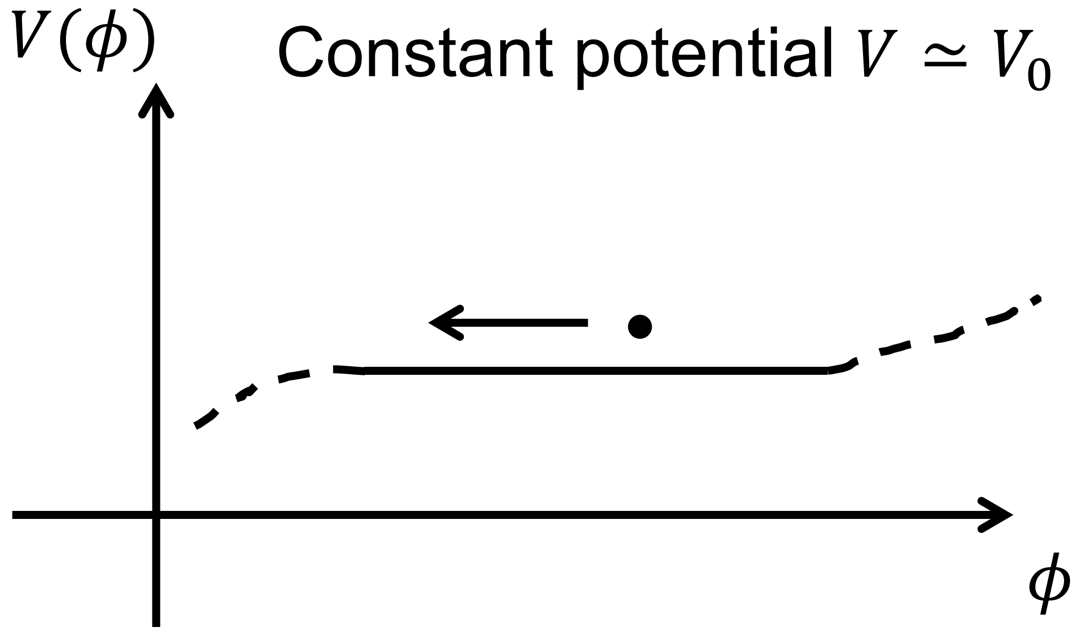
$$\ddot{\phi}/(H\dot{\phi}) \simeq 0 \quad \text{Slow-roll}$$

$$\ddot{\phi}/(H\dot{\phi}) = -3 \quad \text{Ultra slow-roll}$$

$$\ddot{\phi}/(H\dot{\phi}) = \beta \text{ (constant)} \quad \text{Constant-roll}$$

Ultra slow-roll inflation

Kinney, gr-qc/0503017



$$\ddot{\phi} = -3H\dot{\phi} \quad \Rightarrow \quad \dot{\phi} \propto a^{-3}$$

$$\Rightarrow \quad \epsilon_H \propto a^{-6}$$

Ultra slow-roll inflation

Superhorizon solution

$$\zeta_k = A_k + B_k \int \frac{dt}{a^3 \epsilon_H}$$

Constant mode

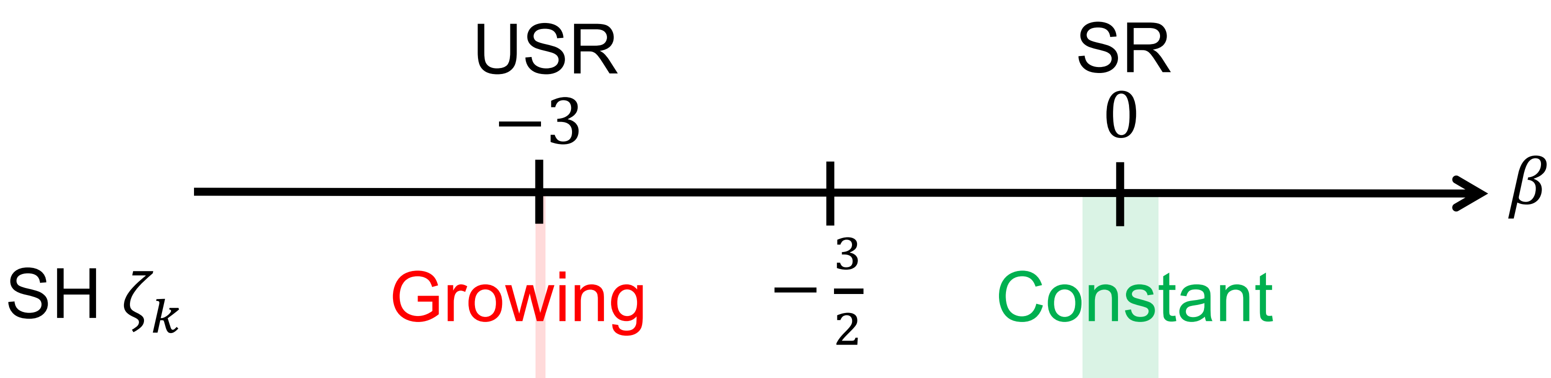
Decaying mode

Growing mode

Slow-roll $\epsilon_H \simeq$ const. $\ll 1$ Decaying mode

Ultra slow-roll $\epsilon_H \propto a^{-6}$ Growing mode

While $\epsilon_H \ll 1$, $d\ln\epsilon_H / dN = -6$ violates slow roll.



Generalization

Martin, HM, Suyama, 1211.0083

Superhorizon solution

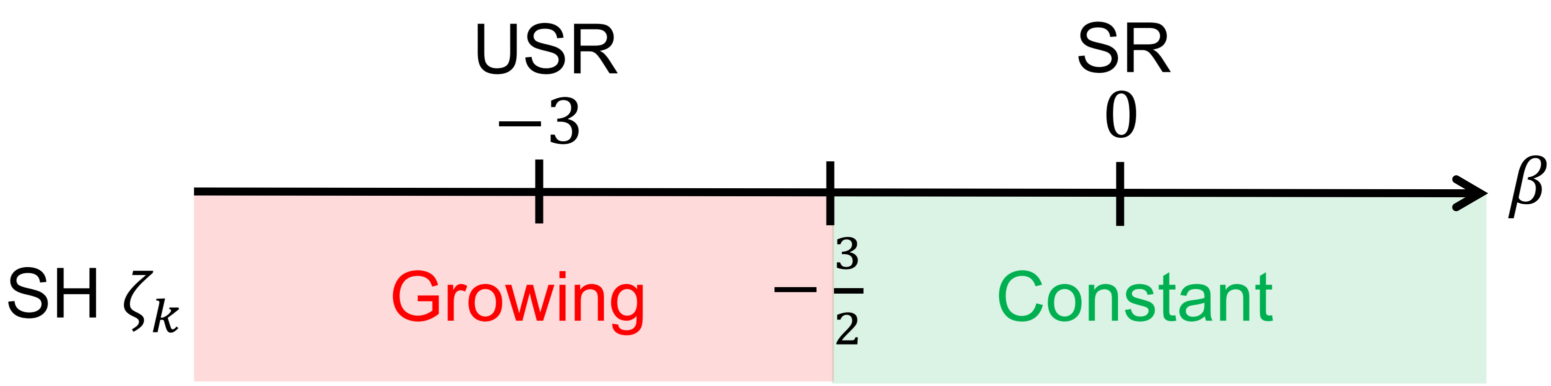
$$\zeta_k = A_k + B_k \int \frac{dt}{a^3 \epsilon_H}$$

Constant mode

Decaying mode

Growing mode

Constant roll $\epsilon_H \propto a^{2\beta}$ $\quad 2\beta \gtrless -3$



HM, Starobinsky, Yokoyama, 1411.5021

Constant-roll potential

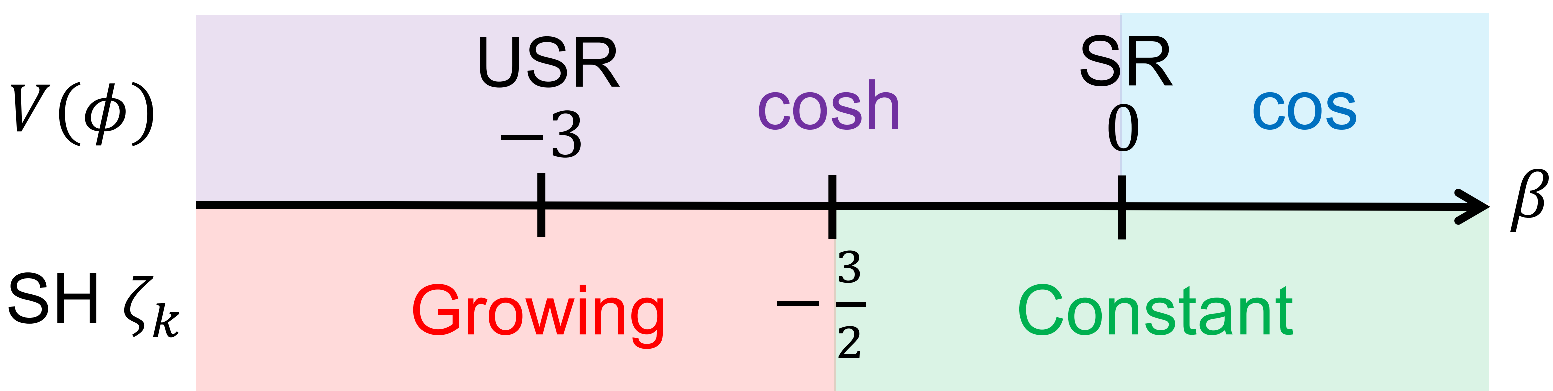
Allows constant-roll evolution as an exact solution

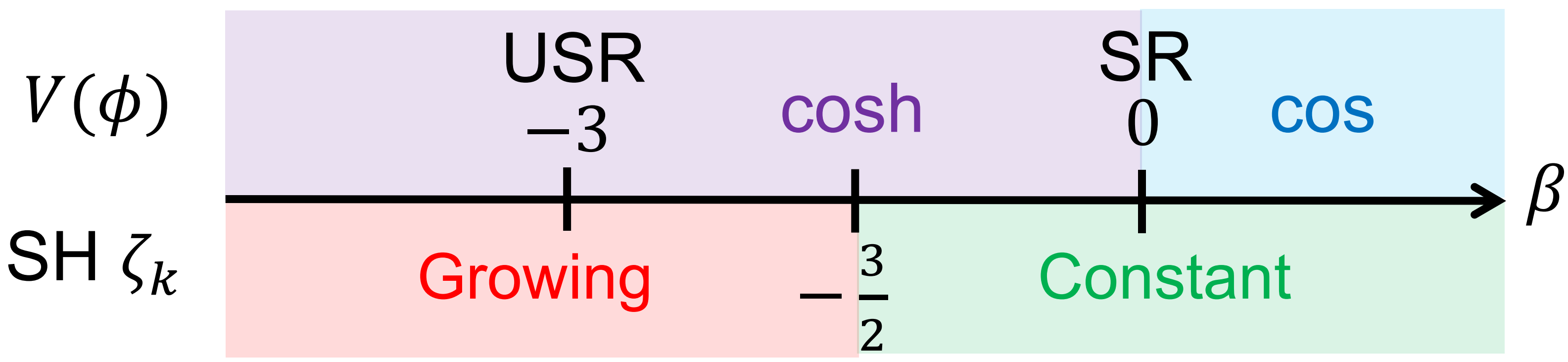
a) $V \propto e^{\sqrt{-2\beta}\phi}$ with $\beta < 0$: Power-law inflation

✗ $r = 8(1 - n_s)$

b) $V \propto \cosh(\sqrt{-2\beta}\phi) + \text{const.}$ with $\beta < 0$

c) $V \propto \cos(\sqrt{2\beta}\phi) + \text{const.}$ with $\beta > 0$





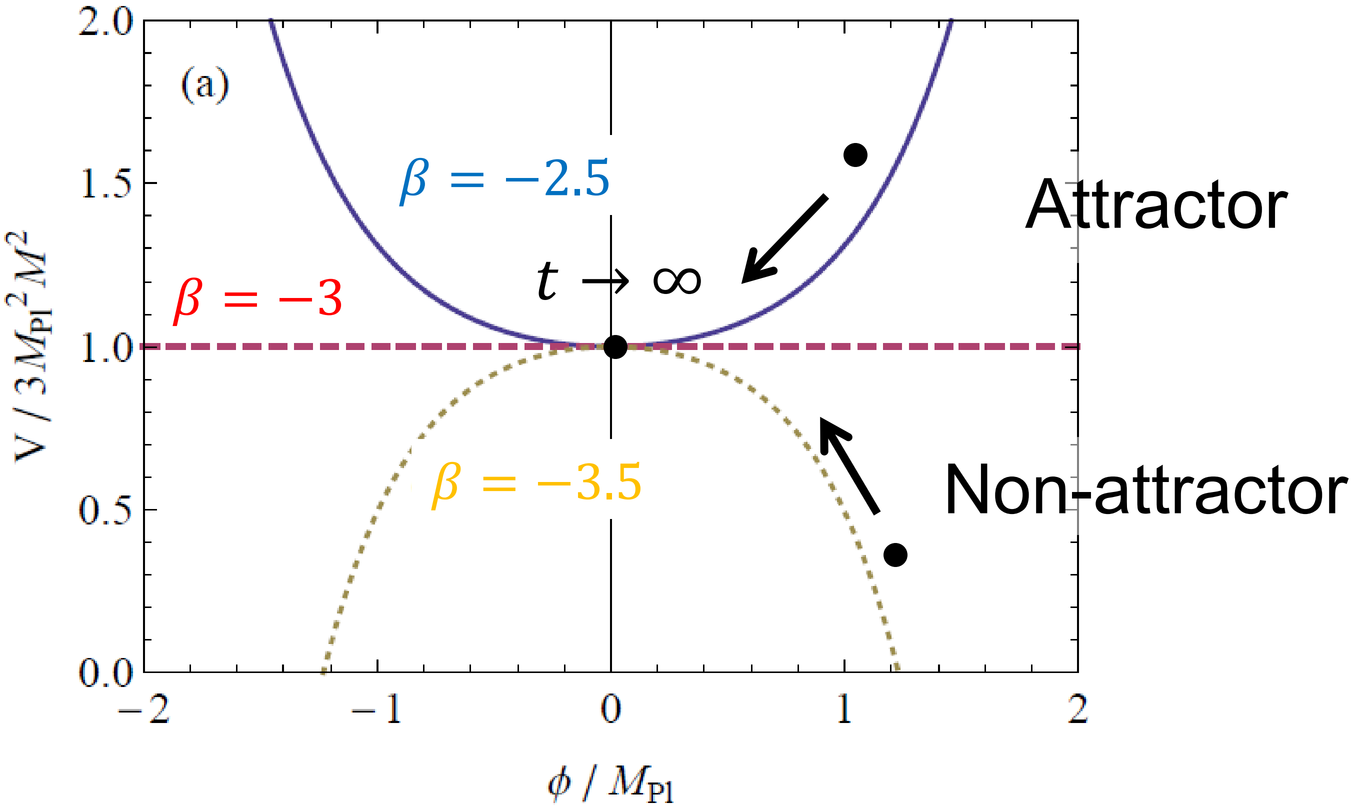
✓ ζ_k frozen on SH

- Attractor
- n_s
- r
- $N \sim 60$

cosh potential ($\beta < 0$)

$$\frac{V(\phi)}{M^2 M_{Pl}^2} = 3\left[1 - \frac{3+\beta}{6}\left\{1 - \cosh\left(\sqrt{-2\beta}\,\frac{\phi}{M_{Pl}}\right)\right\}\right]$$

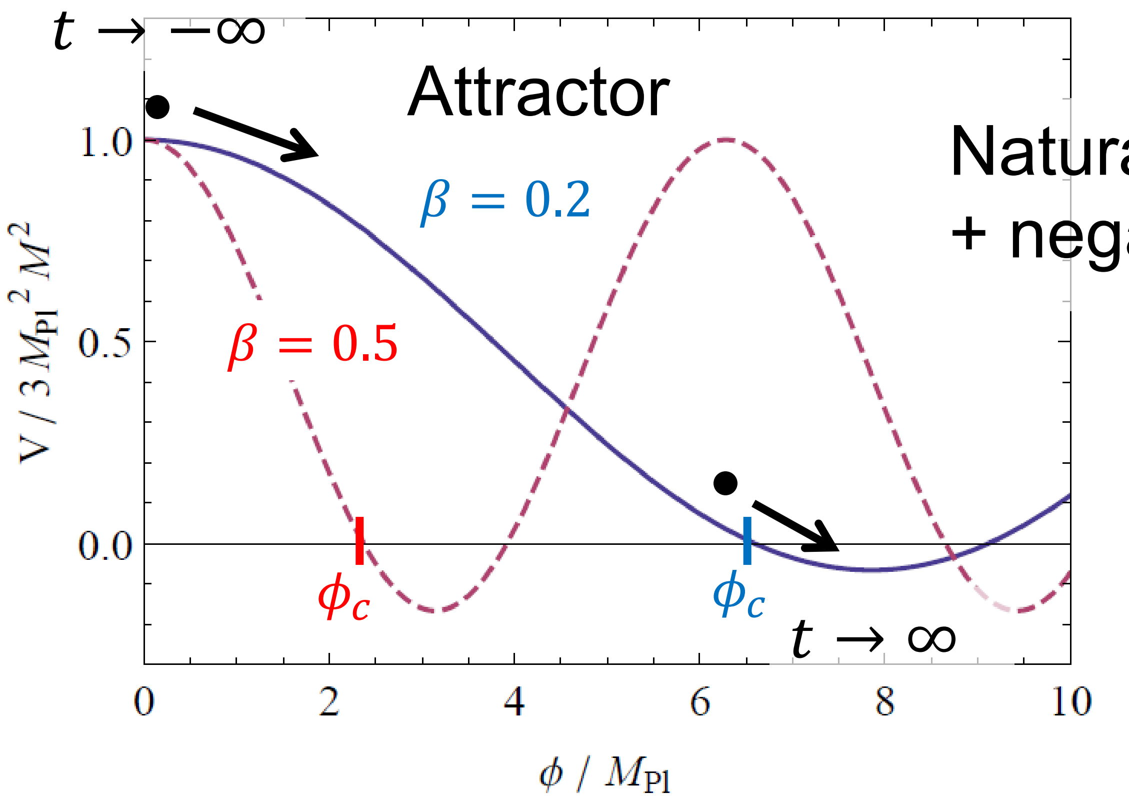
Assume a transition to reheating at $\phi > 0$.



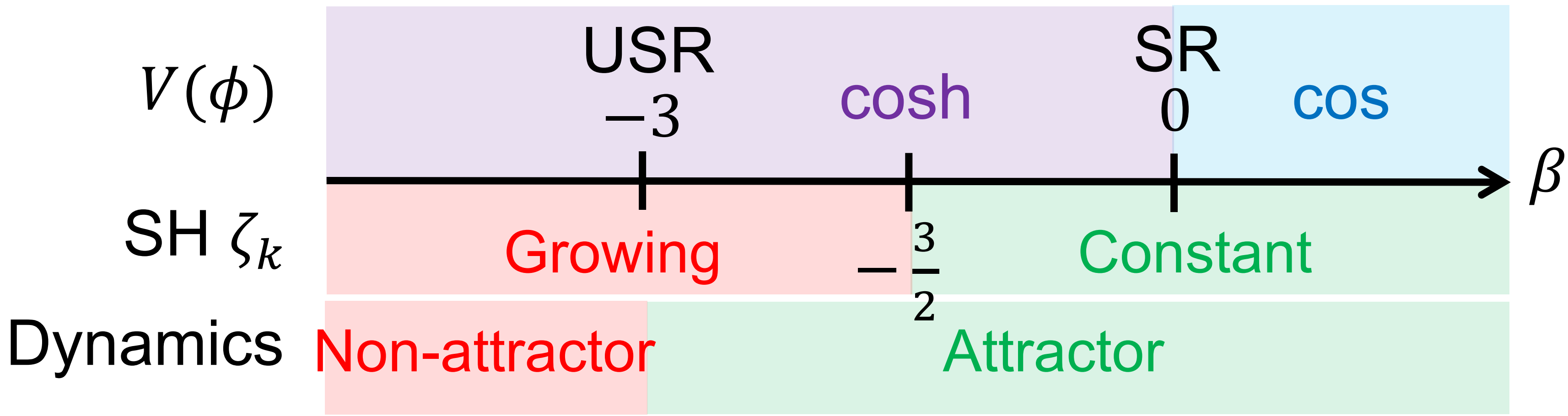
cos potential ($\beta > 0$)

$$\frac{V(\phi)}{M^2 M_{Pl}^2} = 3\left[1 - \frac{3+\beta}{6}\left\{1 - \cos\left(\sqrt{2\beta}\frac{\phi}{M_{Pl}}\right)\right\}\right]$$

Assume a transition to reheating at $\phi < \phi_c$.



Natural inflation
+ negative CC



✓ ζ_k frozen on SH

✓ Attractor

- n_s
- r
- $N \sim 60$

Curvature perturbation

Mukhanov-Sasaki equation

$$v_k'' + \left(k^2 - \frac{z''}{z}\right) v_k = 0$$

Using analytic solutions for cosh and cos potentials,

$$\frac{z''}{z} \to \frac{\nu^2 - 1/4}{\tau^2} \quad (t \to \pm\infty)$$

where $\nu \equiv |\beta + 3/2|$ without slow-roll approximation.

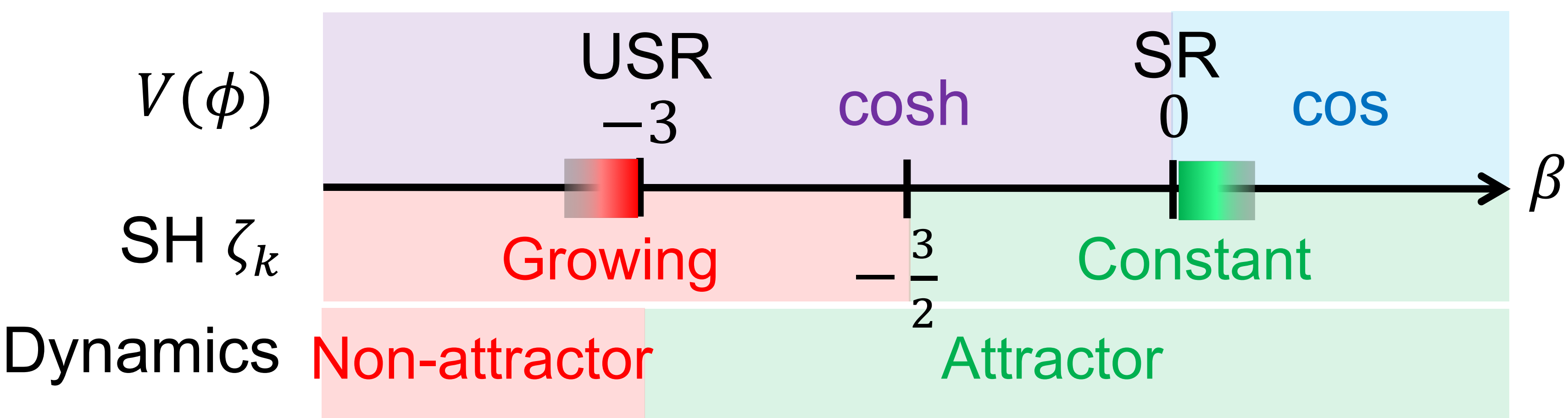
Curvature perturbation

Spectral index

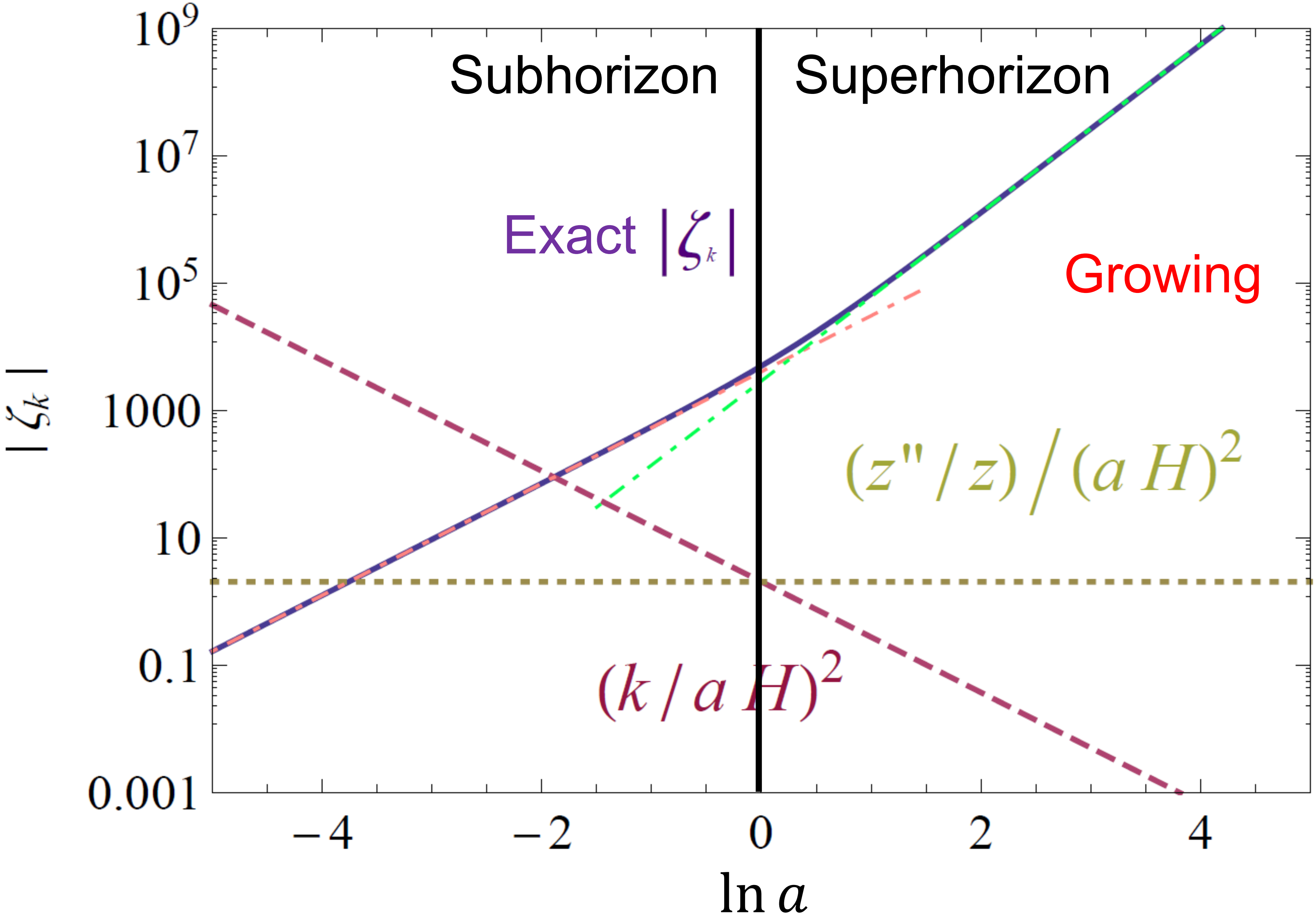
$$n_s - 1 = 3 - 2\nu = 3 - |2\beta + 3|$$

For slightly red-tilted spectrum e.g. $n_s = 0.96$,

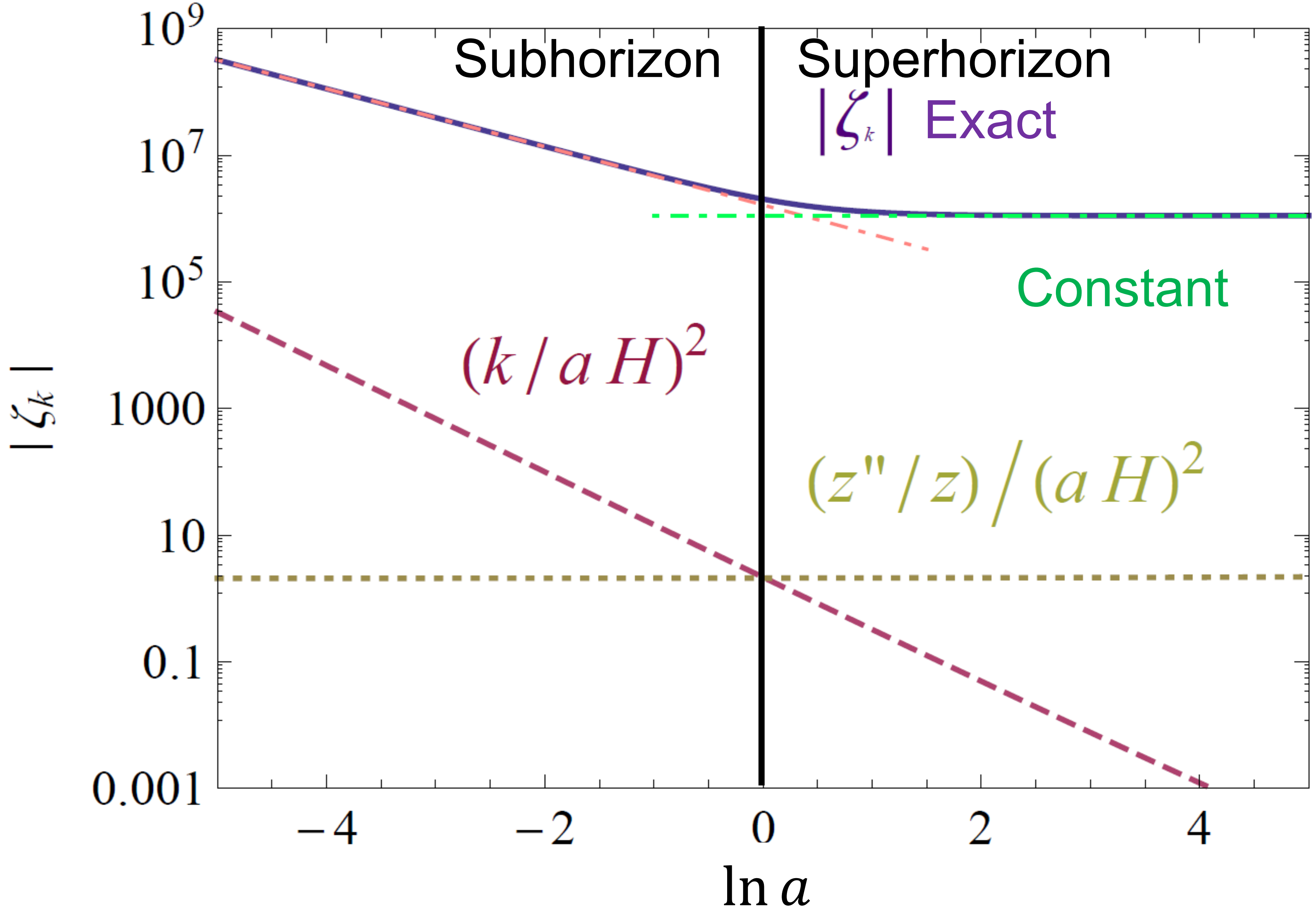
$$\beta = -3.02 \;\text{ or }\; 0.02$$

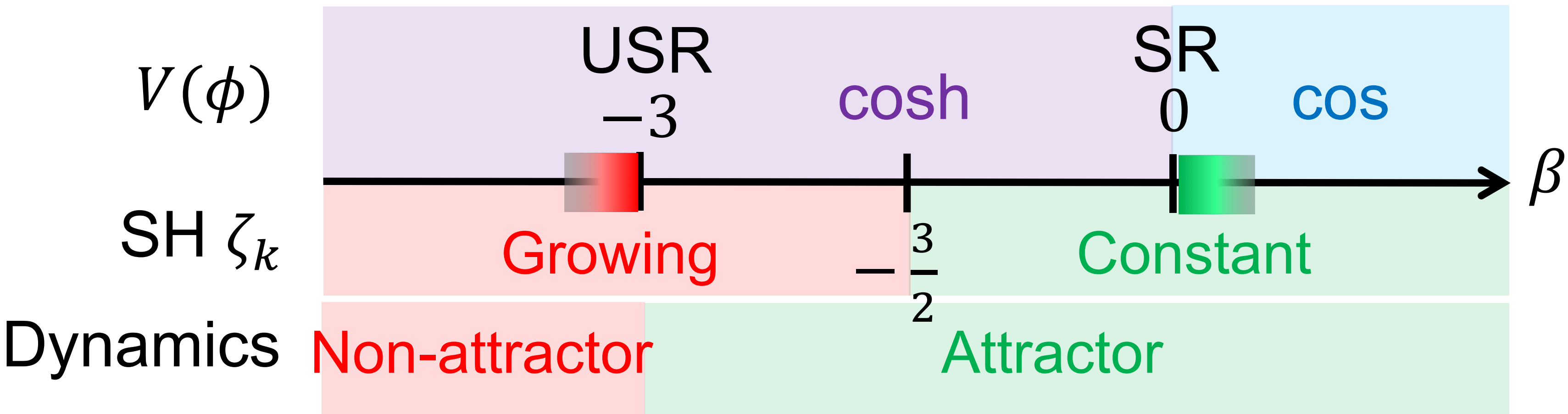


cosh potential $\beta = -3.02$



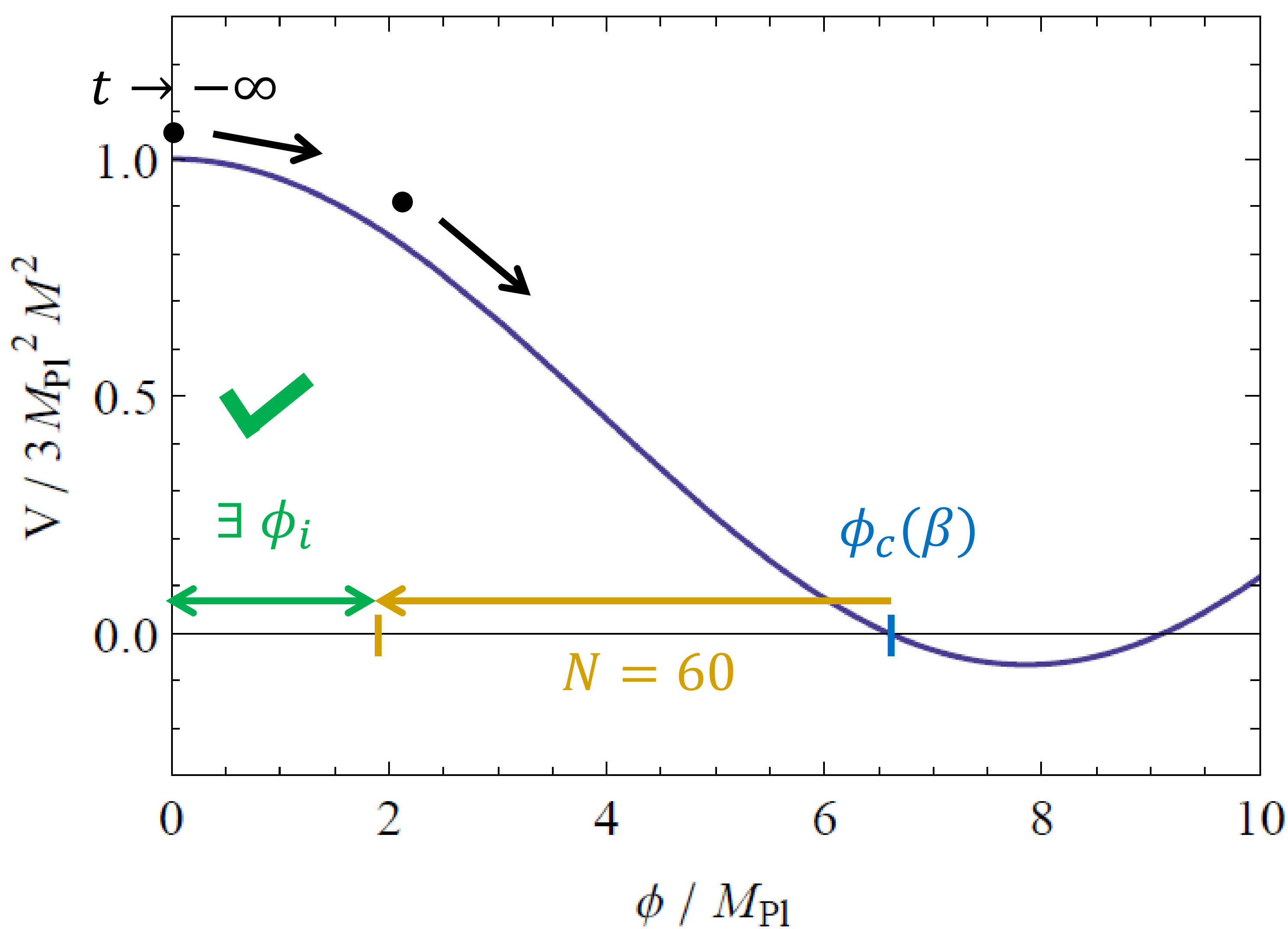
cos potential $\beta = 0.02$





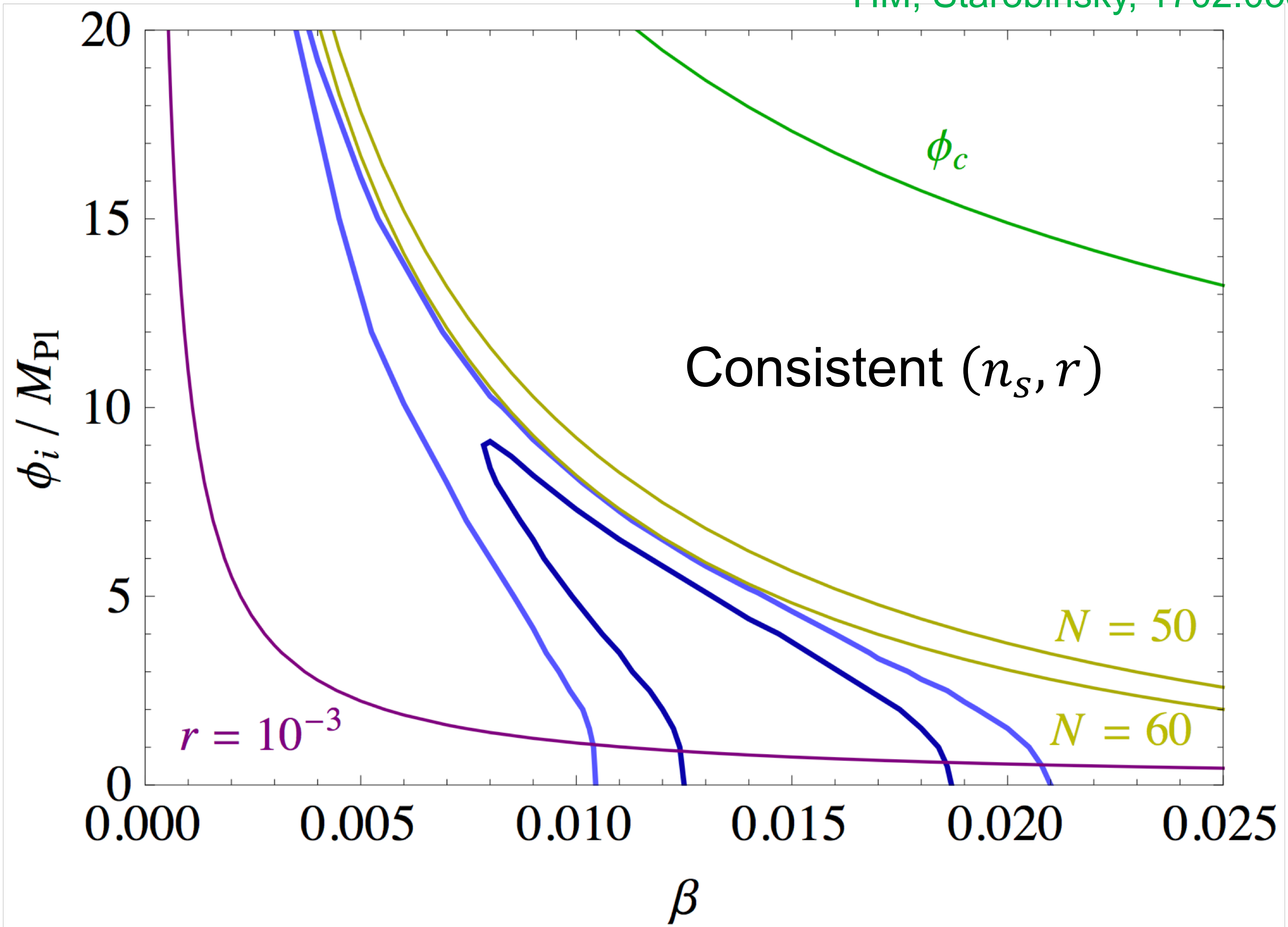
- ✓ ζ_k frozen on SH
- ✓ Attractor
- ✓ n_s
- r
- $N \sim 60$

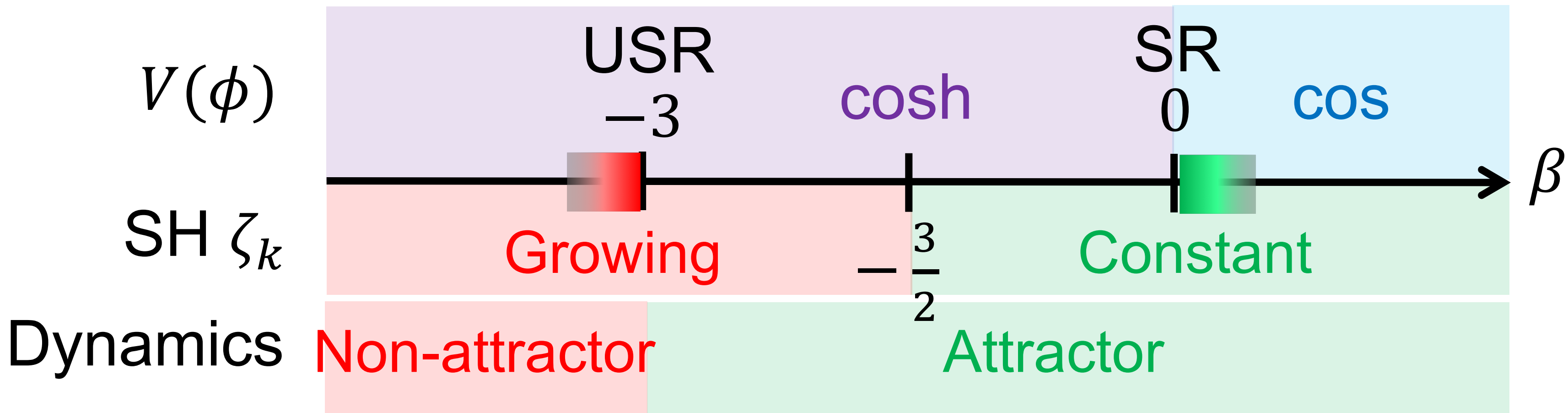
Sufficient number of efolds



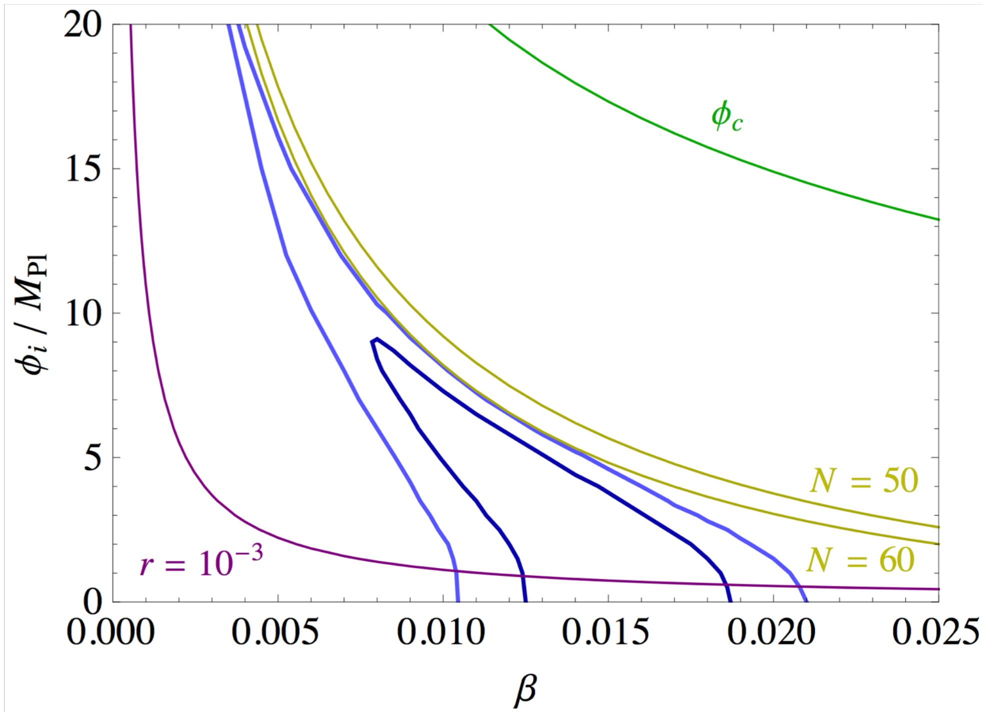
Observational constraint

HM, Starobinsky, 1702.05847





- ✓ ζ_k frozen on SH
- ✓ Attractor
- ✓ n_s
- ✓ r
- ✓ $N \sim 60$



$f(R)$ constant-roll inflation

HM, Starobinsky, 1704.08188

$$S = \int d^4x \sqrt{-g}\,\frac{f(R)}{2}$$

Constant-roll condition in JF (different from $\ddot{\phi} = \beta H \dot{\phi}$)

$$\ddot{F} = \beta H \dot{F}$$

$$(F \equiv df/dR)$$

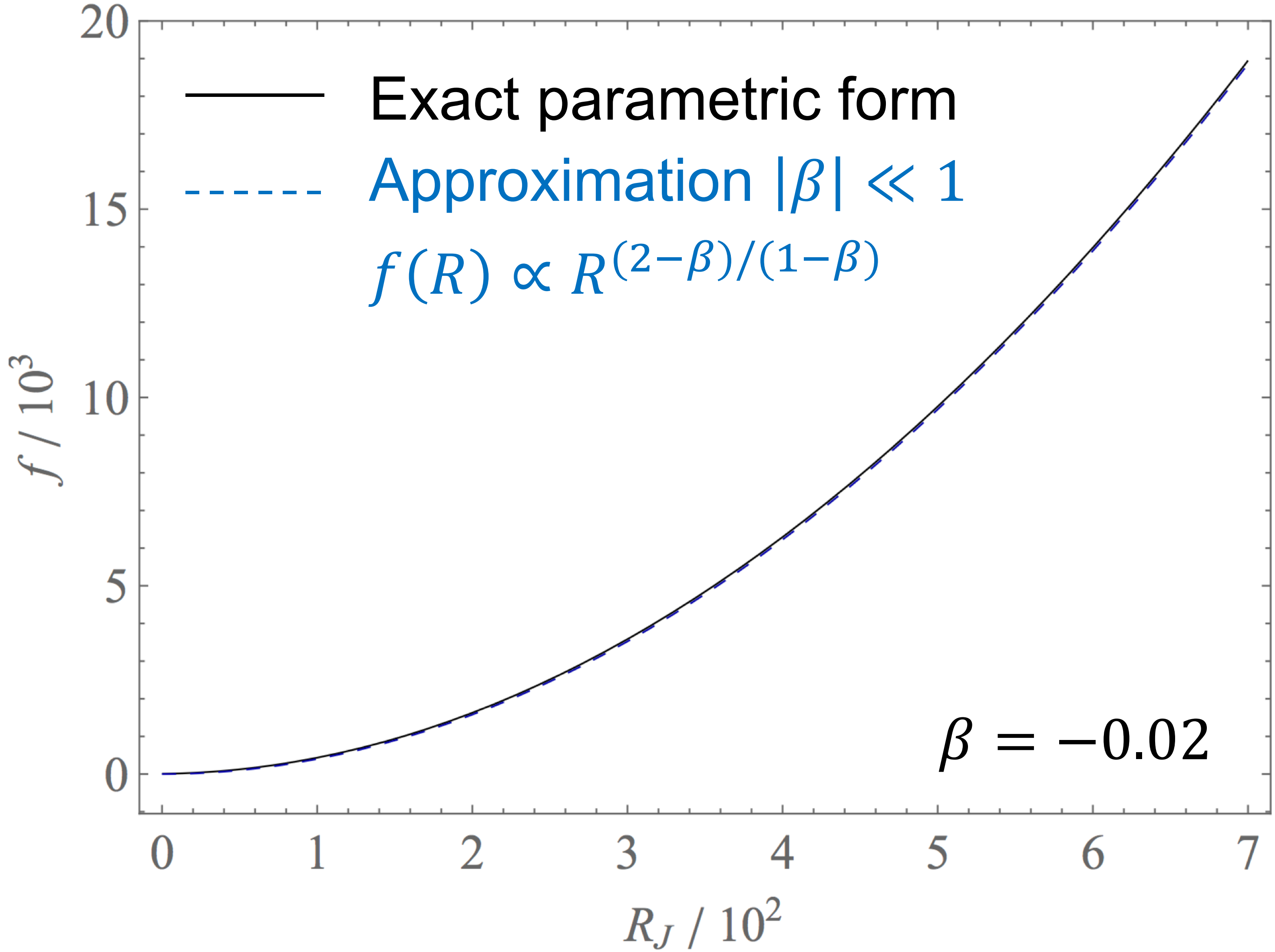
Analytic solution for

- ✓ $f(R)$ (parametric form)
- ✓ $V(\phi)$
- ✓ $\phi(t_E)$

} Einstein frame $g^E_{\mu\nu} = F g^J_{\mu\nu},\ F = e^{\sqrt{\frac{2}{3}}\phi}$

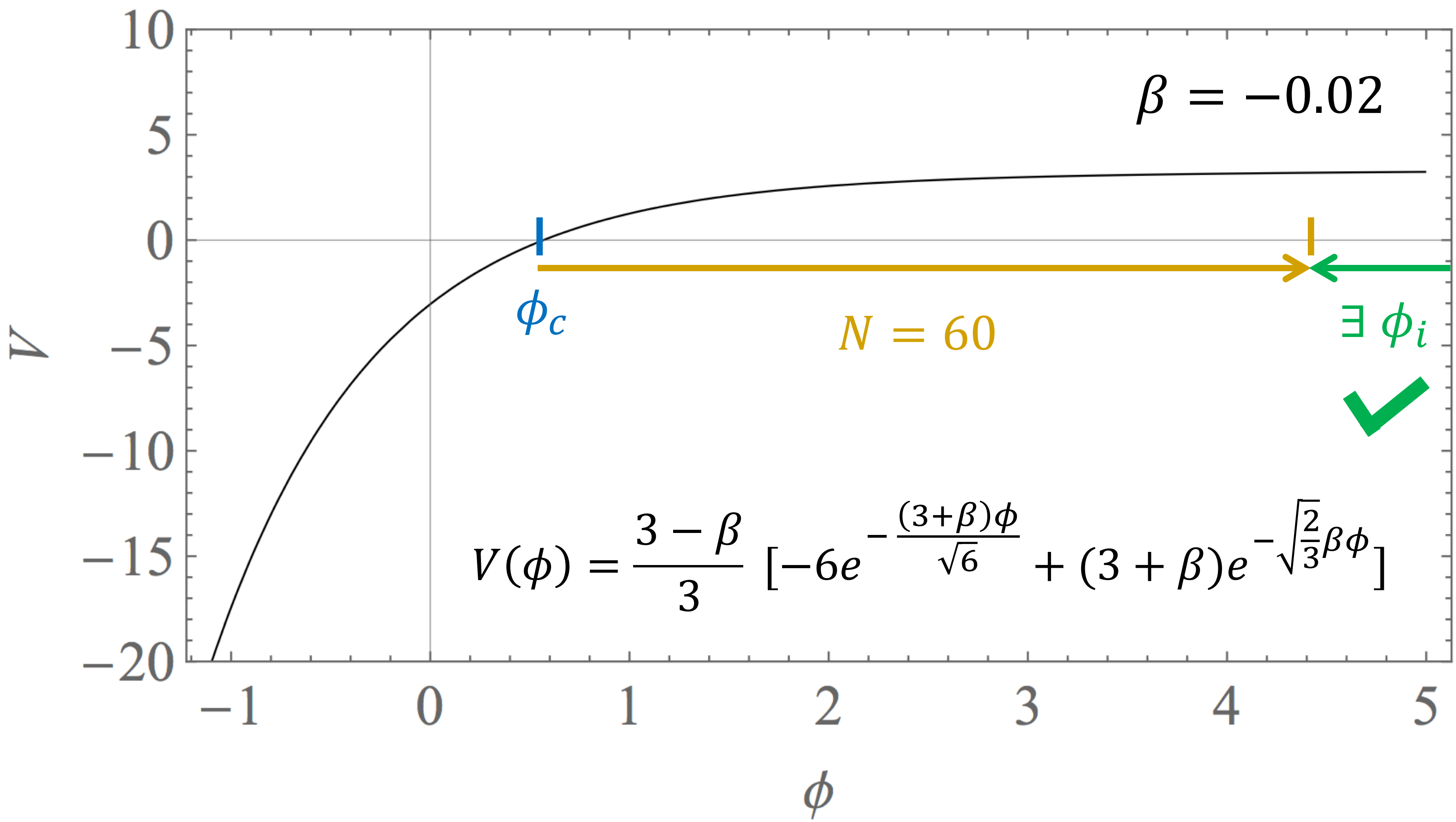
$f(R)$

HM, Starobinsky, 1704.08188



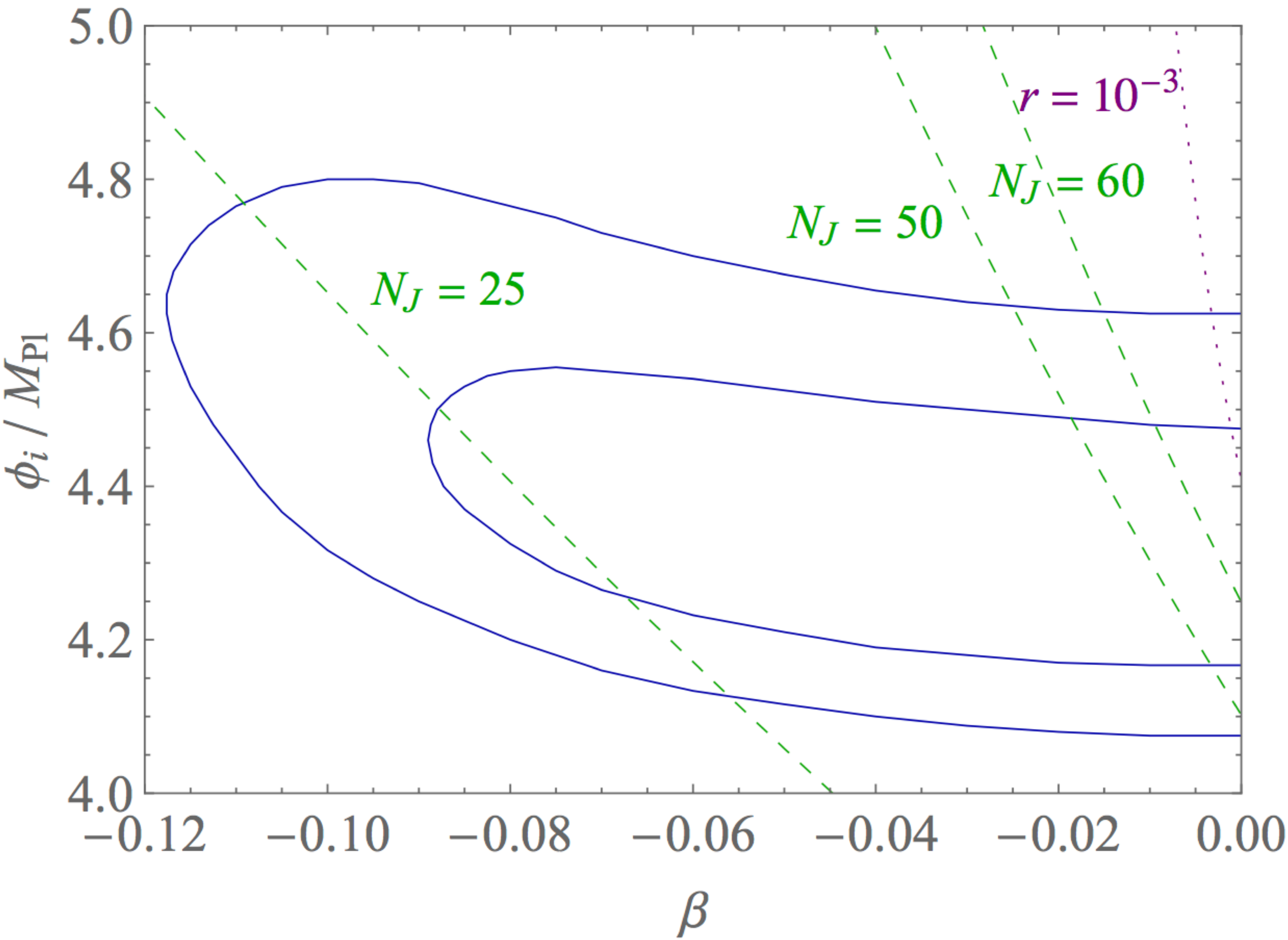
Einstein frame potential

HM, Starobinsky, 1704.08188



Observational constraint

HM, Starobinsky, 1704.08188



Summary

Generalizing SR and USR we explored constant roll

$$\ddot{\phi} = \beta H \dot{\phi} \quad \text{or} \quad \ddot{F} = \beta H \dot{F}$$

which allows analytic solutions of

$$V(\phi), \phi(t), H(t), a(t) \ \text{ and } \ f(R)$$

✓ Attractor

✓ ζ_k frozen on superhorizon scales

✓ $N \sim 60$

✓ n_s & r

Constant roll + obs. constraint

→ Slow roll is preferred.

$\beta \sim$	0.02	Canonical
	-0.02	$f(R)$
	± 0.04	ST