ILCにおけるトップクォークとゲージ粒子Z/γの異常結合探索手法の開発研究

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導入

ILC (International Linear Collider)とは

トップクォークとは

トップクォークとZ/γの結合

全角度情報を用いた新たな探索手法

本研究の目的

ILC (International Linear Collider)とは

電子陽電子衝突の線型加速器

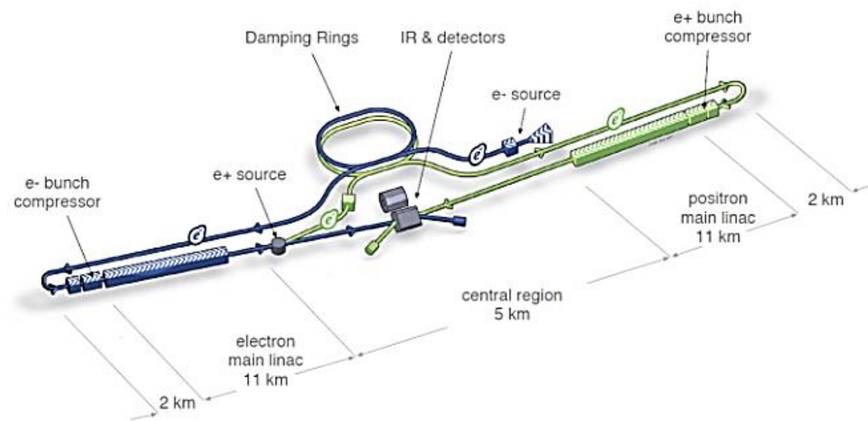
設計値(TDR, 2013)：

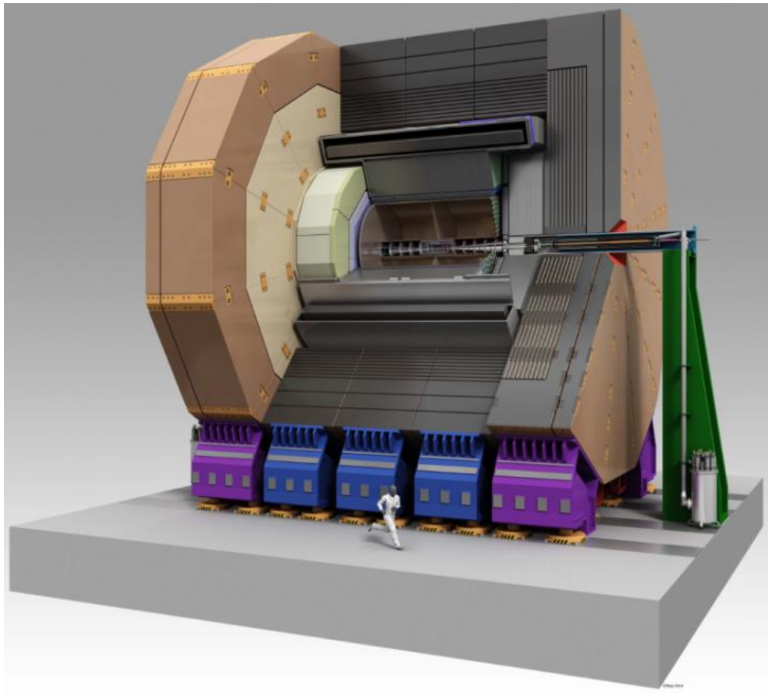
- $\sqrt{s}$ = 250-**500** GeV → 1 TeV
- 全長 : 31 km → 50 km
- ビームの偏極を操作可能

測定器：**ILD**, SiD

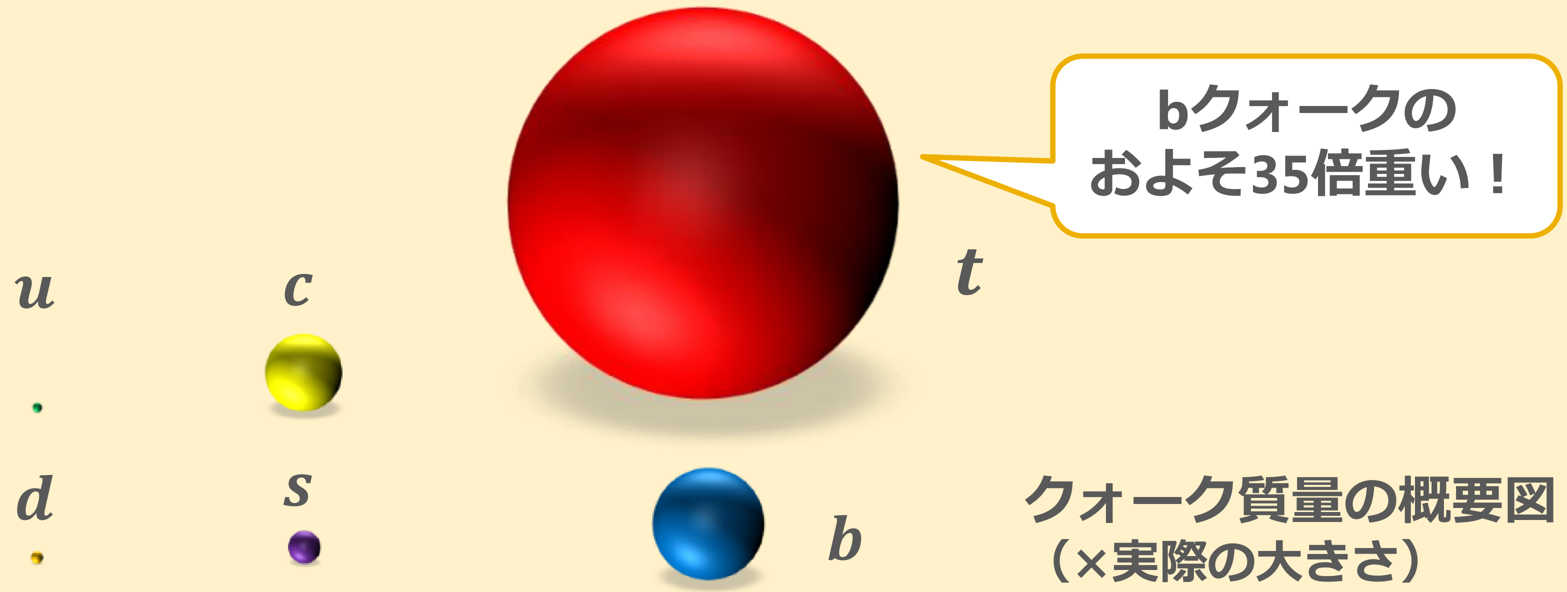
物理目的：

- ヒッグス粒子と**トップクォーク**の精密測定を通じた新物理探索
- 新粒子探索





トップクォークとは



クォーク質量の概要図
（×実際の大きさ）

第三世代のup typeのクォーク

電荷	質量	崩壊幅	寿命
+2/3 e	**~173 GeV**	**~1.5 GeV**	**~10^{-25} s**

- ほぼ100% bW に崩壊
- ハドロン化しない！

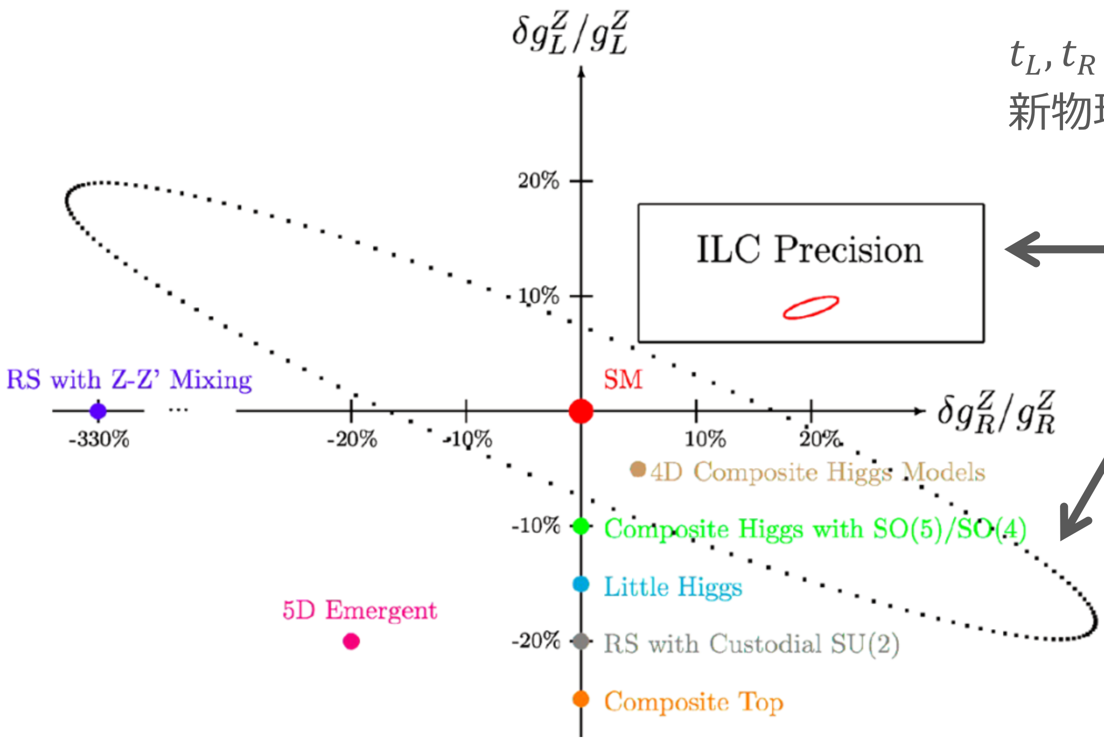
トップクォークとZ/γの結合

トップクォークは他のクォーク・レプトンより非常に重い

→電弱対称性破れの物理と深く関係していると予想される

→トップクォークとZ/γの結合(ttZ/γ結合)に新物理の効果が現れる！

t_L, t_RとZの結合定数についての
新物理模型における標準模型からのずれ

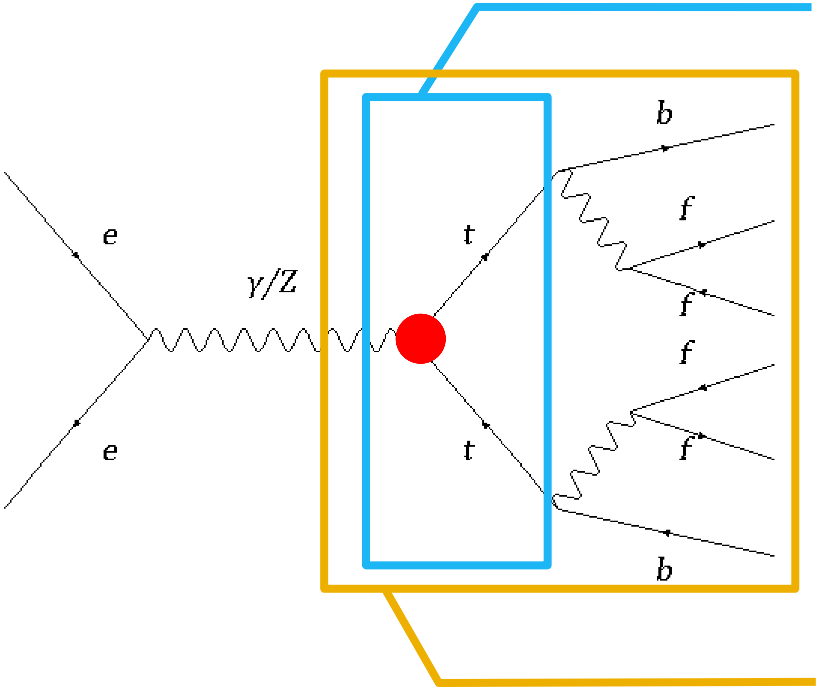


ILC, 500 fb-1
Eur.Phys.J. C75 (2015) no.10, 512

LHC, 3000 fb-1
Phys.Rev.D63 (2006) 034016

LHCの精度の10倍以上！
新物理の証拠を発見可能

全角度情報を用いた新たな探索手法



先行研究では二つの観測量

- σ：全断面積
- A_{FB}：トップの前後非対称度

▌ $e^-e^+ \to t\bar{t}$ 過程に関連した観測量

崩壊過程($t \to bW^+ \to bf\bar{f}$)も

ttZ/γ結合に関する情報を持っている

▌ トップはハドロン化する前に崩壊

▌ 崩壊粒子の角度分布はトップのスピンに依存する

全角度情報を用いることでより高精度の測定が期待できる！

本研究の目的

目的

ILD検出器のフルシミュレーション研究によって、

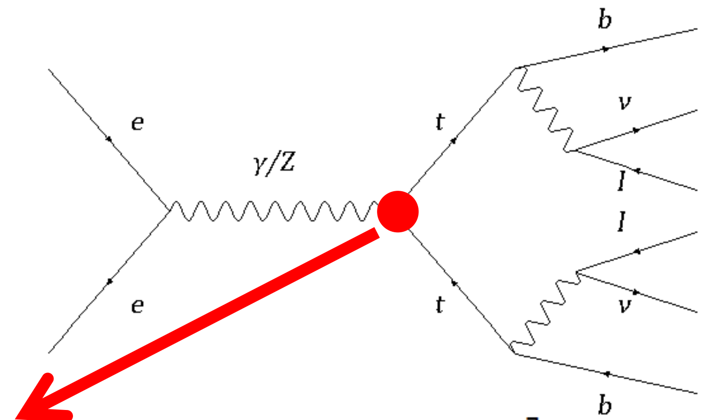
全角度情報を用いたトップクォークとZ/γの異常結合探索手法を開発する

- Di-leptonic 終状態の再構成手法の開発

  $e^-e^+ \to t\bar{t} \to bW^+\bar{b}W^- \to bl^+\nu\bar{b}l^-\bar{\nu}$

  - 最も多くの角度情報が再構成可能

- 全角度情報を用いた解析手法の開発



$$\Gamma_\mu^{ttX}(k^2,q,\bar{q}) = ie\left[\gamma_\mu\left(F_{1V}^X(k^2) + \gamma_5 F_{1A}^X(k^2)\right) + \frac{\sigma_{\mu\nu}}{2m_t}(q+\bar{q})^\nu\left(iF_{2V}^X(k^2) + \gamma_5 F_{2A}^X(k^2)\right)\right]$$

CPC　CPV

10個の形状因子$F^{Z/\gamma}$をパラメータとして測定する。

(結合定数g_L, g_Rとは次の関係；$g_L = F_{1V} - F_{1A}, g_R = F_{1V} + F_{1A}$)

本研究のセットアップ

シミュレーションのセットアップ

信号事象：Di-leptonic 終状態

シミュレーションのセットアップ

イベント生成 : WHIZARD, Pythia

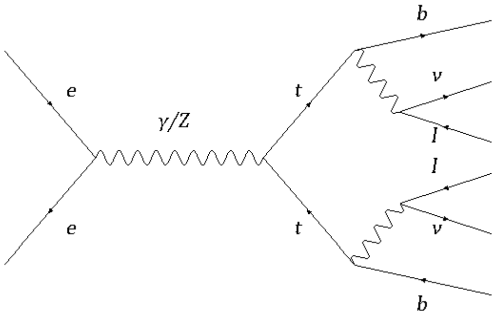
検出器シミュレーション : Mokka, Marlin (TDR, ILD)

シミュレーションのパラメータは TDR (2013)に準拠

重心系エネルギー	$\sqrt{s}$	500 GeV
ビーム偏極	(P_{e^-}, P_{e^+})	(-0.8, +0.3) / (+0.8, -0.3)
積分ルミノシティ	L	250 fb^{-1} / 250 fb^{-1}
トップクォークの質量	m_t	174 GeV
その他の物理パラメータ		SM-LO

信号事象：Di-leptonic 終状態

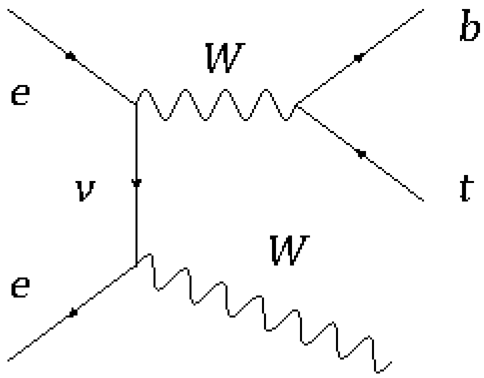
信号事象 : $e^- e^+ \to t\bar{t} \to bW^+ \bar{b} W^- \to b l^+ \nu \bar{b} l^- \nu$



二つのW粒子が $l\nu$ に崩壊する過程（10%程度）

- 二つのニュートリノを含むため再構成が難しい
  → Full simulation による初の解析
- 荷電レプトンの角度情報を用いることができる

主な背景事象



Single top production

- $e^- e^+ \to b l^+ \nu \bar{b} l^- \nu$ (tbW^*など、干渉の効果を考慮)
- $e^- e^+ \to q\bar{q} l^- l^+$ (主に $e^- e^+ \to ZZ \to q\bar{q} l^- l^+$)
- $e^- e^+ \to b l \nu b q q'$ (主に $e^- e^+ \to t\bar{t} \to b l \nu b q q'$)

解析

全角度分布を用いる場合の観測量

解析手法

最適な観測量の導入

Binned FitによるFの測定

先行研究との比較

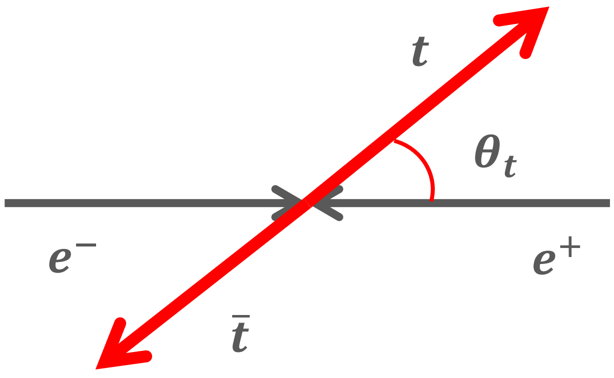
全角度分布を用いる場合の観測量

トップクォークの角度分布のみの場合

θ_t ：e^-e^+静止系の t の角度

$|M(\cos\theta_t)|^2 = A(1+\cos\theta_t)^2 + B(1-\cos\theta_t)^2 + C\sin^2\theta_t$

→ 観測量は最大3つ存在する（A, B, C）



全角度情報の場合

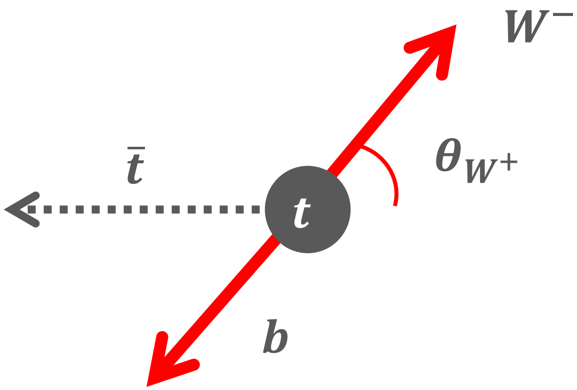
θ_t ：e^-e^+静止系の t の角度

$\theta_{W^\pm}, \phi_{W^\pm}$ ：$t(\bar{t})$ 静止系の$W^\pm$の角度

$\theta_{l^\pm}, \phi_{l^\pm}$ ：$W^\pm$ 静止系の$l^\pm$の角度

$|M(\cos\theta_t, \cos\theta_{W^+}, \phi_{W^+}, \cos\theta_{W^-}, \phi_{W^-}, \cos\theta_{l^+}, \phi_{l^+}, \cos\theta_{l^-}, \phi_{l^-})|^2$

→膨大な観測量が得られる！しかしどうやって扱えば良い？



解析手法

信号事象の角度分布は理想的には以下の式に従う。

$$\left|M(\cos\theta_t, \cos\theta_{W^+}, \phi_{W^+}, \cos\theta_{W^-}, \phi_{W^-}, \cos\theta_{l^+}, \phi_{l^+}, \cos\theta_{l^-}, \phi_{l^-}; \underline{F^{Z/\gamma}})\right|^2$$

解析手法の候補

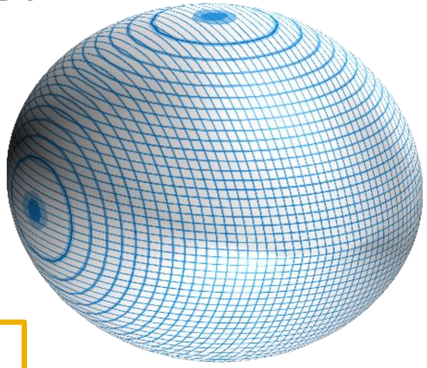
1．$|M|^2$をPDFとしてUnbinned fit

→実験の効果(背景事象・検出器等)によってPDFと合わない ✕

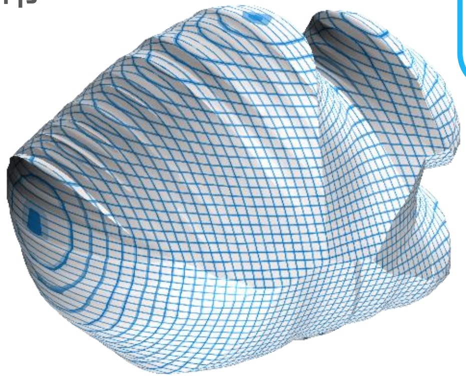
→新しくPDFを定義する必要がある △

- 歪んでいる
- どうやって モデル化する？

理想的なPDF



実際の分布



イメージ図

解析手法

信号事象の角度分布は理想的には以下の式に従う。

$$\left|M(\cos\theta_t, \cos\theta_{W^+}, \phi_{W^+}, \cos\theta_{W^-}, \phi_{W^-}, \cos\theta_{l^+}, \phi_{l^+}, \cos\theta_{l^-}, \phi_{l^-}; \underline{F^{Z/\gamma}})\right|^2$$

解析手法の候補

1．$|M|^2$をPDFとしてUnbinned fit

→実験の効果(背景事象・検出器等)によってPDFと合わない ✕

→新しくPDFを定義する必要がある △

2．9次元の分布$(\cos\theta_t, \cos\theta_{W^+}, \phi_{W^+}, \cos\theta_{W^-}, \phi_{W^-}, \cos\theta_{l^+}, \phi_{l^+}, \cos\theta_{l^-}, \phi_{l^-})$をBinned fit

→高い測定精度のためには大量のビンが必要（2分割でも2^9個） △

解析手法

信号事象の角度分布は理想的には以下の式に従う。

$$\left|M(\cos\theta_t, \cos\theta_{W^+}, \phi_{W^+}, \cos\theta_{W^-}, \phi_{W^-}, \cos\theta_{l^+}, \phi_{l^+}, \cos\theta_{l^-}, \phi_{l^-}; \underline{F^{Z/\gamma}})\right|^2$$

解析手法の候補

1．$|M|^2$をPDFとしてUnbinned fit

→実験の効果(背景事象・検出器等)によってPDFと合わない ×

→新しくPDFを定義する必要がある △

2．9次元の分布$(\cos\theta_t, \cos\theta_{W^+}, \phi_{W^+}, \cos\theta_{W^-}, \phi_{W^-}, \cos\theta_{l^+}, \phi_{l^+}, \cos\theta_{l^-}, \phi_{l^-})$をBinned fit

→高い測定精度のためには大量のビンが必要（2分割でも2^9個） △

3．1次元の分布をBinned fit

→実験の効果を取り入れるのも容易 ○

→どの分布を使うかが重要（$\cos\theta_t$なら先行研究と同じ）

全角度情報を含んだ1次元の分布を導入する！

最適な観測量の導入

確率振幅を$F^{Z/\gamma}$で展開する

$$|M|^2(\Phi;F) = \left(1 + \sum_i \omega_i(\Phi)\delta F_i + \sum_{ij} \tilde{\omega}_{ij}(\Phi)\delta F_i \delta F_j\right)|M^{SM}|^2(\Phi;F^{SM})$$

$$\omega_i = \left.\frac{\partial |M|^2(\Phi)}{\partial F_i}\right|_{\delta F=0} \cdot \frac{1}{|M^{SM}|^2(\Phi)}, \quad \tilde{\omega}_{ij} = \left.\frac{\partial^2 |M|^2(\Phi)}{\partial F_i \partial F_j}\right|_{\delta F=0} \cdot \frac{1}{|M^{SM}|^2(\Phi)}, \quad \delta F_i = F_i - F_i^{SM}$$

Φ は9個の角度のベクトル表記、 F は10個のF_iのベクトル表記

1．の方法(Unbinned fit)の結果はωのみで表される（1パラメータの場合）

$$\delta F_i \simeq \frac{\overline{\omega}_i - \overline{\omega}_i^{SM}}{\overline{\omega_i^2}} \pm \frac{1}{\sqrt{N\overline{\omega_i^2}}}$$

($\overline{\omega}$はωの平均、$\overline{\omega}^{SM}$は標準模型での期待値)

ω_iは全角度情報をF_iの測定に最適に変換した観測量

Binned FitによるFの測定

各F_iについて1パラメータフィット

以下のχ^2を最小にするF_iを求める。

$$\chi^2(\delta F_i) = \sum_{b=1}^{N_{bin}} \left(\frac{n_b^{\mathrm{Data}} - n_b^{\mathrm{Sim.}}(\delta F_i)}{\sqrt{n_b^{\mathrm{Data}}}} \right)^2$$

n_b^{Data} : b番目のビンのデータのイベント数

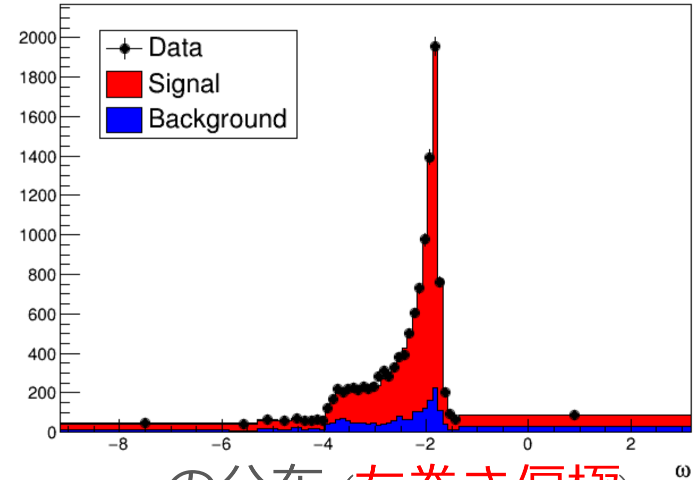
$n_b^{\mathrm{Sim.}}$: b番目のビンのシミュレーションのイベント数

（今回はDataもシミュレーションサンプル）

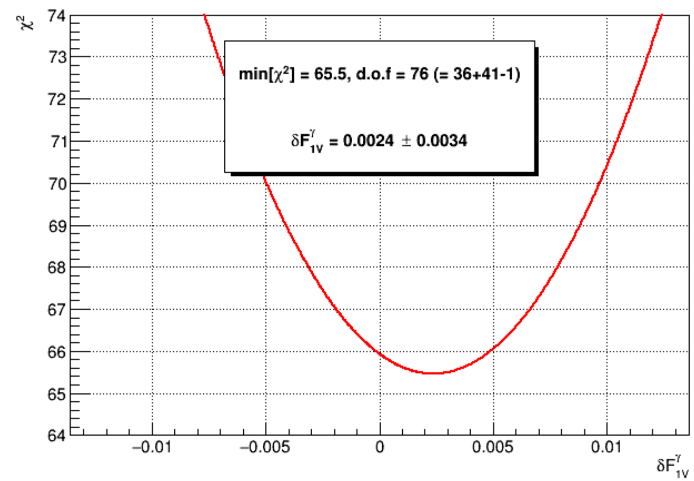
結果： $\delta F_{1V}^{\gamma} = 0.0024 \pm 0.0034\ (\mathrm{CL} = 80\%)$

- 測定結果が入力値に一致
- フィットに成功していることを確認

(例) F_{1V}^{γ} の測定



ω の分布 (左巻き偏極)



$\chi^2(\delta F_{1V}^{\gamma})$ のプロット

先行研究との比較

	This study σ_{stat}	Previous (1) $\sigma_{stat} \times \sqrt{6}$	Previous (1) σ_{stat}
F_{1V}^{γ}	± 0.0034 ←	± 0.0049	± 0.002
F_{1V}^{Z}	± 0.0061 ←	± 0.0073	± 0.003
F_{1A}^{γ}	± 0.0082	---	---
F_{1A}^{Z}	± 0.0133 ←	± 0.0171	± 0.007
F_{2V}^{γ}	± 0.0028	± 0.0024	± 0.001
F_{2V}^{Z}	± 0.0049	± 0.0049	± 0.002

	This study σ_{stat}	Previous (2) $\sigma_{stat} \times \sqrt{6}$	Previous (2) σ_{stat}
ReF_{2A}^{γ}	± 0.012	± 0.012	± 0.005
ReF_{2A}^{Z}	± 0.018	± 0.017	± 0.007
ImF_{2A}^{γ}	± 0.011 ←	± 0.015	± 0.006
ImF_{2A}^{Z}	± 0.019 ←	± 0.024	± 0.010

先行研究と本研究では

信号事象数に6倍程度の差がある

- 信号事象数の差を考慮すると、測定精度を最大10-40%程度改善する可能性
- 本手法を他の終状態に応用することで、精度の向上が期待出来る。

(*) 先行研究は、一部マルチパラメータフィットの結果。ただし、相関による統計誤差への影響は10%以下である。

(1) Eur.Phys.J. C75 (2015) no.10, 512

(2) Eur.Phys.J. C78 (2018) no.2, 155

まとめ

まとめ

まとめ

目的　ILD検出器のフルシミュレーション解析による
全角度情報を用いたttZ/γ異常結合探索手法の開発

- 力学的再構成によってDi-leptonic過程の全終状態を再構成
- ω分布による全角度情報を用いた、形状因子Fの測定手法を開発
- 先行研究の測定精度を最大10-40 %程度改善する可能性

今後の展望

- 他の終状態に本手法の応用
- 複数の形状因子Fのマルチパラメータフィット

Backup

ILC (International Linear Collider)

TDR (Technical Design Report), 2013

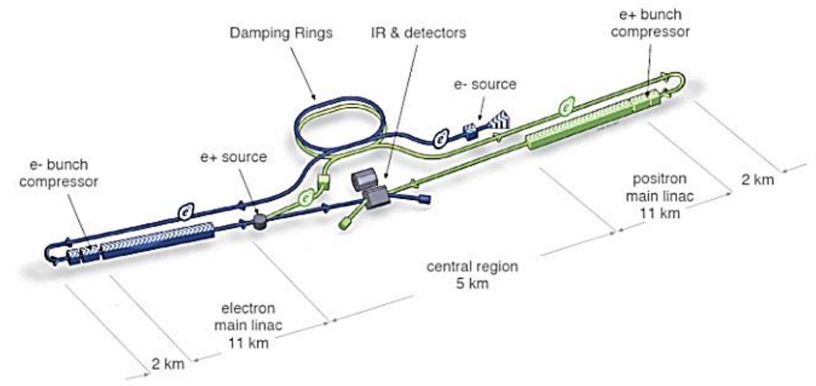
- $\sqrt{s}$ = 250-500 GeV → 1 TeV
- Length : 31 km → 50 km

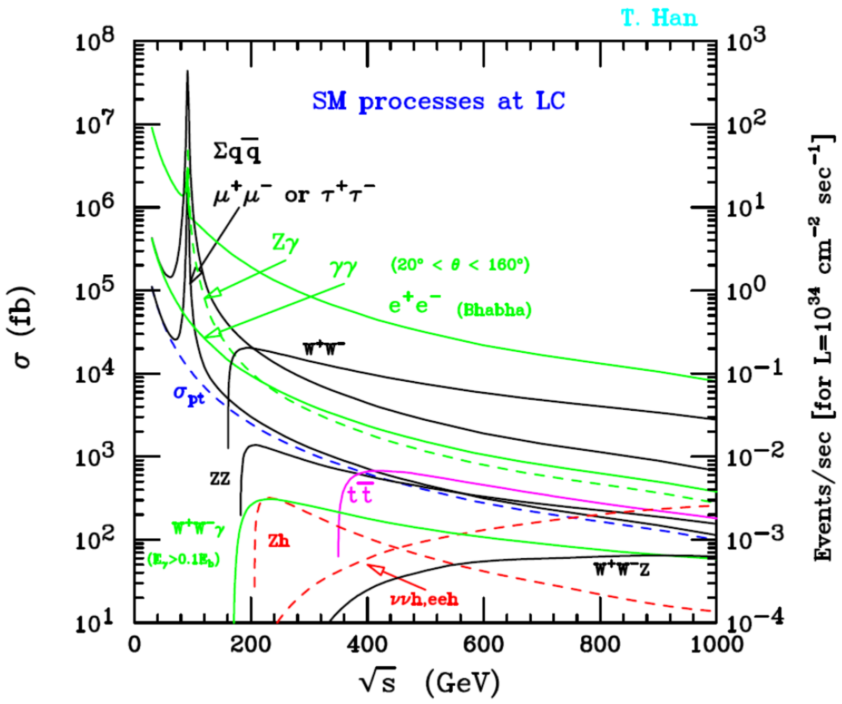
ILC250 (Staging Plan), 2017

- $\sqrt{s}$ = 250 GeV
- Length : 20 km

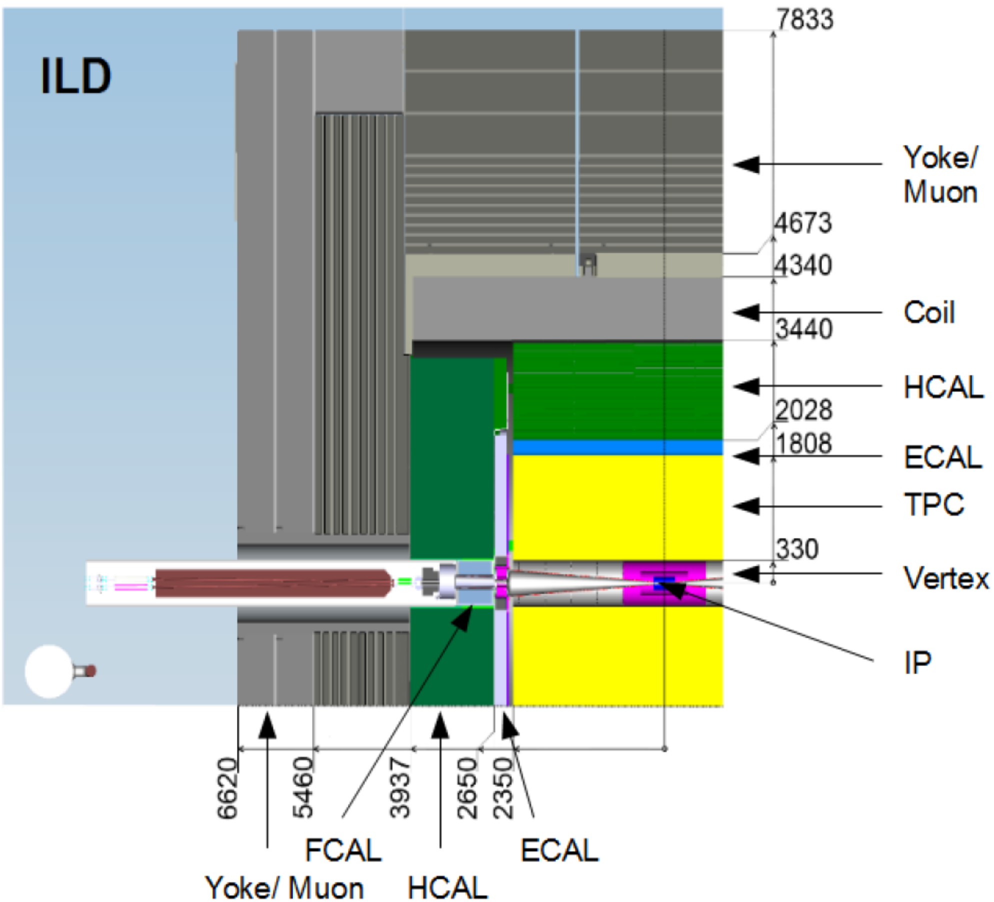
Physics Motivation

- Precise measurement of Higgs boson and Top quark
- New physics search





ILD (International Large Detector)



The ILD is composed of

- Vertex detector
- TPC
- ECAL
- HCAL
- Yoke / Muon detector
- Forward detectors

The reconstruction process uses all aspects of the ILC

Signal Reconstruction

Reconstruction process

Algorithm of the kinematical reconstruction

Combination of mu and b-jet

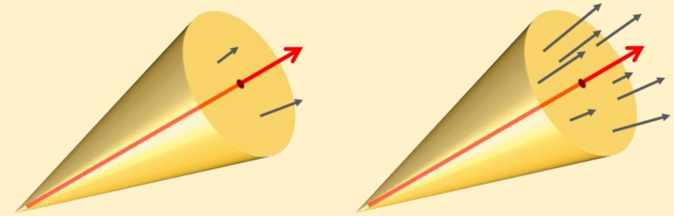
Event selection

Reconstruction Process

Reconstruct all final state particles, $b\bar{b}\mu^-\mu^+\nu\bar{\nu}$.

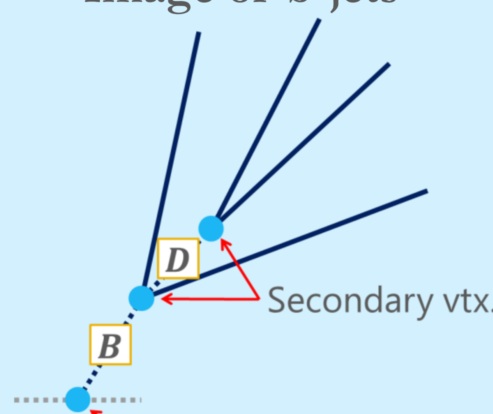
1. Selection of μ^+ and μ^-

- μ^-, μ^+ are isolated from other particles
- Extract isolated muons as final state muons



2. Jet clustering and b-tagging

- Cluster jet particles corresponding to $b, \bar{b}$
- B, D meson moves ~100 μm before the decay
- Assess the "b-likeness" from the vertex information (such as # of vtx. and distance between IP and vtx.)



Reconstruction Process

3. Kinematical Reconstruction

- $\nu, \bar{\nu}$ are not detectable at the ILD detector.
- To recover them, impose the following constraints
  - Initial state constraints : $E_{\text{total}} = 500\ \text{GeV}, \vec{P}_{\text{total}} = \vec{0}\ \text{GeV}$
  - Mass constraints : $m_{t,\bar{t}} = 174\ \text{GeV}, m_{W^{\pm}} = 80.4\ \text{GeV}$
- γ of the ISR/Beamstrahlung deteriorates the initial state condition. Assume the γ is along the beam direction (z-axis).

Unknowns

$\vec{P}_{\nu}, \vec{P}_{\bar{\nu}}, P_{\gamma,z}$: 7

Constraints

$E_{\text{total}}, \vec{P}_{\text{total}},$

$m_t, m_{\bar{t}}, m_{W^+}, m_{W^-}$: 8

Algorithm of the Kinematical Reconstruction

- Introduce 4 free parameters : $\vec{P}_{\nu}, P_{\gamma,z}$

  $\vec{P}_{\bar{\nu}}$ can be computed using the initial momentum constraints

$$\vec{P}_{\bar{\nu}} = -\vec{P}_{\text{vis.}} - \vec{P}_{\nu} - \vec{P}_{\gamma}, \qquad \left(\vec{P}_{\text{vis.}} = \vec{P}_{b} + \vec{P}_{\bar{b}} + \vec{P}_{\mu^+} + \vec{P}_{\mu^-}\right)$$

- Define the likelihood function :

$$L_0\left(\vec{P}_{\nu}, P_{\gamma,z}\right) = BW(m_t)BW(m_{\bar{t}})BW(m_{W^+})BW(m_{W^-})Gaus(E_{\text{total}})$$

- To correct the energy resolution of b-jets, add 2 parameters, $E_b, E_{\bar{b}}$, with the resolution functions to L_0 :

$$L\left(\vec{P}_{\nu}, P_{\gamma,z}, E_b, E_{\bar{b}}\right) = L_0 \times Res(E_b, E_b^{\text{meas.}})Res\left(E_{\bar{b}}, E_{\bar{b}}^{\text{meas.}}\right)$$

  Define $q\left(\vec{P}_{\nu}, P_{\gamma,z}, E_b, E_{\bar{b}}\right) = -2\log L + \text{Const.}$

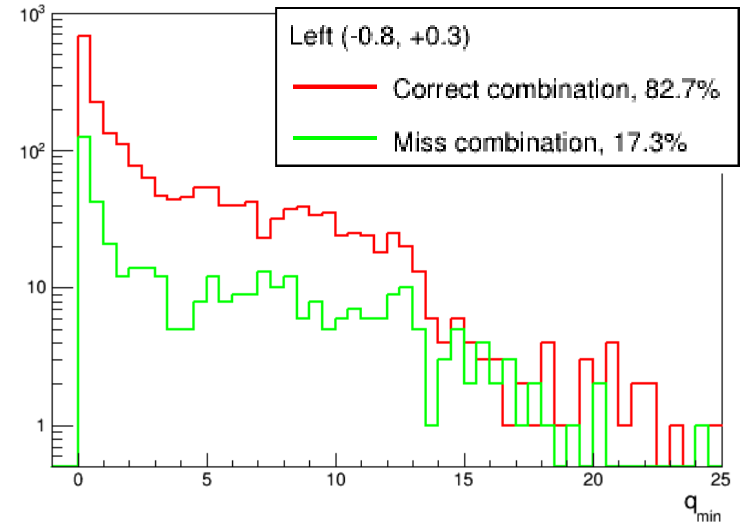
  (scaled as the minimum of each component ($BW(m_t)$, etc) is equal to 0)

Combination of μ and b-jet

Choice of a combination of μ and b-jet

There are two candidates for the combination

- Select one having smaller q, defined as $\boldsymbol{q}_{\mathbf{min}}$
- Fraction of correct combination is ~83%

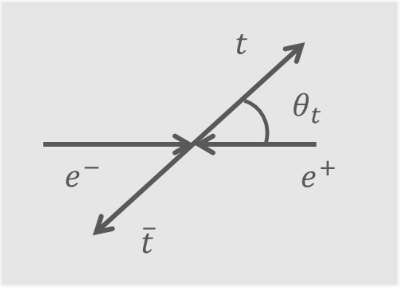


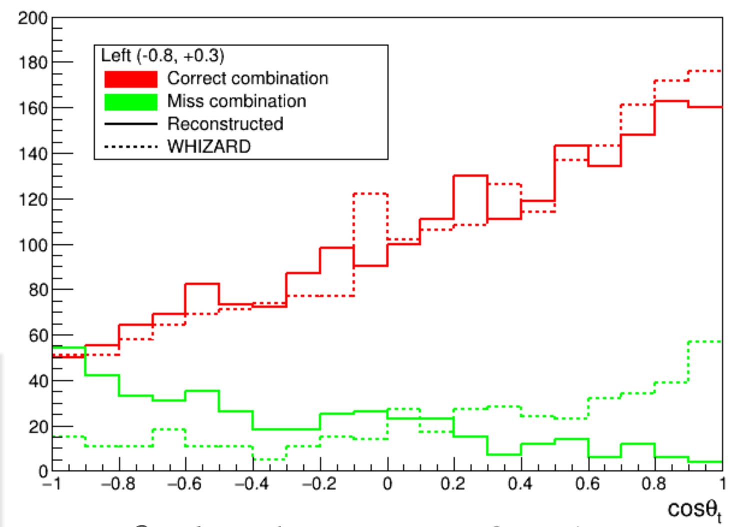
q_{min} distribution (Left polarization)

$\cos\theta_t$ distribution (Rec vs. MC Truth)

- Correct combination: OK !
- Miss combination: Disagree with the MC truth.

Need to estimate an effect of the miss combination for the analysis.





$\cos\theta_t$ distribution (Left polarization)

Event Selection

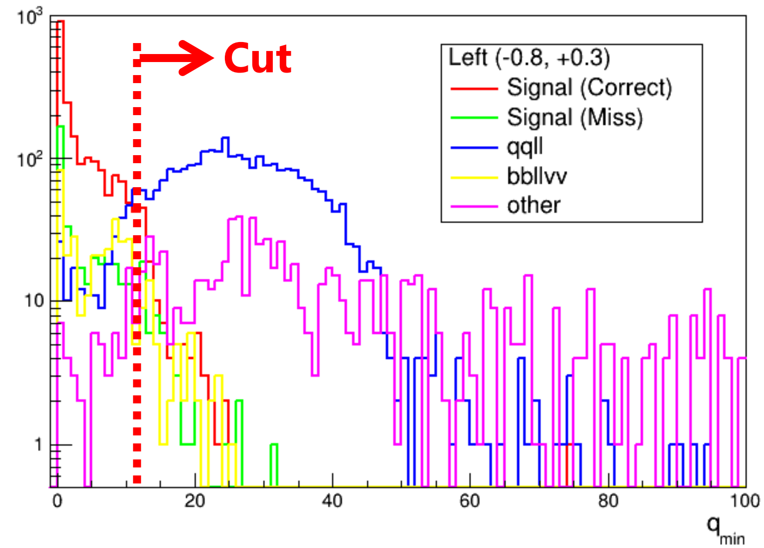
- Quality cut :

$q_{\min}$ means the quality of reconstruction.
Useful to suppress the backgrounds.

Criteria are optimized for the significance,

$$S = \frac{N_{\text{signal}}}{\sqrt{N_{\text{signal}} + N_{\text{background}}}}$$

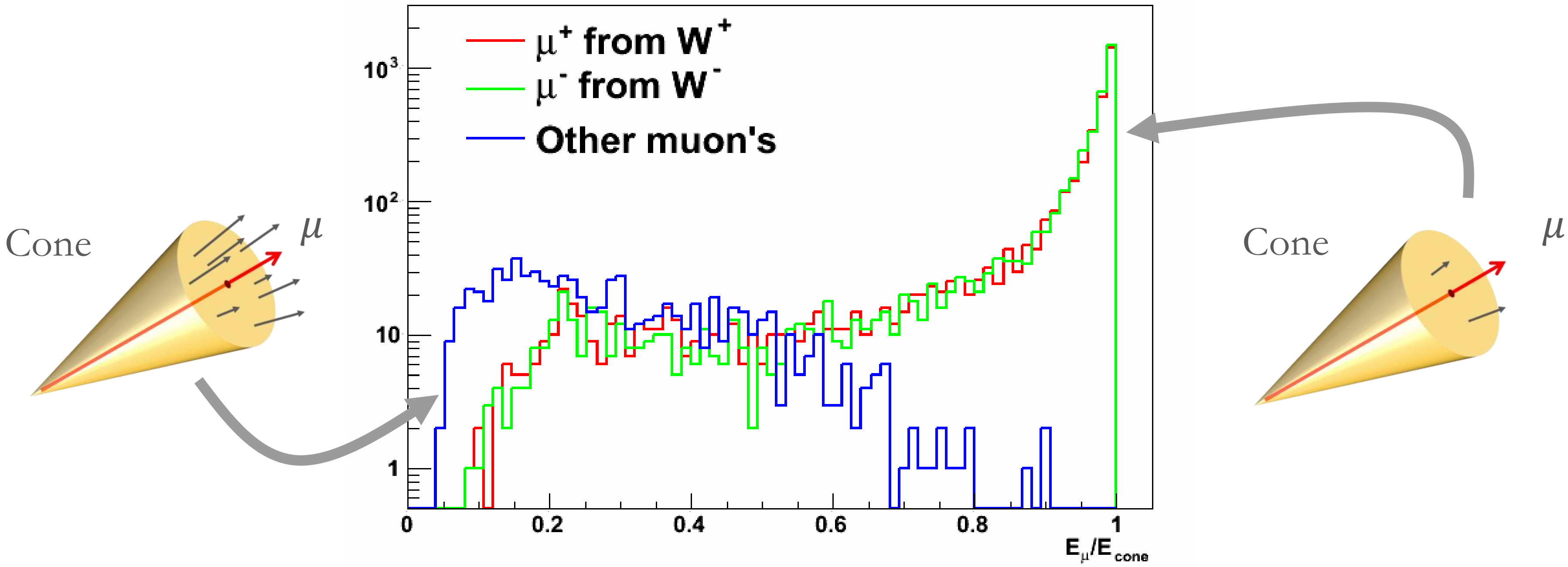
$q_{\min}$ distribution (Left polarization)



Left Polarization **Cut Criteria**	**Signal** $bb\mu\mu\nu\nu$	tt	**except for** tt	**All bkg.**	$qqll$	$bbll\nu\nu$
No cut	2837			8410633	91478	23312
$N_{\mu^-} = 1$ & $N_{\mu^+} = 1$	2618			327488	13827	387
b-tag cut	2489	2215	273	4143	2943	363
Quality cut ($q_{\min} < 11.5$**)**	2396	2103	195	624	258	313

(*) Separate signals into $t\bar{t}$ and the other process from WHIZARD information

Isolated muon finder



Energy ratio between μ and a cone

$R = E_\mu / E_{cone}$ is a quantity to evaluate how isolated the muon is.

(E_{cone} : total energy of particles in the cone)

μ from W boson is more isolated than other μ

Isolated muon finder

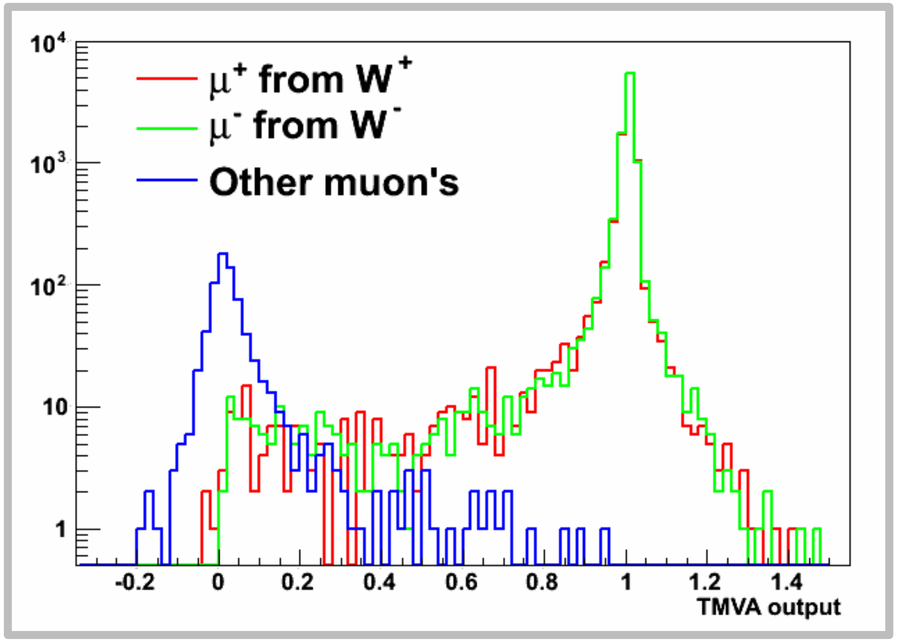
Quantities

$R = E_\mu / E_{cone}$, $E_{cone,neutral}$, $E_{cone,charged}$

$\cos\theta = \frac{P_\mu \cdot P_{cone}}{|P_\mu| \times |P_{cone}|}$, ΔE_{ECAL}, ΔE_{Yoke}, …

TMVA

Multi variable analysis tool



Jet clustering

General strategy

Merge a pair of particles whose "**Distance**" is the smallest until a condition meets "**Criteria**"

"**Distance**"

Durham algorithm : $Y_{ij} = 2\frac{\min\left[E_i^2, E_j^2\right](1-\cos\theta_{ij})}{E_{vis}^2}$, θ_{ij} : angle between P_i and P_j

k_t algorithm : $d_{ij} = \min\left[{p_T}_i^2, {p_t}_j^2\right]\frac{R_{ij}}{R}$ or $d_{iB} = {p_t}_i^2, R_{ij}^2 = (\eta_i - \eta_j)^2 + (\phi_i - \phi_j)^2$

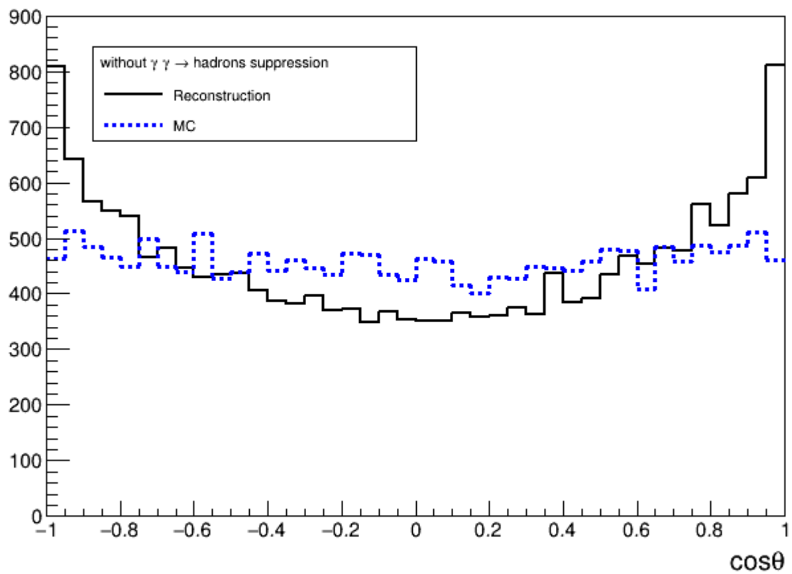
η : pseudo rapidity, ϕ azimuthal angle

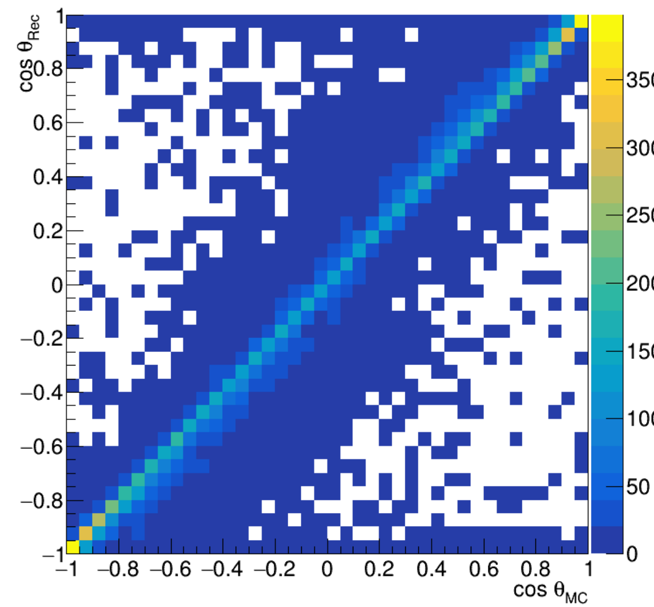
"**Criteria**"

- Number of remaining particles is equal to N_{Req}
- The smallest distance is smaller than D_{Req}

$\gamma\gamma \to$ hadrons rejection

$b, \bar{b}$ are reconstructed from the rest of particles with LCFIPlus





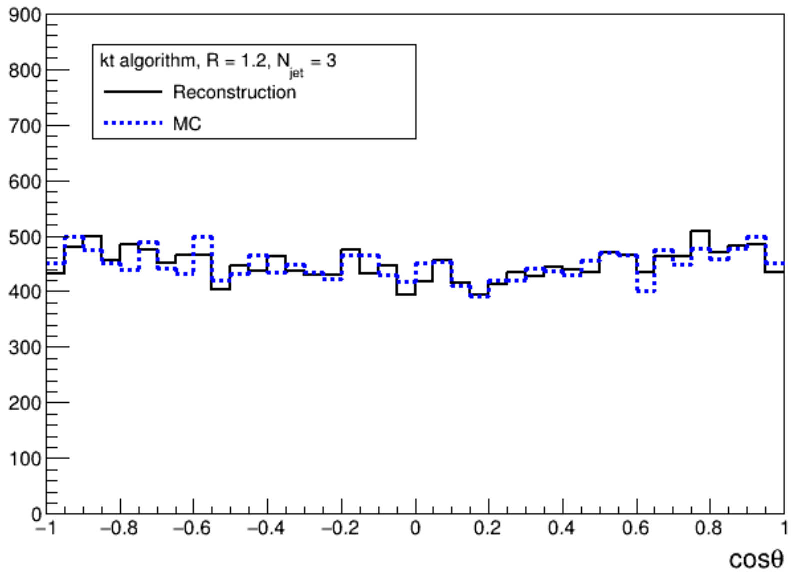
$\cos\theta_{jet}$ distribution

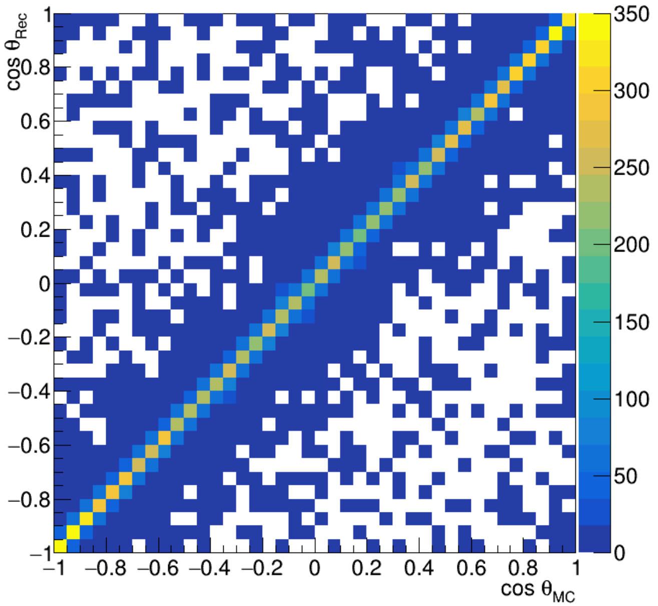
Strongly peaked at very forward region by mistake

$\gamma\gamma \to$ hadrons are emitted along the beam direction

$\gamma\gamma \rightarrow$ hadrons rejection

Eliminate particles close to beam direction rather than other particles with kt algorithm.





$\cos\theta_{jet}$ distribution

Good agreement between Rec and MC

b-tagging with LCFIPlus

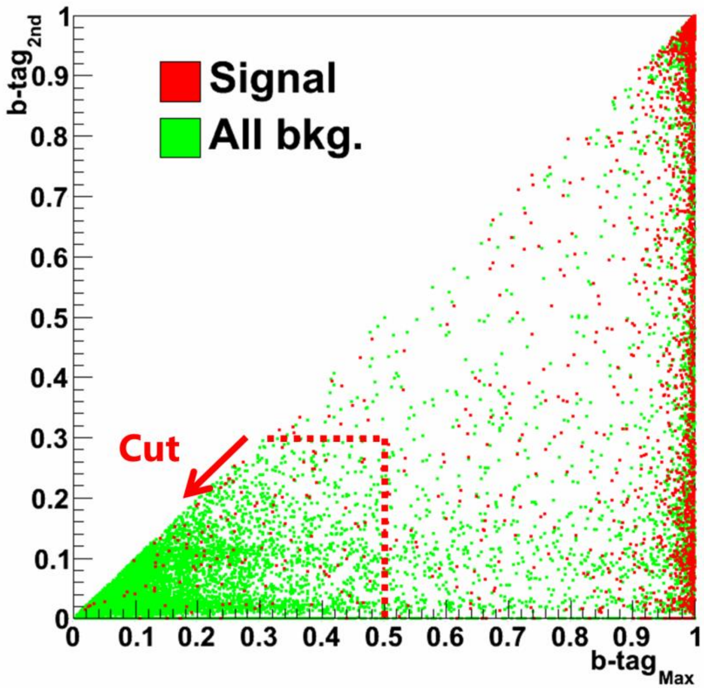
b-tag is TMVA output indicating “b-likeness” of a jet obtained by the LCFIPlus(*).

- $\text{b-tag}_{\text{Max}}$: the largest b-tag
- $\text{b-tag}_{\text{2nd}}$: the 2^{nd} largest b-tag

- Signal has large $\text{b-tag}_{\text{Max}}$
- Many of bkg. have small $\text{b-tag}_{\text{Max}}$ and $\text{b-tag}_{\text{2nd}}$

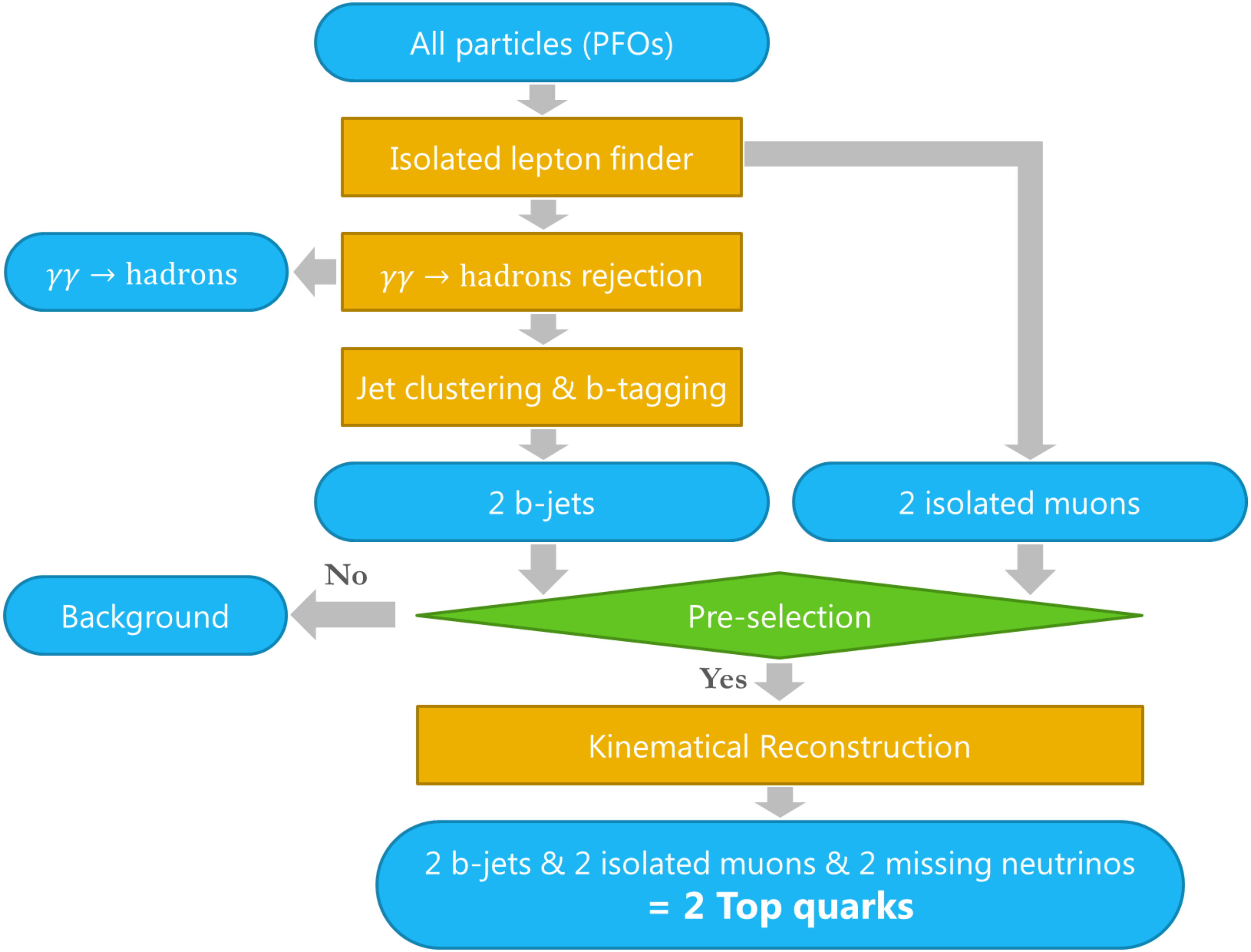
$$b - tag_{Max} > 0.5 \text{ or } b - tag_{2nd} > 0.3$$

(*) A software package of Marlin for the multi-jet analysis.



b-tag value (Left polarization)

Flow of Reconstruction



Kinematical Reconstruction

$$BW(x; m, \Gamma) \propto \frac{1}{1 + \left(\frac{x^2 - m^2}{m\Gamma}\right)^2}$$

$$Gaus(x; \mu, \sigma) \propto \exp\left[-\left(\frac{x - \mu}{\sqrt{2}\sigma}\right)^2\right]$$

Detail definition of L_0 is

$$L_0\left(\vec{P}_\nu, P_{\gamma,Z}\right) = BW(m_t; 174{,}5)BW(m_{\bar{t}}; 174{,}5) \cdot BW(m_{W^+}; 80.4{,}5)BW(m_{W^-}; 80.4{,}5)Gaus(E_{\text{total}}; 500{,}0.39)$$

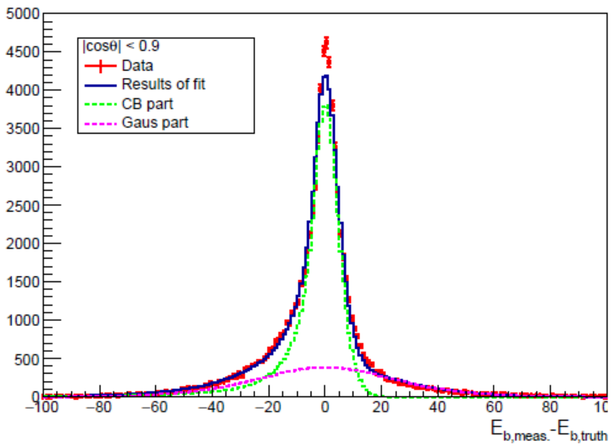
- Larger value for Γ than theoretical value is set because of detector effects
- σ is caused by the Beam energy spread.

Energy resolution of b-jet

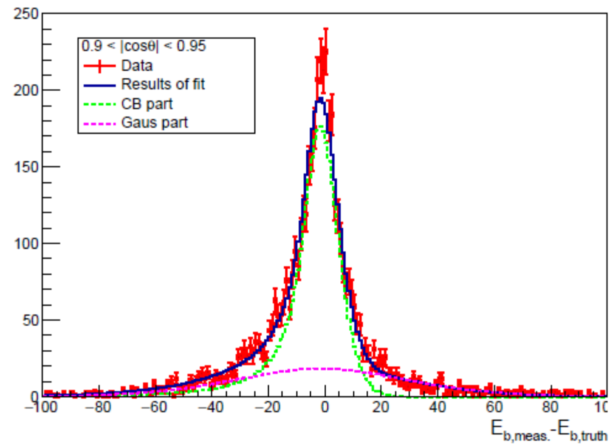
Estimate the energy resolution of b-jet with the following $Res(E_b, E_b^{\mathrm{meas.}})$;

$$Res(E_b, E_b^{\mathrm{meas.}}) = (1-f)CB(\Delta E_b; \alpha, n, \mu_{CB}, \sigma_{CB}) + f * Gaus(\Delta E_b; \mu_{Gaus}, \sigma_{Gaus})$$

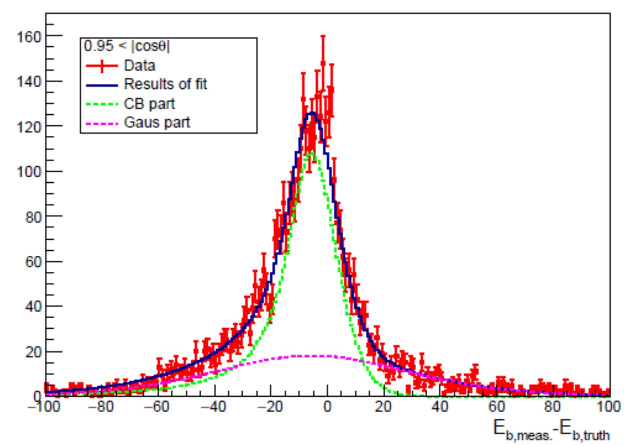
Divide into 3 regions ; $|\cos\theta| = (0, 0.9), (0.9, 0.95), (0.95, 1)$



(a) $|\cos\theta| < 0.9$



(b) $0.9 < |\cos\theta| < 0.95$



(c) $0.95 < |\cos\theta|$

Crystal Ball function

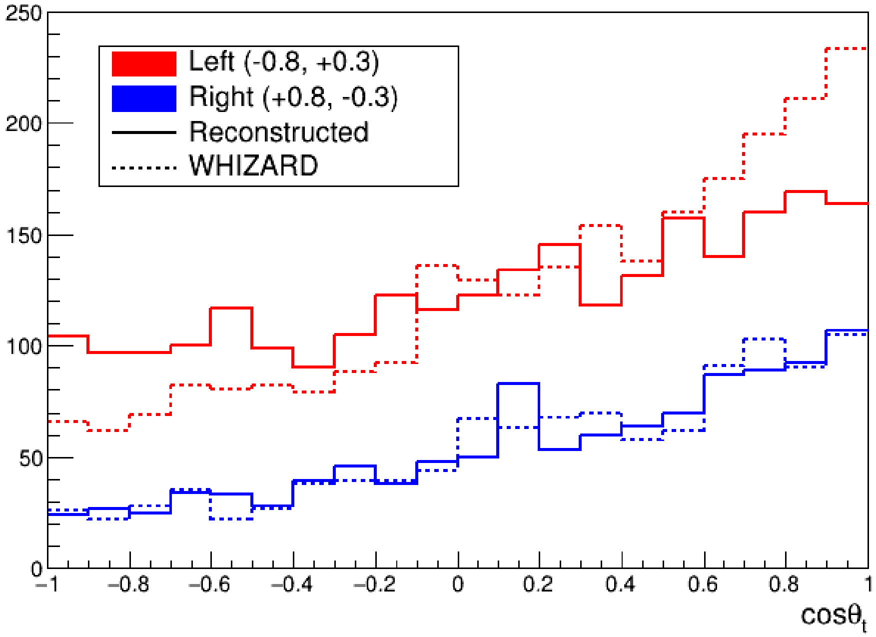
Crystal Ball function consists of a Gaussian core portion and power-law tail.

$$CB(x;\alpha,n,\bar{x},\sigma) = N \cdot \begin{cases} \exp\left(-\frac{(x-\bar{x})^2}{2\sigma^2}\right) & \frac{x-\bar{x}}{\sigma} > -\alpha \\ A \cdot \left(B - \frac{x-\bar{x}}{\sigma}\right)^{-n} & \frac{x-\bar{x}}{\sigma} \leq -\alpha \end{cases}$$

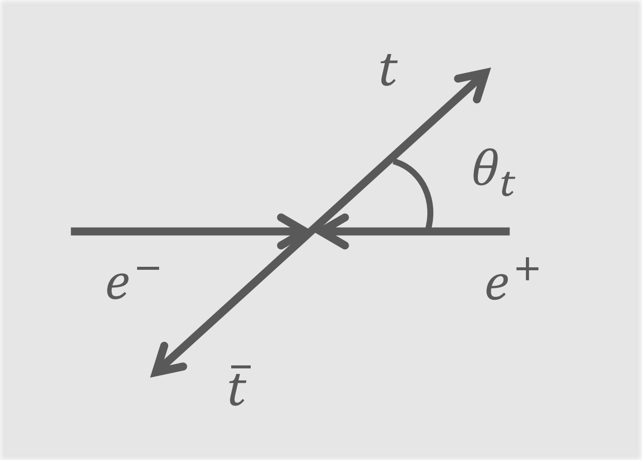
$$\begin{aligned} A &= \left(\frac{n}{|\alpha|}\right)^n \cdot \exp\left(-\frac{|\alpha|^2}{2}\right) \\ B &= \frac{n}{|\alpha|} - |\alpha| \\ N &= \frac{1}{\sigma(C+D)} \\ C &= \frac{n}{|\alpha|} \cdot \frac{1}{n-1} \cdot \exp\left(-\frac{|\alpha|^2}{2}\right) \\ D &= \sqrt{\frac{\pi}{2}}\left(1 + \operatorname{erf}\left(\frac{|\alpha|}{\sqrt{2}}\right)\right) \end{aligned}$$

Results of Reconstruction

Top quark polar angle distribution, $\cos\theta_t$



Definition of θ_t

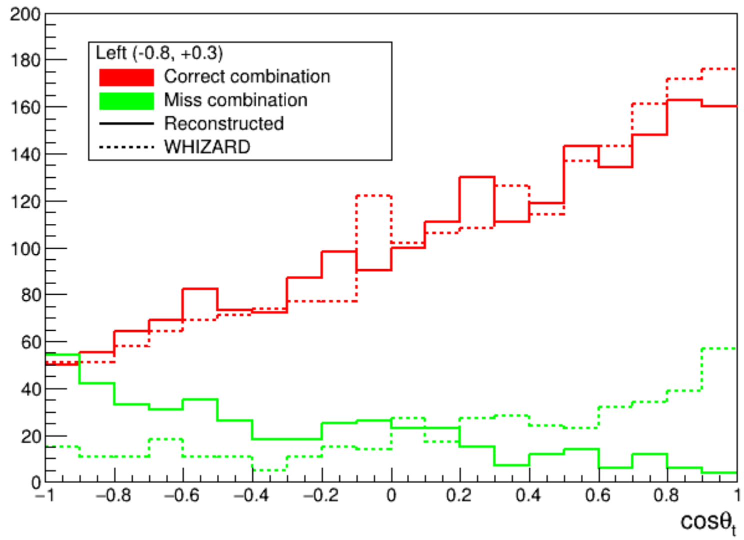


Considerable migration occurs in the Left polarization case

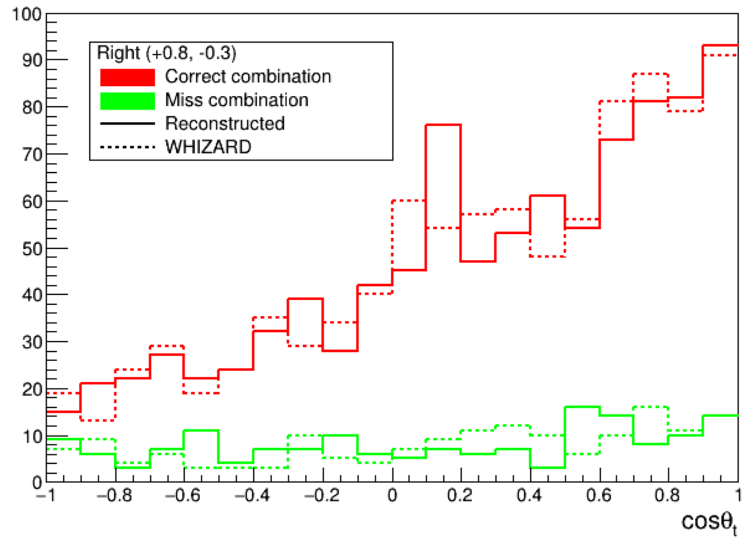
Some events pass from forward to backward because of the miss combination of μ and b-jet.

Dependence from the beam polarization

$\cos\theta_t$ distribution (Left polarization)



$\cos\theta_t$ distribution (Right polarization)



Left polarization

Reconstructed distribution of miss combination is very different from the MC truth.

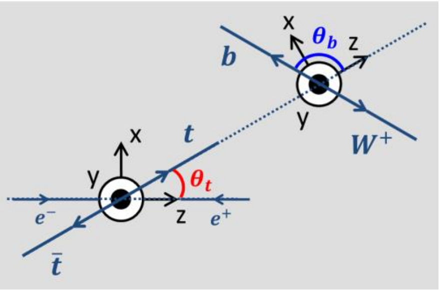
Right polarization

Similar distribution can be reconstructed even when the miss combination is selected.

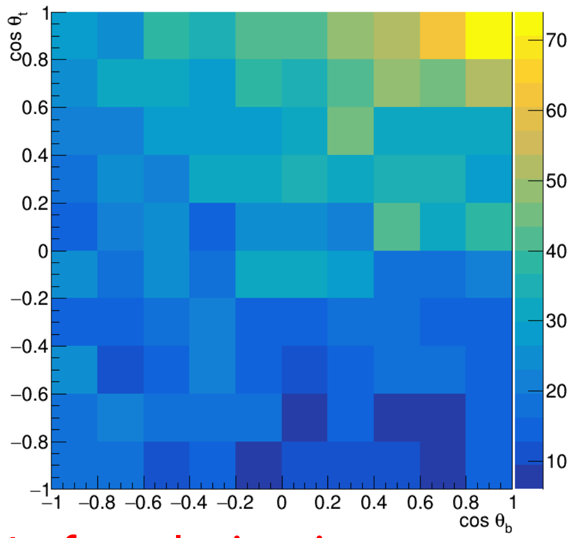
Dependence from the beam polarization

$\cos\theta_b \simeq 1$ → b-jets are energetic

→ **Migration effect is strong**



$\cos\theta_t$ vs. $\cos\theta_b$ (Left polarization)

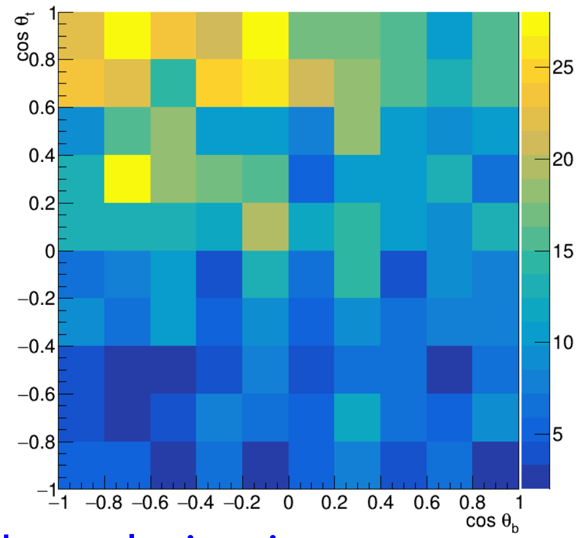


Left polarization

Peak at $\cos\theta_t \simeq 1$ & $\cos\theta_b \simeq 1$

→ Migration is asymmetry

$\cos\theta_t$ vs. $\cos\theta_b$ (Right polarization)



Right polarization

Peak at $\cos\theta_t \simeq 1$ & $\cos\theta_b \simeq -1$

→ Migration is symmetry

Cut table (Right Polarization)

Right Polarization **Cut Criteria**	**Signal** $b b \mu \mu \nu \nu$	$t t$	**except for** $t t$	**All bkg.**	$q q l l$	$b b l l \nu \nu$
No cut	1261			3751175	46344	10117
$N_{\mu^-} = 1$ & $N_{\mu^+} = 1$	1170			230260	6987	189
b-tag cut	1097	1046	79	2118	1468	181
Quality cut ($q_{\min} < 12.5$)	1046	976	70	297	132	151

Criteria of $t\bar{t}$:

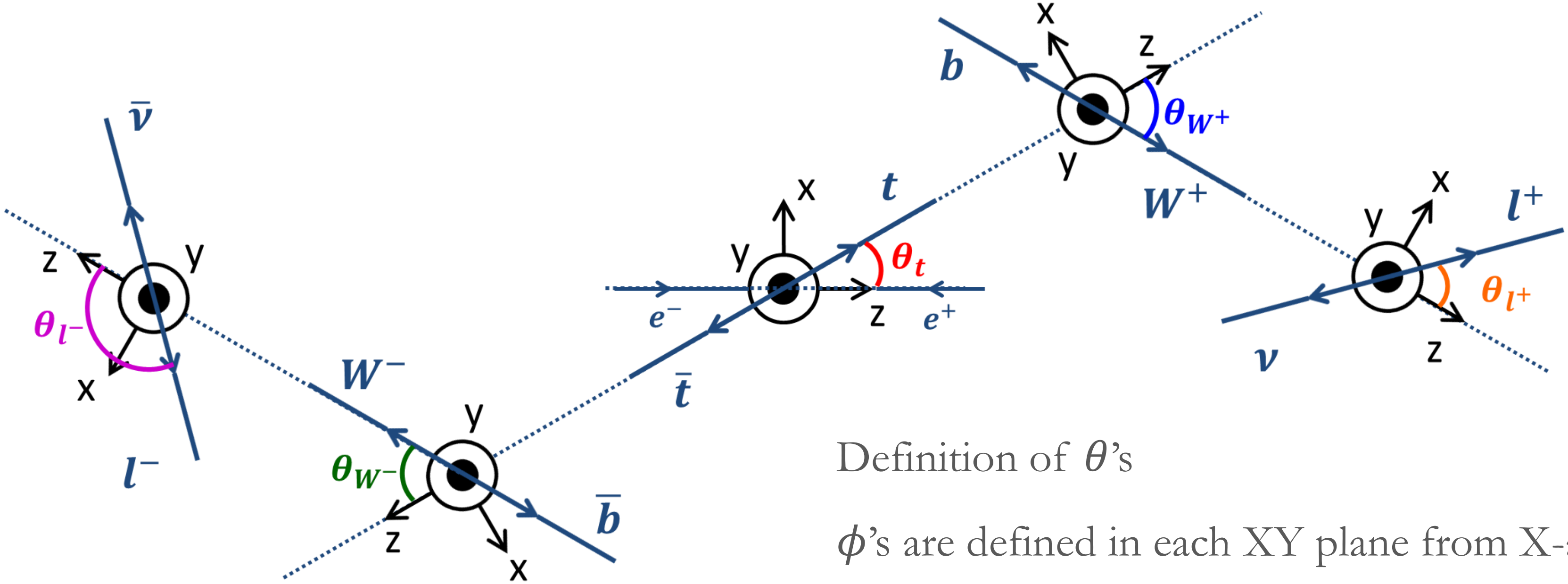
$$\left|M_{b\mu^+\nu} - 174\right| < 15 \,\&\left|M_{\bar{b}\mu^-\bar{\nu}} - 174\right| < 15$$

J. Fuster *et al.* Eur. Phys. J. C **75**, 223 (2015)

The amplitude of the di-leptonic process

The amplitude of the di-leptonic process is a function of 9 angles.

$$|M|^2(\cos\theta_t, \cos\theta_{W^+}, \phi_{W^+}, \cos\theta_{W^-}, \phi_{W^-}, \cos\theta_{l^+}, \phi_{l^+}, \cos\theta_{l^-}, \phi_{l^-}; F)$$



Definition of θ's

ϕ's are defined in each XY plane from X-axis

It is difficult to handle the 9-dimention phase space

→ Expand the amplitude in the form factors, F

Matrix element method

- Based on the unbinned likelihood method. The likelihood function is computed from the amplitude.

  → Full kinematics are used = The most sensitive method in principle.

- Fit results are almost consistent with SM values.
  - ~1.5 σ biases are observed for several form factors

$$
\begin{pmatrix}
\delta\tilde{F}^{\gamma}_{1V,\text{fit}} \\
\delta\tilde{F}^{Z}_{1V,\text{fit}} \\
\delta\tilde{F}^{\gamma}_{1A,\text{fit}} \\
\delta\tilde{F}^{Z}_{1A,\text{fit}} \\
\delta\tilde{F}^{\gamma}_{2V,\text{fit}} \\
\delta\tilde{F}^{Z}_{2V,\text{fit}} \\
\mathcal{R}e\ \delta\tilde{F}^{\gamma}_{2A,\text{fit}} \\
\mathcal{R}e\ \delta\tilde{F}^{Z}_{2A,\text{fit}} \\
\mathcal{I}m\ \delta\tilde{F}^{\gamma}_{2A,\text{fit}} \\
\mathcal{I}m\ \delta\tilde{F}^{Z}_{2A,\text{fit}}
\end{pmatrix}
=
\begin{pmatrix}
+0.0031 \pm 0.0130 \\
-0.0334 \pm 0.0231 \\
-0.0314 \pm 0.0192 \\
+0.0241 \pm 0.0301 \\
-0.0146 \pm 0.0366 \\
-0.0650 \pm 0.0592 \\
+0.0214 \pm 0.0241 \\
-0.0131 \pm 0.0415 \\
-0.0086 \pm 0.0255 \\
+0.0081 \pm 0.0360
\end{pmatrix}
$$

Correlation coefficient for $\tilde{F}$

$$V_C = \begin{pmatrix} +1.000 & -0.141 & +0.027 & +0.085 & \underline{+0.598} & -0.067 & -0.026 & -0.018 & -0.006 & -0.012 \\ -0.141 & +1.000 & +0.093 & +0.066 & -0.028 & \underline{+0.606} & -0.033 & -0.065 & +0.002 & +0.012 \\ +0.027 & +0.093 & +1.000 & -0.082 & +0.012 & +0.034 & -0.038 & -0.096 & -0.024 & -0.013 \\ +0.085 & +0.066 & -0.082 & +1.000 & +0.003 & +0.033 & -0.075 & -0.040 & +0.005 & -0.027 \\ +0.598 & -0.028 & +0.012 & +0.003 & +1.000 & -0.107 & +0.037 & -0.038 & -0.021 & +0.019 \\ -0.067 & +0.606 & +0.034 & +0.033 & -0.107 & +1.000 & -0.064 & +0.006 & +0.013 & -0.020 \\ -0.026 & -0.033 & -0.038 & -0.075 & +0.037 & -0.064 & +1.000 & -0.103 & -0.013 & +0.045 \\ -0.018 & -0.065 & -0.096 & -0.040 & -0.038 & +0.006 & -0.103 & +1.000 & +0.047 & +0.004 \\ -0.006 & +0.002 & -0.024 & +0.005 & -0.021 & +0.013 & -0.013 & +0.047 & +1.000 & -0.074 \\ -0.012 & +0.012 & -0.013 & -0.027 & +0.019 & -0.020 & +0.045 & +0.004 & -0.074 & +1.000 \end{pmatrix}$$

- Correlation coefficient between $\tilde{F}_{1V}^{Z/\gamma}$ and $\tilde{F}_{2V}^{Z/\gamma}$ is about 0.6
- The others are less than 0.15

Correlation coefficient for F

$$
V_C = \begin{pmatrix}
+1.000 & -0.356 & -0.140 & +0.276 & -0.970 & +0.313 & -0.049 & +0.065 & -0.089 & +0.097 \\
-0.356 & +1.000 & +0.173 & -0.215 & +0.281 & -0.971 & +0.053 & -0.038 & +0.113 & -0.066 \\
-0.140 & +0.173 & +1.000 & -0.273 & +0.113 & -0.133 & +0.038 & -0.045 & +0.051 & -0.009 \\
+0.276 & -0.215 & -0.273 & +1.000 & -0.233 & +0.188 & -0.055 & +0.037 & -0.033 & +0.051 \\
-0.970 & +0.281 & +0.113 & -0.233 & +1.000 & -0.254 & +0.046 & -0.063 & +0.099 & -0.104 \\
+0.313 & -0.971 & -0.133 & +0.188 & -0.254 & +1.000 & -0.047 & +0.040 & -0.120 & +0.085 \\
-0.049 & +0.053 & +0.038 & -0.055 & +0.046 & -0.047 & +1.000 & -0.287 & +0.036 & -0.036 \\
+0.065 & -0.038 & -0.045 & +0.037 & -0.063 & +0.040 & -0.287 & +1.000 & -0.059 & +0.024 \\
-0.089 & +0.113 & +0.051 & -0.033 & +0.099 & -0.120 & +0.036 & -0.059 & +1.000 & -0.229 \\
+0.097 & -0.066 & -0.009 & +0.051 & -0.104 & +0.085 & -0.036 & +0.024 & -0.229 & +1.000
\end{pmatrix}
$$

- Correlation coefficient between $F_{1V}^{Z/\gamma}$ and $F_{2V}^{Z/\gamma}$ is about 0.97
- The others are less than 0.36

Goodness of fit for the MEM

Expectation value of ω when the fit results are assigned should be equal to mean of reconstructed ω distribution

$$\chi^2_{\mathrm{GoF},k}(\delta F_{\mathrm{fit}}) = \frac{\left(\langle\omega_k\rangle - \Omega_k(\delta F_{\mathrm{fit}})\right)^2}{\left(\langle\omega_k^2\rangle - \langle\omega_k\rangle^2\right)/N_{data}}$$

$$\tilde{\chi}^2_{\mathrm{GoF},kl}(\delta F_{\mathrm{fit}}) = \frac{\left(\langle\tilde{\omega}_{kl}\rangle - \tilde{\Omega}_{kl}(\delta F_{\mathrm{fit}})\right)^2}{\left(\langle\tilde{\omega}_{kl}^2\rangle - \langle\tilde{\omega}_{kl}\rangle^2\right)/N_{data}}$$

Some χ^2_{GoF} have large value (6~10).

→ Goodness of fit for the MEM is bad.

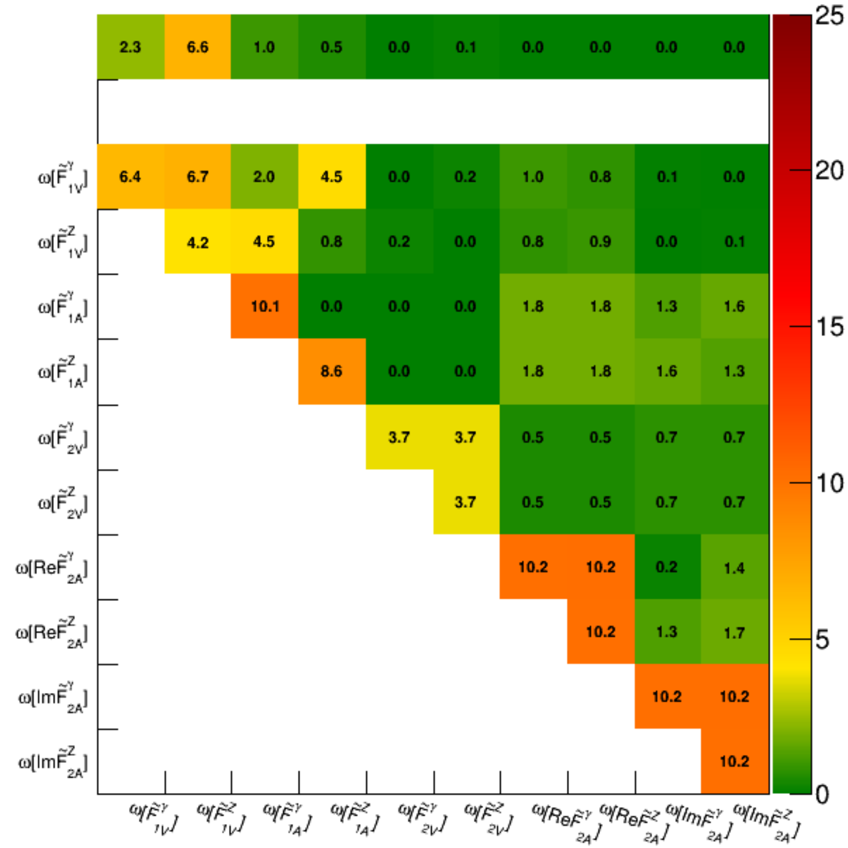


Table of $\chi^2_{\mathrm{GoF}}, \tilde{\chi}^2_{\mathrm{GoF}}$ (Left polarization)

Reweighting (Template-like) Technique

Binned likelihood method : $\chi^2(\delta F) = \sum_{i=1}^{N_{bin}} \left(\frac{n_i^{\text{Data}} - n_i^{\text{Sim.}}(\delta F)}{\sqrt{n_i^{\text{Data}}}} \right)^2$

$n_i^{\text{Sim.}}(\delta F)$ is obtained from the large full simulation

Reweighting technique :

Produce a sample using SM value, then change the weight of events.

$$\begin{aligned} n_i^{\text{Sim.}}(\delta F) &= n_i^{\text{Sim.,sig}}(\delta F) + n_i^{\text{Sim.,bkg}} \\ &= n_i^{\text{Sim.,sig}}(0)(1\langle\omega\rangle_i \delta F + \langle\widetilde{\omega}\rangle_i \delta F^2) + n_i^{\text{Sim.,bkg}} \\ &\simeq n_i^{\text{Sim.,sig}}(0)(1 + \langle\omega\rangle_i \delta F) + n_i^{\text{Sim.,bkg}} \end{aligned}$$

Template technique : Produce many samples using different parameters

Overestimate of goodness of fit

We don't have enough statistics for the background events for now.

$$\chi^2(\delta F) = \sum_{i=1}^{N_{bin}} \left(\frac{n_i^{\text{Data}} - n_i^{\text{Sim.}}(\delta F)}{\sqrt{n_i^{\text{Data}}}} \right)^2 \rightarrow \sum_{i=1}^{N_{bin}} \left(\frac{n_i^{\text{Data,Sig.}} - n_i^{\text{Sim.,Sig.}}(\delta F)}{\sqrt{n_i^{\text{Data}}}} \right)^2$$

When $n_i^{\text{Data,Sig.}} = \alpha n_i^{\text{Data}}$ $(\alpha < 1)$

$$\sum_{i=1}^{N_{bin}} \left(\frac{n_i^{\text{Data,Sig.}} - n_i^{\text{Sim.,Sig.}}(\delta F)}{\sqrt{n_i^{\text{Data}}}} \right)^2 = \alpha \sum_{i=1}^{N_{bin}} \left(\frac{n_i^{\text{Data,Sig.}} - n_i^{\text{Sim.,Sig.}}(\delta F)}{\sqrt{n_i^{\text{Data,Sig}}}} \right)^2$$

$$\equiv \alpha \chi^2_{\text{Sig}}$$

$\min[\chi^2_{\text{Sig}}]$ obeys chi-square distribution of $n.d.f. = N_{bin} - N_{\text{para}}$

→ $\chi^2(\delta F)$ may be $1/\alpha$ times larger if backgrounds are included in $\chi^2(\delta F)$