# Dark energy and Gravitational waves

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## Dark energy

Acceleration of the universe

$$\rho_\Lambda = \frac{\Lambda}{8\pi G} = M_{pl}^2 \Lambda \sim M_{pl}^2 H_0^2$$

$$H_0 = 2.13 \times 10^{-42} h\mathrm{GeV} \sim (10^{23}\mathrm{km})^{-1}$$

$$M_{pl} \sim 10^{19}\mathrm{GeV}$$

## Gravitational waves

Detections by LIGO

$$f_{LIGO} \sim 10 - 100 \text{ Hz}$$

$$\ell_{LIGO} \sim 10^3 - 10^4 \text{ km}$$

$$p_{LIGO} \sim 10^{-22} - 10^{-23}\mathrm{GeV}$$

$$\frac{p_{GW}}{H_0} \sim 10^{20}$$

# Example: scalar-tensor theories

- Scalar tensor theory containing up to second derivatives

$$S = \int d^4x \sqrt{-g} \mathcal{L}\left[\phi, \nabla_\mu \phi, \nabla_\mu \nabla_\nu \phi;\ g_{\mu\nu}\right]$$

$$\mathcal{L}_{\rm tot} = \sum_{i=1}^{5} \mathcal{L}_i + \mathcal{L}_R$$

$$\mathcal{L}_1[A_1] = A_1(\phi,\, X)\phi_{\mu\nu}\phi^{\mu\nu}\,,$$
$$\mathcal{L}_2[A_2] = A_2(\phi,\, X)(\Box\phi)^2\,,$$
$$\mathcal{L}_3[A_3] = A_3(\phi,\, X)(\Box\phi)\phi^{\mu}\phi_{\mu\nu}\phi^{\nu}$$
$$\mathcal{L}_4[A_4] = A_4(\phi,\, X)\phi^{\mu}\phi_{\mu\rho}\phi^{\rho\nu}\phi_{\nu}\,,$$
$$\mathcal{L}_5[A_5] = A_5(\phi,\, X)(\phi^{\mu}\phi_{\mu\nu}\phi^{\nu})^2\,,$$

$$\phi_{\mu\nu} = \nabla_\mu \nabla_\nu \phi \text{ and } X = \phi^\mu \phi_\mu \qquad \phi_\mu = \nabla_\mu \phi$$

$$\mathcal{L}_R[G] = G(\phi,\, X)R$$

# Theoretical consistency

- 2+1 degrees of freedom to avoid Ostrogradsky ghost

  Horndeski theory with 2nd order equation of motion: free function $G$

$$A_1 = -A_2 = -2G_X \, , \quad A_3 = A_4 = A_5 = 0 \qquad L = GR - 2G_{,X}(\phi_{\mu\nu}\phi^{\mu\nu} - (\Box\phi)^2)$$

  Horndeski 1974, Deffayet et.al. 1103.3260, Kobayashi et.al. 1105.5723

- Degenerate higher order scalar tensor (DHOST) theory

  degeneracy condition: free functions $(G, A_1 \text{ and } A_3)$ Langlois, Noui 1510.06930,1512.06820

$$\begin{aligned} A_4 &= \frac{1}{8(G - A_1 X)^2}\left[4G\left(3(A_1 - 2G_X)^2 - 2A_3 G\right) - A_3 X^2 (16 A_1 G_X + A_3 G)\right. \\ &\quad \left. + 4X\left(3A_1 A_3 G + 16 A_1^2 G_X - 16 A_1 G_X^2 - 4A_1^3 + 2A_3 G G_X\right)\right] \\ A_5 &= \frac{1}{8(G - A_1 X)^2}(2A_1 - A_3 X - 4G_X)\left[A_1(2A_1 + 3A_3 X - 4G_X) - 4A_3 G\right] . \end{aligned}$$

# Disformal transformation

- Bekenstein's "two geometries"

$$\sqrt{-g}\,\mathcal{L}_{\rm vacuum}[g,\{\chi_a\}] + \sqrt{-\tilde{g}}\,\mathcal{L}_{\rm matter}[\tilde{g},\{\psi_b\}]$$

vacuum = Horndeski + $\tilde{g}_{\mu\nu} = C(X) g_{\mu\nu} + D(X)\phi_\mu \phi_\nu$ = DHOST

Crisostomi, KK, Tasinato 1602.03119, Achour, Langlois, Noui 1602.08398,
Achour, Crisostomi, KK, Langlois, Noui, Tasinato 1610.07467

- This procedure can be applied to other theories

  cf. massive gravity

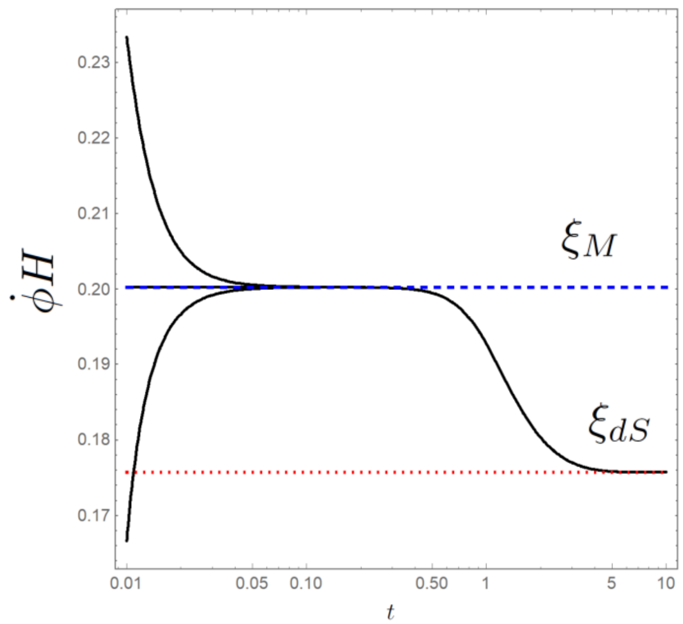
$$\tilde{g}_{\mu\nu} = \bar{C}([\gamma^n]) g_{\mu\nu} + \bar{D}_{ab}([\gamma^n]) \nabla_\mu \phi^a \nabla_\nu \phi^b \qquad \gamma^\mu{}_\nu \equiv \eta_{ab} \nabla^\mu \phi^a \, \nabla_\nu \phi^b$$

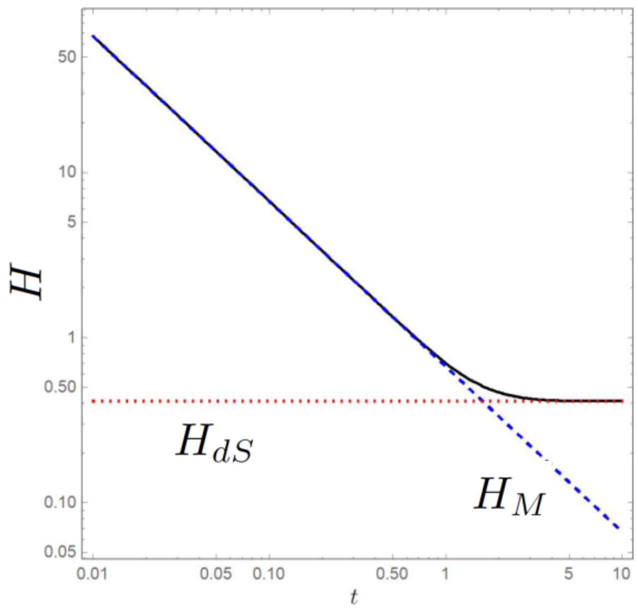
Gümrükçüoğlu & KK 1902.01391

# Dark energy model

- Accelerated expansion is driven by kinetic energy Crisostomi and KK arXiv:1712.06556

tracker solution $\dot{\phi}H = \xi \qquad G_2 = c_2 X\,, \quad G_3 = \frac{c_3}{\Lambda_3^3}X\,, \quad G = \frac{M_P^2}{2} + \frac{c_4}{\Lambda_3^6}X^2\,, \quad A_3 = -\frac{8c_4}{\Lambda_3^6} - \frac{\beta}{\Lambda_3^6}$




$$\Lambda_3^3 = H_0^2 M_{\rm pl}$$

generalisation

$$\frac{\dot{\phi}}{H^p} = \alpha$$

Frusciante, Kase, KK, Tsujikawa, Vernieri arXiv:1812.05204

# Gravitational waves constraints

- Hulse-Taylor binary

$$\frac{\dot{P}}{\dot{P}_{\rm GR}} = 0.997 \pm 0.002$$

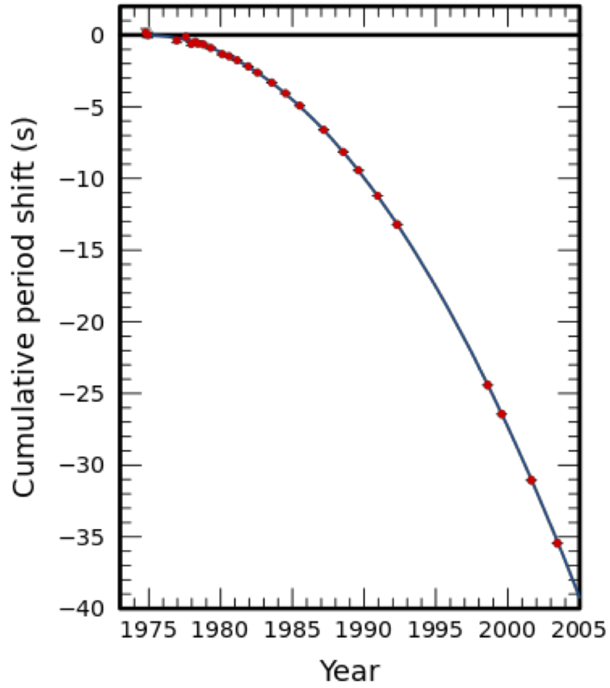
- GW170817/GRB 170817A LIGO collaboration et.al 1710.05834

The observed time delay between GWs and gamma-ray bursts was $(+1.74 \pm 0.05)$ s

The distance to the source was estimated as $43.8^{+2.9}_{-6.9}{\rm Mpc}$

$$-3 \times 10^{-15} < \frac{c_{\rm GW}}{c} - 1 < 7 \times 10^{-16}$$

The lower bound assumes gamma-ray was emitted 10s after GW




cf. gravitational Charenkov radiation

$$-2 \times 10^{-15} < c_{\rm GW}/c - 1$$

Moore & Nelson hep-th/0106220

# Gravitational waves constraints

- Gravitational wave speed $c_{GW}^2 = \frac{G}{G - XA_1}$

de Rham et.al. 1604.08638

$$\mathcal{L}_1[A_1] = A_1(\phi,\, X)\phi_{\mu\nu}\phi^{\mu\nu} \supset \dot{h}_{ij}\dot{h}^{ij}$$
$$\mathcal{L}_2[A_2] = A_2(\phi,\, X)(\Box\phi)^2\,,$$
$$\mathcal{L}_3[A_3] = A_3(\phi,\, X)(\Box\phi)\phi^\mu\phi_{\mu\nu}\phi^\nu\,,$$
$$\mathcal{L}_4[A_4] = A_4(\phi,\, X)\phi^\mu\phi_{\mu\rho}\phi^{\rho\nu}\phi_\nu\,,$$
$$\mathcal{L}_5[A_5] = A_5(\phi,\, X)(\phi^\mu\phi_{\mu\nu}\phi^\nu)^2\,,$$

$c_{\rm GW} = c$ ➡ $A_1 = 0$

$$A_2 = 0\,, \qquad A_5 = \frac{A_3}{2G}(4G_X + A_3 X)\,,$$
$$A_4 = -\frac{1}{8G}\left[8A_3 G - 48G_X^2 - 8A_3 G_X X + A_3^2 X^2\right]$$

this excludes Horndeski theory with $G(X)$ free functions $(G \text{ and } A_3)$

- Disformal transformation from Horndeski

$$\tilde{g}_{\mu\nu} = C(X)g_{\mu\nu} + D(X)\phi_\mu\phi_\nu$$

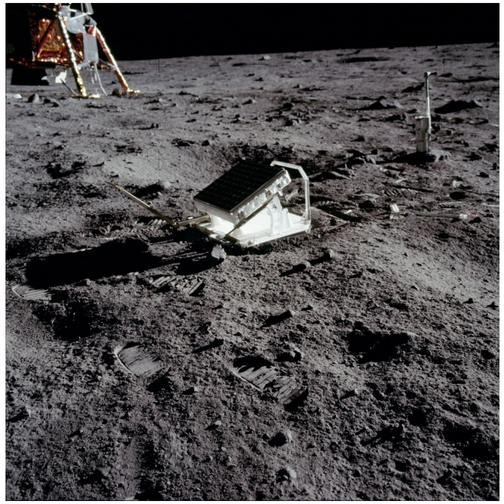
matter coupled to $\tilde{g}_{\mu\nu}$ $c_{\rm GW} \neq c$ ➡ $c_{\rm GW} = \tilde{c}$

# Tests of General Relativity in the solar system

- Solar system tests

$$ds^2 = -\left(1 - 2U + 2\beta U^2\right) dt^2 + \left(1 - 2\gamma U\right) \delta_{ij} dx^i dx^j$$

$$\gamma - 1 < (2.1 \pm 2.3) \times 10^{-5} \qquad \beta - 1 = (-4.1 \pm 7.8) \times 10^{-5}$$



- Lunar Laser Ranging

$$|\delta\theta| = \left|\pi r \frac{d}{dr}\left[r^2 \frac{d}{dr}\left(\frac{\varepsilon}{r}\right)\right]\right| < 2.4 \times 10^{-11}, \quad \varepsilon = \frac{\delta\Psi}{\Psi}$$

$$\frac{\dot{G}_N}{G_N} = (2 \pm 7) \times 10^{-13} \ \text{per year}$$

# Vainshtein mechanism

- Vainshtein mechanism $G \sim M_p^2,\ XA_3 \sim XA_4 \sim X^2A_5 \sim M_p\Lambda_3^{-3}$

  a spherically symmetric object with mass $M(r)$

Crisostomi and KK arXiv:1711.06661
Langlois et.al. arXiv:1711.07403

$r \ll r_V \qquad r_V = (M/M_p\Lambda_3^3)^{1/3}$

$$\Phi' = \frac{G_N M}{r^2} + \frac{\Upsilon_1 G_N}{4} M'',$$
$$\Psi' = \frac{G_N M}{r^2} - \frac{5\Upsilon_2 G_N}{4r} M' + \Upsilon_3 G_N M''$$

$$G_N = \left[8\pi\left(2G - 2XG_X - 3A_3X^2/2\right)\right]^{-1}$$

$$\Upsilon_1 = -\frac{(4G_X - XA_3)^2}{4A_3G},$$
$$\Upsilon_2 = \frac{8G_X X}{5G},$$
$$\Upsilon_3 = -\frac{-16G_X^2 + A_3^2X^2}{16A_3G}$$

The Vainshtein mechanism works to restore GR outside matter but it is broken inside matter source  Kobayashi et.al. 1411.4130

# Constraints

- Stellar structure KK & Sakstein 1502.06872, Saito et.al. 1503.01448, Saketsin 1510.05964

$$-2/3 < \Upsilon_1 < 1.6$$

upper limit: stars are less compact and a minimum mass for hydrogen burning is larger than GR. The bound comes from the lightest red dwarfs white dwarfs also give an upper bound <1.4 Saltas et.al. 1803.0054

lower limit: stable stars cannot be formed

- Hulse-Taylor pulsar constraints $G_{\rm GW} = G \neq G_N$

$$\frac{G_{\rm GW}}{G_N} - 1 = \frac{2XG_{4X}}{G_4} + \frac{3A_3X^2}{2G_4} \qquad -7.5 \times 10^{-3} < \frac{G_{\rm GW}}{G_N} - 1 < 2.5 \times 10^{-3}$$

Beltran et.al. 1507.05047 Dima and Vernizzi arXiv:1712.04731

# Quantum corrections

- Strong coupling problem    Luty, Porrati, Rattazzi hep-th/0303116   Nicolis, Rattazzi hep-th/0404159

  The Vainshtein mechanism replies on non-linear interactions

  $$S_{\rm non-linear} = -\int d^4x \frac{1}{\Lambda_3^3} \Box\phi(\partial\phi)^2$$

  $$\Lambda_3 = (M_{\rm pl} H_0^2)^{1/3} \sim 10^{-22}{\rm GeV} \sim (10^3{\rm km})^{-1} \sim p_{\rm LIGO}$$

  At the energy scale of $\Lambda_3$ quantum corrections are not suppressed

  $$S \propto -(1/\Lambda_3^6) \int d^4x \phi \Box^4 \phi$$

  The theory loses it predictability (we need to know a UV completion of the theory)

# Gravitational wave decay

Creminelli, Lewandowski, Tambalo, Vernizzi 1809.03484

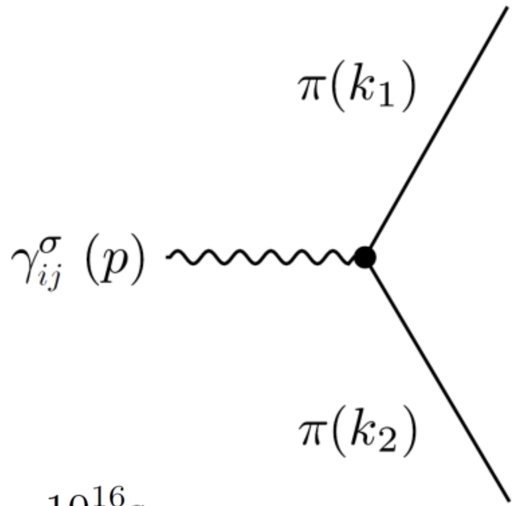
- Interactions between gravitational waves and dark energy field

$$L_{\gamma\pi\pi} = \frac{1}{\Lambda_*^3}\ddot{\gamma}_{ij}^{(c)}\partial_i\pi^{(c)}\partial_j\pi^{(c)}$$

decay rate $c_s^2 < 1$

$$\Gamma_{\gamma\to\pi\pi} = \frac{p^7(1-c_s^2)^2}{480\pi c_s^7\Lambda_*^6}$$

$$\Lambda_* \sim \Lambda_3, \quad p_{LIGO} \sim \Lambda_3, \quad \Gamma \sim \Lambda_3 \sim 10^{-3}\text{s} \ll 40\text{Mpc}/c \sim 10^{16}\text{s}$$




- DHOST $A_3 = 0$ free function: $G$ (this is not Horndeski)

# Current status of scalar tensor theory

- Survived theory   Creminelli, Lewandowski, Tambalo, Vernizzi 1809.03484

$$L = G_2(\phi, X) + G_3(\phi, X)\Box\phi + G(\phi, X)R + \frac{6G_{,X}(\phi, X)^2}{G(\phi, X)}\phi^\mu\phi_{\mu\nu}\phi_\lambda\phi^{\lambda\nu}$$

cubic Horndeski $G = G(\phi)$ + conformal transformation $\tilde{g}_{\mu\nu} = C(X)g_{\mu\nu}$

- Lower cut-off   de Rham and Melville 1806.09417

The theory may not be trustable up to $p_{LIGO} \sim \Lambda_3$ depending the UV completion

The Lorentz invariant completion will likely result in $c_{\rm GW} = c.$

no constraints on dark energy from GW observations by LIGO
(but we loose predictability)

# Other constraints from GW propagation

- GW propagation    Nishizawa 1710.04825, Ezquiaga & Zumalacarregui 1807.09241,

$$h''_{ij} + (2+\nu)\mathcal{H}h'_{ij} + (c_g^2 k^2 + m^2 a^2)h_{ij} = \Pi_{ij}$$

modified friction term $\nu \propto \frac{dG_{GW}(\eta)}{d\eta}$

this changes the luminosity distance measured by GWs

$$\frac{d_L^{\rm gw}(z)}{d_L^{\rm em}(z)} = \exp\left[\frac{1}{2}\int_0^z \frac{\nu}{1+z'}dz'\right]$$

$$-75.3 \le \nu \le 78.4$$

Arai & Nishizawa 1711.03776

mass term

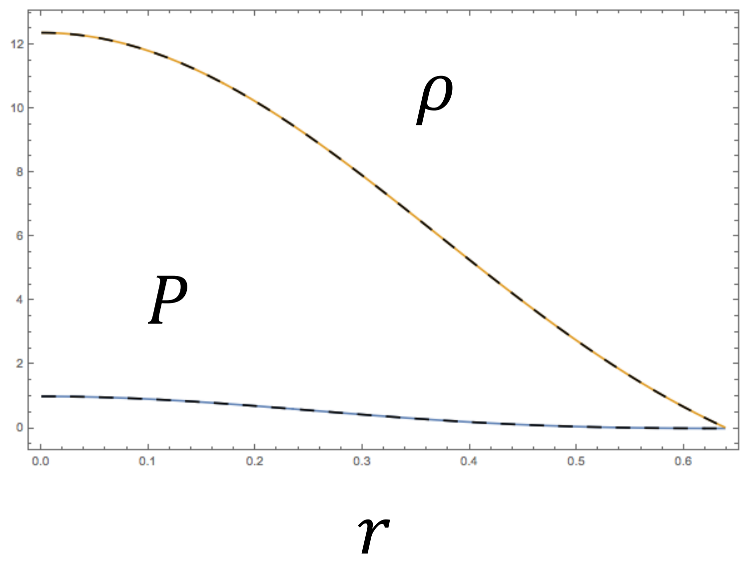
$$m_g \le 7.7 \cdot 10^{-23}\,{\rm eV}/c^2$$

LIGO collaboration

# Generation of GWs

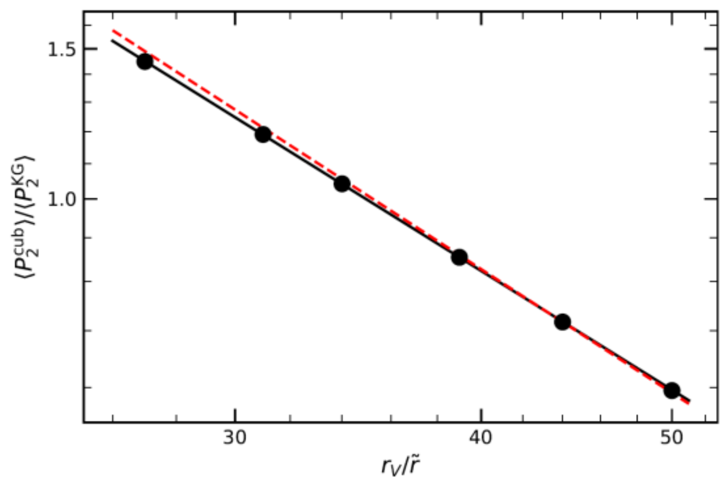
- Cubic galileon theory - Vainshtein mechanism is highly effective

neutron stars Ogawa, Kobayashi, KK




Poster by Ogawa

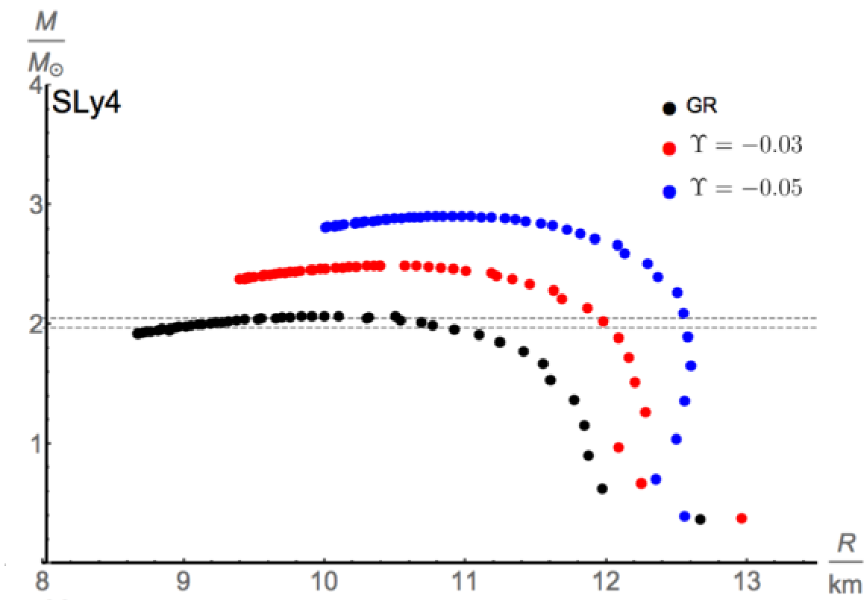
GW radiation Dar et.al. 1808.02165



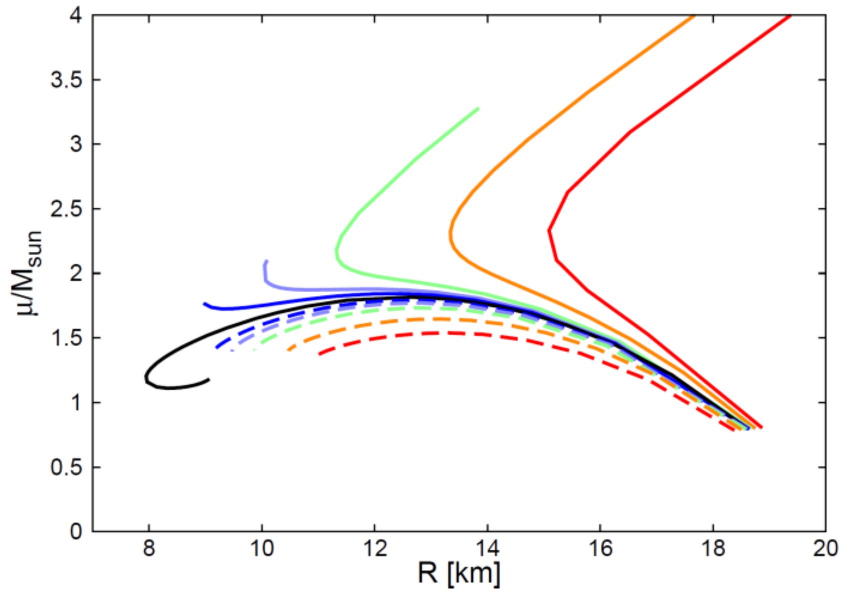

$$\left.\frac{P_2^{\rm cubic}}{P_2^{\rm GR}}\right|_{\rm numeric} \propto (\Omega_{\rm p})^{-2.49} (r_{\rm v})^{-1.44}$$

# Neutron stars

- Vainshtein breaking

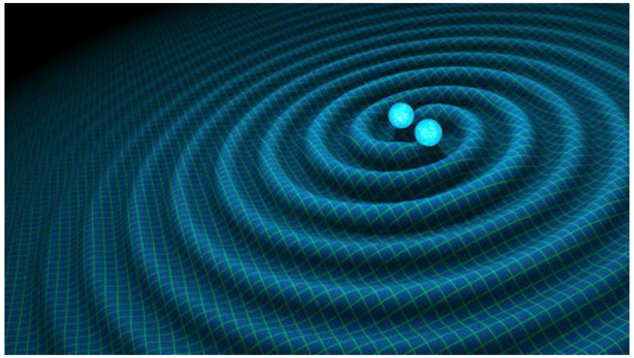


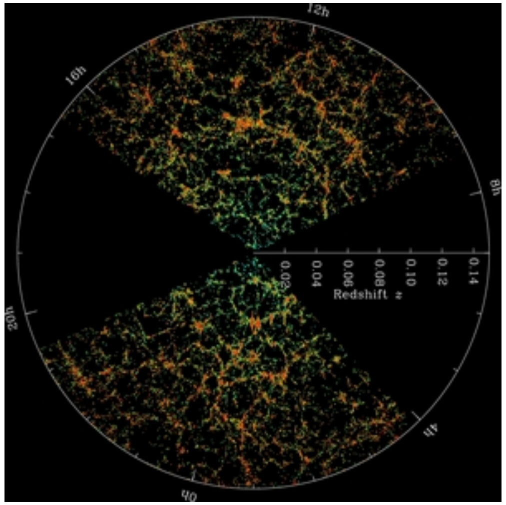

Babichev, KK, Langlois,Saito, Sakstein 1611.01062
Sakstein, Babichev, KK, Langlois,Saito 1612.04263




Kobayashi & Hiramatsu 1803.10510

# Summary






$$H_0 = 2.13 \times 10^{-42} h \mathrm{GeV} \sim (10^{23} \mathrm{km})^{-1}$$

$$f_{LIGO} \sim 10 - 100 \ \mathrm{Hz}$$

$$\ell_{LIGO} \sim 10^3 - 10^4 \ \mathrm{km}$$

$$p_{LIGO} \sim 10^{-22} - 10^{-23} \mathrm{GeV}$$