Hunting for Statistical Anisotropy in Tensor Modes with B-mode observations

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Collaboration with
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"Hunting for Statistical Anisotropy in Tensor Modes with B-mode observations"
PRD 98 (2018) 083522 [arXiv:1808.08044]

Introduction

Inflation

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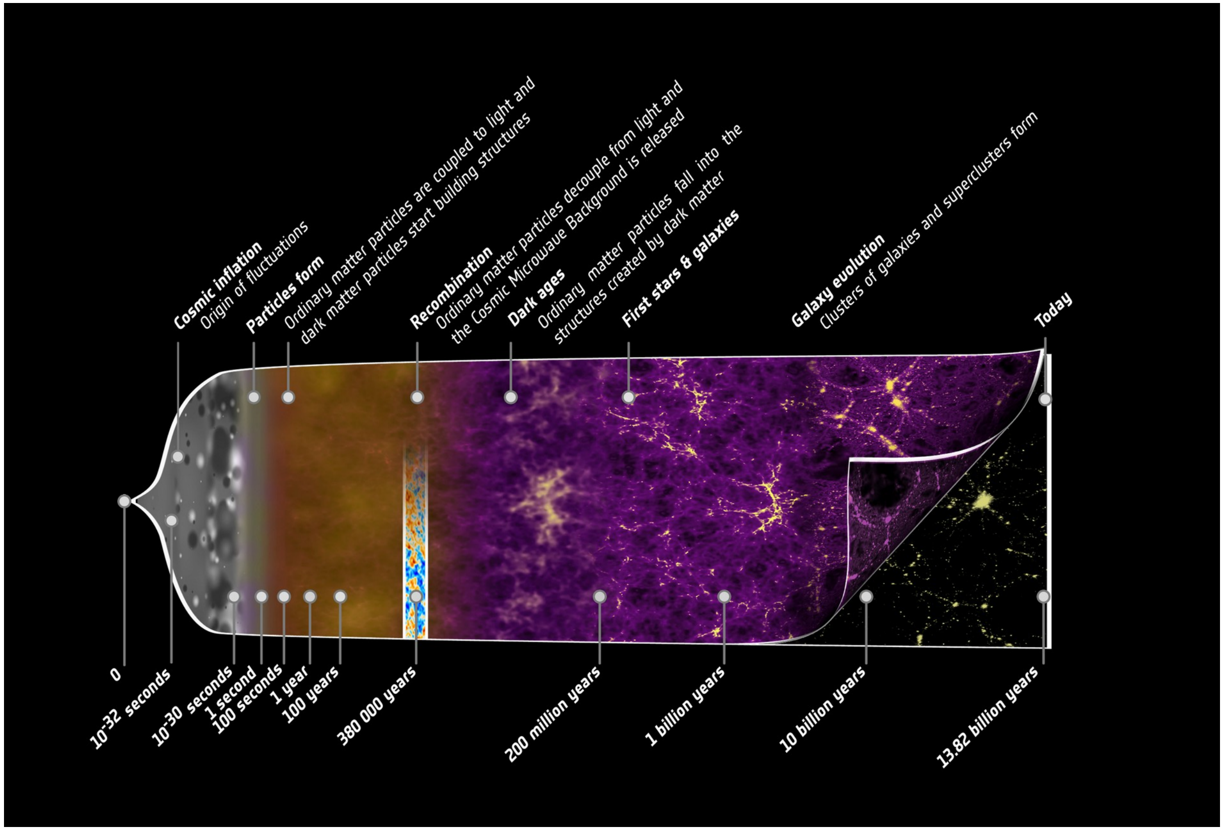

Spacetime fluctuations during inflation

Scalar-mode $\zeta(\boldsymbol{x}, \eta) \longrightarrow \Phi, \delta_X \longrightarrow$ Large-scale structure

CMB anisotropies

Tensor-mode $h_{ij}(\boldsymbol{x}, \eta) \longrightarrow$ Gravitational waves

CMB polarisation anisotropy

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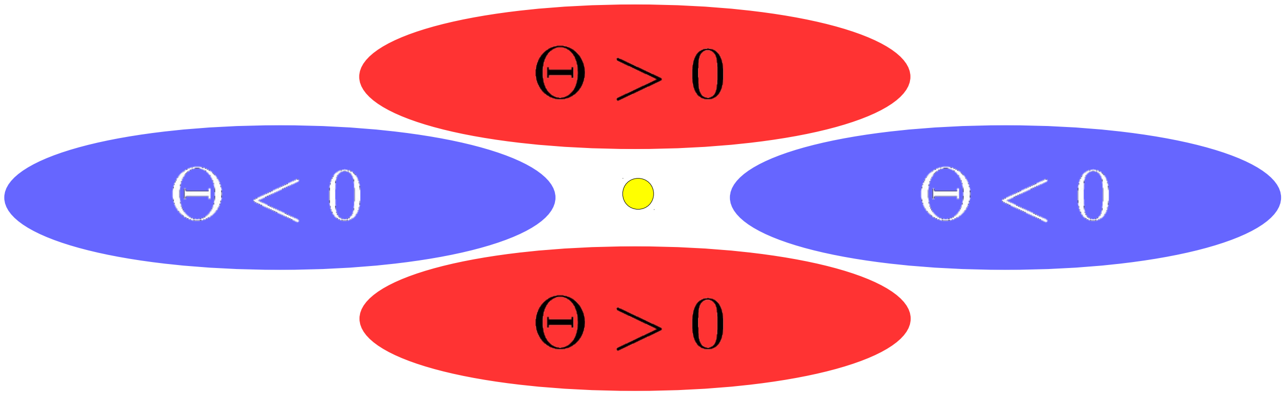
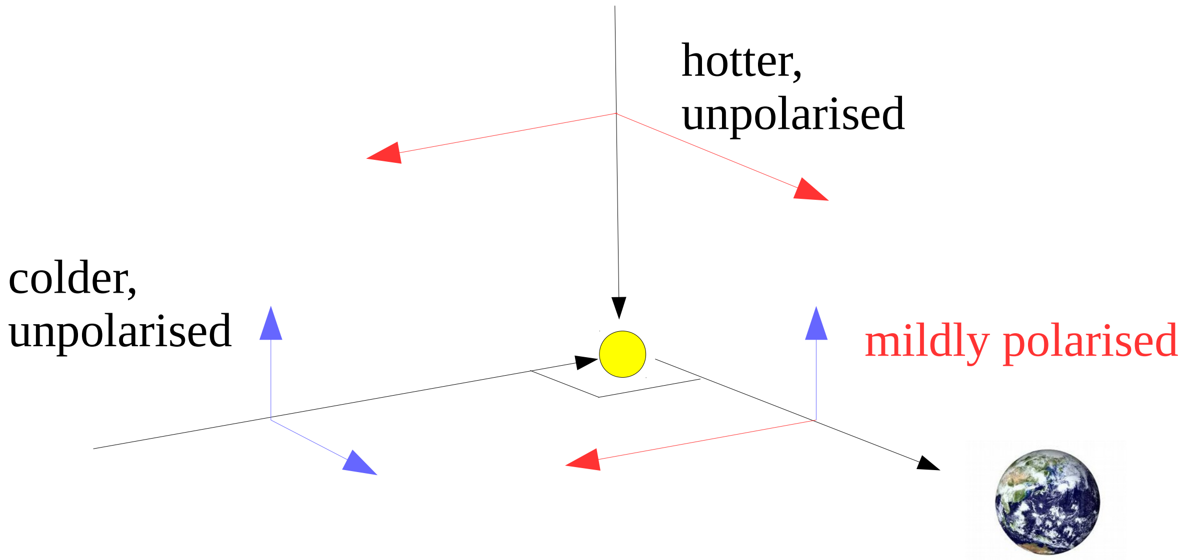
Consider the local quadrupole anisotropy...



Temperature anisotropy produces anisotropy of polarisations.

(See http://background.uchicago.edu/~whu/intermediate/Polarization/polar1.html)



CMB polarisation anisotropy

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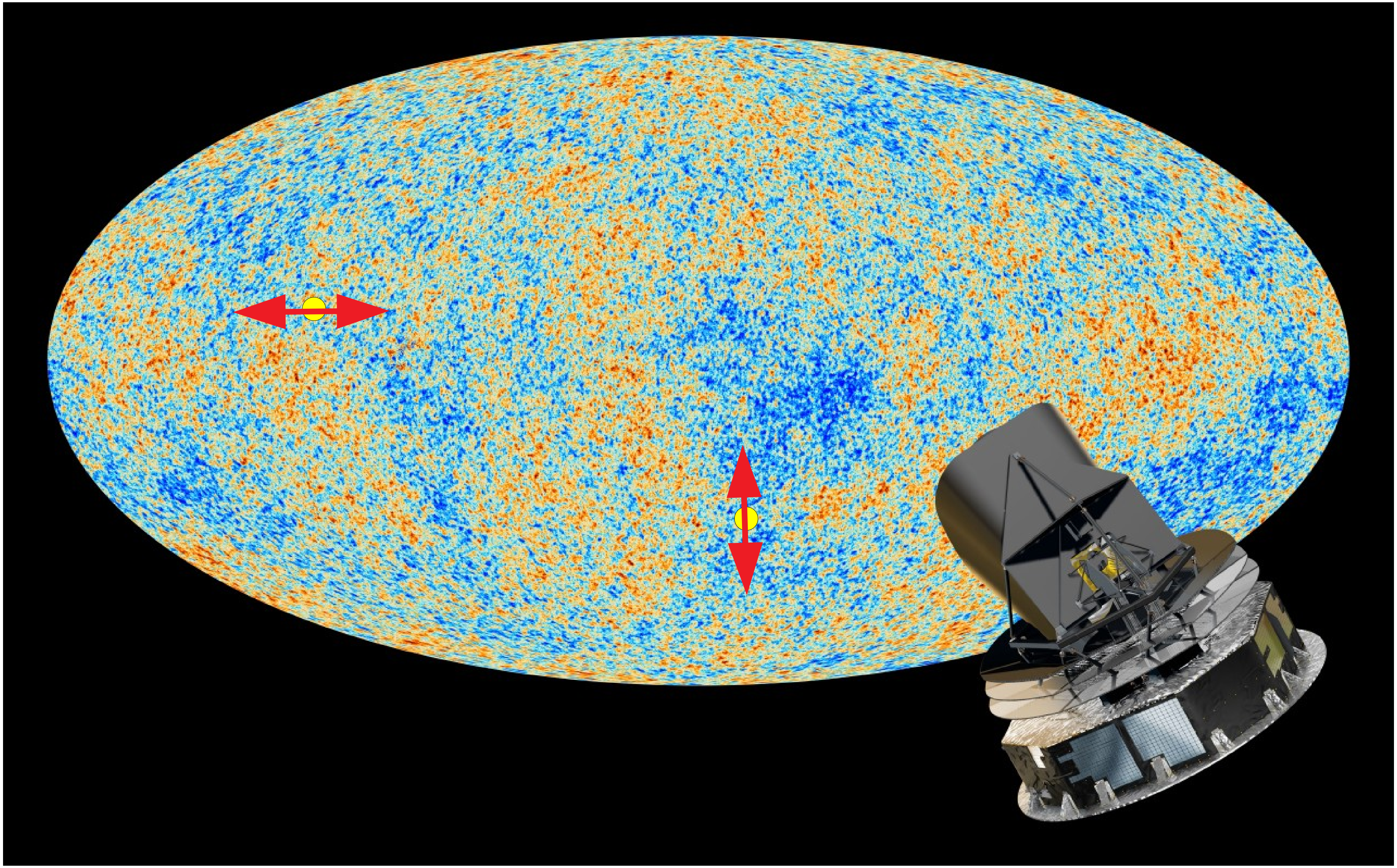
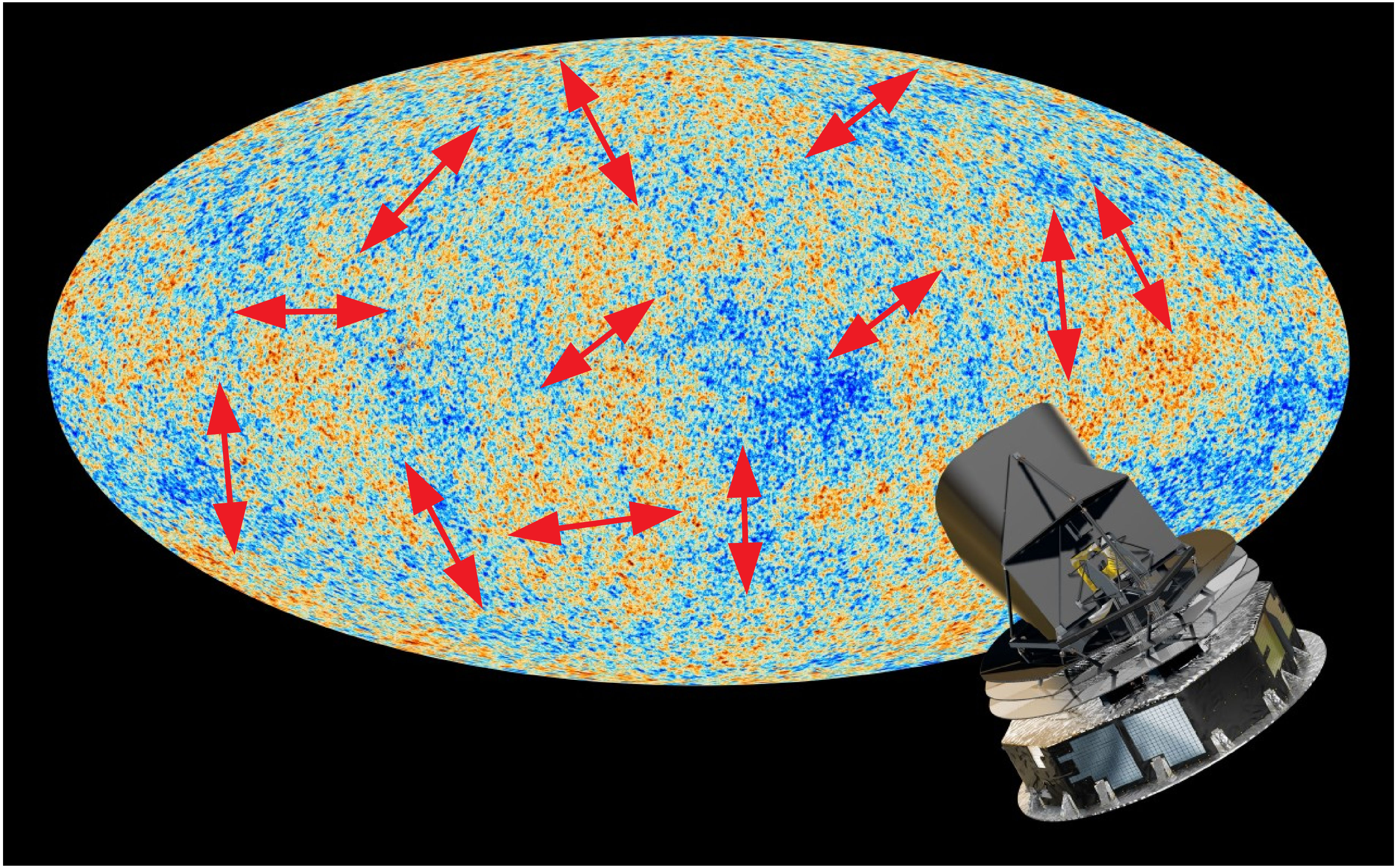


CMB polarisation anisotropy



CMB E/B-mode polarisation

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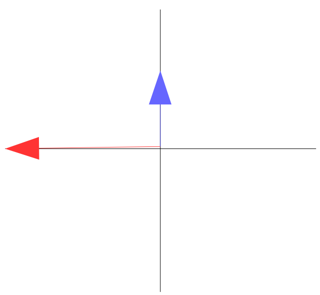
Polarisation is quantified by Stokes parameters

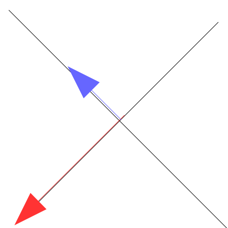
$$\begin{aligned} I &= |E_x|^2 + |E_y|^2 && \text{Strength} \\ Q &= |E_x|^2 - |E_y|^2 && \\ U &= 2\mathrm{Re}(E_x E_y^*) && \text{Linear pol.} \\ V &= -2\mathrm{Im}(E_x E_y^*) && \text{Circular pol.} \end{aligned}$$

$$I \rightarrow \Theta_{\ell m}$$

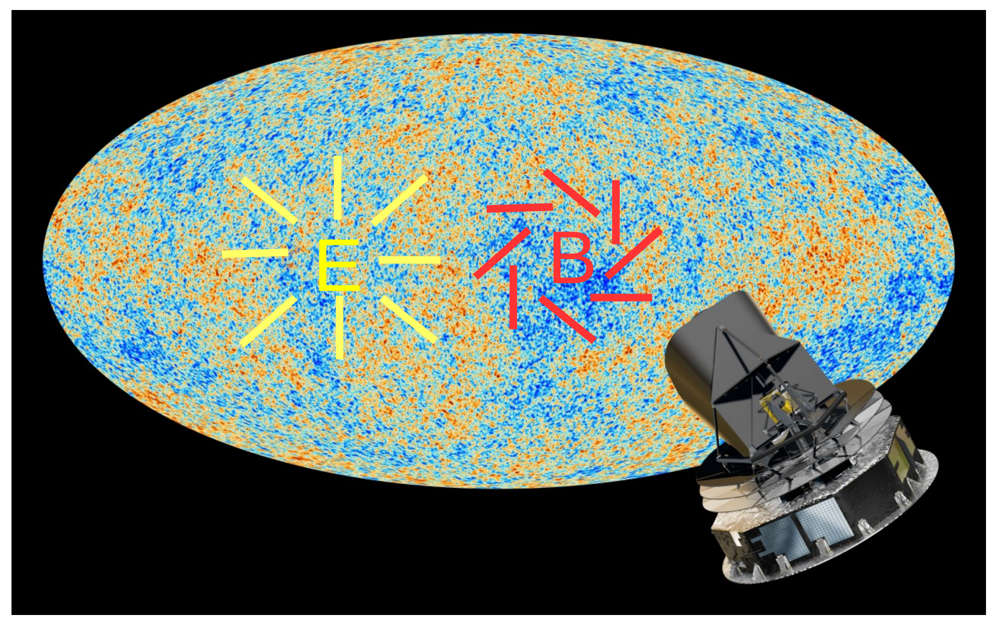
$$Q \pm iU \rightarrow E_{\ell m} \pm iB_{\ell m}$$

(expanded by $Y_{\ell m}$)



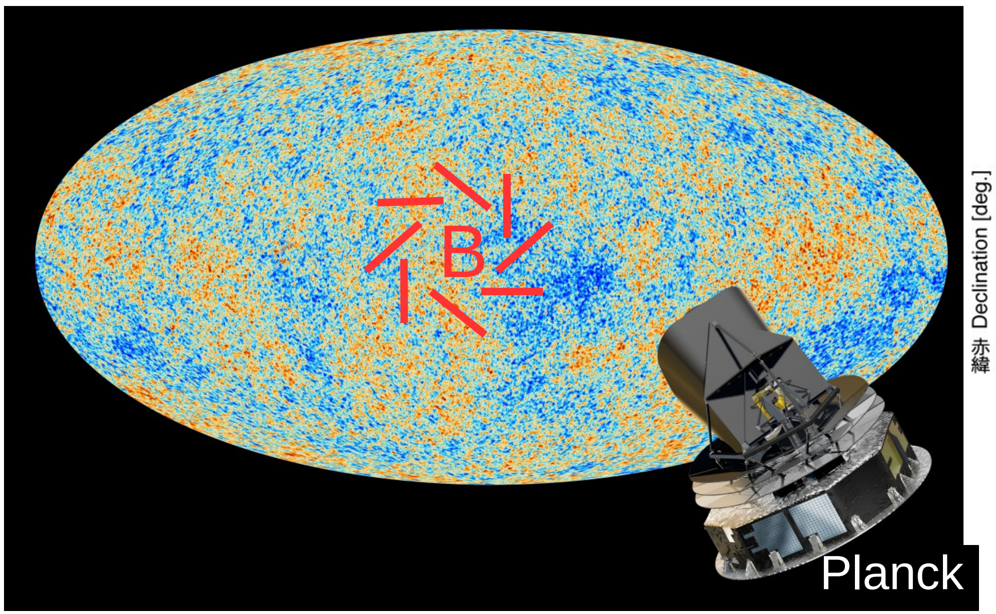


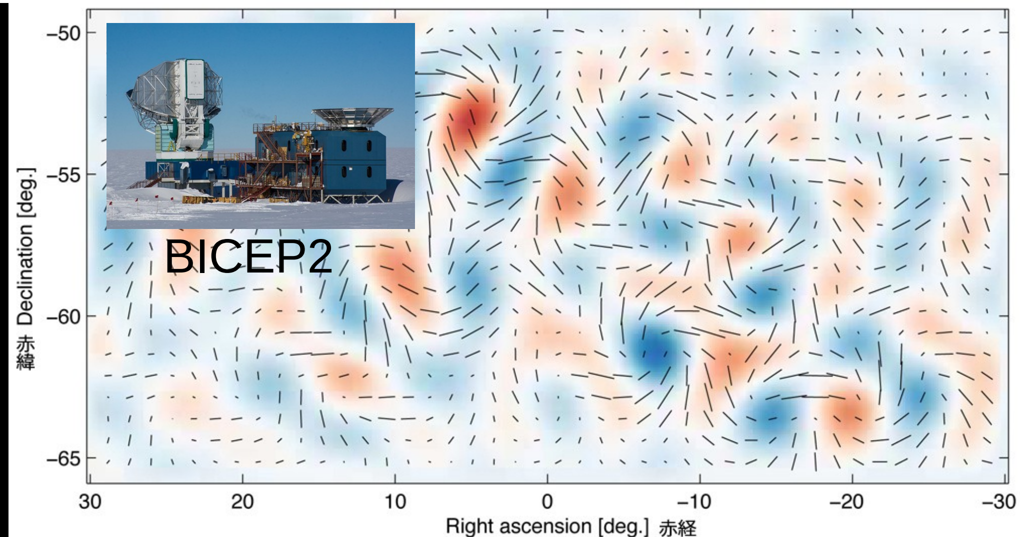
Scalar	E-mode	
Vector	E-mode	B-mode
Tensor	E-mode	B-mode



CMB B-mode

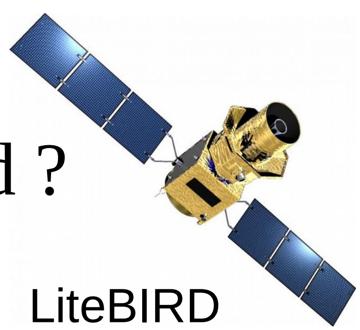
- A polarisation mode of photons $(Q, U) \to (E_{\ell m}, B_{\ell m})$
- Generated by gravitational waves (tensor perturbations)
- No detections so far, but possibly to be done in the (near) future





BICEP2 collaboration, PRL 112 (2014) 241101

Can we know how the primordial GWs are generated ?
Inflation ? Or, other possibilities ?



Anisotropy of tensor modes

Parameterisation of primordial spectrum

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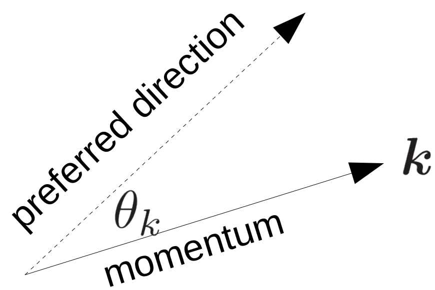
Anisotropy of primordial tensor power spectrum

$$\langle h_{ij}(\boldsymbol{k})h^{ij}(\boldsymbol{k}')\rangle := (2\pi)^3 P_h(\boldsymbol{k})\delta^{(3)}(\boldsymbol{k}-\boldsymbol{k}')$$

$$P_h(\boldsymbol{k}) = P_h(k)\sum_{LM} q_{LM}\left(\frac{k}{k_0}\right)^{\gamma} Y_{LM}(\hat{k})$$

$$= P_h(k)\sum_{n\geq 0} g_n \left(\frac{k}{k_0}\right)^{\gamma} \cos^n\theta_{\hat{k}}$$



Some models predict such an anisotropy,

U(1) gauge field : $(g_0, g_2, g_4, g_6) = (1, -1, 1, -1)$
Fujita, Obata, Tanaka, S.Yokoyama, JCAP 1807 (2018) 023

2-form field : $(g_0, g_2, g_4, g_6) = (1, 1, -2, 1)$
Obata, Fujita, arXiv:1808.00548

Massive spin-s : $(g_0, g_2, g_4, g_6) = (1, -{}_sC_1, {}_sC_2, -{}_sC_3)$
(curvature perturbation)
Kehagias, Riotto, JCAP 1707 (2017) 046

Angular power spectrum

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As a result, B-mode fluctuations with different ℓ's are correlated with each other,

$$C^{BB}_{\ell_1 m_1;\ell_2 m_2}(\gamma) = \frac{2}{\pi} i^{\ell_2-\ell_1}(-1)^{m_1} \sum_{LM} \delta^{\mathrm{even}}_{\ell_1+\ell_2+L} \mathcal{G}^{-m_1 m_2 M;-220}_{\ell_1 \ell_2 L} q_{LM} C^{BB}_{\ell_1\ell_2}(\gamma)$$

Shiraishi, Mota, Ricciardone, Arroja, JCAP 1407 (2014) 047

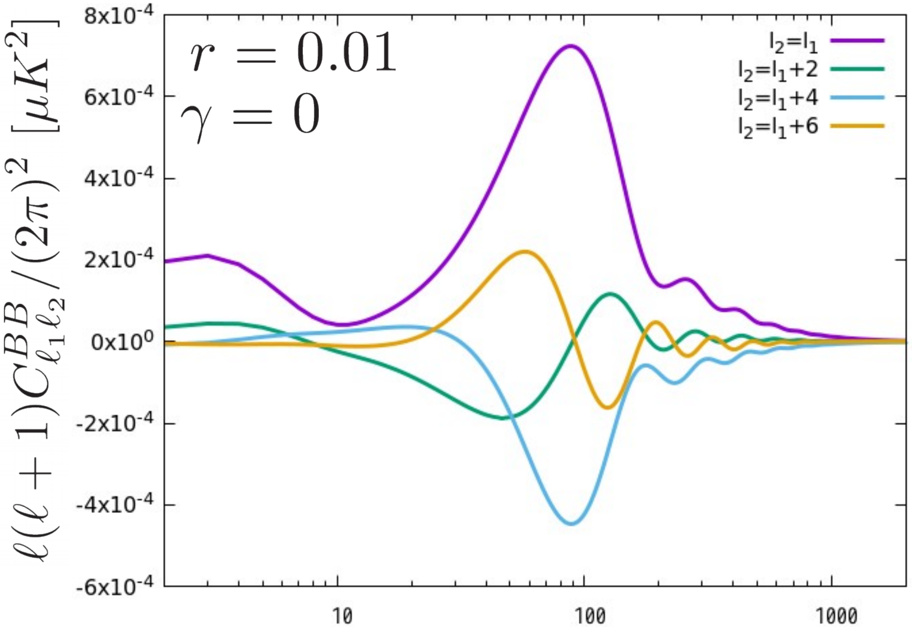
with

$$C^{BB}_{\ell_1\ell_2}(\gamma) = \frac{2}{\pi}\int dk\, k^2 P_h(k) T^{(B)}_{\ell_1}(k) T^{(B)}_{\ell_2}(k) \left(\frac{k}{k_0}\right)^{\gamma}$$

If no anisotropies $(q_{LM} = \delta_{L0}\delta_{M0})$,

$$C^{BB}_{\ell_1 m_1;\ell_2 m_2} = \delta_{\ell_1\ell_2}\delta_{m_1 m_2} C^{BB}_{\ell_1}$$

Transfer function given by CMB2nd
with Planck 2015 results



Boltzmann solver "CMB2nd"

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CMB2nd ver. 1.101

* Angular power spectra, $C_\ell^{\Theta\Theta}, C_\ell^{\Theta E}, C_\ell^{EE}, C_\ell^{BB}$, from Scalar/Vector/Tensor Perturbations, which are consistent to CAMB results with O(0.1)% error.

* TT/TE/EE/BB power with primordial anisotropies

* Lensed bispectra $\widehat{B}^{XYZ,s_1s_2s_3}_{L_1L_2L_3}$ as well as the lensed power spectra $\widehat{C}^{XY,s}_L$ including curl-mode can be computed.

TH, Yamauchi in preparation, arXiv:190[4-9]\.[0-9]+

* Signal-to-noise ratio, $\frac{S}{N} = \frac{1}{\sqrt{(F_{ii})^{-1}}}$, where F_{ij} is the Fisher matrix,

$$F_{ij} = \sum_{L_1L_2L_3} \frac{\widehat{B}^i_{L_1L_2L_3}\widehat{B}^j_{L_1L_2L_3}}{\Delta_{L_1L_2L_3}C^{XX}_{L_1}C^{YY}_{L_2}C^{ZZ}_{L_3}} \quad (i,j = XYZ)$$

* Compute $f_{\rm NL}$ parameters for local/equilateral/orthogonal/folded templates

* EFT of Type-I DHOST is ready for implementation. **TH in preparation, arXiv:190[34]\.[0-9]+**

* Would be open in near future ?

Setup

Fiducial power spectrum + anisotropic component

$$P_h(k)\sum_{n\geq 0} g_n \left(\frac{k}{k_0}\right)^{\gamma} \cos^n\theta_{\hat{k}}$$

$$\begin{cases} \gamma = 0 \qquad g_0, q_{0M} = 1 \\ P_h(\boldsymbol{k}) = P_h(k) + P_h(k)\sum_{n\geq 2} g_n \cos^n\theta_{\hat{k}} \\ \gamma \neq 0 \qquad g_0, q_{0M} = \text{arbitrary} \\ P_h(\boldsymbol{k}) = P_h(k) + P_h(k)\sum_{n\geq 0} g_n \left(\frac{k}{k_0}\right)^{\gamma} \cos^n\theta_{\hat{k}} \end{cases}$$

$$\Downarrow$$

$$C^{BB(\mathrm{obs})}_{\ell_1 m_1;\ell_2 m_2}(\gamma) = C^{BB}_{\ell_1}\delta_{\ell_1\ell_2}\delta_{m_1 m_2} + C^{BB}_{\ell_1 m_1;\ell_2 m_2}(\gamma)$$

observed

fiducial
(flat, vacuum fluc.)

anisotropic

How precisely can we detect ?

Power spectra depend on anisotropy parameters :

$$C^{BB}_{\ell_1 m_1;\ell_2 m_2}(q_{0M}, q_{2M}, q_{4M}, q_{6M})$$

How precisely can we determine these parameters ? → Fisher information matrix

For isotropic case,

$$F_{ij} = \sum_{\ell} \left[\widetilde{\boldsymbol{C}}_\ell\right]^{-1} \left(\frac{\partial \boldsymbol{C}_\ell}{\partial \theta_i}\right) \left[\widetilde{\boldsymbol{C}}_\ell\right]^{-1} \left(\frac{\partial \boldsymbol{C}_\ell}{\partial \theta_j}\right)$$

$$\widetilde{\boldsymbol{C}}_\ell := \boldsymbol{C}_\ell(\theta_i = \theta_i^{\mathrm{fid}}) + \boldsymbol{N}_\ell$$

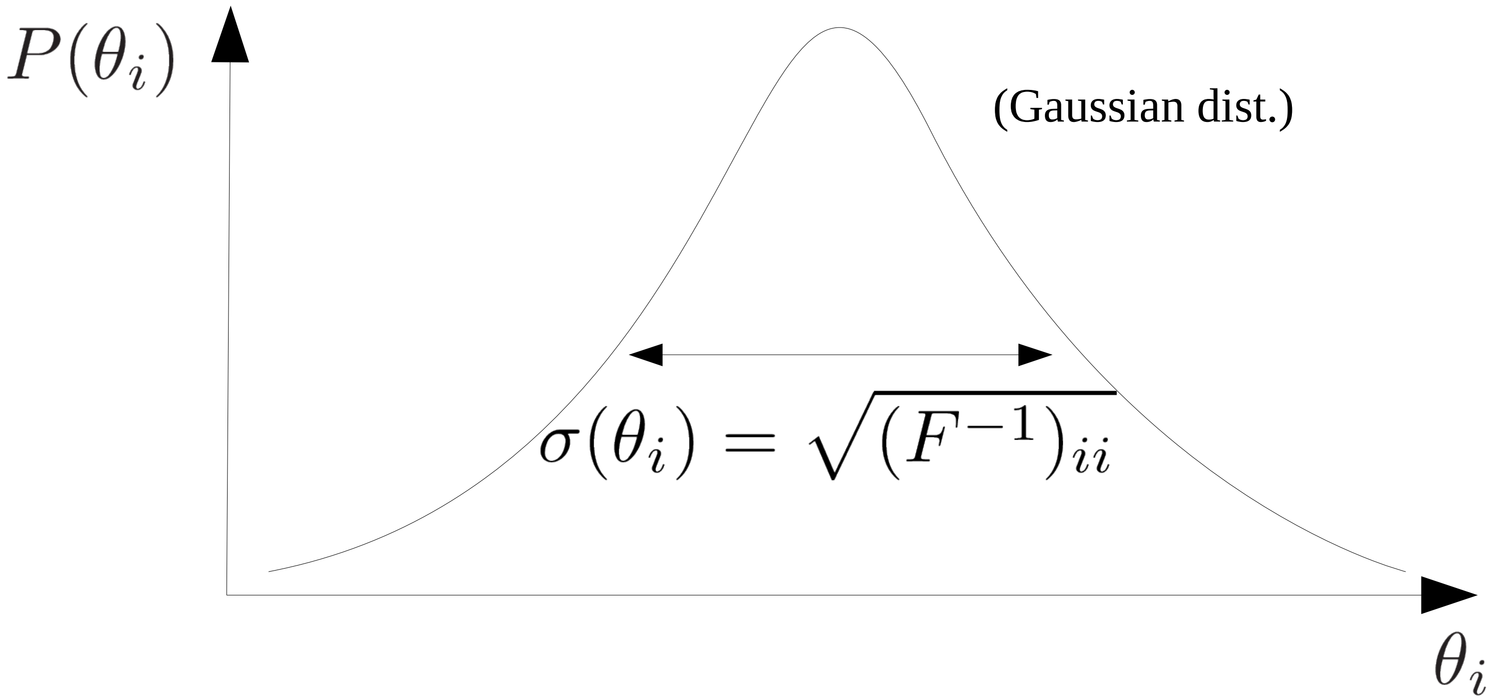
$$\boldsymbol{C}_\ell := \begin{pmatrix} C_\ell^{\Theta\Theta} & C_\ell^{\Theta E} & 0 \\ C_\ell^{\Theta E} & C_\ell^{EE} & 0 \\ 0 & 0 & C_\ell^{BB} \end{pmatrix} \quad \boldsymbol{N}_\ell := \begin{pmatrix} \mathcal{N}_\ell^{\Theta\Theta} & 0 & 0 \\ 0 & \mathcal{N}_\ell^{EE} & 0 \\ 0 & 0 & \mathcal{N}_\ell^{BB} \end{pmatrix}$$

Covariance matrix | Detector noise

For B-mode,

$$F_{ij} = \frac{1}{2}\sum_{\ell} \frac{2\ell+1}{C_\ell^{BB(\mathrm{fid})} + \mathcal{N}_\ell^{BB}} \left(\frac{\partial C_\ell^{BB}}{\partial \theta_i}\right)\left(\frac{\partial C_\ell^{BB}}{\partial \theta_j}\right)$$

The uncertainty in measuring θ_i is given by the inverse of Fisher matrix,



"Reduced Fisher matrix"

For anisotropic B-mode,

$$\{\theta_i\} = \{q_{LM}\} \quad : \quad F_{LM;L'M'} = \delta_{LL'}\delta_{MM'} F_L^{BB}$$

$$\{\theta_i\} = \{g_n\} \quad : \quad F_{pq} = \sum_{LM} \frac{\partial q_{LM}}{\partial g_p} \frac{\partial q_{LM}}{\partial g_q} F_L^{BB}$$

with

$$F_L^{BB} = \frac{1}{4\pi} \sum_{\ell_1 \ell_2} (2\ell_1 + 1)(2\ell_2 + 1) \begin{pmatrix} \ell_1 & \ell_2 & L \\ -2 & 2 & 0 \end{pmatrix}^2 \frac{(C^{BB}_{\ell_1 \ell_2})^2}{\widetilde{C}^{BB}_{\ell_1} \widetilde{C}^{BB}_{\ell_2}}$$

$$\widetilde{C}^{BB}_\ell = C^{BB}_\ell(\{\theta_i\} = 0) + \mathcal{N}^{BB}_\ell$$

fiducial spectrum : $q_{LM} = 0$ or $g_n = 0$

Noise model

Detector noise is represented by a white noise,

$$\mathcal{N}_\ell^{BB} = N_\ell^{BB} e^{\ell^2 \sigma_b^2}$$

Noise spectrum : $$N_\ell^{BB} = \left(\frac{\pi}{10800} \frac{w_{BB}^{-1/2}}{\mu\mathrm{K\ arcmin}} \right)^2 \ \mu\mathrm{K}^2\ \mathrm{str}$$

Beam width : $$\sigma_b = \frac{\pi}{10800} \frac{\theta_{\mathrm{FWHM}}}{\mathrm{arcmin}} \frac{1}{\sqrt{8 \ln 2}}$$

We choose

$w_{BB}^{-1/2} =$	$63.1\ \mu\mathrm{K\ arcmin}$	Planck
	$5.0\ \mu\mathrm{K\ arcmin}$	LiteBIRD
	$1.0\ \mu\mathrm{K\ arcmin}$	CMB-S4
$\theta_{\mathrm{FWHM}} =$	$30\ \mathrm{arcmin}$	LiteBIRD

Note : we assume the complete de-lensing.

Results

$$\sigma_{g_n} = \sqrt{(F^{-1})_{nn}} \quad \text{with} \quad \gamma = 0 \,, r = 0.01$$

		CMB-S4	LiteBIRD	Planck
	CVL	1.0	5.0	63.1
g_2	1.93×10^{-3}	5.98×10^{-2}	2.03×10^{-1}	3.25
g_2	8.23×10^{-3}	2.66×10^{-1}	9.25×10^{-1}	1.89×10^{1}
g_4	1.24×10^{-2}	3.90×10^{-1}	1.35	2.94×10^{1}
g_2	2.33×10^{-2}	6.29×10^{-1}	1.47	1.92×10^{1}
g_4	8.27×10^{-2}	2.25	4.67	3.23×10^{1}
g_6	6.46×10^{-2}	1.80	3.67	1.11×10^{1}
q_{2M}	4.98×10^{-3}	1.26×10^{-1}	4.10×10^{-1}	9.52
q_{4M}	4.28×10^{-3}	1.53×10^{-1}	5.59×10^{-1}	1.27×10^{1}
q_{6M}	5.71×10^{-3}	1.50×10^{-1}	2.66×10^{-1}	7.55×10^{-1}

Note : As for the anisotropy of curvature perturbations,

$$|g_2^{(S)}| \leq 2.56 \times 10^{-2}$$

Planck Collaboration, A&A 594 (2016) A20

Results

$$\sigma_{g_n} = \sqrt{(F^{-1})_{nn}} \quad \text{with} \quad \gamma = 0\,, r = 0.001$$

	CVL	CMB-S4 1.0	LiteBIRD 5.0	Planck 63.1
g_2	1.93×10^{-3}	1.25×10^{-1}	7.37×10^{-1}	2.37×10^{1}
g_2	8.23×10^{-3}	5.61×10^{-1}	3.96	1.39×10^{2}
g_4	1.24×10^{-2}	8.13×10^{-1}	6.08	2.15×10^{2}
g_2	2.33×10^{-2}	1.10	4.20	1.41×10^{2}
g_4	8.27×10^{-2}	3.78	8.23	2.33×10^{2}
g_6	6.46×10^{-2}	3.03	4.58	7.31×10^{1}
q_{20}	4.98×10^{-3}	2.49×10^{-1}	1.95	6.68×10^{1}
q_{40}	4.28×10^{-3}	3.34×10^{-1}	2.60	9.98×10^{1}
q_{60}	5.71×10^{-3}	2.33×10^{-1}	3.13×10^{-1}	4.98

Note : As for the anisotropy of curvature perturbations,

$$|g_2^{(S)}| \leq 2.56 \times 10^{-2}$$

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Results

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$$P_h(\boldsymbol{k}) = P_h(k) \sum_n g_n \left(\frac{k}{k_0}\right)^\gamma \cos^n \theta_{\hat{k}}$$

$\gamma = -\frac{1}{2}, r = 0.01$

	CVL	CMB-S4 1.0	LiteBIRD 5.0	Planck 63.1
g_0	1.23×10^{-2}	8.26×10^{-2}	1.96×10^{-1}	1.42
g_2	3.36×10^{-2}	2.18×10^{-1}	5.26×10^{-1}	3.96
g_0	1.59×10^{-2}	1.22×10^{-1}	3.23×10^{-1}	2.38
g_2	1.06×10^{-1}	9.23×10^{-1}	2.62	1.94×10^{1}
g_4	1.17×10^{-1}	1.05	2.99	2.22×10^{1}
g_0	2.02×10^{-2}	1.27×10^{-1}	3.25×10^{-1}	2.38
g_2	2.82×10^{-1}	1.19	2.74	1.95×10^{1}
g_4	7.93×10^{-1}	2.49	3.84	2.31×10^{1}
g_6	5.75×10^{-1}	1.66	1.77	4.66
q_{0M}	1.82×10^{-2}	1.39×10^{-1}	3.07×10^{-1}	1.90
q_{2M}	3.55×10^{-2}	2.30×10^{-1}	5.56×10^{-1}	4.18
q_{4M}	3.17×10^{-2}	2.83×10^{-1}	8.09×10^{-1}	5.99
q_{6M}	3.92×10^{-2}	1.13×10^{-1}	1.20×10^{-1}	3.18×10^{-1}

$\gamma = -1, r = 0.01$

	CVL	CMB-S4 1.0	LiteBIRD 5.0	Planck 63.1
g_0	5.28×10^{-2}	9.67×10^{-2}	1.31×10^{-1}	5.93×10^{-1}
g_2	1.43×10^{-1}	2.61×10^{-1}	3.58×10^{-1}	1.64
g_0	7.23×10^{-2}	1.56×10^{-1}	2.23×10^{-1}	1.04
g_2	5.14×10^{-1}	1.25	1.85	8.65
g_4	5.76×10^{-1}	1.42	2.11	9.91
g_0	7.38×10^{-2}	1.56×10^{-1}	2.24×10^{-1}	1.04
g_2	6.02×10^{-1}	1.29	1.88	8.69
g_4	1.10	1.72	2.33	1.02×10^{1}
g_6	6.91×10^{-1}	7.08×10^{-1}	7.17×10^{-1}	1.88
q_{0M}	8.08×10^{-2}	1.49×10^{-1}	1.88×10^{-1}	8.21×10^{-1}
q_{2M}	1.51×10^{-1}	2.76×10^{-1}	3.79×10^{-1}	1.73
q_{4M}	1.56×10^{-1}	3.84×10^{-1}	5.71×10^{-1}	2.68
q_{6M}	4.70×10^{-2}	4.82×10^{-2}	4.89×10^{-2}	1.28×10^{-1}

Note : As for the aniositropy of curvature perturbations,

$$|g_2^{(S)}(\gamma = -1)| \leq 1.56 \times 10^{-1}$$

Planck Collaboration, A&A 594 (2016) A20

Note : With $\gamma = -2$, $(\sigma_{q_{0M}}, \sigma_{q_{2M}}) = (30, 58)$ has been reported whereas we obtained $(\sigma_{q_{0M}}, \sigma_{q_{2M}}) = (30.1, 59.6)$

Shiraishi, Mota, Ricciardone, Arroja, JCAP 1407 (2014) 047

Part II summary

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- There are several generation mechanisms of GWs at the early Universe.

- Some mechanisms generate anisotropic GWs, so is it possible to get such anisotropic signals with future missions of B-mode observations ?

- To constrain the anisotropy parameter up to g_4 , we need CMB-S4.