

# **Vertex algebras, factorization algebras and applications**

**Tuesday 17 July 2018 - Saturday 21 July 2018**

**Lecture Hall(1F), Kavli IPMU**

## **Programme**

## Vertex Algebras, Factorization Algebras and Applications Schedule

### ---Tuesday, July 17 ---

10:00-11:00

Sergei Gukov (Caltech U)

11:30-12:30

Boris Feigin (Higher School of Economics, Moscow / Kyoto U)

14:30-15:30

Emily Cliff (Univ. of Illinois at Urbana-Champaign)

16:00-17:00

Yakov Kremnitzer (Univ. of Oxford)

### ---Wednesday, July 18 ---

10:00-11:00

Tomoyuki Arakawa (RIMS, Kyoto U)

11:30-12:30

Andrew Linshaw (Denver U)

14:30-15:30

Alexander Braverman (U of Toronto)

16:00-17:00

Thomas Creutzig (U of Alberta)

### ---Thursday, July 19 ---

09:30-10:30

Toshiro Kuwabara (Tsukuba U)

11:00-12:00

Kazuya Kawasetsu □U of Melbourne)

14:00-15:00

Quoc Ho (Institute of Science and Technology, Austria)

15:30-17:00

<<MS Seminar>> Sam Raskin (U of Texas at Austin)

### ---Friday, July 20 ---

09:30-10:30

Takahiro Nishinaka (Ritsumeikan U)

11:00-12:00

Benjamin Hennion (U of Paris-Sud)

14:00-15:00

Dennis Gaitsgory (Harvard U)

15:30-16:00

Lin Chen (Harvard U)

16:00-16:30

David Yang (Harvard U)

16:30-17:00

Yifei Zhao (Harvard U)

**---Saturday, July 21---**

09:30-10:30

Eric Vasserot (U of Paris Diderot)

10:45-11:45

Vadim Schechtman (U of Toulouse)

12:00-13:00

John Francis (Northwestern U)

## **"Gluing manifolds and Vertex Algebras" by Sergei Gukov**

Given a smooth 4-manifold  $M_4$ , what algebras act on (generalized) cohomology of instantons on  $M_4$ ? One can tackle this question either directly or take an equivalent route suggested by the (conjectural) existence of super-conformal theory in six dimensions. The latter does not have enough supersymmetry to be defined on an arbitrary 6-manifold, but can be made fully topological on a 4-manifold and holomorphic in the resulting two dimensions. Such topological+holomorphic twist of 6d fivebrane theory leads to a large class of vertex algebras labeled by smooth 4-manifolds, in such a way that different cutting and gluing operations on 4-manifolds lead to their counterparts in  $\text{VOA}[M_4]$  and equivalent ways of constructing 4-manifolds manifest themselves as equivalences of VOAs. This talk is based on the recent and ongoing work with Boris Feigin.

## **"Vertex algebras describing the blowing-up of the 4-manifold" by Boris Feigin**

We describe some strange class of vertex algebras and formulate conjectures about the corresponding tensor categories of representations of quantum groups.

## **"Universal chiral algebras and universal factorization algebras" by Emily Cliff**

In this talk we study bridges between the different perspectives offered by vertex algebras, chiral algebras, and factorization algebras. I introduce a notion of étale pullback for factorization spaces and algebras, which allows me to define categories of universal factorization spaces/algebras in any dimension. These families of factorization algebras are equivalent to universal chiral algebras in the same dimension. In particular, when working over curves, both notions are equivalent to quasi-conformal vertex algebras. I discuss examples of universal families of dimension one already appearing in the literature, and also new examples in higher dimensions coming from Hilbert schemes.

## **"Differentiable Chiral Algebras " by Yakov Kremnitzer**

I will explain how to define O-modules and D-modules in the setting of differentiable pre-stacks. Using this I will describe a theory of differentiable chiral and factorization algebras and how to use a differentiable version of the Beilinson-Drinfeld Grassmannian in order to construct chiral algebras. This is joint work with Dennis Borisov and Jack Kelly.

## **"Higgs branch conjecture for class S theory" by Tomoyuki Arakawa**

Rastelli et. al has constructed a map from 4d  $N=2$  SCFTs to VOAs in such a way that the Schur index of a 4d  $N=S$  SCFT coincides with the character of the corresponding VOA. Rastelli and Beem have further conjectured that the Higgs branch of a 4d  $N=2$  SCFT should coincide with the associated variety of the corresponding VOA.

In my talk we confirm the conjecture of Rastelli and Beem for the class S theory.

## **"Vertex algebras defined over commutative rings and $W_{\infty}$ -algebras" by Andrew Linshaw**

I will discuss vertex algebras defined over commutative rings, and as special cases the universal  $W_{\infty}$ -algebras of types  $W(2,3,4,\dots)$  and  $W(2,4,6,\dots)$ , which are defined over the polynomial ring in two variables. The existence and uniqueness of these algebras was conjectured in the physics literature, and was recently established in my papers arXiv:1710.02275 and arXiv:1805.11031 (joint with S. Kanade). All one-parameter vertex algebras of type  $W(2,3,\dots,N)$  or  $W(2,4,\dots,2N)$  for some  $N$  satisfying some mild hypotheses, can be obtained as quotients of these algebras. This includes the principal  $W$ -algebras of types A, B, and C, as well as many others arising as cosets of affine vertex algebras inside larger structures. Each of these one-parameter vertex algebras corresponds to a certain curve in the plane, and the intersection points of these curves give rise to nontrivial

isomorphisms between these vertex algebras. Finally, we will describe some remarkable infinite families of such curves whose singular points and pairwise intersection points are all rational.

## **"Factorisation categories and 3-dimensional quantum field theories" by Alexander Braverman**

The purpose of this talk is to present a suggestion describing the (factorisation) categories of line operators in a wide class of topologically twisted 3d  $N=4$  quantum field theories (all the relevant notions will be discussed) and to discuss some interesting equivalences of categories which this proposal together with 3-dimensional mirror symmetry implies. Connections to local geometric Langlands correspondence and to series of my joint works with Finkelberg and Nakajima about Coulomb branches of 3d  $N=4$  gauge theories will be discussed.

## **"Equivalences at admissible level" by Thomas Creutzig**

Equivalences of braided tensor categories of modules of  $W$ -algebras of simply-laced Lie algebras at admissible levels can be deduced using what we call a  $W$ -algebra translation functor together with a coset realization of the  $W$ -algebra. I want to explain how this goes and how the findings relate to physics and quantum geometric Langlands.

## **"Vertex algebras associated with hypertoric varieties" by Toshiro Kuwabara**

Hypertoric varieties are known as an example of conical symplectic singularities and their resolutions. Using BRST reduction, we construct a sheaf of ( $\hbar$ -adic) vertex algebras over a family of Poisson deformations of a hypertoric variety. A conformal vector is constructed explicitly, and we obtain a vertex operator algebra as a vertex algebra of global sections.

As certain special cases, the construction gives localization of affine  $W$ -algebras of subregular type  $A$  of level  $-N+1$ , and one of simple affine VOA of type  $A$  of level  $-1$ .

## **"Relaxed highest-weight modules over affine vertex operator algebras" by Kazuya Kawasetsu**

In this talk, we classify and compute characters of  $N$ -gradable simple weight modules over non-integrable affine vertex operator algebras using theory of relaxed highest-weight modules and Mathieu's coherent families. The results have significant applications in the Creutzig-Ridout Verlinde formula of non-integrable affine vertex operator algebras. As an example, we compute (Grothendieck) fusion rules for affine  $\mathfrak{sl}(3)$  vertex operator algebra of level  $-3/2$  using the Verlinde type formula.

This is based on joint works with David Ridout and Simon Wood.

## **"Densities and stability via factorization homology" by Quoc Ho**

Using factorization homology, we develop a uniform and conceptual approach for treating homological stability, homological densities, and arithmetic densities for configuration spaces (and generalizations thereof) in algebraic geometry. This categorifies and generalizes the coincidences appearing in the work of Farb-Wolfson-Wood, and in fact, provides a conceptual understanding of these coincidences. Our computation of the stable homological densities also yields rational homotopy types which answer a question posed by Vakil-Wood. Our approach hinges on the study of homological stability of cohomological Chevalley complexes, which is of independent interest.

## **"Affine Beilinson-Bernstein at the critical level for $GL_2$ " by Sam Raskin**

There has long been interest in Beilinson-Bernstein localization for the affine Grassmannian (or affine flag variety). First, Kashiwara-Tanisaki treated the so-called negative level case in the 90's. Some ten years later, Frenkel-Gaitsgory (following work of Beilinson-Drinfeld and Feigin-Frenkel) formulated a conjecture at the critical level and made some progress on it. Their conjecture is more subtle than its negative level counterpart, but also more satisfying.

We will review the necessary background from representation theory of Kac-Moody algebras at critical level, formulate the Frenkel-Gaitsgory conjecture, and outline a proof for  $GL_2$ .

## **"Gelfand--Fuks cohomology for algebraic varieties" by Benjamin Hennion**

This is joint work with Mikhail Kapranov and Anton Koroshkin.

Given a smooth affine algebraic variety over the complex numbers, we prove that the Chevalley-Eilenberg cohomology of its Lie algebra of global vector fields is a topological invariant of the underlying complex manifold and is finite dimensional in every degree. The proof uses methods from factorization homology.

In this talk, we will first explain the case of smooth real manifolds as studied in the 70's (Gelfand, Fuks, Bott--Segal, Haefliger, Guillemin, ...). We will show how to transpose those methods to complex algebraic varieties.

## **" $C_2$ -cofinite VOAs from BRST reduction" by Takahiro Nishinaka**

I will talk about an infinite series of conjecturally  $C_2$ -cofinite conformal VOAs with negative central charge, which are obtained by a BRST reduction of the tensor product of simple affine vertex

algebras. The simplest example in the series is conjectured to be isomorphic to one of the doublet algebras studied by B. Feigin, E. Feigin and I. Tipunin. I will also give a conjectural formula for their characters. All these conjectures arise from the study of Argyres-Douglas theories in physics.

## "The factorization algebra that encodes the quantum group" by Dennis Gaitsgory

The fundamental local equivalence for quantum geometric Langlands is a conjecture that states that the Kazhdan-Lusztig category (for a group  $G$  at level  $\kappa$ ) is equivalent to the twisted Whittaker category on the affine Grassmannian (for the Langlands dual group  $G^L$  and the dual level  $\kappa^L$ ). Since the relationship between  $G$  and  $G^L$  is expressed combinatorially, in order to prove this conjecture one has to introduce a combinatorial object that encodes (or at least approximates) both categories. In this talk we will describe such combinatorial object. It comes in the guise of a factorization algebra, denoted  $\Omega_q$ . This factorization algebra has many remarkable features. On the one hand, it encodes the quantum group (attached to  $G$ , with quantum parameter  $q$  expressible in terms of  $\kappa$ ): namely  $U_q(\mathfrak{n}^+)^{\text{Lus}}$  identifies with hyperbolic cohomology of  $\Omega_q$ . On the other hand,  $\Omega_q$ , viewed as a geometric object, can be described explicitly in terms of the Cartan matrix and  $\kappa$ , which makes it amenable for quantum Langlands type comparison.

## "Semi-infinite cohomology vs quantum group cohomology" by Lin Chen

Via the Kazhdan-Lusztig equivalence, we relate various semi-infinite cohomology functors to cohomology of various versions of quantum Borels.

## "The master chiral algebra and Langlands duality" by David Yang

Following Gaitsgory, we define the master chiral algebra and discuss some evidence that it provides the equivalence expected in geometric Langlands.

## "A relation between K-theory and factorization structures" by Yifei Zhao

In this talk, we will explain an equivalence between two kinds of data attached to a smooth algebraic curve and a reductive group. The first is K-theoretic, and the second has to do with the factorization structure of the affine Grassmannian. Both of these data appear in the study of the metaplectic Langlands theory. This is joint work with James Tao, building on a construction of Dennis Gaitsgory.

**by Eric Vasserot**

## **"Three lines and a bialgebra" by Vadim Schechtman**

We will discuss Laplacians and a Lefschetz type decomposition of linear algebra data for perverse sheaves over hyperplane arrangements.

In the factorizable case these objects are closely related to braided bialgebras.

Joint work in progress with M.Kapranov.

**by John Francis**