Statistical anisotropy in the cosmic microwave background spectral distortions

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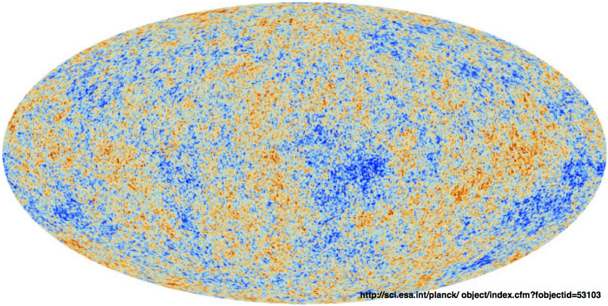
What I did

Measurements of the cosmic microwave background (CMB) spectral y-distortion anisotropy offer a test for the statistical isotropy of the primordial density perturbations on $0.01 \lesssim k\mathrm{Mpc} \lesssim 1$.

*Questions

1. What is/Why statistical anisotropy?
2. What is the spectral (y-)distortion?
3. Why y distortion is sensitive to SA on $0.01 \lesssim k\mathrm{Mpc} \lesssim 1$?

Anisotropies in the CMB



10^{-5} of CMB anisotropies: Gaussian, adiabatic, almost scale invariant.

$$\frac{\delta T}{T} \sim \zeta$$

*ζ is a random realization on top of the isotropic background.

Statistically anisotropic perturbations

*Background isotropy: common but non trivial assumption.

d

e.g.) Anisotropic inflation: a vector field leads to the preferred direction **d**. (e.g. Soda 2012)

$$P_\zeta(\mathbf{k}_1) \to P_\zeta(|\mathbf{k}_1|)\left(1 + g_*(\mathbf{d}\cdot\hat{k}_1)^2\right)$$

Statistically anisotropic perturbations

*More generally, for spin-L fields with a preferred direction $\mathbf{d}$

[Franciolini, Kehagias, Riotto (2017)]

$$P_\zeta(\mathbf{k}) \to P_\zeta(k) \sum_{L=0}^{\infty} (-i)^{2L} (2(2L)+1) A_{2L}(k) P_{2L}(\mathbf{d} \cdot \hat{k})$$

*Test of SI $\to$ test of matter contents during inflation!
*Can we test it by the spectral distortions to the CMB?

Implicit assumption at linear perturbation theory

*Starting point for the CMB = Boltzmann equation

$$\frac{df_{\rm CMB}}{d\eta} = \mathcal{C}_{\rm C}[f_{\rm CMB}] + \mathcal{C}_{\rm DC}[f_{\rm CMB}] + \mathcal{C}_{\rm BR}[f_{\rm CMB}] + \cdots$$

*Implicit assumptions of the linear theory:

- Up to ζ^1
- Up to $(T_{\rm e}/m_{\rm e})^0$
- (Local) Blackbody ansatz

*NLO solution → More complicated non-equilibrium physics.

But, NLO ansatz is simplified if

- Ignore the DC & BR ($z < 2 \times 10^6$)
- Up to ζ^2
- Up to $(T_\mathrm{e}/m_\mathrm{e})^1$

*Second-order ansatz:[Stebbins, Pitrou 2014, **AO** 2017]

$$f_\mathrm{CMB}(\eta, \mathbf{x}, p\mathbf{n}) = (\text{Local BB}) + y(\eta, \mathbf{x}, \mathbf{n})\mathcal{Y}(p)$$

CMB spectrum at second-order

*y-distortion is a NLO correction to the Local BB spectrum.

$$y \sim \mathcal{O}(\zeta^2) \text{ or } \mathcal{O}\left(\frac{T_\mathrm{e}}{m_\mathrm{e}}\right)$$

*y ensemble average

$$\langle y^{\mathbf{d}}(\mathbf{n})\rangle = \int \frac{d^3\mathbf{k}}{(2\pi)^3} P_\zeta^{\mathbf{d}}(\mathbf{k}) \mathcal{W}_y(\mathbf{k},\mathbf{n}) \sim 10^{-9}$$

*Is y anisotropy sensitive to A_L?

$$A_L \overset{?}{\to} \langle y_{LM}\rangle = \int d\mathbf{n} Y^*_{LM}(\mathbf{n})\langle y^{\mathbf{d}}(\mathbf{n})\rangle$$

y-distortion monopole

Well-known monopole expression:

$$\langle y_{00} \rangle = \sqrt{4\pi} \int_{\eta_i}^{\eta_0} d\eta(-\dot{\tau}) \int \frac{dk}{k} \frac{k^3}{2\pi^2} A_0 P_\zeta(k) \left[3\Theta_{1g}^2 + \frac{9}{2}\Theta_2^2 + \cdots \right]$$
$$+ \sqrt{4\pi} \int_{\eta_i}^{\eta_0} d\eta(-\dot{\tau}) \frac{T_\mathrm{e} - T_0(1+z)}{m_\mathrm{e}}$$

- y-distortion from acoustic damping
- Sunyaev-Zel'dovich effect

Quadrupole acoustic source

I found

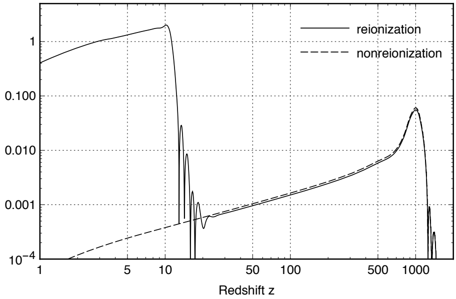
$$\left\langle S_{2m}^{(\text{ac.})} \right\rangle = 4\pi Y_{2m}^*(\mathbf{d}) \int \frac{dk}{k} \frac{k^3}{2\pi^2} (-i)^2 A_2 P_\zeta \left[\frac{33}{50} \Theta_{1g}^2 + \frac{9}{14} \Theta_2^2 + \cdots \right],$$

Then,

$$\langle y_{2m} \rangle = \int_{\eta_i}^{\eta} d\bar{\eta} \left[-\dot{\tau} e^{-\frac{9}{10}[\tau - \tau(\eta)]} \right] \left\langle S_{2m}^{(\text{ac.})} \right\rangle ,$$

Heating rate

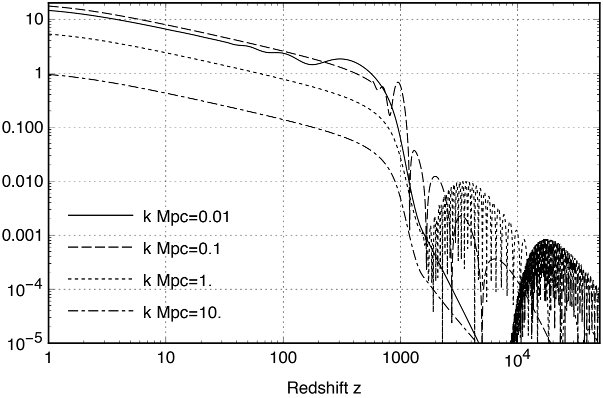
*Heating rate $d\langle y_{2m}\rangle/d\ln z$ in units of $4\pi A_2 A_\zeta Y_{2m}(\mathbf{d})$.



- Reionization contribution is dominant
- $z \gg 10^3$ is suppressed

Why reionization?

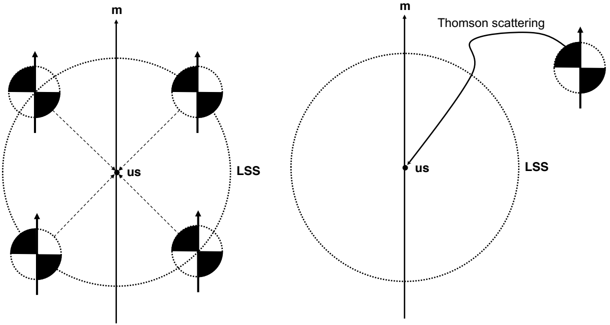
Evolution of heat conduction Θ_{1g}:



$$\left\langle S_{2m}^{(\mathrm{ac.})} \right\rangle = 4\pi Y_{2m}^{*}(\mathbf{d}) \int \frac{dk}{k} \frac{k^3}{2\pi^2} (-i)^2 A_2 P_\zeta \left[\frac{33}{50} \Theta_{1g}^2 + \frac{9}{14} \Theta_2^2 + \cdots \right],$$

Why $z \gg 10^3$ is suppressed?

*Last scattering is essential.



For observations

*We don't know **d** at the beginning.
→ $\langle y_{\ell m} \rangle \propto Y_{\ell m}(\mathbf{d})$ is coordinate dependent.

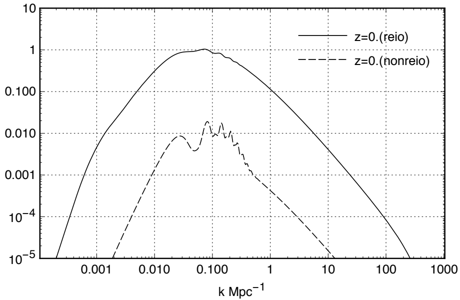
$$\alpha_\ell \equiv \frac{1}{4\pi} \sqrt{\frac{4\pi}{2\ell+1} \sum_{m=-\ell}^{\ell} |\langle y_{\ell m} \rangle|^2}$$

*For $L = 2$,

- With reionization: $\alpha_2 = 6.8|A_2| \times 10^{-9}$
- Without reionization: $\alpha_2^{\mathrm{NR}} = 6.0|A_2| \times 10^{-11}$

Quadrupole scale dependence

*$d\alpha_2/d\ln k$ in Fourier space in units of $|A_2|A_\zeta$



- kMpc < 0.01: Long modes
- $1 < k$Mpc: Silk damping exponentially suppresses

Extension to higher spin fields

*Extension to higher L is straight forward. For $L = 4$,

$$\langle y_{4m} \rangle \Big|_{\eta=\eta_0} = 4\pi Y^*_{4m}(\mathbf{d}) \int_{\eta_i}^{\eta_0} d\eta g \int \frac{dk}{k} \frac{k^3}{2\pi^2} A_4 P_\zeta \left[\frac{5}{7} \Theta_2^2 + \cdots \right],$$

*The amplitude is

$$\alpha_4 = 1.2|A_4| \times 10^{-11}.$$

Preferred direction reconstruction

Once we get α_ℓ, we can reconstruct the spherical harmonics as

$$Y_{\ell m} = (4\pi)^{-1} \langle y_{\ell m} \rangle \alpha_\ell^{-1}$$

*We can find the direction **d**.

Summary

1. Test of SI = Test of particle contents during inflation.
2. y-distortion is a NLO correction to the local BB of the CMB.
3. I computed $\langle y_{2m} \rangle$. This is zero for SI but nonzero for L=2 SA.
4. Easy to extend to higher SA.

Future dirrections

1. Astrophysical contaminations
   e.g.) SZ effects from galaxies, halos
2. Forecast for actual observational projects, like PIXIE or PRISM.

Cosmic variance

*Cosmic variance?

$$\frac{\delta\alpha_\ell}{\alpha_\ell} \stackrel{?}{=} \sqrt{\frac{2}{2\ell+1}}$$

*For, $\ell = 2$, observed y_{2m} are 5.
*Huge cosmic variance like C_ℓ?

$$\langle |a_{\ell m}^{\text{theory}}|^2 \rangle \leftrightarrow \frac{1}{2\ell+1} \sum_{m=-\ell}^{\ell} |a_{\ell m}^{\text{obs.}}|^2$$

Cosmic variance for nonlinear observables

*For linear observables

$$\left(\frac{\delta T}{T}\right)_{\mathbf{k}} \sim \zeta_{\mathbf{k}}$$

*# of observables = # of samples

Cosmic variance for nonlinear observables

*For nonlinear observables

$$y_{\mathbf{k}} \sim \int \frac{d^3k_1 d^3k_2}{(2\pi)^6}(2\pi)^3\delta^{(3)}(\mathbf{k}-\mathbf{k}_1-\mathbf{k}_2)\mathcal{W}_y(\mathbf{k}_1,\mathbf{k}_2)\zeta_{\mathbf{k}_1}\zeta_{\mathbf{k}_2}$$

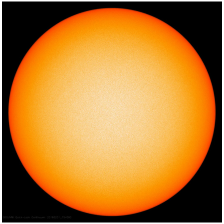
*$\mathcal{W}_y$ is open around $k\mathrm{Mpc} = 0.1$.
*Present horizon scale $k\mathrm{Mpc} = 3000$

# of samples=$(3000 \cdot 0.1)^2 \sim 10^5$, if it's isotropic.

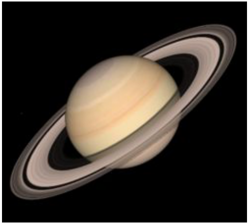
[Pajer Zaldarriaga 2013]

$$\langle y^{\text{theory, iso.}}\rangle = \int \frac{d\mathbf{n}}{4\pi} y^{\text{obs.}}(\mathbf{n}).$$

Cosmic variance



$SO(3)$



$SO(2)$

*We identify only the different azimuthal angle ϕ as the different quantum realization.

$$\# \text{ of samples} \sim 3000 \cdot 0.1 = 300.$$

*Cosmic variance is its square root

$$\delta\alpha_2/\alpha_2 \text{ is typically } \sim 10\%.$$