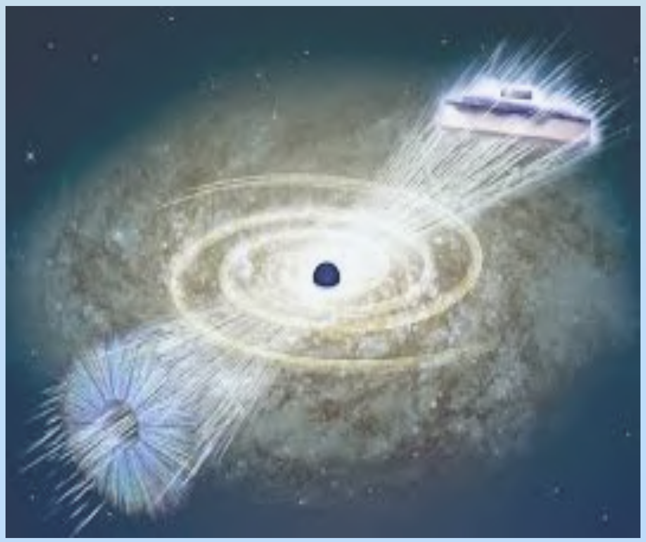
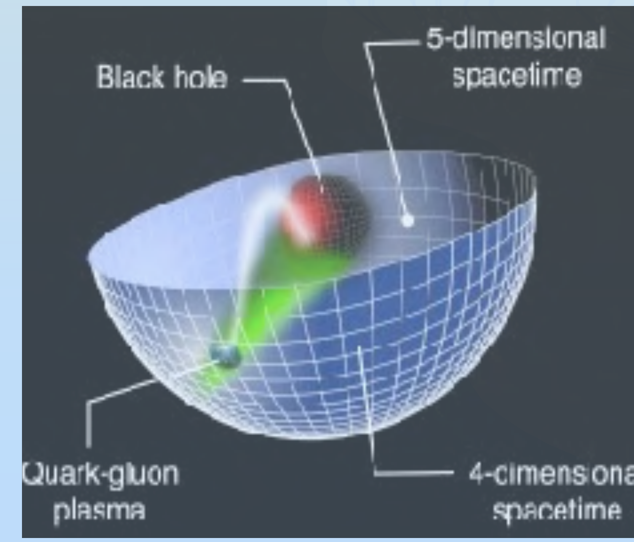
Emergent Dark Universe
and the Swampland Criteria



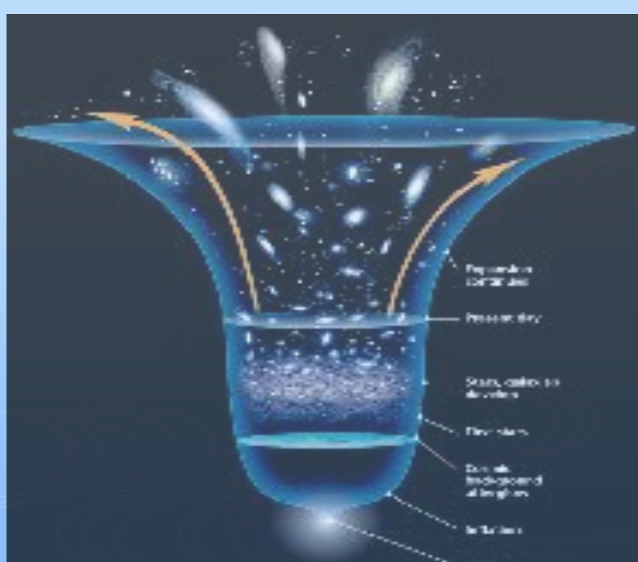
by **Yun-Long Zhang** **(張 雲龍)**

ユン ロン　チャン

Yukawa Institute for Theoretical Physics
@ YITP, Kyoto, Japan

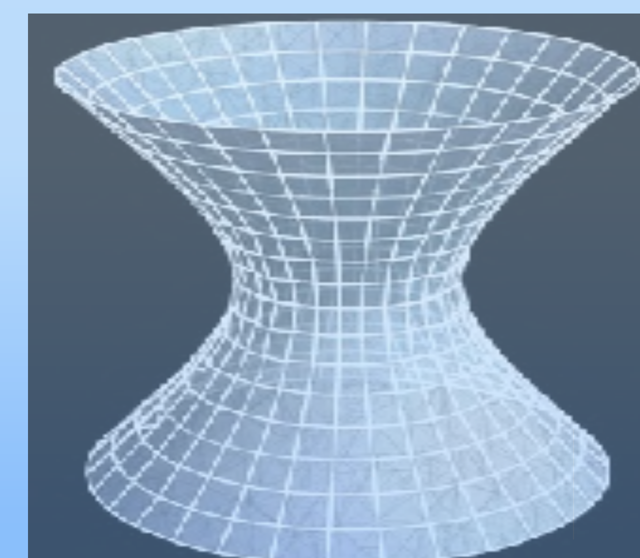


$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} - \frac{1}{L}\left(\mathcal{K}_{\mu\nu} - \mathcal{K}g_{\mu\nu}\right) = \kappa_4 T_{\mu\nu}$$



Ref: JHEP 1810 (2018) 009 [arXiv: 1712.09326]
by: Rong-Gen Cai (ITP/ Beijing)
Sichun Sun (NTU/ Taipei)
Yun-Long Zhang (APCTP/ Pohang)

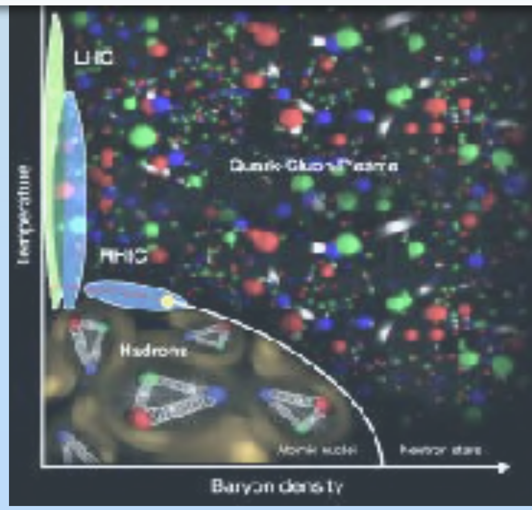
& Fitting with SNIa [arXiv: 1812.11105]
with Bum-Hoon Lee (Sogang U./ Seoul)
Sunly Khimphun (Phnom Penh)
Gansukh Tumurtushaa (IBS/Daejeon)



Motivation — Holographic Hydrodynamics

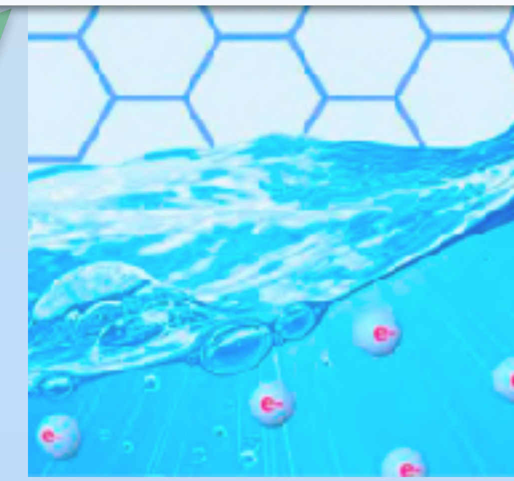
Quark Gluon Plasma

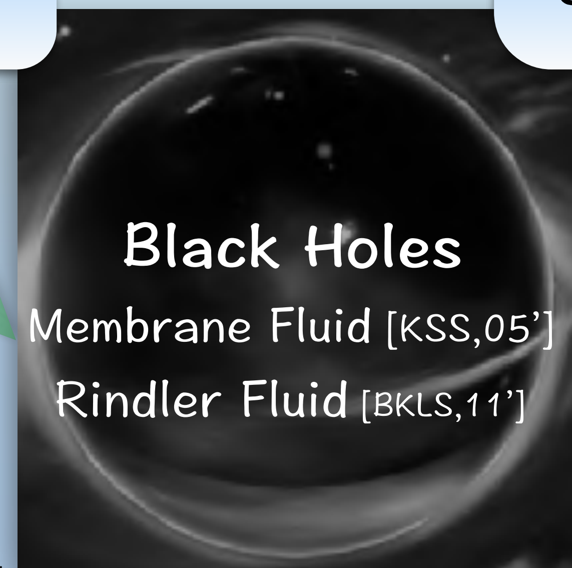
RHIC ['08] & LHC ['16]



Quantum Critical Liquid

Graphene ['09] & Semi-Metal['16]





Black Holes

Membrane Fluid [KSS,05']

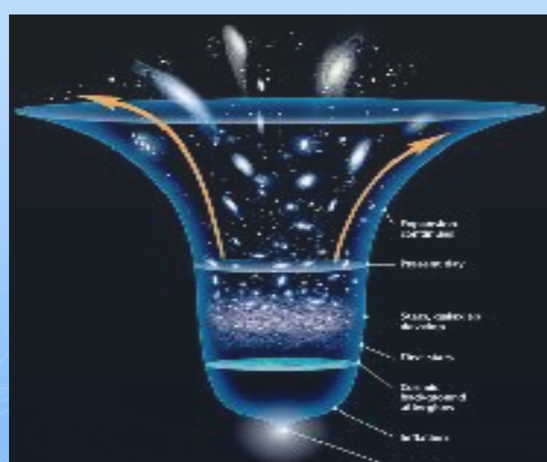
Rindler Fluid [BKLS,11']

$$\frac{\eta}{s} \simeq \frac{1}{4\pi}\frac{\hbar}{k_B}$$

$$\tau_c^{-1} \simeq \frac{k^2}{4\pi T_c}$$

$$\frac{H^2}{H_0^2} \simeq \frac{\Omega_B}{a^3} + \sqrt{\Omega_\Lambda\left(\frac{H^2}{H_0^2} + \frac{\Omega_I}{a^4}\right)}$$

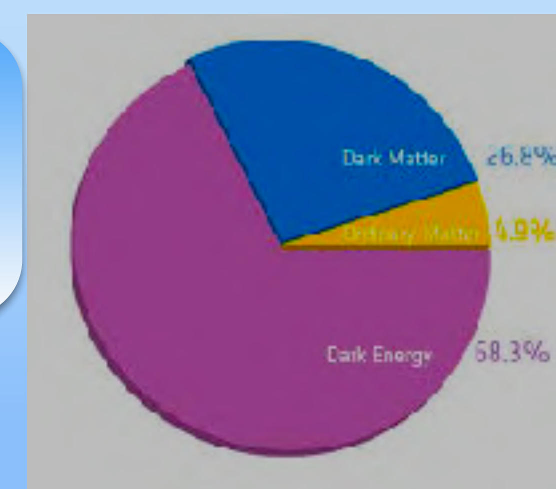
$$\Omega_D^2 \simeq \frac{1}{2}\Omega_\Lambda(\Omega_D - \Omega_B)$$



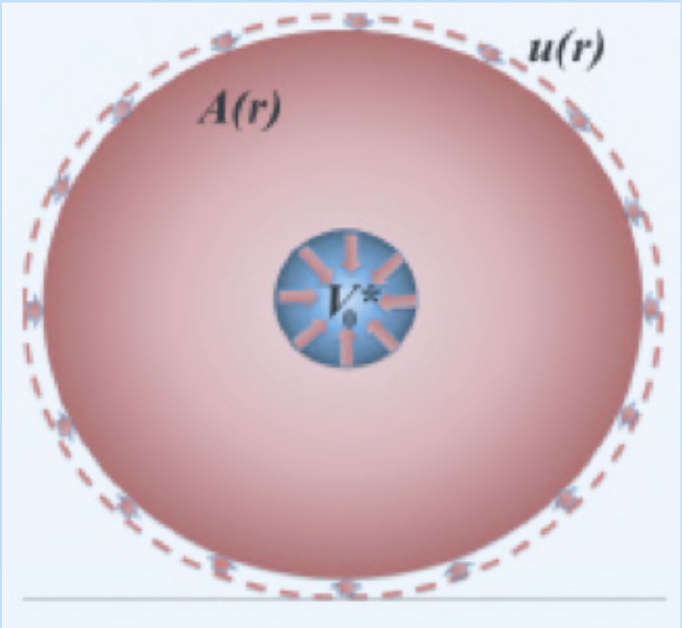
Cosmological Fluid ['00,'17]

Dark Energy & Matter & Radiation

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} - \frac{1}{L}\left(\mathcal{K}_{\mu\nu} - \mathcal{K}g_{\mu\nu}\right) = \kappa_4 T_{\mu\nu}$$



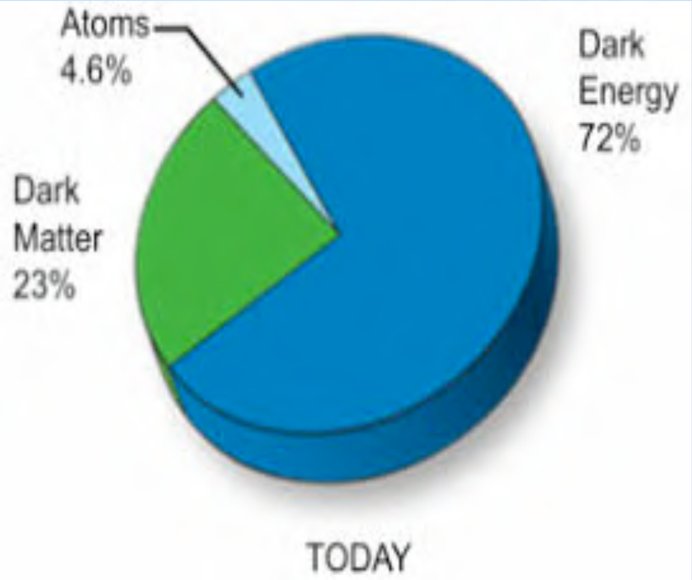
Motivation— Verlinde's Gravity & Dark Universe



Entropy Gravity = GR (2010)

Area Law & Einstein's Gravity

Volume Law & Modified Gravity?



$$\int_0^r \frac{G M_D^2(r')}{r'^2} dr' = \frac{M_B(r) a_0 r}{6}.$$

Emergent Gravity
Dark Matter? (2016)

WHY NOW? — Prof. Y. Nomura

$$\rho_{DM} \simeq \rho_{DE}$$

$$a_0 = cH_0$$

Any relation between Dark Matter & Dark Energy?

Tully–Fisher relation

Cluster of galaxies

Parameters in **LCDM**

$$v_f^4 \simeq a_0 G_N M_B$$

$$\overline{\rho}_D^2(r) = \left(4 - \overline{\beta}_B(r)\right) \frac{a_0}{8\pi G} \frac{\overline{\rho}_B(r)}{r}$$

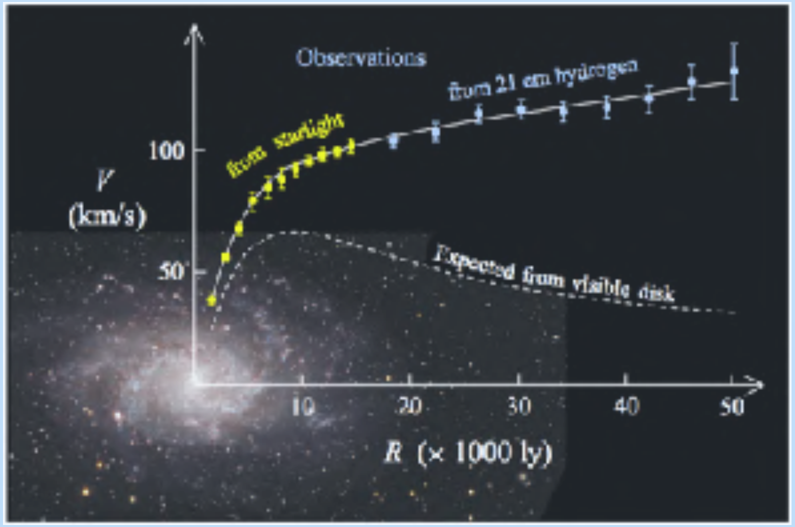
$$\Omega_D^2 = \frac{4}{3}\Omega_B$$

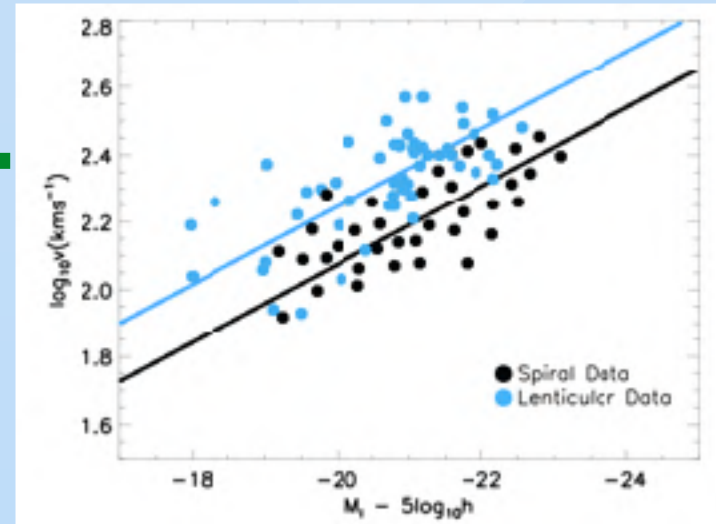
No Covariant Equations of Motion!

Refs: Erik Verlinde: JHEP 1104 **(2011)** 029 [1001.0785]
& SciPost Phys. 2 **(2017)** no.3, 016 [1611.02269]

From Observation to Milgrom's MOND

— Modified Newton Dynamics



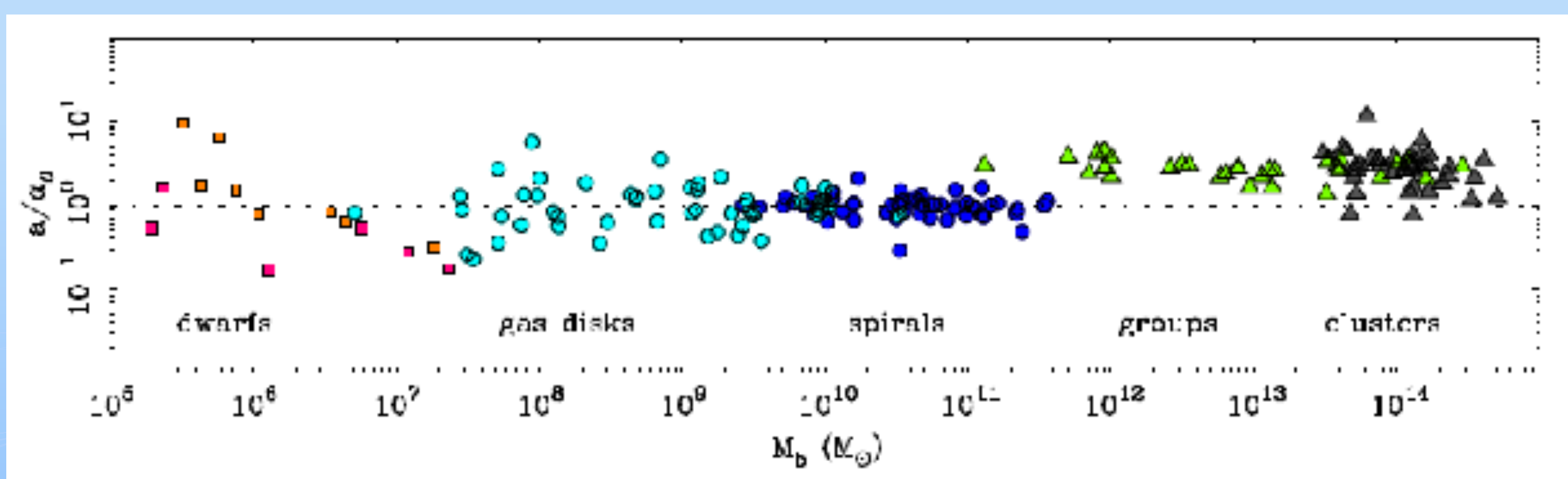


Dark Matter

Why $a_0 \simeq \sqrt{\Lambda}$

Galaxy Rotation Curve (1970s) → **Tully–Fisher Relation (1977)** → **Milgrom's MOND (1983)**

$$v_f^4 \simeq a_0 G_N M_B$$



The acceleration parameter of extragalactic systems

$$F_N = G_N \frac{M_B m}{r^2\, \mu(a/a_0)}$$

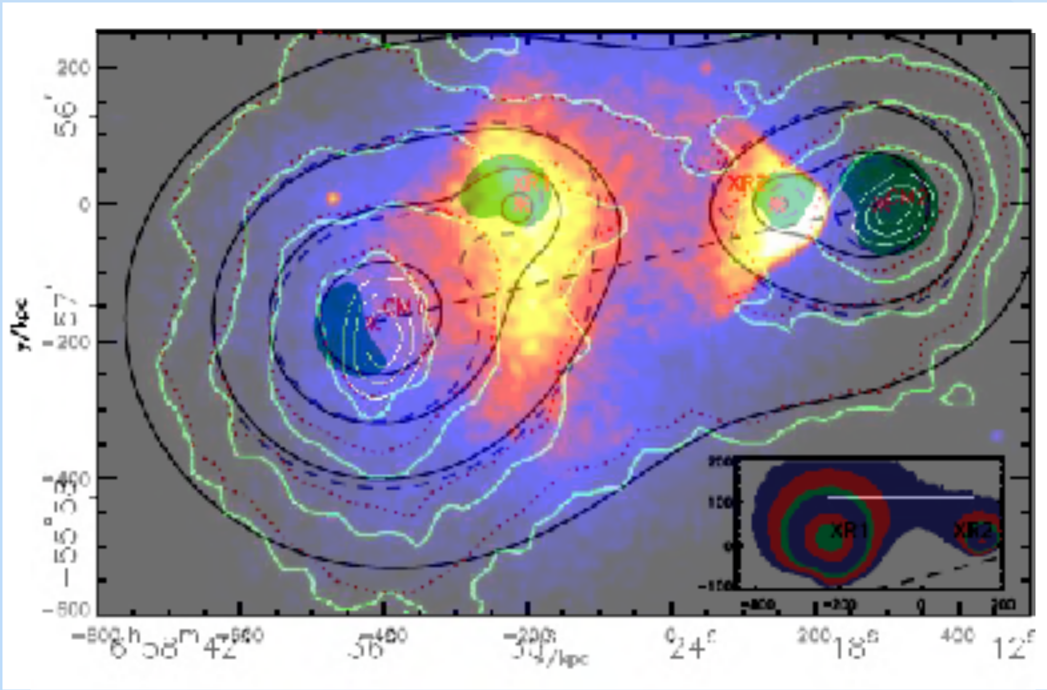
$$a \ll a_0, \quad \mu(a/a_0) \to 1/r$$

$$a \gg a_0, \quad \mu(a/a_0) \to 1$$

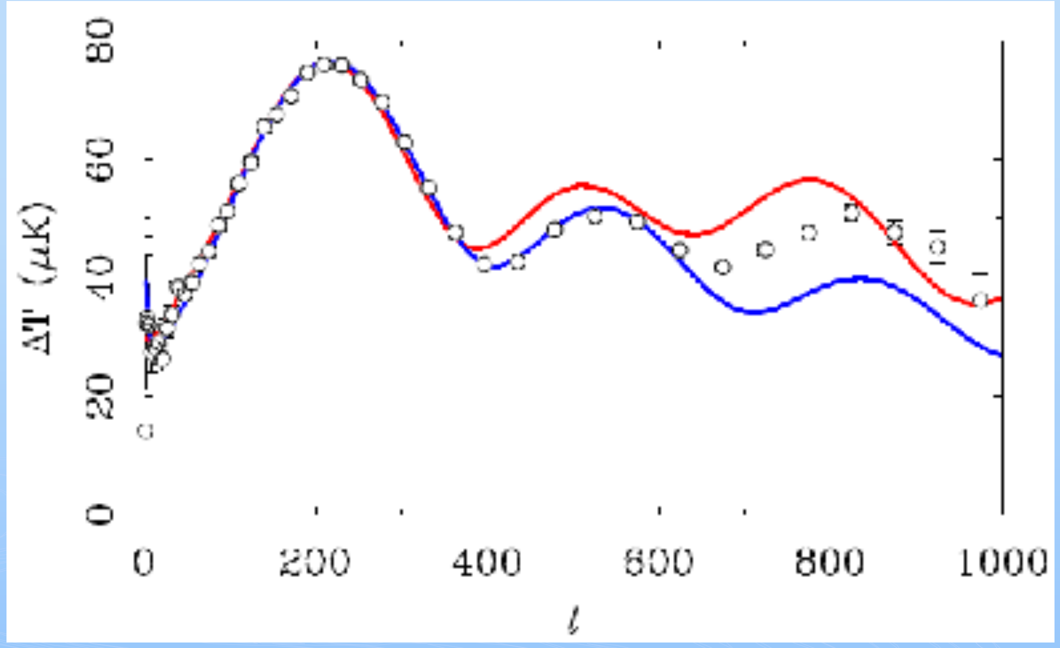
Refs: Famaey & McGaugh, Living Rev.Rel. 15 (2012) 10

30 years after MOND

Milgrom @ Weizmann $\nabla \cdot \left[\mu\left(\frac{|\nabla\Phi|}{a_0}\right)\nabla\Phi\right] = 4\pi G\rho$



Bullet Clusters



Acoustic Power Spectrum of CMB

Table 2: Observational tests of MOND.

Observational Test	Successful	Promising	Unclear	Problematic
Rotating Systems				
solar system			X	
galaxy rotation curve shapes	X			
surface brightness $\propto \Sigma \propto a^2$	X			
galaxy rotation curve fits	X			
fitted M_*/L	X			
Tully–Fisher Relation				
baryon based	X			
slope	X			
normalization	X			
no size nor Σ dependence	X			
no intrinsic scatter	X			
Galaxy Disk Stability				
maximum surface density	X			
spiral structure in LSBGs	X			
thin & bulgeless disks		X		
Interacting Galaxies				
tidal tail morphology		X		
dynamical friction			X	
tidal dwarfs	X			
Spheroidal Systems				
star clusters			X	
ultrafaint dwarfs			X	
dwarf Spheroidals	X			
ellipticals	X			
Faber Jackson relation	X			
Clusters of Galaxies				
dynamical mass				X
mass–temperature slope	X			
velocity (bulk & collisional)		X		
Gravitational Lensing				
strong lensing	X			
weak lensing (clusters & LSS)			X	
Cosmology				
expansion history			X	
geometry			X	
big bang nucleosynthesis	X			
Structure Formation				
galaxy power spectrum			X	
empty voids		X		
early structure		X		
Background Radiation				
first:second acoustic peak	X			
second:third acoustic peak				X
detailed fit				X
early re-ionization	X			

Refs: Famaey & McGaugh, Living Rev.Rel. 15 (2012) 10

Constrains on MOND from Gravitational waves

1) The Speed of gravitational waves ~ c

Constraint of energy loss rate from ultra-high energy cosmic rays

2) Linear equations of motion in the weak-field limit

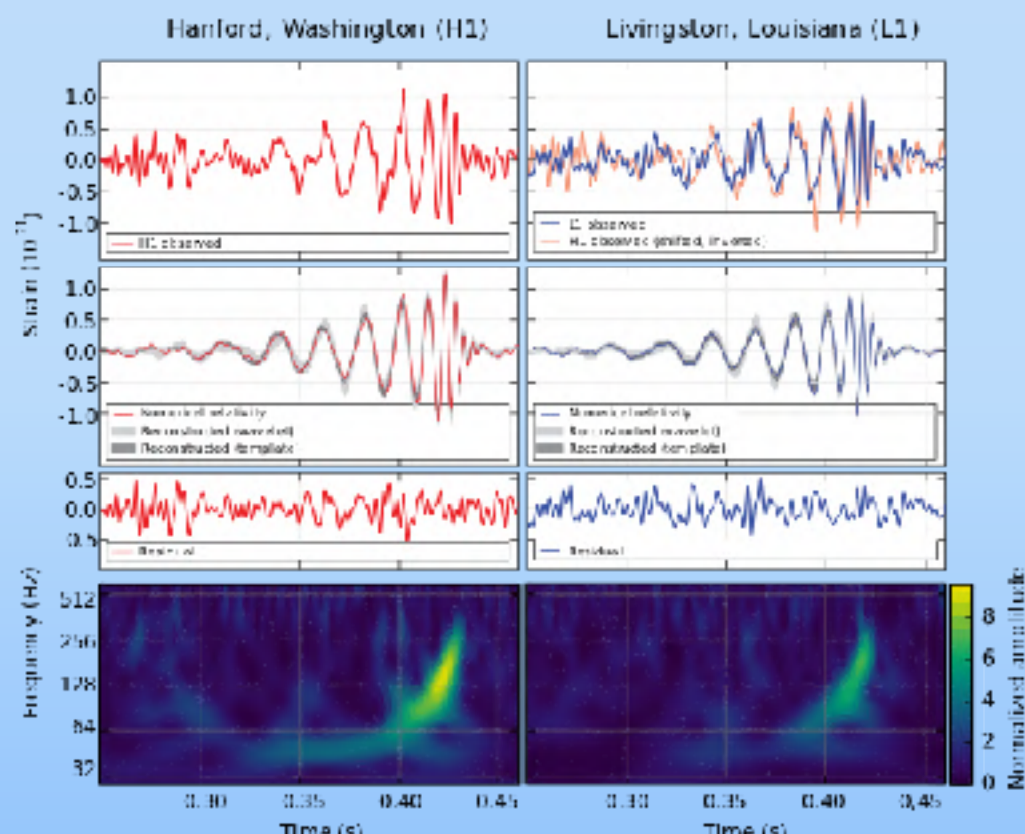
The observed gravitational waveforms from LIGO are consistent with Einstein's gravity

$$S = \frac{1}{16\pi G}\int d^4x\sqrt{g}\,\big[R + \mathcal{M}^2\mathcal{F}(\tfrac{\mathcal{K}}{\mathcal{M}^2}) + \lambda(A^2{+}1)\big] + S_{\rm mat}$$

Einstein-Aether theory (2004, Bekenstein)

$$\begin{aligned} R_{\mu\nu} - \tfrac{1}{2}Rg_{\mu\nu} \;&=\; \mathcal{T}_{\mu\nu} + 8\pi G T^{\rm mat}_{\mu\nu}, \\ \nabla_\alpha[\mathcal{F}' J^{\alpha}_{\ \beta}] - \mathcal{F}' y_\beta \;&=\; 2\lambda A_\beta, \end{aligned}$$

$$\begin{aligned} \mathcal{T}_{\alpha\beta} \;&=\; \tfrac{1}{2}\nabla_\sigma\{\mathcal{F}'[J_{(\alpha}{}^{\sigma}A_{\beta)} - J^{\sigma}{}_{(\alpha}A_{\beta)} - J_{(\alpha\beta)}A^\sigma]\} \\ &-\; \mathcal{F}' Y_{\alpha\beta} + \tfrac{1}{2}g_{\alpha\beta}\mathcal{M}^2\mathcal{F} + \lambda A_\alpha A_\beta, \end{aligned}$$



Refs: Chesler & Loeb, PRL, 119, 031102 (**2017**) [1704.05116]

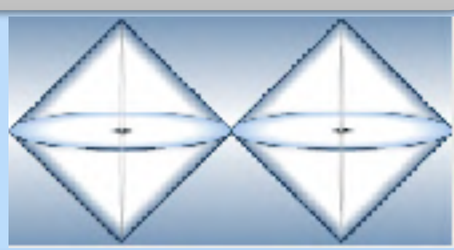
From Verlinde's Gravity to Dark Universe

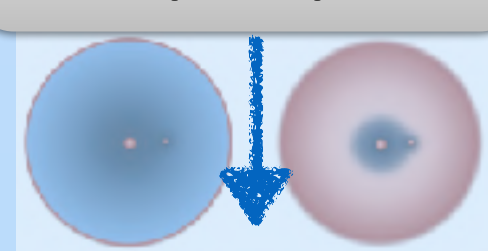
Gravitational quantity		Elastic quantity		Correspondence
Newtonian potential	Φ	displacement field	u_i	$u_i = \Phi n_i / a_0$
gravitational acceleration	g_i	strain tensor	ε_{ij}	$\varepsilon_{ij} n_j = -g_i / a_0$
surface mass density	Σ_i	stress tensor	σ_{ij}	$\sigma_{ij} n_j = \Sigma_i a_0$
mass density	ρ	body force	b_i	$b_i = -\rho\, a_0 n_i$
point mass	m	point force	f_i	$f_i = -m\, a_0 n_i$

Entropy Gravity = GR (2010)

Emergent Gravity (2016)

$$\int_0^r \frac{G M_D^2(r')}{r'^2} dr' = \frac{M_B(r) a_0 r}{6}.$$





Tully–Fisher relation

Cluster of galaxies

Parameters in **LCDM**

$$g_D(r) = \sqrt{a_M g_B(r)}$$

$$a_M = \frac{a_0}{6}$$

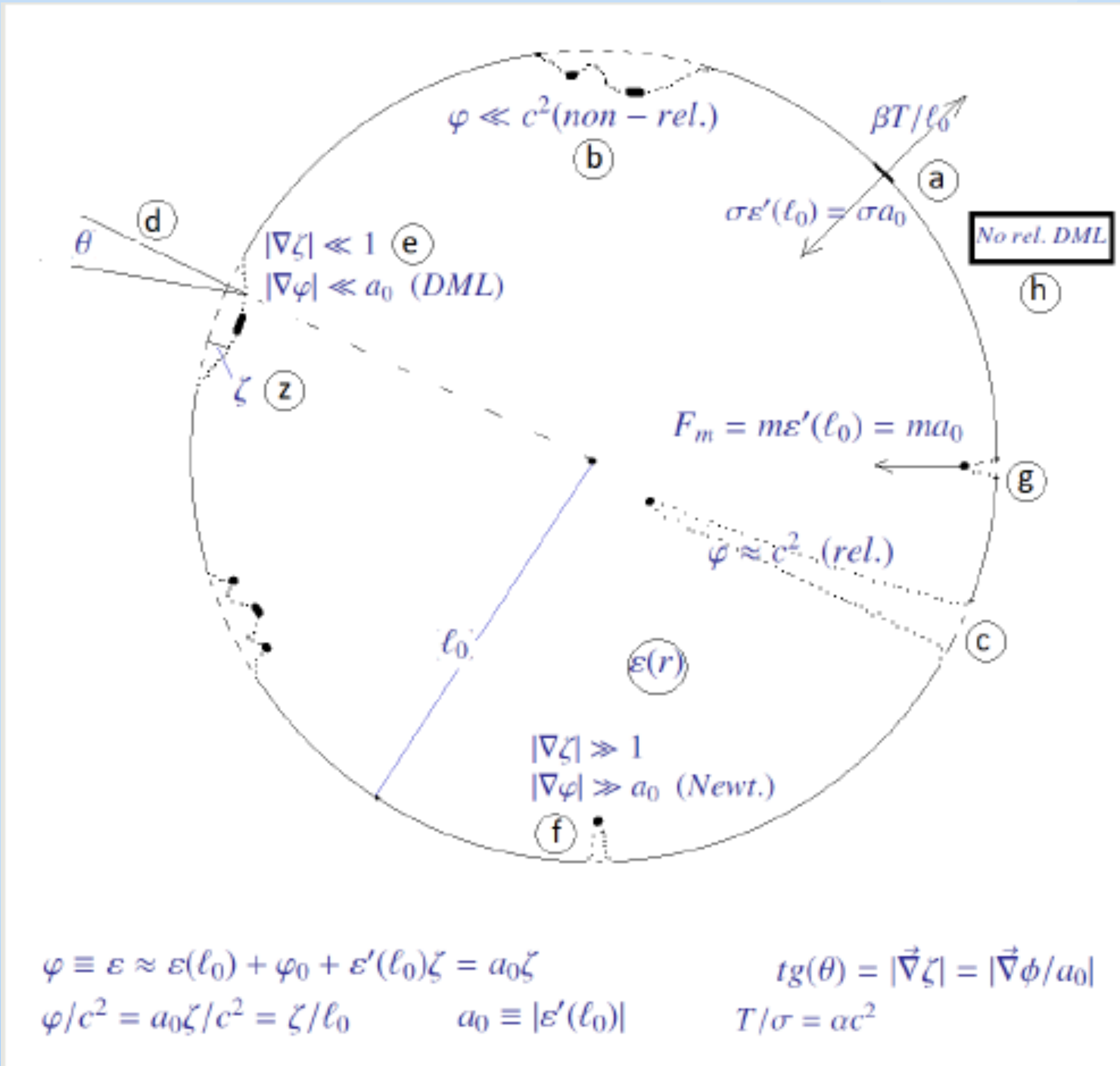
$$\bar{\rho}_D^2(r) = \left(4 - \bar{\beta}_B(r)\right) \frac{a_0}{8\pi G} \frac{\bar{\rho}_B(r)}{r}$$

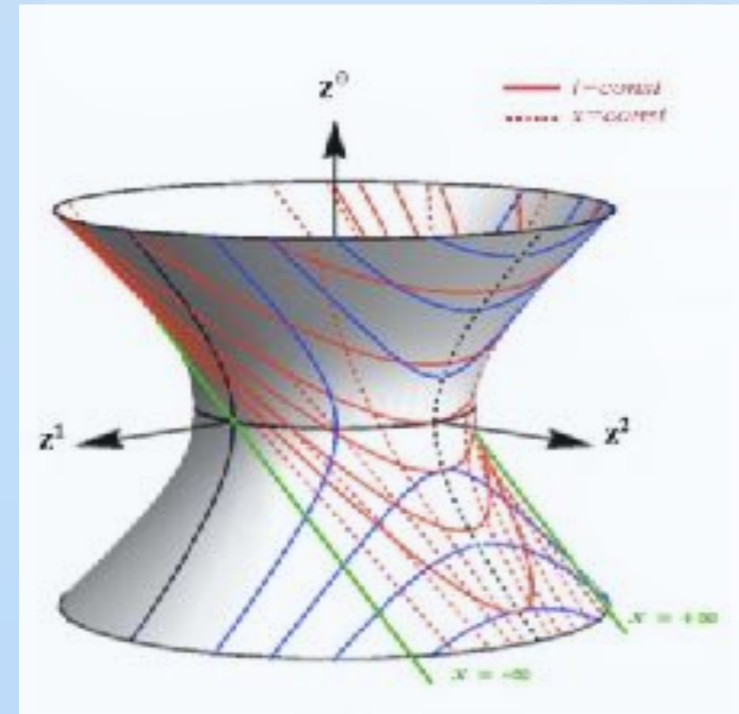
$$a_0 = cH_0$$

$$\Omega_D^2 = \frac{4}{3}\Omega_B$$

No Covariant Equations of Motion!

MOND from a Brane-World Picture?





$$F_{\text{Entropy}} = m\left[\sqrt{a^2 + a_0^2} - a_0\right]$$

$$= \begin{cases} ma_N, & a \gg a_c \\ m\sqrt{a_N a_c}, & a \ll a_c \end{cases}$$

Ho-Minic-Ng, Phys.Lett. B693 (2010) 567-570

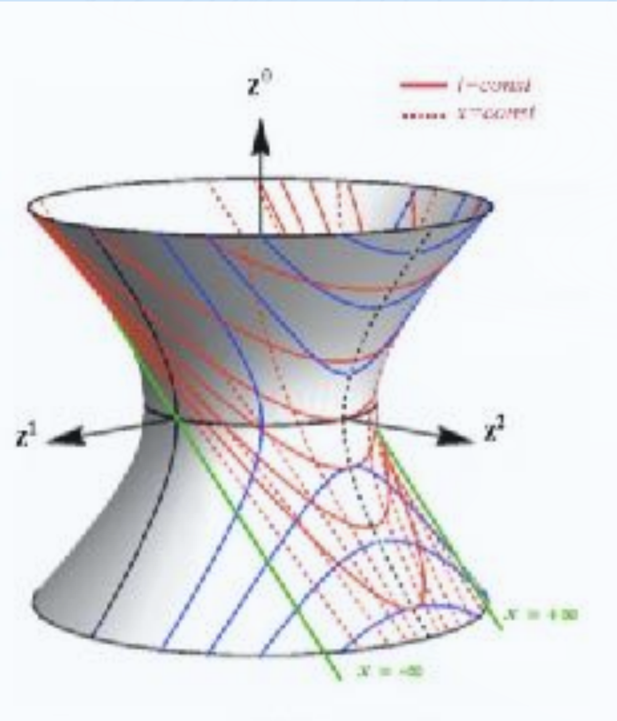
$$a_0 \simeq 2\pi a_c$$

$$F_{\text{Milgrom}} = ma = \begin{cases} ma_N, & a \gg a_c \\ m\sqrt{a_N a_c}, & a \ll a_c \end{cases}$$

Schematics of the brane dynamics —updated by Milgrom (v2: Mar 3, 2019)

Flat Bulk/de-Siter Universe

1) Holographic Stress Tensor — Dark Sectors



$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = \kappa_4 T_{\mu\nu} + \kappa_4 \langle \mathcal{T} \rangle_{\mu\nu}, \qquad \langle \mathcal{T} \rangle_{\mu\nu} \equiv \frac{1}{\kappa_4 L} \left(\mathcal{K}_{\mu\nu} - \mathcal{K} g_{\mu\nu} \right)$$

Modified Einstein equations

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} - \frac{1}{L} \left(\mathcal{K}_{\mu\nu} - \mathcal{K} g_{\mu\nu} \right) = \kappa_4 T_{\mu\nu}$$

$$\langle \mathcal{T} \rangle^{\Lambda}_{\mu\nu} = -\frac{\Lambda}{\kappa_4} g_{\mu\nu}, \qquad L = \frac{\kappa_5}{\kappa_4}$$

Hamiltonian Constraint

$$0 = 2 \mathcal{G}^{(5)}_{AB} \mathcal{N}^A \mathcal{N}^B \equiv \mathcal{K}^2 - \mathcal{K}_{\mu\nu} \mathcal{K}^{\mu\nu} - R.$$

$$\frac{\langle \mathcal{T} \rangle^2}{3} - \langle \mathcal{T} \rangle_{\mu\nu} \langle \mathcal{T} \rangle^{\mu\nu} = -\frac{\kappa_4}{(\kappa_5)^2} \frac{1}{2} \left(T + \langle \mathcal{T} \rangle \right).$$

2) Embedding in higher dimensions — Brane World Models

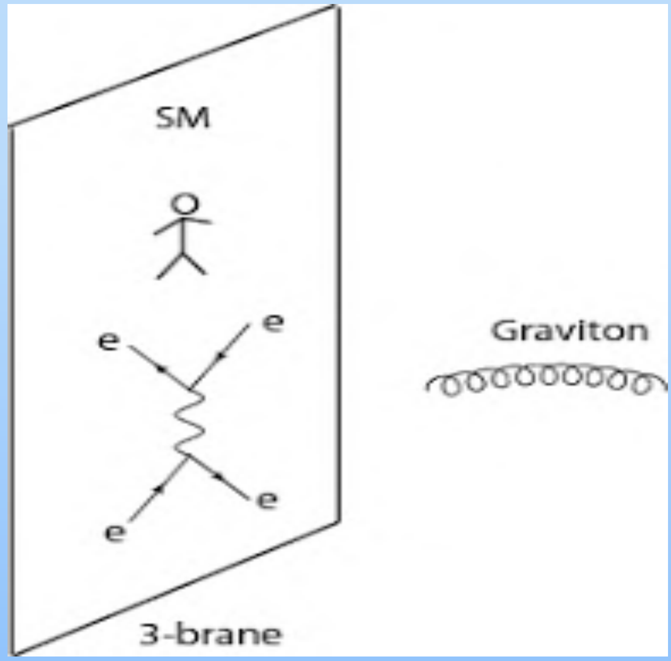
$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = \mathcal{T}^{\mathcal{M}}_{\mu\nu} + T^{B}_{\mu\nu},$$

$$\mathcal{T}^{\mathcal{M}}_{\mu\nu} \equiv (\mathcal{K} g_{\mu\sigma} - \mathcal{K}_{\mu\sigma}) \mathcal{K}^{\sigma}{}_{\nu} + \mathcal{M}_{\mu\nu} - \frac{1}{2} \left(\mathcal{K}^2 - \mathcal{K}_{\rho\sigma} \mathcal{K}^{\rho\sigma} \right) g_{\mu\nu},$$

$$\mathcal{M}_{\mu\nu} \equiv g_{\mu}{}^{M} g_{\nu}{}^{N} R^{(d+1)}_{MN} - g_{\mu}{}^{M} \mathcal{N}^P g_{\nu}{}^{N} \mathcal{N}^Q R^{(d+1)}_{MPNQ}.$$

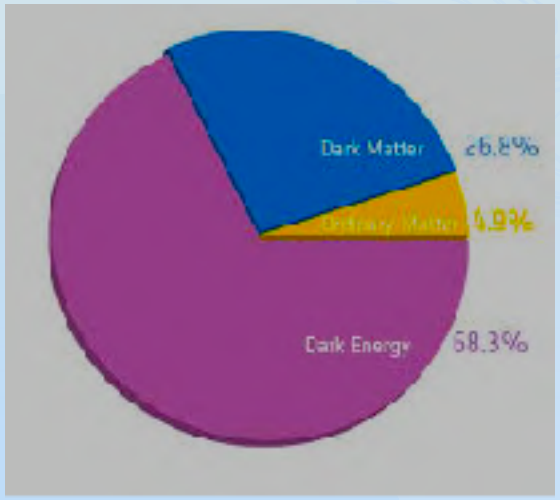
[Maeda, Mukohyama, Sasaki, Shiromizu, …, ['99, '10]

Ref: 1106.2476 [Living Rev. '10] Modified Gravity and Cosmology



Hamiltonian Constraint & Add-on matters

$$\frac{\langle \mathcal{T}\rangle^2}{3} - \langle \mathcal{T}\rangle_{\mu\nu}\langle \mathcal{T}\rangle^{\mu\nu} = -\frac{\tilde{\rho}_\Lambda c^2}{3}\left[T + \langle \mathcal{T}\rangle\right].$$



$$T^{\mu\nu} = T_B^{\mu\nu} + T_R^{\mu\nu}, \qquad T_B^{\mu\nu} = (\rho_B)u^\mu u^\nu, \quad T_R^{\mu\nu} = (\rho_R)u^\mu u^\nu + p_R h^{\mu\nu},$$

$$\langle \mathcal{T}\rangle^{\mu\nu} = \langle \mathcal{T}\rangle_\Lambda^{\mu\nu} + \langle \mathcal{T}\rangle_D^{\mu\nu}, \qquad \langle \mathcal{T}\rangle_\Lambda^{\mu\nu} = -(\rho_\Lambda c^2)g^{\mu\nu}, \quad \langle \mathcal{T}\rangle_D^{\mu\nu} = (\rho_D)u^\mu u^\nu + p_D h^{\mu\nu}.$$

$$\boxed{\rho_D^2 = \frac{\rho_\Lambda}{2(1+3\tilde{w}_D)}\left[\rho_D(1-3\tilde{w}_D) - \rho_B\right],} \qquad \tilde{w}_D \equiv \frac{p_D}{\rho_D c^2}\,.$$

$$\tilde{\Omega}_\Lambda \equiv \rho_\Lambda/\rho_c, \quad \tilde{\Omega}_D \equiv \rho_D/\rho_c, \quad \tilde{\Omega}_B \equiv \rho_B/\rho_c,$$

$$\frac{H(t)^2}{H_0^2} = \Omega_\Lambda + \frac{\Omega_D}{a(t)^3} + \frac{\Omega_B}{a(t)^3} \equiv \tilde{\Omega}_\Lambda + \tilde{\Omega}_D + \tilde{\Omega}_B.$$

$$\boxed{\tilde{\Omega}_D^2 = \frac{\tilde{\Omega}_\Lambda}{2(1+3\tilde{w}_D)}\left[\tilde{\Omega}_D(1-3\tilde{w}_D) - \tilde{\Omega}_B\right].}$$

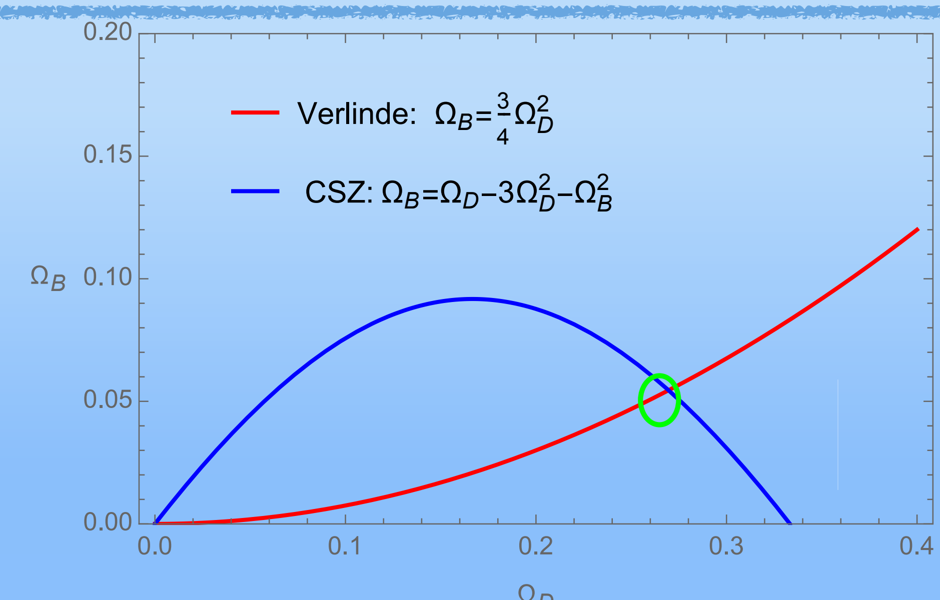
$$\frac{H(t)^2}{H_0^2} \simeq \frac{\Omega_B}{a(t)^3} + \Omega_\Lambda^{1/2}\left[\frac{H(t)^2}{H_0^2} + \frac{\Omega_I}{a(t)^4}\right]^{1/2}$$

Late Time Evolution

$$\Omega_\Lambda \simeq 0.685, \quad \Omega_D \simeq 0.265, \quad \Omega_B \simeq 0.050,$$

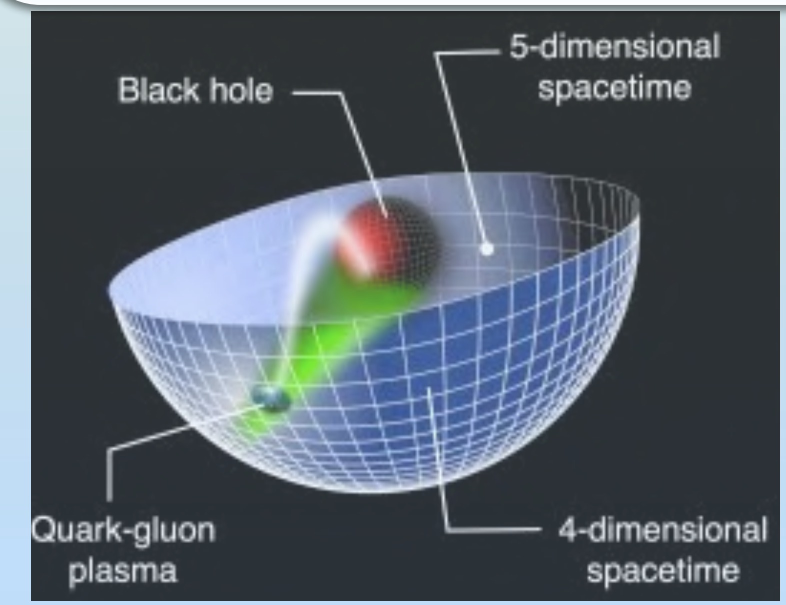
$$w_D \simeq 0$$

$$\boxed{\begin{aligned} \delta_{CSZ} &\equiv \Omega_D^2 - \frac{1}{2}\Omega_\Lambda(\Omega_D - \Omega_B) \simeq -0.003\,, \\ \delta_V &\equiv \Omega_D^2 - \frac{4}{3}\Omega_B \simeq 0.004\,. \end{aligned}}$$



Hamilton Constraint Equation & Emergent de-Sitter Universe

AdS/CMT Duality: Geometry & Quantum Matters

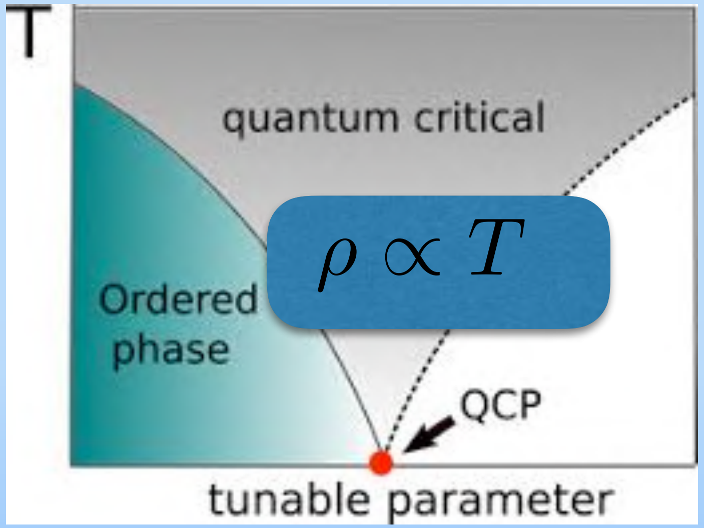


$$Z_{CFT} = \langle e^{S_{CFT}} \rangle \overset{AdS/CFT}{\simeq} e^{S_{AdS}}$$

Holographic Condensed Matters

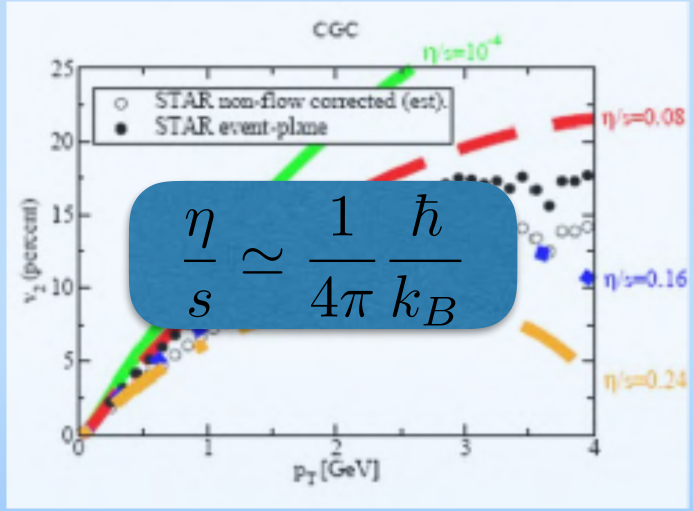
$$\langle \mathcal{T} \rangle^{d}_{\mu\nu} = -\frac{2}{\sqrt{-g}} \frac{\delta(\mathcal{S}_5)}{\delta g^{\mu\nu}} = \frac{1}{\kappa_5} \left(\mathcal{K}_{\mu\nu} - \mathcal{K} g_{\mu\nu} \right).$$

Holographic Strange Metal (2008-)



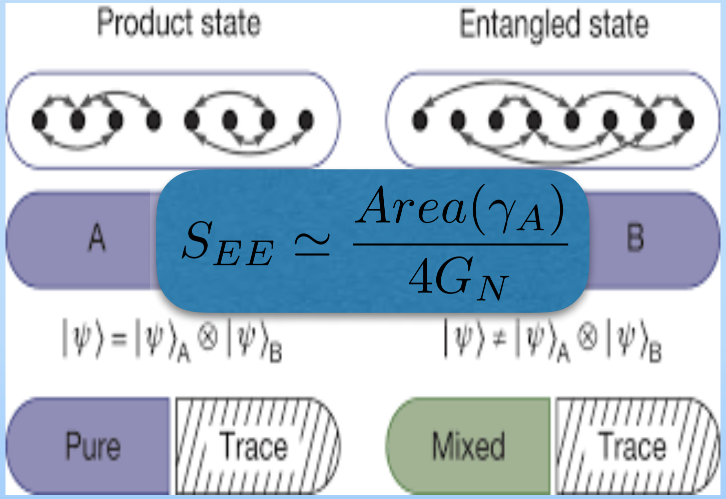
$$\rho \propto T$$

Holographic Hydrodynamics (2003-)



$$\frac{\eta}{s} \simeq \frac{1}{4\pi} \frac{\hbar}{k_B}$$

Holographic Entanglement (2006-)



$$S_{EE} \simeq \frac{Area(\gamma_A)}{4G_N}$$

ICTP Dirac Medal 2018

Subir Sachdev (Quantum & Phase)

Dam Thanh Son (Nuclear & Fluid)

Xiao-Gang Wen (Entanglement & Order)

Figures Credit: Google figures

Moving the Holographic Screen to the Finite Cutoff

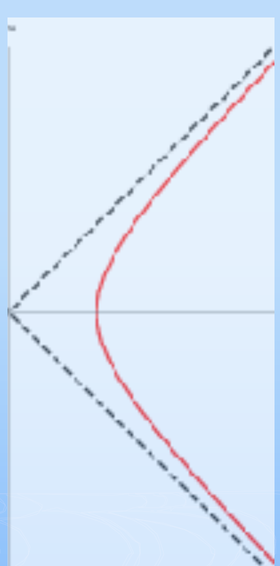
Extremal Charged BH

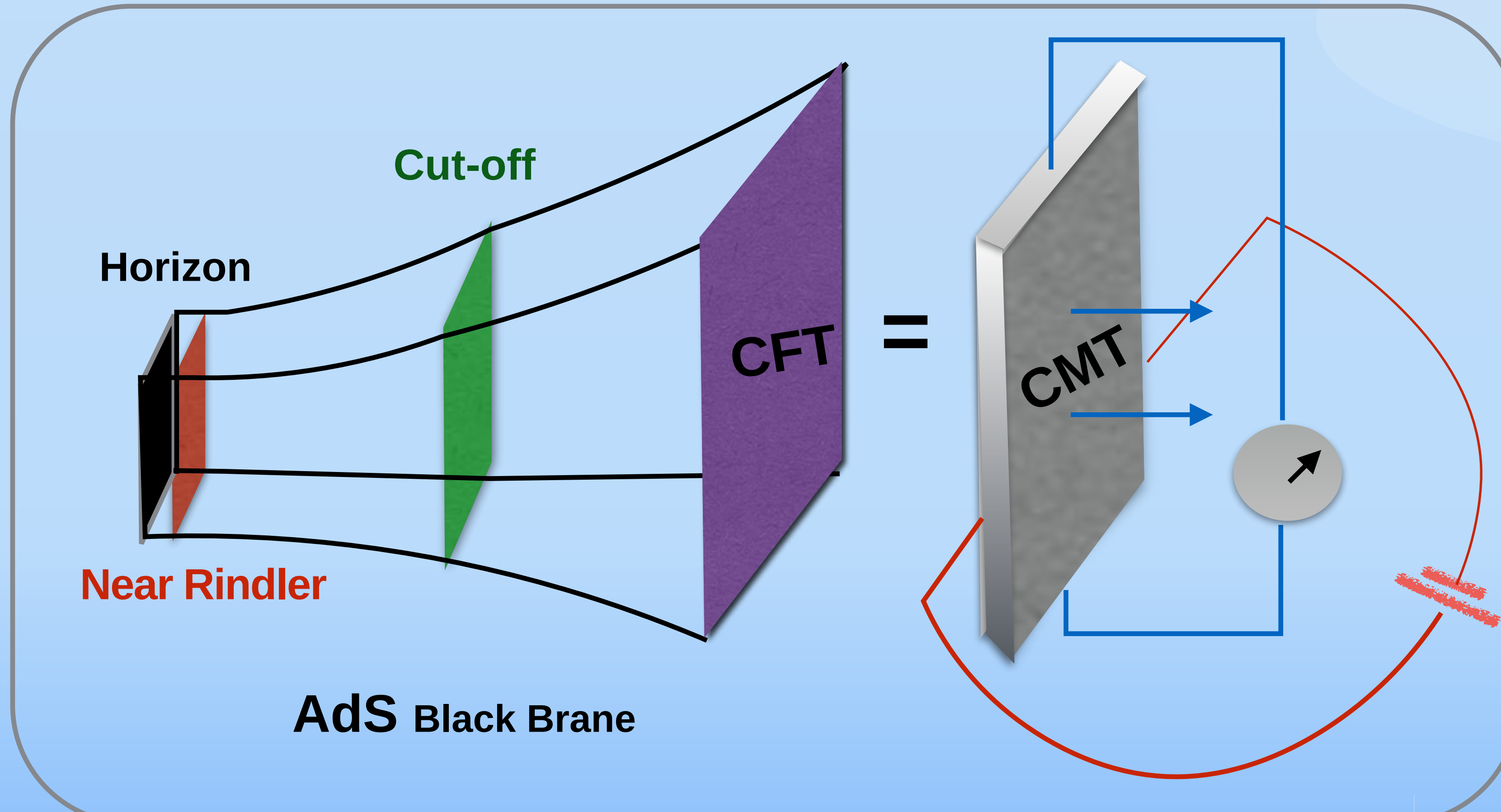
$AdS_2/CFT_1 \times R_p$ & Non-Fermi Liquid ← Near Horizon — **Cutoff AdS / Effective CMT?** — Near Boundary → **AdS /CFT&CMT**

Finite Temperature

Rindler Space/ Special CMT

Membrane Black Holes





Wilsonian Approach to Fluid/Gravity Duality [Bredberg, Keeler, Lysov, Strominger, JHEP 1103(2011)141]

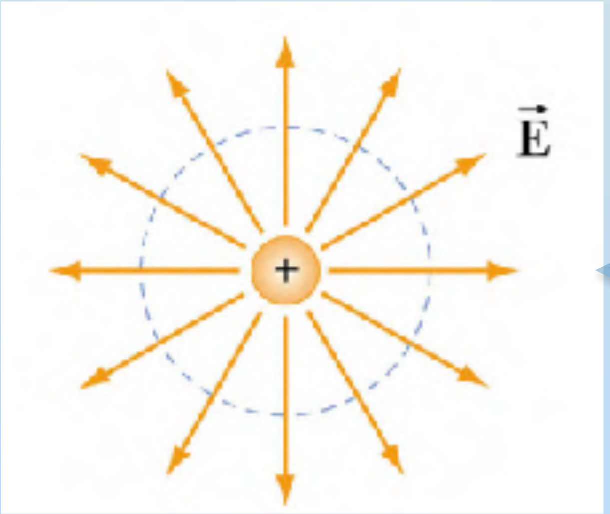
Non-Relativistic Fluid Dual to Asymptotically AdS Gravity at Finite Cutoff Surface [Cai, Li, Zhang, JHEP 1107(2011)027]

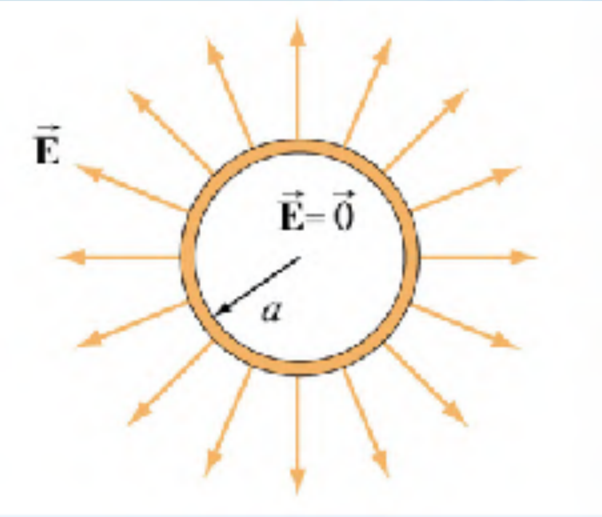
Toy Duality: Field & Surface Matter

I. Electromagnetism

Electric Field &

Surface Charge

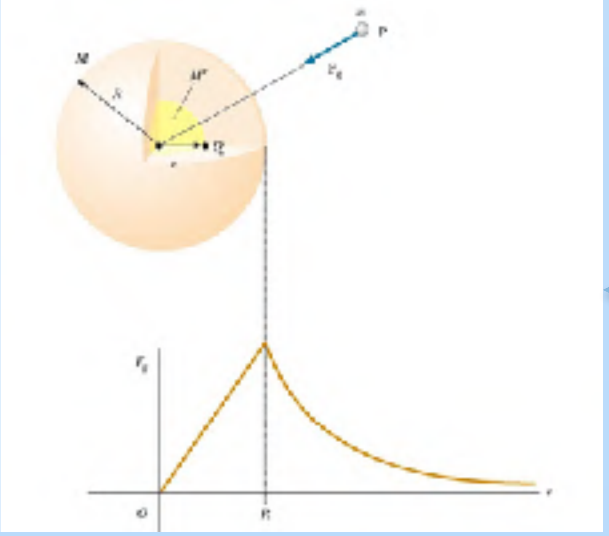


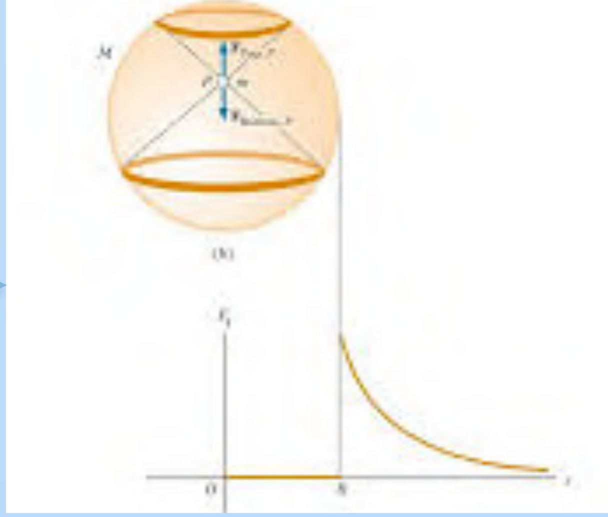


II. Newton Gravity

Massive Star &

Surface Shell

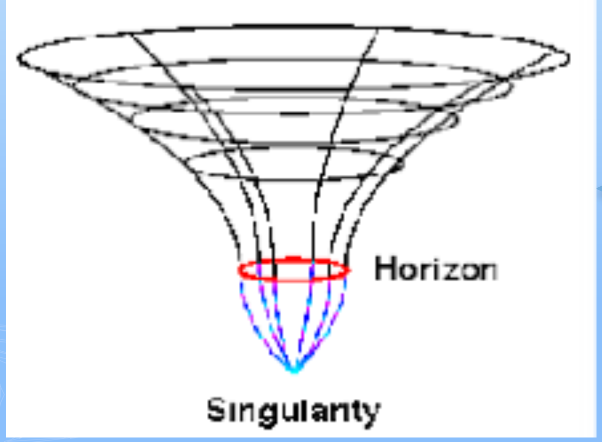


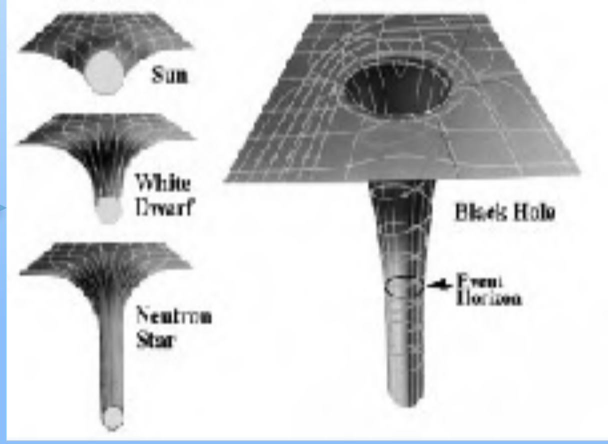


III. Einstein Gravity

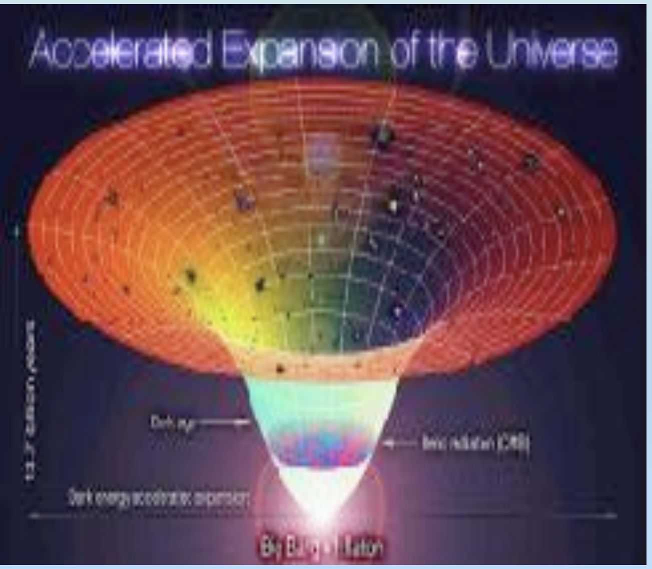
Black Hole &

Surface Fluid





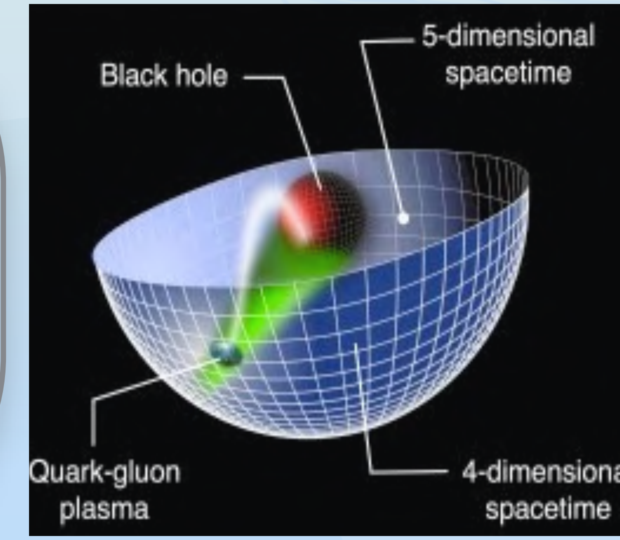
Flat Bulk/FRW Universe



The Total Action

$$\mathcal{S}_{tot} = \int_{\mathcal{H}} \mathrm{d}^4x\sqrt{-g}\left(\frac{1}{2\kappa_4}R + \mathcal{L}_m\right) + \mathcal{S}_5\,,$$

$$\mathcal{S}_5 \equiv \int_{\mathcal{M}} \mathrm{d}^5x\sqrt{-\tilde{g}}\left(\frac{1}{2\kappa_5}\mathcal{R}\right) + \int_{\mathcal{H}} \mathrm{d}^4x\sqrt{-g}\,\frac{1}{\kappa_5}\mathcal{K},$$



Einstein Field Equations

$$\frac{1}{\kappa_4}G_{\mu\nu} = T^m_{\mu\nu} + \langle\mathcal{T}\rangle^d_{\mu\nu},$$

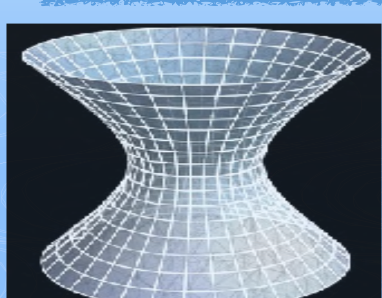
Holographic Dark Fluid?

Stress Energy Tensors

$$T^m_{\mu\nu} \equiv -\frac{2}{\sqrt{-g}}\frac{\delta(S_m)}{\delta g^{\mu\nu}}, \qquad \langle\mathcal{T}\rangle^d_{\mu\nu} = -\frac{2}{\sqrt{-g}}\frac{\delta(\mathcal{S}_5)}{\delta g^{\mu\nu}} = \frac{1}{\kappa_5}\left(\mathcal{K}_{\mu\nu} - \mathcal{K}g_{\mu\nu}\right)$$

In the Bulk => Modified GR

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} - \frac{\kappa_4}{\kappa_5}\left(\mathcal{K}_{\mu\nu} - \mathcal{K}g_{\mu\nu}\right) = \kappa_4 T^m_{\mu\nu}$$



Hamilton Constraint Equation
&Emergent de-Sitter Universe

$$\tilde{\Omega}^2_D = \frac{\tilde{\Omega}_\Lambda}{2(1+3\tilde{w}_D)}\left[\tilde{\Omega}_D(1-3\tilde{w}_D) - \tilde{\Omega}_B\right].$$

Flat Bulk/Dark Fluid?

Induced Metric $\quad \mathrm{d}s_4^2 = g_{\mu\nu}\mathrm{d}x^\mu \mathrm{d}x^\nu = -c^2\mathrm{d}t^2 + a(t)^2\left[\mathrm{d}r^2 + r^2\mathrm{d}\Omega_2\right].$

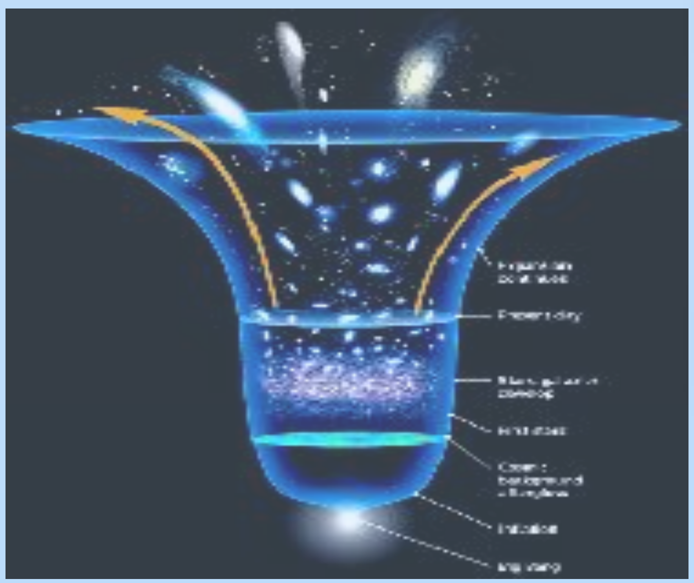
Bulk Metric

$$\mathrm{d}s_5^2 = \tilde{g}_{AB}\mathrm{d}x^A\mathrm{d}x^B = \mathrm{d}y^2 + \tilde{g}_{\mu\nu}\mathrm{d}x^\mu\mathrm{d}x^\nu.$$

$$= \mathrm{d}y^2 - \mathbf{n}(y,t)^2\ c^2\mathrm{d}t^2 + \mathbf{a}(y,t)^2\left[\mathrm{d}r^2 + r^2\mathrm{d}\Omega_2\right]$$

$$\mathbf{a}(y,t)^2 = a(t)^2 + y^2\frac{\dot{a}(t)^2}{c^2} \pm 2y\sqrt{a(t)^2\frac{\dot{a}(t)^2}{c^2} + I},$$

$$\mathbf{n}(y,t) = \frac{\partial_t\mathbf{a}(y,t)}{\dot{a}(t)}.$$



Holographic Stress energy Tensor

$$\langle\mathcal{T}\rangle^d_{\mu\nu} = -\frac{2}{\sqrt{-g}}\frac{\delta(\mathcal{S}_5)}{\delta g^{\mu\nu}} = \frac{1}{\kappa_5}\left(\mathcal{K}_{\mu\nu} - \mathcal{K}g_{\mu\nu}\right).$$

$$(\kappa_5 c^2)\rho_d = 3\sqrt{\frac{H(t)^2}{c^2} + \frac{I}{a(t)^4}},$$

$$(\kappa_5)p_d = -\frac{\frac{\dot{H}(t)+3H(t)^2}{c^2} + \frac{I}{a(t)^4}}{\sqrt{\frac{H(t)^2}{c^2} + \frac{I}{a(t)^4}}}.$$

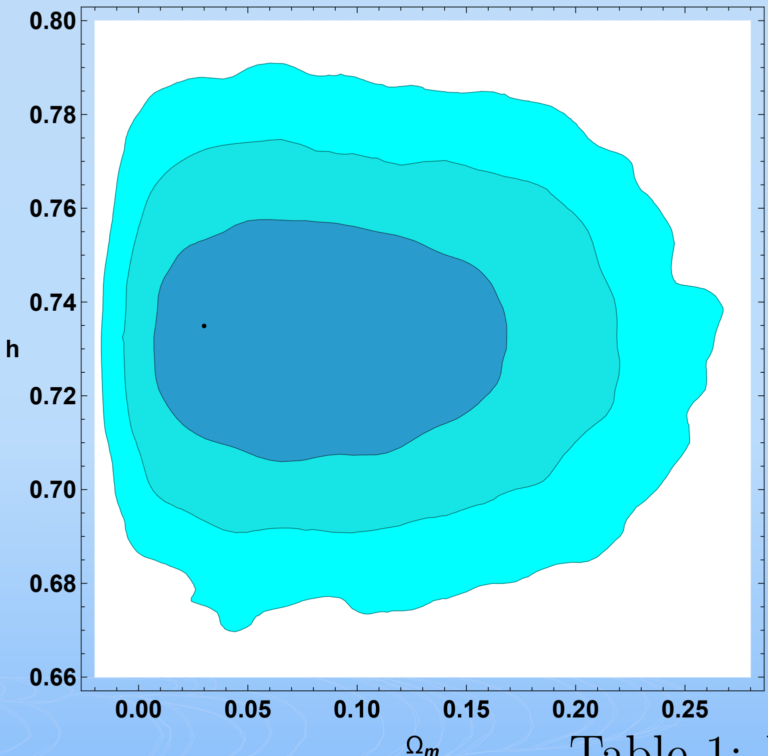
holographic Emergent Dark Universe (hEDU) & SNIa Data

$$\text{LCDM}: \frac{H(z)^2}{H_0^2} = \Omega_\Lambda + \Omega_m(1+z)^3$$

Friedmann Equations

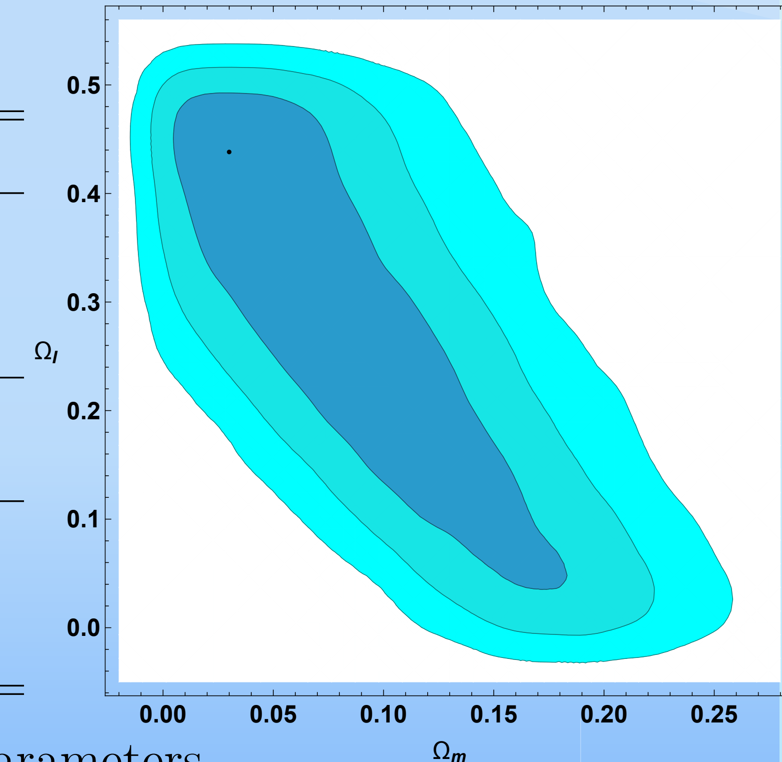
$$\text{sDGP}: \frac{H(z)^2}{H_0^2} = \frac{\Omega_\Lambda}{2} + \Omega_m(1+z)^3 + \frac{\Omega_\Lambda}{2}\sqrt{1+\frac{4\Omega_m}{\Omega_\Lambda}(1+z)^3}$$

$$\text{hEDU}: \frac{H(z)^2}{H_0^2} = \frac{\Omega_\Lambda}{2} + \Omega_m(1+z)^3 + \frac{\Omega_\Lambda}{2}\sqrt{1+\frac{4\Omega_m}{\Omega_\Lambda}(1+z)^3+\frac{4\Omega_I}{\Omega_\Lambda}(1+z)^4}$$

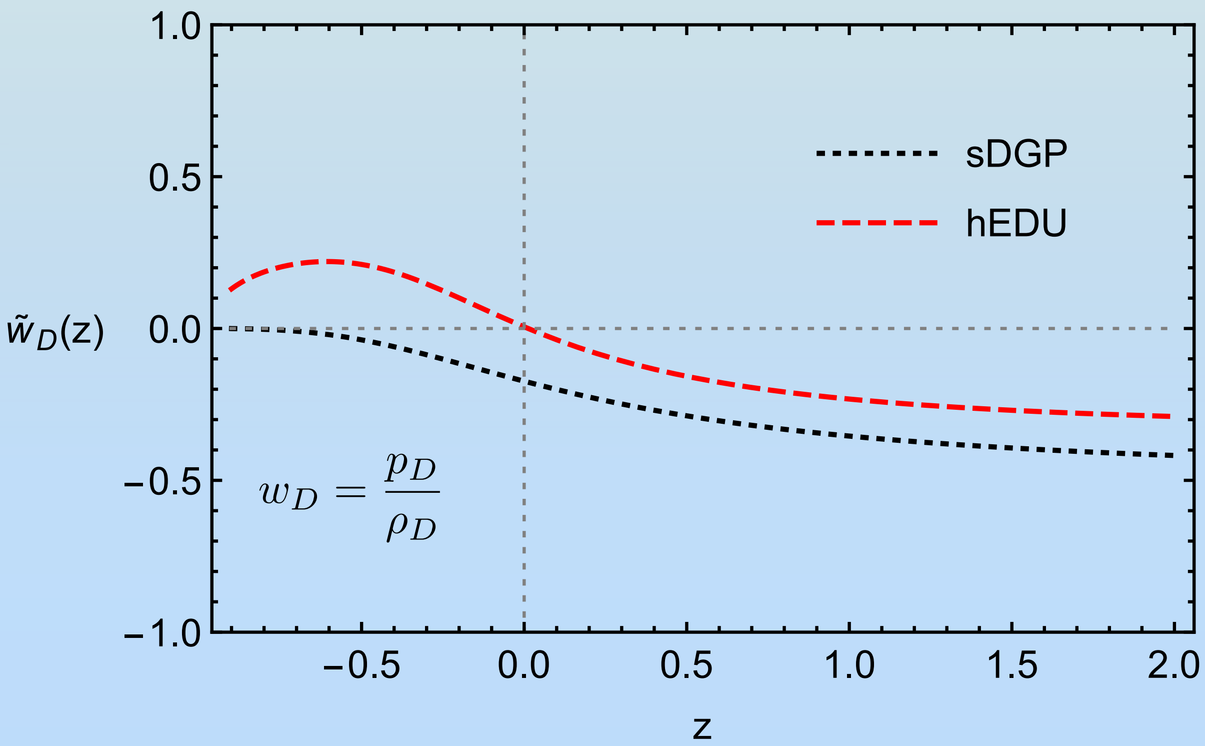


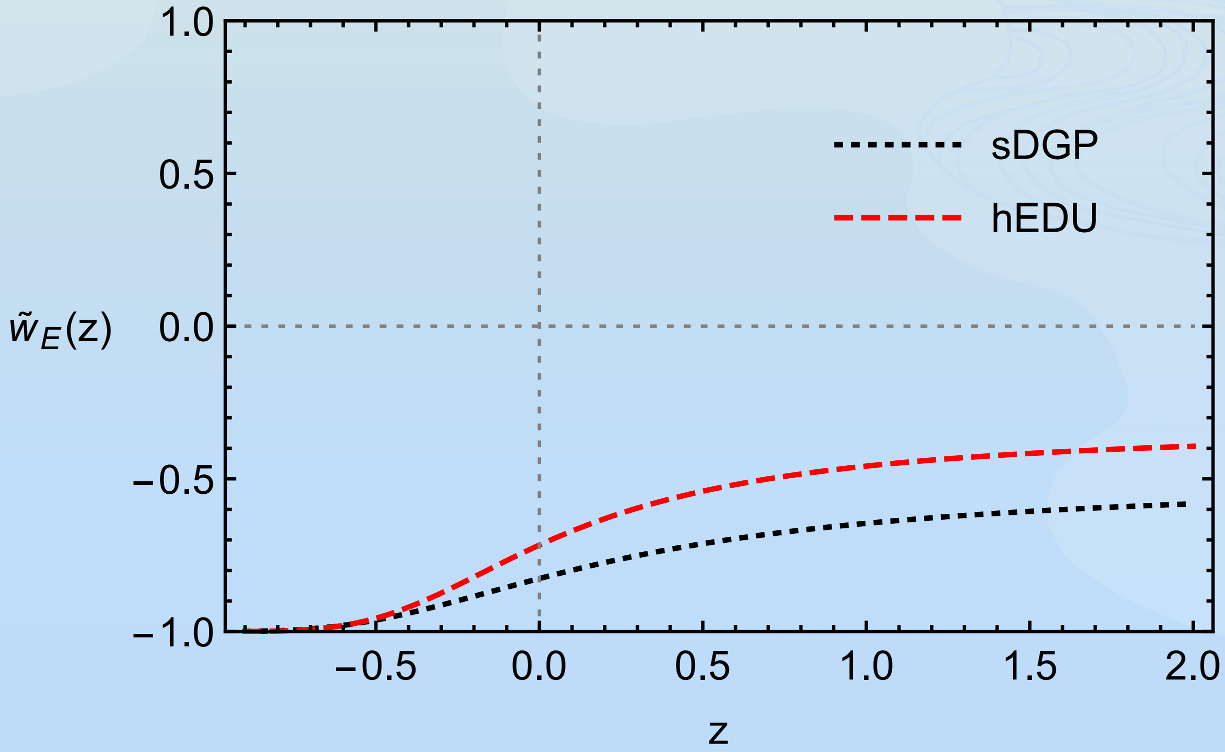
Parameters	LCDM	hEDU
h	0.7330 ± 0.0180	0.7349 ± 0.0179
Ω_m	0.2969 ± 0.0352	0.0299 ± 0.0515
Ω_I	—	0.4382 ± 0.1317
α	0.1403 ± 0.0068	0.1409 ± 0.0068
β	3.1081 ± 0.0892	3.1144 ± 0.0896
$\chi^2_{\rm min}$	695.063	694.321
ΔAIC	0	1.258
ΔBIC	0	5.866

Table 1: Fitting values and uncertainties of the cosmological parameters.



Late-Time Evolution of hEDU Model





Friedmann Equation

$$\frac{H(z)^2}{H_0^2} = \frac{\Omega_\Lambda}{2} + \Omega_m(1+z)^3 + \frac{\Omega_\Lambda}{2}\sqrt{1 + \frac{4}{\Omega_\Lambda}\left[\Omega_m(1+z)^3 + \Omega_I(1+z)^4\right]}$$

Hamiltonian Constraint of FRW Hypersurface

$$\boxed{\tilde{\Omega}_D^2 = \frac{\tilde{\Omega}_\Lambda}{2(1+3\tilde{w}_D)}\left[\tilde{\Omega}_D(1-3\tilde{w}_D) - \tilde{\Omega}_B\right].}$$

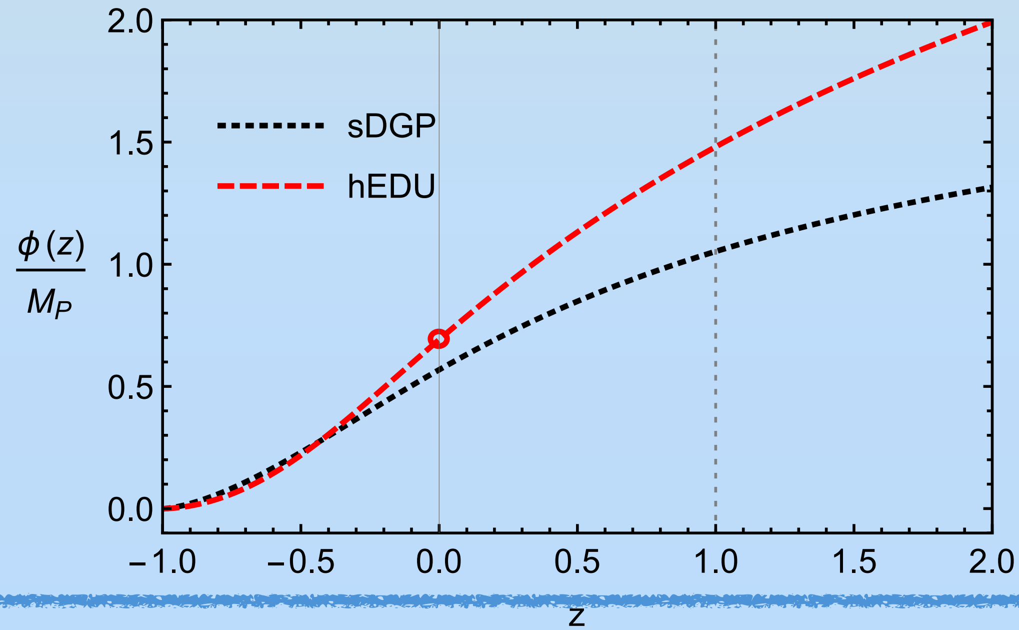
$$\tilde{\Omega}_\Lambda = \Omega_\Lambda, \qquad \tilde{\Omega}_D = \Omega_\mathcal{H}(t) - \Omega_\Lambda, \qquad \tilde{\Omega}_B \equiv \frac{\Omega_M}{a(t)^3} = \frac{H(t)^2}{H_0^2} - \Omega_\mathcal{H}(t),$$

$$\tilde{w}_D = -1 - \frac{1}{3H(t)}\frac{\dot{\Omega}_\mathcal{H}(t)}{\Omega_\mathcal{H}(t) - \Omega_\Lambda}, \qquad \Omega_\mathcal{H}(t) \equiv \frac{\rho_\mathcal{H}}{\rho_c} = \Omega_\Lambda^{1/2}\left[\frac{H(t)^2}{H_0^2} + \frac{\Omega_I}{a(t)^4}\right]^{1/2}.$$

Reconstruct the Effective Potential

Total Action
$$\mathcal{S}_{tot}=\int_{\mathcal{H}}\mathrm{d}^4x\sqrt{-g}\left(\frac{1}{2\kappa_4}R+\mathcal{L}_m+\frac{1}{2\kappa_5}\mathcal{K}\right)+\int_{\mathcal{M}}\mathrm{d}^5x\sqrt{-\tilde{g}}\left(\frac{1}{2\kappa_5}\mathcal{R}\right)$$

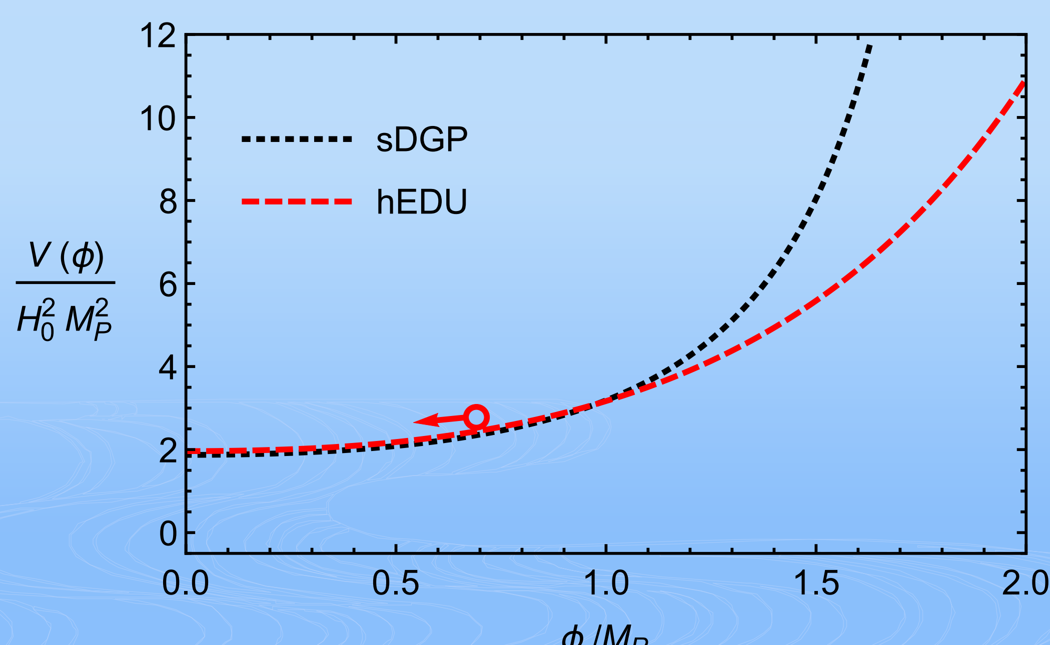
Effective Action
$$\mathcal{S}_{tot}=\int\mathrm{d}^4x\sqrt{-g}\Big[\frac{1}{2\kappa_4}R+\mathcal{L}_m-\frac{1}{2}(\partial\phi)^2-V(\phi)\Big].$$



$$\boxed{\begin{aligned}V[\phi(t)]&=\frac{1}{2}\left[\rho_d(t)-p_d(t)\right],\\ \dot{\phi}(t)&=-\sqrt{\rho_d(t)+p_d(t)}.\end{aligned}}$$

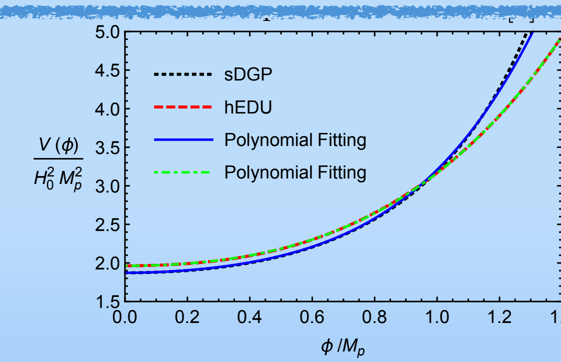
$$\rho_d(t)=\rho_c\sqrt{\Omega_\Lambda}\sqrt{\frac{H(t)^2}{H_0^2}+\frac{\Omega_I}{a(t)^4}},$$
$$p_d(t)=-\frac{\dot{\rho}_d}{3H(t)}-\rho_d\,,$$

Friedmann Equation $\dfrac{H(t)^2}{H_0^2}\simeq\dfrac{\Omega_B}{a(t)^3}+\Omega_\Lambda^{1/2}\left[\dfrac{H(t)^2}{H_0^2}+\dfrac{\Omega_I}{a(t)^4}\right]^{1/2}$



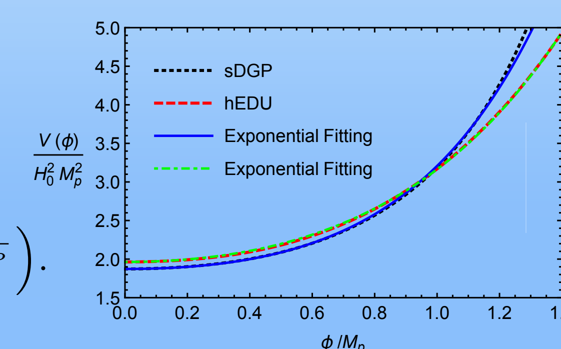
Polynomial fittings

$$\frac{V(\phi)}{H_0^2M_P^2}=\frac{\Lambda_0}{H_0^2}+\frac{h_2}{2}\frac{\phi^2}{M_P^2}+\frac{h_3}{3!}\frac{\phi^3}{M_P^3}+$$



Exponential fittings

$$\frac{V(\phi)}{M_P^2}=\frac{\Lambda_0}{\lambda_++\lambda_-}\left(\lambda_+e^{-\lambda_-\frac{\phi}{M_P}}+\lambda_-e^{\lambda_+\frac{\phi}{M_P}}\right).$$

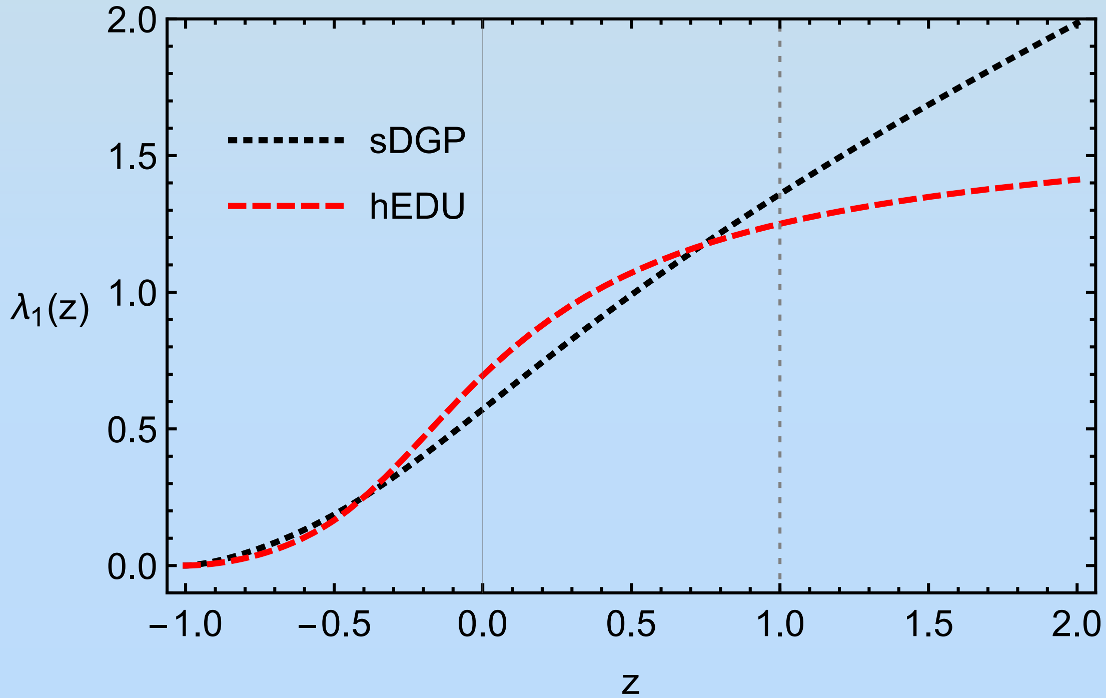


Checking the parameters in Swampland Criteria

Swampland Criteria

$$\text{Criterion 1}: \quad \frac{|\Delta\phi|}{M_P} \leq d_0, \qquad d_0, c_1, c_2 \sim \mathcal{O}(1)$$

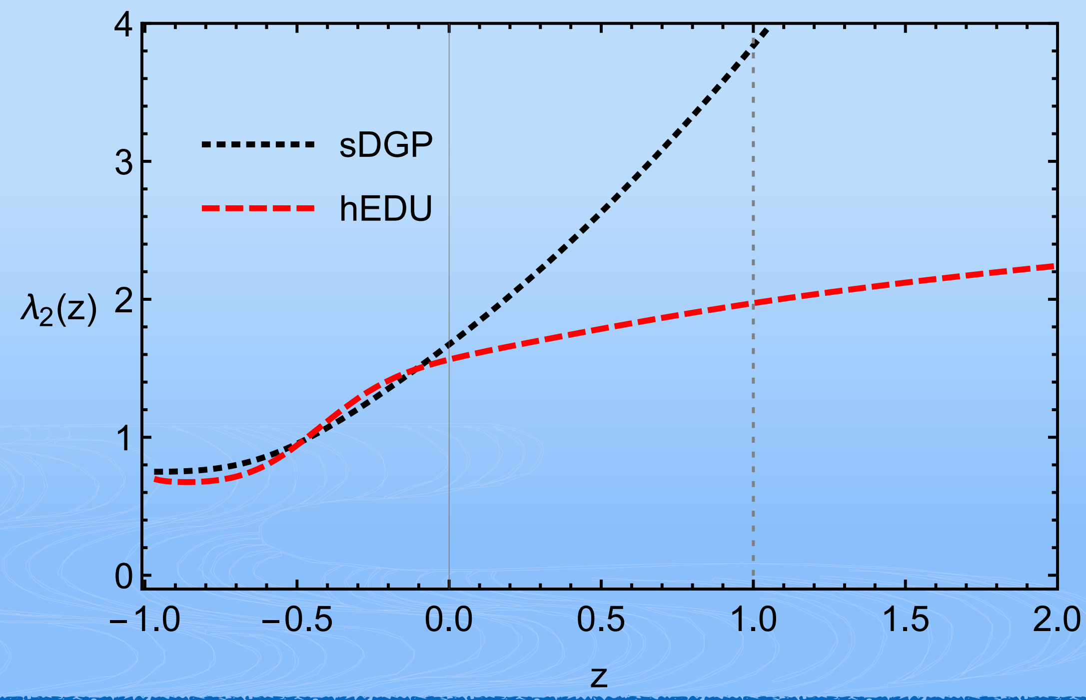
$$\text{Criterion 2}: \quad M_P|\nabla V| \geq c_1 V \quad \text{or} \quad M_P^2 \min[\nabla_i\nabla_j V] \leq -c_2 V,$$



$$\lambda_1 \equiv M_P \frac{V'}{V}, \qquad V' \equiv \frac{\mathrm{d}V(\phi)}{\mathrm{d}\phi} = \frac{\dot{V}(t)}{\dot{\phi}(t)},$$

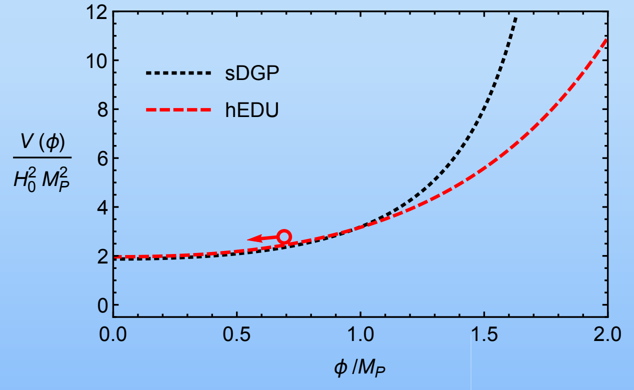
$$\lambda_2 \equiv M_P^2 \frac{V''}{V}, \qquad V'' \equiv \frac{\mathrm{d}}{\mathrm{d}\phi}\frac{\mathrm{d}V(\phi)}{\mathrm{d}\phi} = \frac{1}{\dot{\phi}(t)}\frac{\mathrm{d}}{\mathrm{d}t}\left[\frac{\dot{V}(t)}{\dot{\phi}(t)}\right].$$

$$\frac{H(z)^2}{H_0^2} = \frac{\Omega_\Lambda}{2} + \Omega_m(1+z)^3 + \frac{\Omega_\Lambda}{2}\sqrt{1 + \frac{4}{\Omega_\Lambda}\left[\Omega_m(1+z)^3 + \Omega_I(1+z)^4\right]}$$



$$1 = \Omega_m + \sqrt{\Omega_\Lambda(1+\Omega_I)}$$

Models	Ω_m	Ω_I	Ω_Λ
sDGP	0.21	0	0.62
hEDU	0.03	0.44	0.65

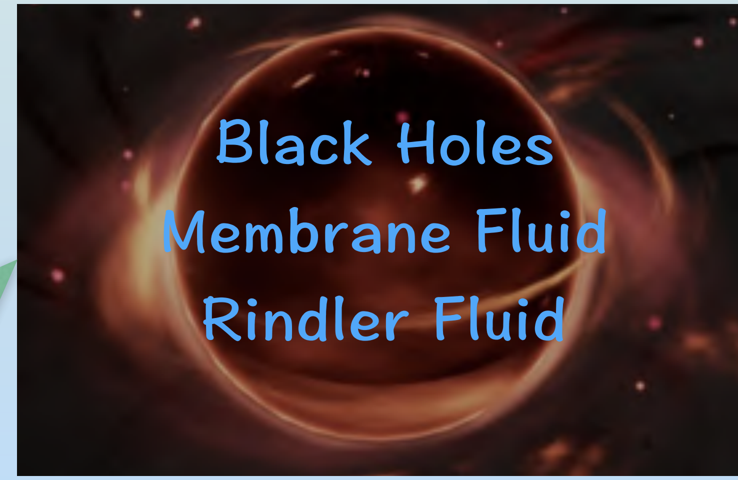


$$M_P^2|\nabla_i\nabla_j V| \geq c_3 V, \qquad c_3 \sim \mathcal{O}(1).$$

Summary & Outlook

$$\frac{\eta}{s} \simeq \frac{1}{4\pi}\frac{\hbar}{k_B}$$

$$\tau_c^{-1} \simeq \frac{k^2}{4\pi T_c}$$



$$\Omega_D^2 \simeq \frac{1}{2}\Omega_\Lambda(\Omega_D - \Omega_B)$$

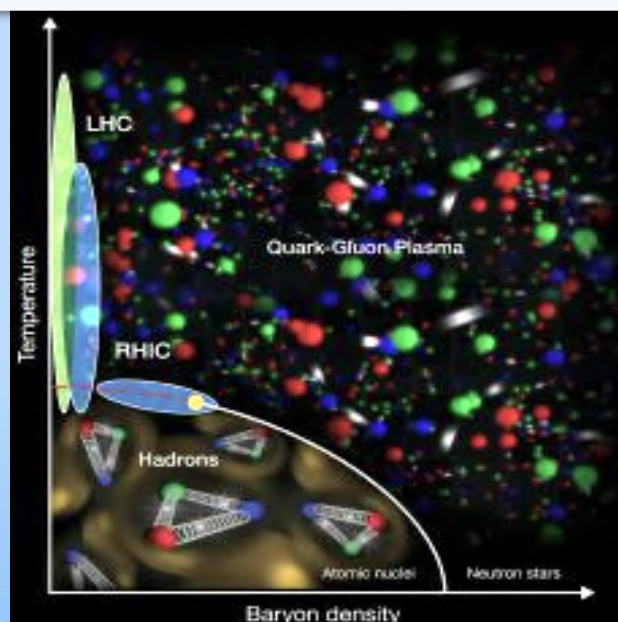
$$\frac{H^2}{H_0^2} \simeq \frac{\Omega_B}{a^3} + \sqrt{\Omega_\Lambda\left(\frac{H^2}{H_0^2} + \frac{\Omega_I}{a^4}\right)}$$

Quark Critical Liquid

Heavy Ion ['00s] & Semi-metal ['10s]

Cosmological Fluid

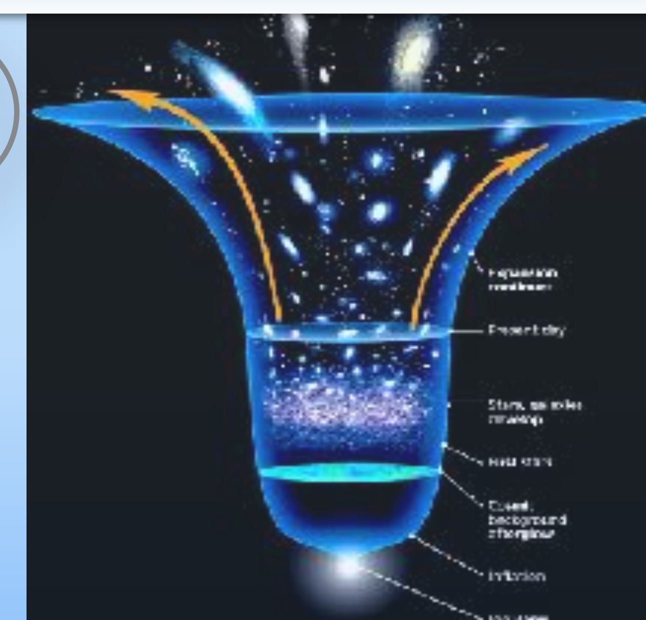
Dark Matter ['70s] & Energy ['90s]



$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} - \frac{1}{L}\left(\mathcal{K}_{\mu\nu} - \mathcal{K}g_{\mu\nu}\right) = \kappa_4 T_{\mu\nu}$$

$$\mathcal{S}_5 = \frac{1}{2\kappa_5}\int_{\mathcal{M}} \mathrm{d}^5x\sqrt{-\tilde{g}}\,\mathcal{R} + \frac{1}{\kappa_5}\int_{\partial\mathcal{M}} \mathrm{d}^4x\sqrt{-g}\,\mathcal{K},$$

$$\mathcal{S}_4 = \frac{1}{2\kappa_4}\int_{\partial\mathcal{M}} \mathrm{d}^4x\sqrt{-g}\,R + \int_{\partial\mathcal{M}} \mathrm{d}^4x\sqrt{-g}\mathcal{L}_M.$$



Thanks for Your Attention!