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Extended Cuscuton

カスクートン(?)

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Aya Iyonaga (Rikkyo Univ.)

Collaborators:

Kazufumi Takahashi (Rikkyo Univ.)

Tsutomu Kobayashi (Rikkyo Univ.)

Scalar-Tensor (S-T) Theories

S-T theories : GR + new scalar fields

#DOFs of single-scalar theories ≦ 2+1= 3

Generalized S-T theories

Horndeski theory Horndeski (1974), Deffayet, et al.(2011), Kobayashi, et al.(2012)

E-L eqs. are at most 2nd-order

beyond Horndeski theory Gleyzes, et al. (2014)

E-L eqs. are higher-order, but after combining with each other, they are at most 2nd-order

Cuscuton Theory

Generalized S-T theories have **3DOFs**,
but some special subclasses have only **2DOFs**

$\longrightarrow$ S-T theories with 2DOFs **"minimally" modify GR**

Cuscuton theory: $S = \int d^4x\sqrt{-g}\left[\frac{R}{2} + \mu^2\sqrt{|X|} - V(\phi)\right]$

Afshordi, et al. (2007)

$X \equiv \partial_\mu\phi\partial^\mu\phi \qquad \mu = const.$

- $\partial_\mu\phi$ is space-like $\longrightarrow$ ϕ has a 2nd-order eq. (**EOM**)
  $\Rightarrow$ **3DOFs**
- $\partial_\mu\phi$ is time-like (we can take the unitary gauge $\phi = \phi(t)$)
  $\longrightarrow$ ϕ has a 1st-order eq. (**constraint**)
  $\Rightarrow$ **2DOFs**

Origin of the "Cuscuton"

Cuscuta

..The parasitic plant

Cuscuta
coils around
other plants

Afshordi, et al. (2007)

Why "cuscuton" ?

In the first paper about cuscuton theory,

..the equation of motion (of a scalar field) does not have any second order time derivatives and the field becomes a nondynamical field, which merely follows the dynamics of the fields that it couples to. Thus we call this field Cuscuton.

Features of Cosmology

Cosmological scalar perturbations(unitary gauge $\phi = \phi(t)$):

$$ds^2 = -N^2 dt^2 + \gamma_{ij}(N^i dt + dx^i)(N^j dt + dx^j)$$

$$N = 1 + \alpha, \quad N_i = \partial_i \psi, \quad \gamma_{ij} = a^2 e^{2\zeta} \delta_{ij}$$

[A] Background eqs.:

$$\begin{cases} \delta S/\delta a \propto 3H^2 + 2\dot{H} + \mu^2 \dot{\phi} - V(\phi) = 0 \\ \delta S/\delta \phi \propto 3\mu^2 H + V'(\phi) = 0 \end{cases}$$

at most $\ddot{a}, \dot{\phi}$

[B] The kinetic term of perturbations vanishes:

$$S^{(2)} = \int dt d^3x \, a N \mathcal{F}_S \times (\partial_i \zeta)^2$$

→ ✓ Scalar DOF: nondynamical & not propagate

(⟶ propagating number of DOF is only 2)

Power-law Inflation with Cuscuton

A. Ito, AI, J. Soda and S. Kim [1902.08663]

Power-law inflation with canonical inflaton:

$$S = \int d^4x\sqrt{-g}\left[\frac{M_{pl}^2}{2}R - \underbrace{\frac{1}{2}\partial_\mu\chi\partial^\mu\chi - U_0\exp\left(u\frac{\chi}{M_{pl}}\right)}_{\text{inflaton}}\right]$$

$$\longrightarrow \quad r \sim 8(1-n_s)\,,\ n_s - 1 \sim -\frac{2}{q} \qquad (a(t) \propto t^q)$$

CMB: $n_s \sim 0.96 \quad \Rightarrow \quad r \sim 0.32$

ruled out by Planck data: $r \leq 0.1$

Power-law Inflation with Cuscuton

A. Ito, AI, J. Soda and S. Kim [1902.08663]

Power-law inflation with canonical inflaton + cuscuton:

$$S = \int d^4x\sqrt{-g}\left[\frac{M_{pl}^2}{2}R \underbrace{- \frac{1}{2}\partial_\mu\chi\partial^\mu\chi - U_0 \exp\left(u\frac{\chi}{M_{pl}}\right)}_{\text{inflaton}} \underbrace{+\mu^2\sqrt{-\partial_\mu\phi\partial^\mu\phi} - \frac{1}{2}m^2\phi^2}_{\text{cuscuton}}\right]$$

$$\longrightarrow \quad r \sim \frac{8}{u^2}(n_s - 1)^2, \; n_s - 1 \sim -\frac{u^2}{2}\left(2 - \frac{3\mu^4}{M_{pl}^2 m^2}\right)$$

CMB: $n_s \sim 0.96 \quad \Rightarrow \quad r \sim \frac{12}{u^2} \times 10^{-3}$

satisfy Planck data if $u^2 \geq 0.12$

Other Features

- ✓ Subclass of Horndeski theory (explain later)
- ✓ Different cosmologies from GR (CMB, matter power spectra) Afshordi, et al. (2007)
- ✓ Cuscuton with $V(\phi) = \frac{1}{2}m^2\phi^2$

  $\Leftrightarrow$ low energy limit of Hořava-Lifshitz theory
  Afshordi (2009), Bhattacharyya, et al. (2016)

..Even if it is a minimal modification of GR, cuscuton theory has interesting features

(Beyond) Horndeski Theories

✓ Cuscuton theory is a subclass of Horndeski theory

(Beyond) Horndeski in the unitary gauge

$$S = \int dt d^3 x N \sqrt{\gamma} \left[A_2 + A_3 K + A_4 \left(K^2 - K_{ij} K^{ij} \right) + B_4 R \right.$$
$$\left. + A_5 \left(K^3 - 3 K K_{ij} K^{ij} + 2 K_{ij} K^{ik} K^j_k \right) + B_5 K^{ij} G_{ij} \right]$$

$A_i = A_i(t, N) \qquad B_i = B_i(t, N)$

(Horndeski ⟶ $A_4 = -B_4 - N B_{4N}, \ A_5 = \frac{N}{6} B_{5N}$ "Horndeski conditions") $\quad B_N = \frac{\partial B}{\partial N}$

Cuscuton theory in Horndeski (unitary gauge) :

$$A_2 = -V(t) + \frac{\sigma(t)}{N}, \quad A_3 = 0, \quad A_4 = -\frac{1}{2}, \quad A_5 = 0,$$

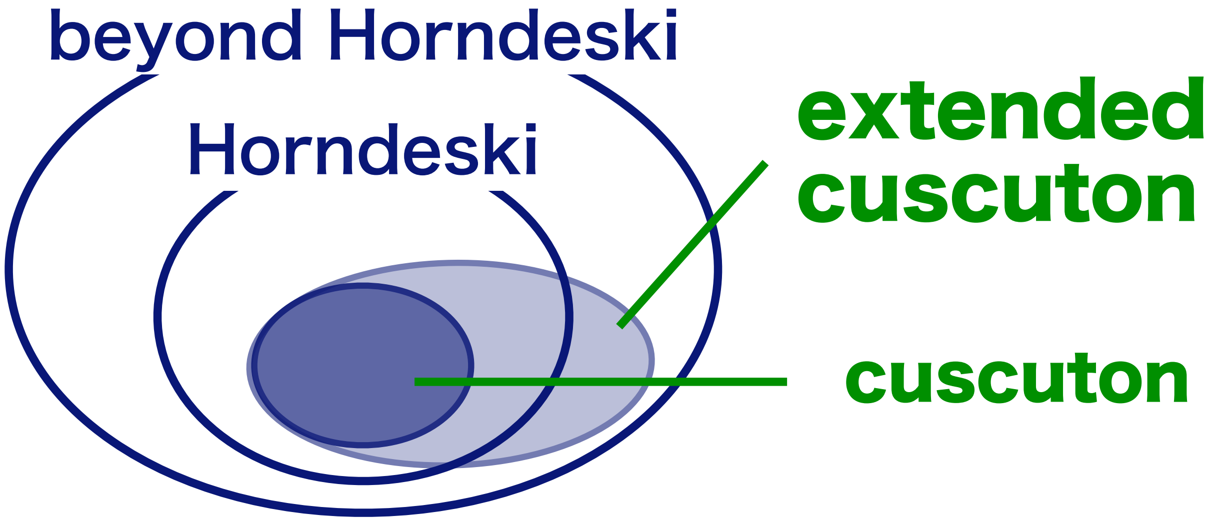
$$B_4 = \frac{1}{2}, \quad B_5 = 0 \qquad \sigma(t) = \mu^2 |\dot{\phi}(t)|$$

Motivation of Our Study

There may be
more generalized cusuton-like theories
(2DOF S-T theories with a time-like $\partial_\mu \phi$)

↓

We considered "extended cuscuton" theory as a part of the beyond Horndeski



Formulation

Determine A_i, B_i in beyond Horndeski theory
to have only 2DOFs in the unitary gauge
by Hamiltonian analysis (or degeneracy of E-L eqs.)

↓

Extended cuscuton

$$A_2 = \mu_2 + \frac{\nu_2}{N} + \frac{2(\mu_4 N + \nu_4)^3}{9N(\mu_5 N + \nu_5)^2}$$

$$A_3 = \mu_3 \pm \frac{2(\mu_4 N + \nu_4)^2}{3(\mu_5 N + \nu_5)^2},$$

$$A_4 = \frac{N(\mu_4 N + \nu_4)}{(\mu_5 N + \nu_5)^2}, \qquad A_5 = \pm\frac{N^2}{(\mu_5 N + \nu_5)^2},$$

$$B_4 = b_0(t) + \frac{b_1(t)}{N}, \quad B_5 = 0$$

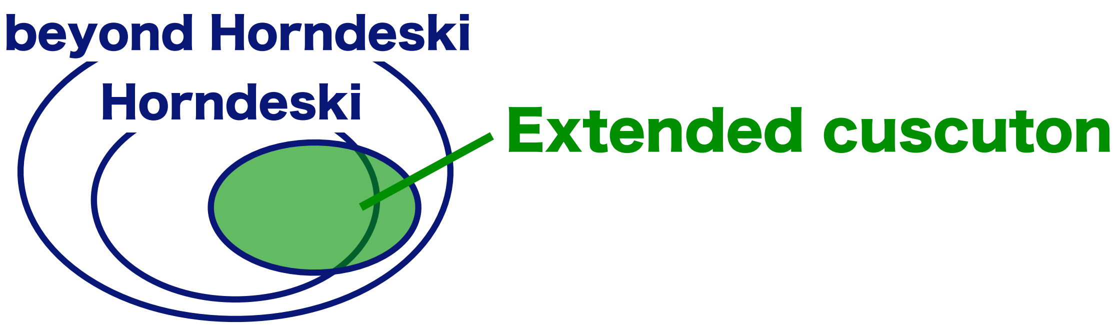
$$u_i = u_i(t)$$

$$v_i = v_i(t)$$

Nature of Extended Cuscuton

- Horndeski conditions are not satisfied

  → **Subclass of the beyond Horndeski**



- In the cosmological setup,

Original cuscuton: Background eqs. are **at most** $\dot{\phi}$

↓ **generalized**

Extended cuscuton: Background eqs. are **at most** $\ddot{\phi}$,
but after combining each other
they are **at most** $\dot{\phi}$ (degenerate)

Cosmology with Matter

Extended cuscuton with matter:

$$\mathcal{L} = \mathcal{L}_{EC} + \mathcal{L}_{\chi}$$

Matter sector: $\mathcal{L}_{\chi} = P(Y), \quad Y \equiv -\frac{1}{2}\partial_{\mu}\chi\partial^{\mu}\chi$

- **Sound speed of** χ **:** $c_{\chi}^2 = \dfrac{P_Y}{P_Y + 2YP_{YY}}$ $\qquad P_Y = \dfrac{\partial P}{\partial Y}$
- $p = P, \quad \rho = 2YP_Y - P$

Cosmological Perturbations

Scalar perturbations (unitary gauge) :

$$N = 1 + \alpha, \quad N_i = \partial_i \psi, \quad \gamma_{ij} = a^2 e^{2\zeta} \delta_{ij}$$

$$\chi = \chi(t) + \delta\chi(t, \vec{x})$$

(Gauge-invariant) density fluctuation :

$$\delta = \frac{\rho + p}{\rho c_\chi^2} \left(\frac{\delta\chi}{\dot{\chi}} - \alpha \right) + 3 \frac{\rho + p}{\rho} \zeta$$

$$\mathcal{L}^{(2)}(\alpha, \psi, \zeta, \delta\chi) \xrightarrow{\delta\chi \to \delta} \mathcal{L}^{(2)}(\alpha, \psi, \zeta, \delta)$$

↓ E-L eqs. for α, ψ, ζ (constraints)

$$\mathcal{L}^{(2)} = a^3 (\mathcal{A} \dot{\delta}^2 + \mathcal{B} \delta^2)$$

✓ **scalar perturbation is only δ without the quasi-static approximation**

Modifications of GR

$$\mathcal{L}^{(2)} = a^3(\mathcal{A}\dot{\delta}^2 + \mathcal{B}\delta^2)$$

Dust limit ($p \to 0,\ c_\chi \to 0$)+subhorizon limit ($k \to \infty$) :

$$\mathcal{A} \to \frac{a^2\rho}{2k^2\Upsilon}, \quad \mathcal{B} \to \frac{a^2\rho^2}{2k^2} \times \ldots$$

$$\left(\Upsilon = \frac{2\mathcal{F}_S\Theta^2 - \overline{\mathcal{G}}_T^2\rho}{2\mathcal{F}_S\Theta^2 - \mathcal{G}_T\left(2\overline{\mathcal{G}}_T - \mathcal{G}_T\right)\rho}, \qquad \mathcal{G}_T, \bar{\mathcal{G}}_T, \mathcal{F}_S, \Theta \text{ : functions of } A_i, B_i, H\right)$$

- Variation with respect to δ :

$$\ddot{\delta} + \left(2H - \frac{\dot{\Upsilon}}{\Upsilon}\right)\dot{\delta} - 4\pi G_{\rm eff}\rho\delta = 0\,,\ \ 4\pi G_{\rm eff} = \Upsilon \times \ldots$$

- Poisson equation :

$$-\frac{k^2}{a^2}\Psi = 4\pi G_{\rm eff}\rho\delta + \dot{\delta} \times \ldots \quad \text{where } \Psi \equiv \alpha + \dot{\psi}$$

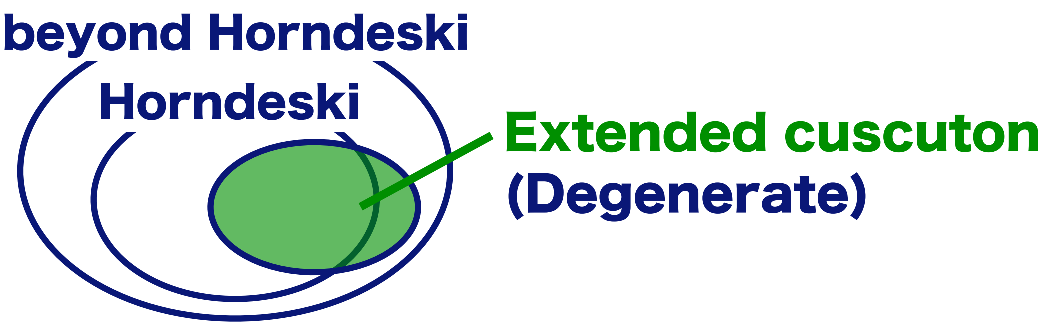
These are modified from GR

Summary

Cuscuton theory : Scalar-tensor theory with **2DOFs**,
with a time-like $\partial_\mu \phi$

↓

We determined **"extended cuscuton"** theory
in beyond Horndeski



Cosmological perturbations with matter :

- Scalar perturbation is only density fluctuation δ
  without any approximation
- EOM for density fluctuation & Poisson equation
  are modified from GR

Future work

- GR

  ↓ disformal transformation:

$$\tilde{g}_{\mu\nu} = A(\phi, X) g_{\mu\nu} + B(\phi, X) \nabla_\mu \phi \nabla_\nu \phi$$

S-T theory with 2DOFs

Are these involved in the extended cuscuton?

- Extension into DHOST theories?
- BH solutions?

BACKUP

Power-law Inflation with Cuscuton

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$$S = \int d^4x\sqrt{-g}\left[\frac{M_{pl}^2}{2}R + \mu^2\sqrt{-\partial_\mu\phi\partial^\mu\phi} - V(\phi) - \frac{1}{2}\partial_\mu\chi\partial^\mu\chi - U(\chi)\right]$$

Flat FLRW sp., $\phi(t)$: cuscuton, $\chi(t)$: inflaton

Consider $U(\chi) = U_0 \exp\left(u\frac{\chi}{M_{pl}}\right),\ V(\phi) = \frac{1}{2}m^2\phi(t)^2$

⟶ **One of the solutions:**

$$H(t) = \frac{p}{t}, \quad \frac{\chi(t)}{M_{pl}} = s\ln M_{pl}t, \quad \phi(t) = \frac{q}{t}$$

with

$$u = -\frac{2}{s}, \quad p = s^2\left(2 - \frac{3\mu^4}{M_{pl}^2m^2}\right)^{-1}, \quad q = -\frac{3\mu^2s^2}{m^2}\left(2 - \frac{3\mu^4}{M_{pl}^2m^2}\right)^{-1}$$

$2 - \frac{3\mu^4}{M_{pl}^2m^2} > 0$ **for accelerating universe**

Power-law Inflation with Cuscuton

A. Ito, AI, J. Soda and S. Kim [1902.08663]

Spectral index & tensor-scalar ratio:

$$n_s - 1 = -\frac{2}{s^2}\left(2 - \frac{3\mu^4}{M_{pl}^2 m^2}\right), \quad r = \frac{8}{s^2}\left(2 - \frac{3\mu^4}{M_{pl}^2 m^2}\right)^2$$