muon g-2 の理論

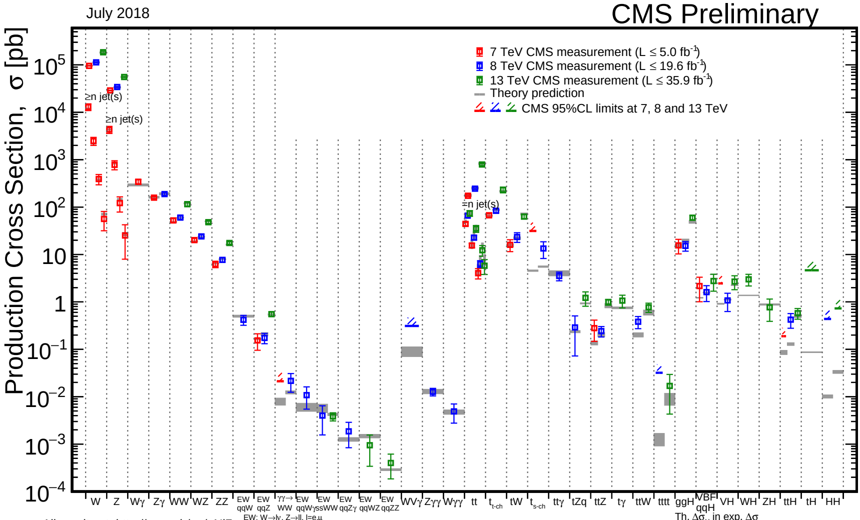
野村大輔 (KEK 素核研)

Flavor Physics Workshop 2018 @ Kavli IPMU

2018年10月31日

素粒子標準模型の現状

LHC data vs 標準模型



All results at: http://cern.ch/go/pNj7

Fig. from CMS TWiki

LHC でのデータと標準模型の予言はとてもよく一致

小林益川理論の実験的検証

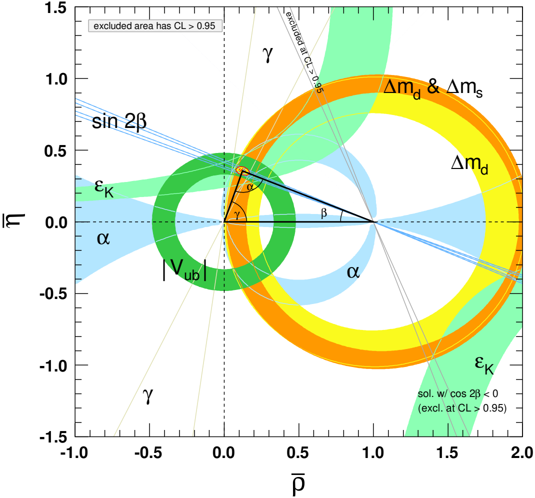


Fig. from Particle Data Book, 2018

CP 非保存の物理の実験データも
小林益川理論（素粒子標準模型の一部）を強く支持

電弱精密測定 vs 標準模型

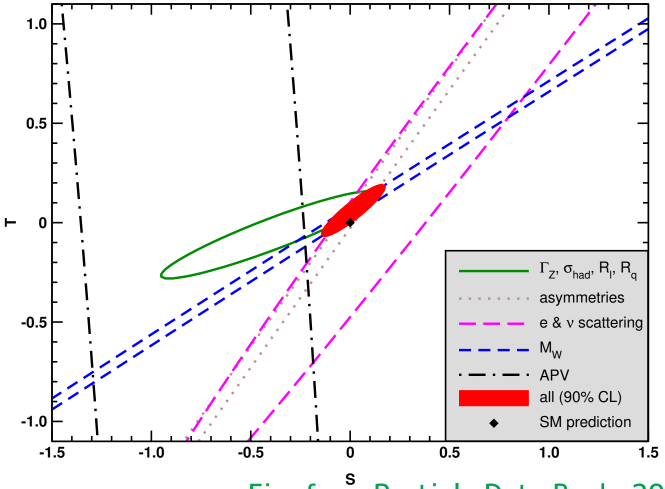


Fig. from Particle Data Book, 2018

$\mathcal{O}(100\mathrm{GeV})$ での精密実験の結果も
標準模型の予言ととてもよく一致

標準模型はこれだけ実験的に成功している模型であるが、それが究極の理論であるとは（おそらく）誰も考えていない。

なぜなら...

標準模型の枠内では説明できないこと

- なぜ３世代か？ なぜ $SU(3)_C \times SU(2)_L \times U(1)_Y$ か？
- many (19) free parameters

gauge couplings	g', g, g_s
vacuum expectation value (VEV)	v
Higgs boson mass	m_H
lepton masses	m_e, m_μ, m_τ
quark masses	$m_u, m_d, m_s, m_c, m_b, m_t$
quark mixing angles	ϕ_1, ϕ_2, ϕ_3
CKM phase	δ
(θ-angle	θ)

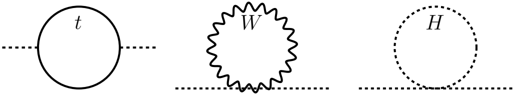
- ニュートリノ質量、ニュートリノ混合行列
- strong CP 問題
- ゲージ階層性問題 (Why $m_{\text{weak}} \ll m_{\text{GUT}}$???)
- ダークマター、ダークエナジー、宇宙のバリオン数の起源？
- 重力が入っていない
- $\vdots$

これらの点を解決するために、**標準模型**を越える物理はきっとある。

TeV スケールにあるかもしれない。

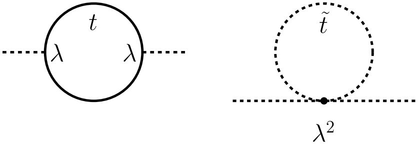
なぜなら...

Hierarchy Problem in the Standard Model



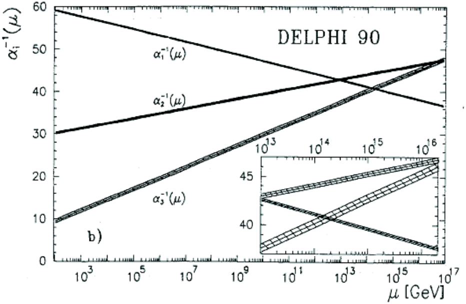
Radiative corrections to $m_{\mathbf{H}}^{\mathbf{2}}$ diverges as $\sim \mathbf{\Lambda}^{\mathbf{2}}$. ⇔ Physical Higgs mass $\sim m_{\mathbf{weak}}^{\mathbf{2}}$. (**Fine-tuning necessary** if $\mathbf{\Lambda} \gg m_{\mathbf{weak}}$)

In **SUSY Standard Models** this is automatically solved since **(softly broken) SUSY** ensures the cancellation of the quad. divergences. For example,

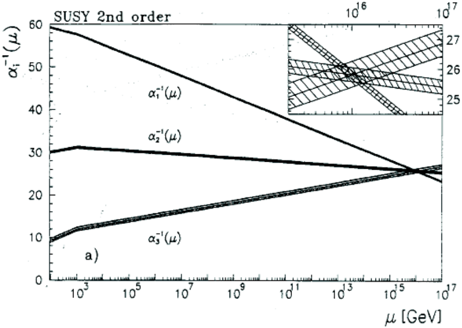


Gauge coupling unification:

SM case



MSSM case

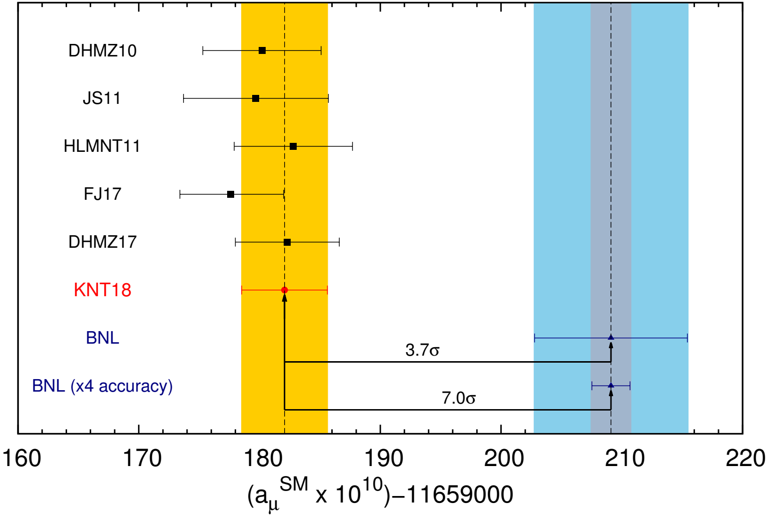


Amaldi-de Boer-Fürstenau '91

SUSY 粒子の寄与によって、m_{SUSY} を境として ゲージ結合定数のスケール依存性(“running”) が変わる

見た目に美しいだけでなく、「電荷の量子化」が説明できる

Muon g-2：新物理のヒント？



$(g\text{-}2)_\mu$：実験と標準模型の予言との間に$\gtrsim 3.5\sigma$ のずれ

新粒子からの寄与？

これだけ標準模型を越える新物理の間接証拠があるのだから、LHC が走り始めるとすぐにでも新粒子が見つかるものと思われていた。

ところがフタを開けてみると、、、

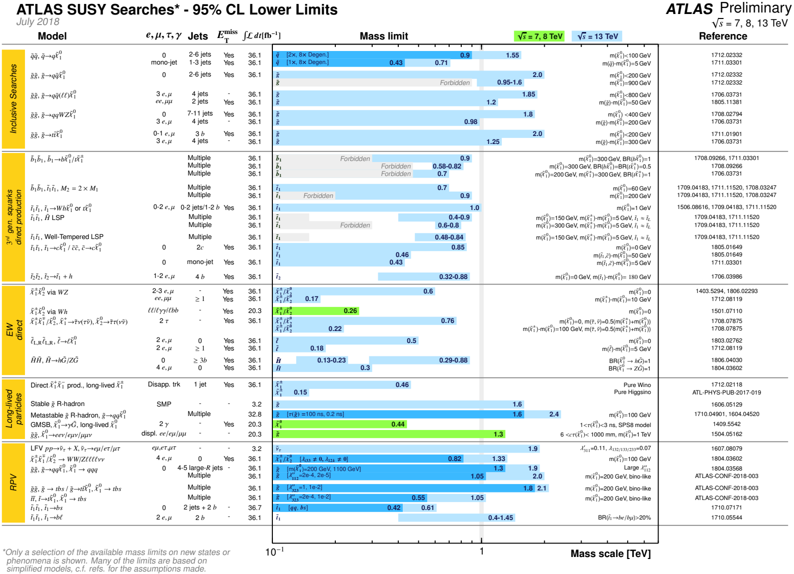


Fig. from ATLAS TWiki

様々な努力に関わらず超対称粒子は未発見

現時点での LHC データからの制限：$m_{\mathrm{SUSY}} \gtrsim 1$ TeV

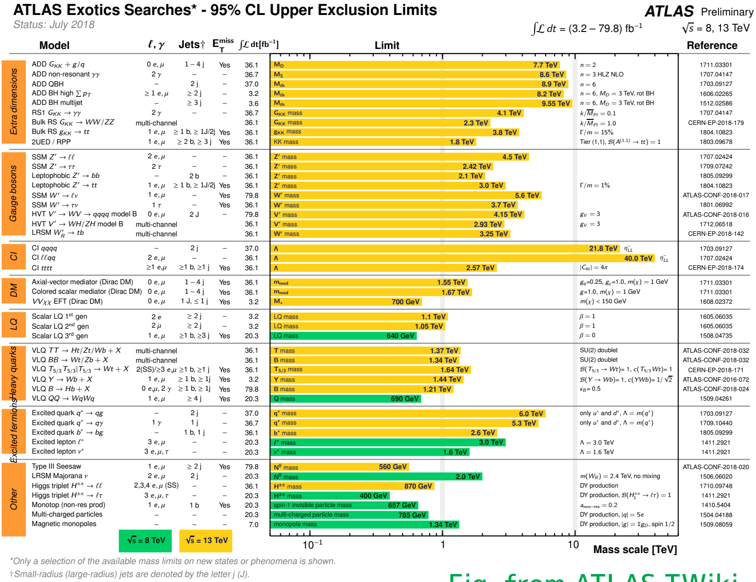


Fig. from ATLAS TWiki

SUSY 以外の新物理が予言する粒子についても同様

現時点での LHC からの制限：$m_{\rm exotics} \gtrsim 1$ TeV

しかし LHC だけで物理がすべてわかるわけではない

- 諸々の不定性 (pdf, backgrounds, hadronic uncertainties, ...) のため、LHC はある程度以上の精密測定には向かない
- 近い将来に到達できるエネルギーには限界が

$\Longrightarrow$ ほかの手段と組み合わせることが必要

- e^+e^- 加速器, 種々の精密測定 (flavor 物理, EDMs, g-2, $0\nu\beta\beta$ 崩壊探索...), 暗黒物質探索, 宇宙物理, ...

$\Longrightarrow$ 精密測定の物理はその好例
とくに muon g-2 はその代表例

Magnetic Moment: Definition

Suppose that there is a point particle f at rest in an external magnetic field. If the interaction Hamiltonian H_{mdm} between f and the magnetic field $\vec{B}$ is given by

$$H_{\text{mdm}} = -\vec{\mu} \cdot \vec{B} \ ,$$

then the vector $\vec{\mu}$ is called the **magnetic dipole moment** of f.

- If f has a non-zero spin, then $\vec{\mu} \propto \overrightarrow{\text{spin}}$
- H_{mdm} is P-even and T-even
  cf:
  EDM $\vec{d}$: $H_{\text{EDM}} = -\vec{d} \cdot \vec{E}$ (P-odd, T-odd)
  anapole $\vec{a}$: $H_{\text{ana}} = -\vec{a} \cdot (\nabla \times \vec{B})$ (P-odd, T-even)

Magnetic dipole moment of a spin-1/2 particle

For a spin-1/2 particle f, in the language of field theory,

$$\langle f(p')|J_\mu^{\text{em}}|f(p)\rangle = \bar{u}_f(p')\Gamma_\mu u_f(p)\ ,$$

$$\Gamma_\mu = F_1(q^2)\gamma_\mu + \frac{i}{2m_f}F_2(q^2)\sigma_{\mu\nu}q^\nu$$
$$- F_3(q^2)\sigma_{\mu\nu}q^\nu\gamma_5 - F_4(q^2)(\gamma_\mu q^2 - 2m_f q_\mu)\gamma_5$$

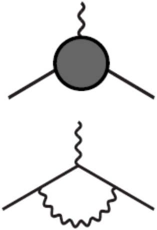
It is known that there are no other independent form factors of a spin-1/2 particle other than $F_1(q^2), \dots, F_4(q^2)$ (See e.g., Nowakowski, Paschos, & Rodriguez, physics/0402058)

$F_1(0) = -eQ_f$	(electric charge)
$F_2(0) = -eQ_f a_f$	(a_f : anomalous magnetic moment)
$F_3(0) = d_f$	(EDM)
$F_4(0) = \tilde{a}_f$	(anapole moment)

Muon g-2: introduction

Lepton magnetic moment $\vec{\mu}$:

$$\boxed{\vec{\mu} = -g\frac{e}{2m}\vec{s}}, \qquad \left(\vec{s} = \frac{1}{2}\vec{\sigma} \text{ (spin)}, \qquad g = 2 + 2F_2(0)\right)$$



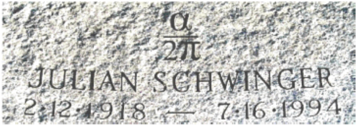
where

$$\overline{u}(p+q)\Gamma^\mu u(p) = \overline{u}(p+q)\left(\gamma^\mu F_1(q^2) + \frac{i\sigma^{\mu\nu}q_\nu}{2m}F_2(q^2)\right)u(p)$$

Anomalous magnetic moment: $a \equiv (g-2)/2$ $(= F_2(0))$

Historically,

- ★ $g = 2$ (tree level, Dirac)
- ★ $a = \alpha/(2\pi)$ (1-loop QED, Schwinger)



Today, still important, since...

- ★ One of the **most precisely measured** quantities:

$$\boxed{a_\mu^{\text{exp}} = 11\ 659\ 208.9(6.3) \times 10^{-10}} \quad [0.5\text{ppm}]$$

(Bennett *et al*)

- ★ **Extremely useful** in **probing/constraining physics beyond the SM**

Muon g-2：先行実験 (1960年以降分)

Experiment	Years	Polarity	$a_\mu \times 10^{10}$	Precision [ppm]	Sensitivity
CERN I	1961	μ^+	11 450 000(220 000)	4300	2-loop QED contrib. (3600 ppm)
CERN II	1962-1968	μ^+	11 661 600(3100)	270	3-loop QED contrib. (260 ppm)
CERN III	1974-1976	μ^+	11 659 100(110)	10	hadronic vacuum polarization contrib. (60 ppm)
CERN III	1975-1976	μ^-	11 659 360(120)	10	
BNL	1997	μ^+	11 659 251(150)	13	
BNL	1998	μ^+	11 659 191(59)	5	4-loop QED contrib. (3.3 ppm)
BNL	1999	μ^+	11 659 202(15)	1.3	electroweak contrib. (1.3 ppm)
BNL	2000	μ^+	11 659 204(9)	0.73	hadronic light-by-light contrib. (0.86 ppm)
BNL	2001	μ^-	11 659 214(9)	0.72	hadronic NLO vacuum pol. contrib. (-0.85 ppm)
Average			11 659 208.0(6.3)	0.54	

Table from BNL-E821 final report, Phys. Rev. D 73 (2006) 072003

muon g-2 実験の歴史は標準模型の検証の歴史でもあり、それは
新物理探索の歴史でもある。そして話は現代に続く。

John Stewart Bell (1928-1990)

CERN 理論部のスタッフ (1960-1990)

主な業績

- 量子力学基礎論
  Bell の不等式, ...
- 場の量子論
  Adler-Bell-Jackiw anomaly, ...
- 素粒子現象論
  $\pi^0 \to \gamma\gamma$, CP violation in K decays, Shifman-Vainshtein-Zakharov QCD sum rules, ...

J. S. Bell
(Photo from Wikipedia)

ほかにも Landau-Lifshitz シリーズのいくつかを英訳（共訳）
「力学」,「量子力学」,「相対論的量子力学」,「媒質中の電気力学」

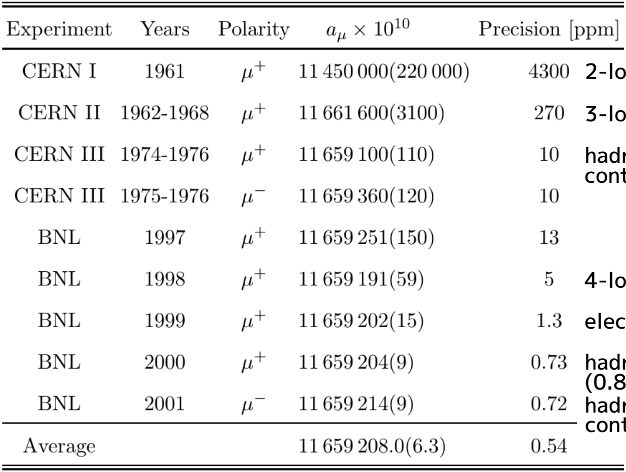
John Stewart Bell (1928-1990)

Bell のあまり知られていない業績：

- “Hadronic vacuum polarization and $g_\mu - 2$”
  by J. S. Bell and E. de Rafael
  Nucl. Phys. B11 (1969) 611
- “Polarized Particles for Accelerator Physicists”
  by J. S. Bell
  CERN preprint 75-11
  Lectures given in Academic Training Program of CERN 1974-1975
  (電磁場中での粒子の polarization の振る舞いを記述する Thomas-BMT 方程式の解説、とくに magic momentum を使った CERN muon g-2 実験の原理の解説)

muon g-2 は50年以上前から hot topic だった

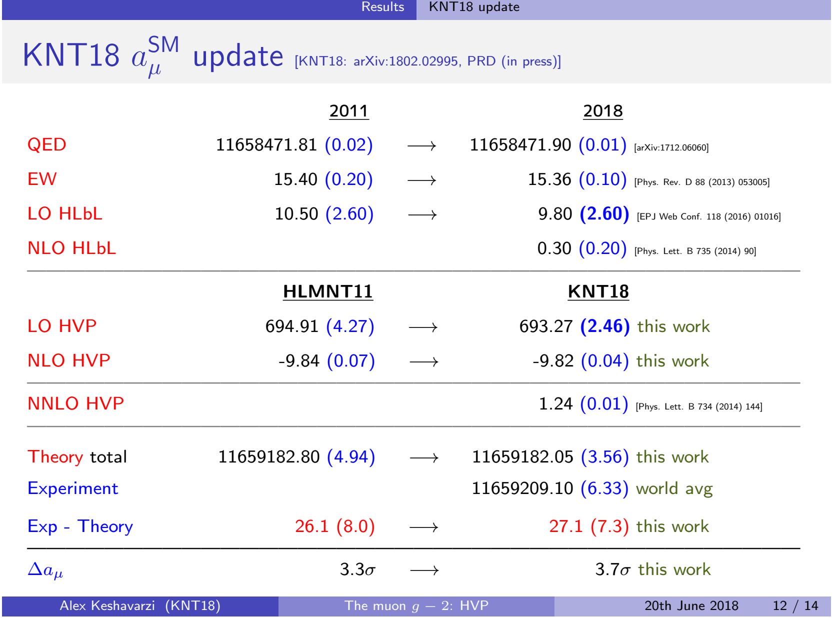
Muon g-2：先行実験 (1960年以降分)



Experiment	Years	Polarity	$a_\mu \times 10^{10}$	Precision [ppm]	Sensitivity
CERN I	1961	μ^+	11 450 000(220 000)	4300	2-loop QED contrib. (3600 ppm)
CERN II	1962-1968	μ^+	11 661 600(3100)	270	3-loop QED contrib. (260 ppm)
CERN III	1974-1976	μ^+	11 659 100(110)	10	hadronic vacuum polarization contrib. (60 ppm)
CERN III	1975-1976	μ^-	11 659 360(120)	10	
BNL	1997	μ^+	11 659 251(150)	13	
BNL	1998	μ^+	11 659 191(59)	5	4-loop QED contrib. (3.3 ppm)
BNL	1999	μ^+	11 659 202(15)	1.3	electroweak contrib. (1.3 ppm)
BNL	2000	μ^+	11 659 204(9)	0.73	hadronic light-by-light contrib. (0.86 ppm)
BNL	2001	μ^-	11 659 214(9)	0.72	hadronic NLO vacuum pol. contrib. (-0.85 ppm)
Average			11 659 208.0(6.3)	0.54	

Table from BNL-E821 final report, Phys. Rev. D 73 (2006) 072003

muon g-2 実験の歴史は標準模型の検証の歴史でもあり、それは新物理探索の歴史でもある。そして話は現代に続く。



KNT18 a_μ^{SM} update [KNT18: arXiv:1802.02995, PRD (in press)]

	2011		2018	
QED	11658471.81 (0.02)	⟶	11658471.90 (0.01)	[arXiv:1712.06060]
EW	15.40 (0.20)	⟶	15.36 (0.10)	[Phys. Rev. D 88 (2013) 053005]
LO HLbL	10.50 (2.60)	⟶	9.80 **(2.60)**	[EPJ Web Conf. 118 (2016) 01016]
NLO HLbL			0.30 (0.20)	[Phys. Lett. B 735 (2014) 90]
	HLMNT11		**KNT18**	
LO HVP	694.91 (4.27)	⟶	693.27 **(2.46)**	this work
NLO HVP	-9.84 (0.07)	⟶	-9.82 (0.04)	this work
NNLO HVP			1.24 (0.01)	[Phys. Lett. B 734 (2014) 144]
Theory total	11659182.80 (4.94)	⟶	11659182.05 (3.56)	this work
Experiment			11659209.10 (6.33)	world avg
Exp - Theory	26.1 (8.0)	⟶	27.1 (7.3)	this work
Δa_μ	3.3σ	⟶	3.7σ	this work

(HVP: Hadronic Vacuum Polarization)
(HLbL: Hadronic Light-by-Light)

Slide by A. Keshavarzi (Liverpool) at 'Muon $g-2$ Workshop' at Mainz, June 18-22, 2018

QED contribution (1)

QED contribution:

$$a_\mu(\text{QED}) = \frac{\alpha}{2\pi} + 0.765857425(17)\left(\frac{\alpha}{\pi}\right)^2 + 24.05050996(32)\left(\frac{\alpha}{\pi}\right)^3$$

$$+ 130.8796(63)\left(\frac{\alpha}{\pi}\right)^4 + 753.3(1.0)\left(\frac{\alpha}{\pi}\right)^5 + \cdots$$

$$= 11658471.895(0.008) \times 10^{-10}\,,$$ (numbers from PDG 2018)

where the uncertainty is dominated by that of α.

- 5-loop calculation! (Aoyama, Hayakawa, Kinoshita & Nio)
- The 4-loop corrections $\simeq 38 \times 10^{-10} \simeq \mathcal{O}(a_\mu(\text{exp}) - a_\mu(\text{SM}))$.
- The 4-loop contribution now fully cross-checked by another group. Mass-independent part by S. Laporta (Phys.Lett. **B772** (2017) 232), and mass-dependent part by A. Kurz et al (Nucl. Phys. **B879** (2014) 1; Phys. Rev. **D92** (2015) 073019; ibid. **D93** (2016) 053017)
- The 5-loop contribution very small ($\simeq 0.5 \times 10^{-10} \ll a_\mu(\text{exp}) - a_\mu(\text{SM})$)

QED contribution (2)

QED contribution to the electron $g-2$:

$$a_e(\text{QED}) = \frac{\alpha}{2\pi} - (0.32847844400\ldots)\left(\frac{\alpha}{\pi}\right)^2 + (1.181234017\ldots)\left(\frac{\alpha}{\pi}\right)^3 - 1.91206(84)\left(\frac{\alpha}{\pi}\right)^4 + 7.79(34)\left(\frac{\alpha}{\pi}\right)^5 + \cdots$$

(coefficients from CODATA 2014)

QED contributions to the muon $g-2$:

$$a_\mu(\text{QED}) = \frac{\alpha}{2\pi} + 0.765857425(17)\left(\frac{\alpha}{\pi}\right)^2 + 24.05050996(32)\left(\frac{\alpha}{\pi}\right)^3 + 130.8796(63)\left(\frac{\alpha}{\pi}\right)^4 + 753.3(1.0)\left(\frac{\alpha}{\pi}\right)^5 + \cdots$$

(coefficients from PDG 2018)

At higher orders, the coefficients of a_μ(QED) are much larger than those of a_e(QED). This happens because ....

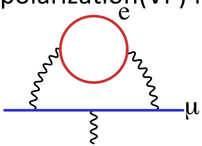
logarithmic enhancement in muon g-2

the logarithmic enhancement $\ln(m_\mu/m_e) \approx 5.3$

note: It does not exist for the lightest lepton, electron.

Two sources of the logarithm

1. Charge renormalization of the vacuum-polarization(VP) function

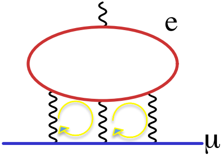


2nd-order VP arises

$$\frac{2}{3}\ln(m_\mu/m_e) - \frac{5}{9} \sim 3$$

"Renormalization Group" estimate

2. Light-by-light scattering diagram



$$\frac{2}{3}\pi^2\ln(m_\mu/m_e) \sim 35$$

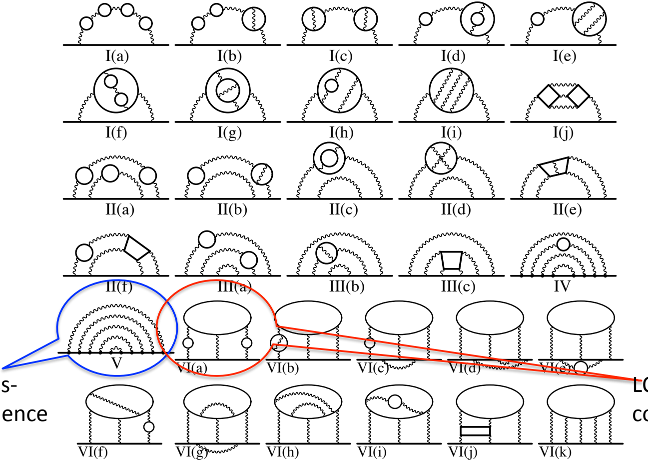
Coulomb photon loops provide the factor π^2

Slide by M. Nio (RIKEN), talk at a RIKEN workshop, March 2, 2016

10th-order contribution

12,672 Feynman vertex diagrams contribute to the 10th order .

They are classified into 32 gauge-invariant subsets over 6 sets.

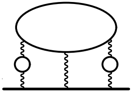


No mass-dependence

LO contribution

Slide by M. Nio (RIKEN), talk at a RIKEN workshop, March 2, 2016

10th-order leading term of $A_2^{(10)}$



VI(a)

The Leading Order(LO) contribution:

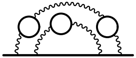
6th-order light-by-light x two 2nd-order vp's

estimate 20 x 3^2 x 6 ways ~ 1080

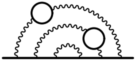
l-by-l 2 vp

Actually, its contribution is 542.760 ± 0.099 > $(\alpha/\pi)^{-1}$ ~430

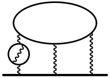
Can also pick up the NLO diagrams:



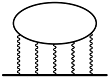
II(a)



III(a)



VI(b)



VI(k)

....

A. L. Kataev, PRD74(2006)073011

The numerical results are consistent

with the renormalization group estimate

The total of 31 subsets of the mass-dependent 10th-order term

$$\boxed{A_2^{(10)}(m_\mu/m_e) = 742.18(87)}$$

Slide by M. Nio (RIKEN), talk at a RIKEN workshop, March 2, 2016

Summary of 8th and 10th-order QED to muon g-2

$$A_2^{(8)}(m_\mu/m_e) = 132.6852\ (60)$$

$$A_2^{(8)}(m_\mu/m_\tau) = 0.042\ 34\ (12)$$

$$A_3^{(8)}(m_\mu/m_e, m_\mu/m_\tau) = 0.062\ 72\ (4)$$

$$A_2^{(10)}(m_\mu/m_e) = 742.18\ (87)$$

$$A_2^{(10)}(m_\mu/m_\tau) = -0.068\ (5)$$

$$A_3^{(10)}(m_\mu/m_e, m_\mu/m_\tau) = 2.011\ (10)$$

QED contributions to the muon g-2 is now firmly established.

Rough estimate of the 12th-order contribution:

6th-order light-by-light x three 2nd-order vp x 10 ways

~ 20 x 3^3 x 10 $(\alpha/\pi)^6$ ~5,000 $(\alpha/\pi)^6$ ~ 0.08 x 10^{-11}

Recall the aimed goal of the on-going experiments ~ 12 x 10^{-11}

Slide by M. Nio (RIKEN), talk at a RIKEN workshop, March 2, 2016

Electroweak Contribution

Electroweak (EW) contribution:

$$a_\mu(\text{EW}) = \underbrace{19.48 \times 10^{-10}}_{\text{1-loop}} + \underbrace{(-4.12(10) \times 10^{-10})}_{\text{2-loop}} + \underbrace{\mathcal{O}(10^{-12})}_{\text{leading log 3-loop}}$$

$$= 15.36(10) \times 10^{-10} \, ,$$ (Number taken from PDG 2018)

where the uncertainty mainly comes from quark loops.

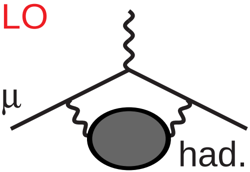
- 1-loop result published by many groups (Bardeen-Gastmans-Lautrup, Altarelli-Cabibbo-Maiani, Jackiw-Weinberg, Bars-Yoshimura, Fujikawa-Lee-Sanda) in 1972, and now a textbook exercise (Peskin & Schroeder's textbook, Problems 6.3 (Higgs) and 21.1 (W, Z))
- 2-loop contribution ($\sim$ 1700 diagrams in the 't Hooft-Feynman gauge) enhanced by $\ln(m_Z/m_\mu)$ and also by a factor of $\mathcal{O}(10)$,

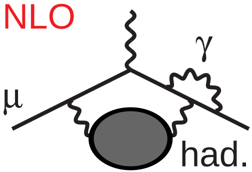
$$a_\mu(\text{EW, 2-loop}) \simeq -10 \left(\frac{\alpha}{\pi}\right) a_\mu(\text{EW, 1-loop}) \left(\ln \frac{m_Z}{m_\mu} + 1\right) \, ,$$

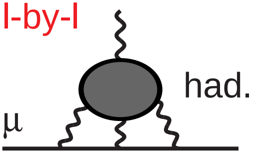
where the factor of 10 appears since many "order one" diagrams accidentally add up coherently.

Hadronic Contributions

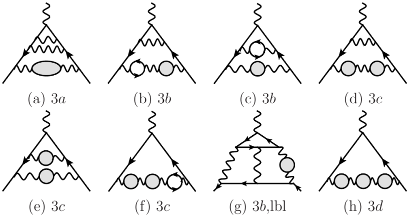
There are several hadronic contributions:



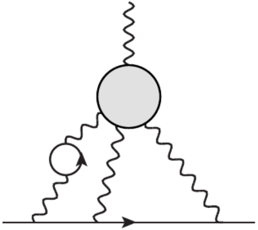




LO: Leading Order (or Vacuum Polarization) Hadronic Contribution
NLO: Next-to-Leading Order Hadronic Contribution
l-by-l: Hadronic light-by-light Contribution


(a) $3a$ (b) $3b$ (c) $3b$ (d) $3c$
(e) $3c$ (f) $3c$ (g) $3b$,lbl (h) $3d$

NNLO Hadronic Contributions

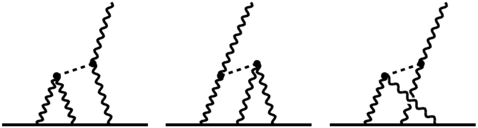


Hadronic l-by-l NLO Contrib.

Modern evaluation of l-by-l contribution

(Melnikov & Vainshtein)

1. First, use the large N_C expansion to find that the leading contribution is the pion pole contribution.

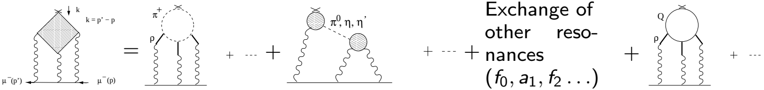


2. Choose the momentum-dependence of the $\pi\gamma\gamma$ coupling (form factor) in such a way that it is consistent with a constraint from QCD (OPE) at the momentum region $q_1^2 \sim q_2^2 \gg q_3^2$. Integrate over the loop momenta.
3. Repeat the above for $\eta, \eta', a_1, \ldots$. Basically that's all for the LO in $1/N_C$.
4. As for NLO in $1/N_C$, it depends on authors which diagram is numerically important.

For example,

$$a_\mu^{\text{lbyl}} = \begin{cases} (10.5 \pm 2.6) \times 10^{-10} & \text{'Glasgow consensus', arXiv:0901.0306} \\ (9.8 \pm 2.6) \times 10^{-10} & \text{'G.c.' w/ correction by Nyffeler, PRD94(2016)053006} \\ (10.2 \pm 3.9) \times 10^{-10} & \text{Nyffeler, arXiv:1710.09742} \end{cases}$$

HLbL in muon $g-2$: summary of selected results (model calculations)



Exchange of other resonances $(f_0, a_1, f_2 \ldots)$

de Rafael '94:

Chiral counting:	p^4	p^6	p^8	p^8
N_C-counting:	1	N_C	N_C	N_C
Contribution to $a_\mu \times 10^{11}$:				
BPP: +83 (32)	-19 (13)	+85 (13)	-4 (3) $[f_0, a_1]$	+21 (3)
HKS: +90 (15)	-5 (8)	+83 (6)	+1.7 (1.7) $[a_1]$	+10 (11)
KN: +80 (40)		+83 (12)		
MV: +136 (25)	0 (10)	+114 (10)	+22 (5) $[a_1]$	0
2007: +110 (40)				
PdRV: +105 (26)	-19 (19)	+114 (13)	+8 (12) $[f_0, a_1]$	+2.3 [c-quark]
N,JN: +116 (39)	-19 (13)	+99 (16)	+15 (7) $[f_0, a_1]$	+21 (3)
	ud.: -45	ud.: $+\infty$		ud.: +60

ud. = undressed, i.e. point vertices without form factors

Pseudoscalars: numerically dominant contribution (according to most models !).

Recall (in units of 10^{-11}): $\delta a_\mu(\text{HVP}) \approx 40$; $\delta a_\mu(\text{exp [BNL]}) = 63$; $\delta a_\mu(\text{future exp}) = 16$

BPP = Bijnens, Pallante, Prades '96, '02; HKS = Hayakawa, Kinoshita, Sanda '96, '98, '02; KN = Knecht, AN '02; MV = Melnikov, Vainshtein '04; 2007 = Bijnens, Prades; Miller, de Rafael, Roberts; PdRV = Prades, de Rafael, Vainshtein '09 (compilation; "Glasgow consensus"); N,JN = AN '09; Jegerlehner, AN '09 (compilation)

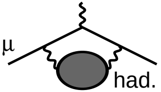
Recent reevaluations of axial vector contribution lead to much smaller estimates than in MV '04: $a_\mu^{\text{HLbL;axial}} = (8 \pm 3) \times 10^{-11}$ (Pauk, Vanderhaeghen '14; Jegerlehner '14, '15). Would shift central values of compilations downwards:

$a_\mu^{\text{HLbL}} = (98 \pm 26) \times 10^{-11}$ (PdRV) and $a_\mu^{\text{HLbL}} = (102 \pm 39) \times 10^{-11}$ (N, JN).

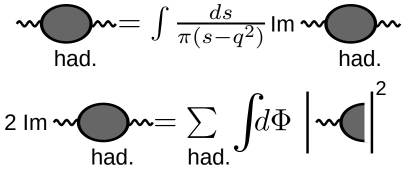
Slide by A. Nyffeler (Mainz) at 'Muon g-2 lbyl Workshop' at Connecticut, March 12-14, 2018

LO Hadronic Vacuum Polarization Contribution

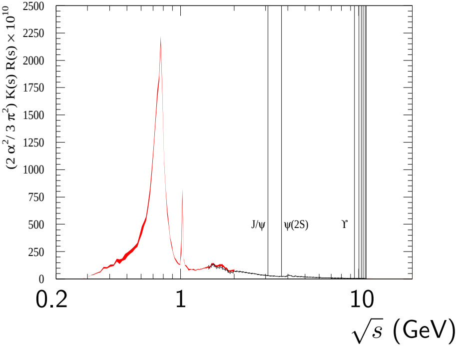
The diagram to be evaluated:



pQCD not useful. Use the dispersion relation and the optical theorem.



$$\boxed{a_\mu^{\mathrm{had,LO}} = \frac{m_\mu^2}{12\pi^3}\int_{s_{\mathrm{th}}}^{\infty} ds\ \frac{1}{s}\hat{K}(s)\sigma_{\mathrm{had}}(s)}$$



- Weight function $\hat{K}(s)/s = \mathcal{O}(1)/s$

$\Longrightarrow$ Lower energies more important

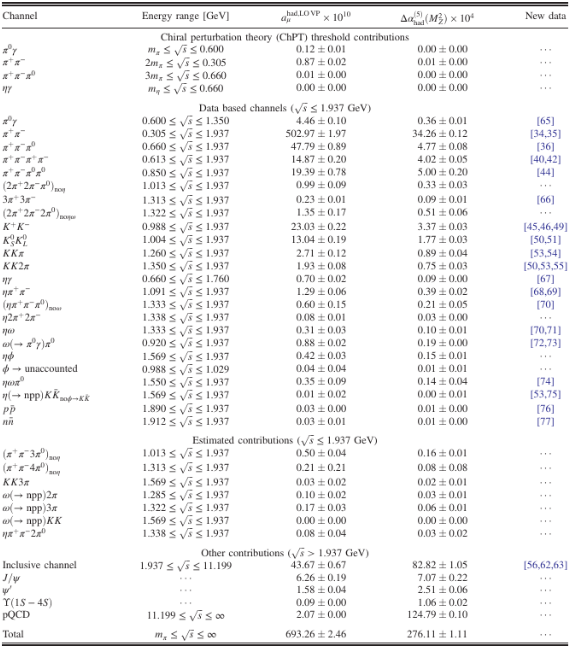
$\Longrightarrow$ $\pi^+\pi^-$ channel: 73% of total $a_\mu^{\mathrm{had,LO}}$

Rough Estimate of the LO Hadronic Contribution

To estimate the LO hadronic contribution, let us take the following toy model for the hadronic cross sections: (from the Melnikov-Vainshtein book)

- At $2m_\pi < \sqrt{s} < m_\rho/\sqrt{2}$, we consider the $\pi^+\pi^-$ contribution assuming scalar QED:
$$\sigma(e^+e^- \to \pi^+\pi^-)(s) = \frac{\pi\alpha^2}{3s}\left(1 - \frac{4m_\pi^2}{s}\right)^{3/2} ,$$
from which we obtain $a_\mu(2\pi) = 40 \times 10^{-10}$.
- Include the ρ, ω and ϕ meson contributions using the narrow width approximation:
$$\sigma(e^+e^- \to V)(s) = \frac{12\pi^2\Gamma_{V\to e^+e^-}}{m_V}\delta\left(s - m_V^2\right) .$$
From this we obtain $a_\mu(\rho + \omega + \phi) = 551 \times 10^{-10}$.
- Include the remaining hadronic spectrum using 3-flavor perturbative QCD:
$$\sigma(e^+e^- \to \text{hadrons})(s) = \frac{4\pi\alpha^2}{3s}N_c \sum_{q=u,d,s} Q_q^2 = \frac{8\pi\alpha^2}{3s}$$
If we use pQCD from $\sqrt{s}$ =1 GeV to infinity, then we get $a_\mu(\text{pQCD}) = 124 \times 10^{-10}$.

In total, we get $a_\mu = 715 \times 10^{-10}$, which is not very far from $693.3(2.5) \times 10^{-10}$ (KNT18). It is not very difficult to obtain a value in the right ballpark.



Channel	Energy range [GeV]	$a_\mu^{\mathrm{had,LO\,VP}} \times 10^{10}$	$\Delta\alpha_{\mathrm{had}}^{(5)}(M_Z^2) \times 10^4$	New data
Chiral perturbation theory (ChPT) threshold contributions				
$\pi^0\gamma$	$m_\pi \le \sqrt{s} \le 0.600$	0.12 ± 0.01	0.00 ± 0.00	$\cdots$
$\pi^+\pi^-$	$2m_\pi \le \sqrt{s} \le 0.305$	0.87 ± 0.02	0.01 ± 0.00	$\cdots$
$\pi^+\pi^-\pi^0$	$3m_\pi \le \sqrt{s} \le 0.660$	0.01 ± 0.00	0.00 ± 0.00	$\cdots$
$\eta\gamma$	$m_\eta \le \sqrt{s} \le 0.660$	0.00 ± 0.00	0.00 ± 0.00	$\cdots$
Data based channels ($\sqrt{s} \le 1.937$ GeV)				
$\pi^0\gamma$	$0.600 \le \sqrt{s} \le 1.350$	4.46 ± 0.10	0.36 ± 0.01	[65]
$\pi^+\pi^-$	$0.305 \le \sqrt{s} \le 1.937$	502.97 ± 1.97	34.26 ± 0.12	[34,35]
$\pi^+\pi^-\pi^0$	$0.660 \le \sqrt{s} \le 1.937$	47.79 ± 0.89	4.77 ± 0.08	[36]
$\pi^+\pi^-\pi^+\pi^-$	$0.613 \le \sqrt{s} \le 1.937$	14.87 ± 0.20	4.02 ± 0.05	[40,42]
$\pi^+\pi^-\pi^0\pi^0$	$0.850 \le \sqrt{s} \le 1.937$	19.39 ± 0.78	5.00 ± 0.20	[44]
$(2\pi^+2\pi^-\pi^0)_{\mathrm{no}\eta}$	$1.013 \le \sqrt{s} \le 1.937$	0.99 ± 0.09	0.33 ± 0.03	$\cdots$
$3\pi^+3\pi^-$	$1.313 \le \sqrt{s} \le 1.937$	0.23 ± 0.01	0.09 ± 0.01	[66]
$(2\pi^+2\pi^-2\pi^0)_{\mathrm{no}\eta\omega}$	$1.322 \le \sqrt{s} \le 1.937$	1.35 ± 0.17	0.51 ± 0.06	$\cdots$
K^+K^-	$0.988 \le \sqrt{s} \le 1.937$	23.03 ± 0.22	3.37 ± 0.03	[45,46,49]
$K_S^0K_L^0$	$1.004 \le \sqrt{s} \le 1.937$	13.04 ± 0.19	1.77 ± 0.03	[50,51]
$KK\pi$	$1.260 \le \sqrt{s} \le 1.937$	2.71 ± 0.12	0.89 ± 0.04	[53,54]
$KK2\pi$	$1.350 \le \sqrt{s} \le 1.937$	1.93 ± 0.08	0.75 ± 0.03	[50,53,55]
$\eta\gamma$	$0.660 \le \sqrt{s} \le 1.760$	0.70 ± 0.02	0.09 ± 0.00	[67]
$\eta\pi^+\pi^-$	$1.091 \le \sqrt{s} \le 1.937$	1.29 ± 0.06	0.39 ± 0.02	[68,69]
$(\eta\pi^+\pi^-\pi^0)_{\mathrm{no}\omega}$	$1.333 \le \sqrt{s} \le 1.937$	0.60 ± 0.15	0.21 ± 0.05	[70]
$\eta 2\pi^+2\pi^-$	$1.338 \le \sqrt{s} \le 1.937$	0.08 ± 0.01	0.03 ± 0.00	$\cdots$
$\eta\omega$	$1.333 \le \sqrt{s} \le 1.937$	0.31 ± 0.03	0.10 ± 0.01	[70,71]
$\omega(\to \pi^0\gamma)\pi^0$	$0.920 \le \sqrt{s} \le 1.937$	0.88 ± 0.02	0.19 ± 0.00	[72,73]
$\eta\phi$	$1.569 \le \sqrt{s} \le 1.937$	0.42 ± 0.03	0.15 ± 0.01	$\cdots$
$\phi \to$ unaccounted	$0.988 \le \sqrt{s} \le 1.029$	0.04 ± 0.04	0.01 ± 0.01	$\cdots$
$\eta\omega\pi^0$	$1.550 \le \sqrt{s} \le 1.937$	0.35 ± 0.09	0.14 ± 0.04	[74]
$\eta(\to \mathrm{npp})K\bar{K}_{\mathrm{no}\phi\to K\bar{K}}$	$1.569 \le \sqrt{s} \le 1.937$	0.01 ± 0.02	0.00 ± 0.01	[53,75]
$p\bar{p}$	$1.890 \le \sqrt{s} \le 1.937$	0.03 ± 0.00	0.01 ± 0.00	[76]
$n\bar{n}$	$1.912 \le \sqrt{s} \le 1.937$	0.03 ± 0.01	0.01 ± 0.00	[77]
Estimated contributions ($\sqrt{s} \le 1.937$ GeV)				
$(\pi^+\pi^-3\pi^0)_{\mathrm{no}\eta}$	$1.013 \le \sqrt{s} \le 1.937$	0.50 ± 0.04	0.16 ± 0.01	$\cdots$
$(\pi^+\pi^-4\pi^0)_{\mathrm{no}\eta}$	$1.313 \le \sqrt{s} \le 1.937$	0.21 ± 0.21	0.08 ± 0.08	$\cdots$
$KK3\pi$	$1.569 \le \sqrt{s} \le 1.937$	0.03 ± 0.02	0.02 ± 0.01	$\cdots$
$\omega(\to \mathrm{npp})2\pi$	$1.285 \le \sqrt{s} \le 1.937$	0.10 ± 0.02	0.03 ± 0.01	$\cdots$
$\omega(\to \mathrm{npp})3\pi$	$1.322 \le \sqrt{s} \le 1.937$	0.17 ± 0.03	0.06 ± 0.01	$\cdots$
$\omega(\to \mathrm{npp})KK$	$1.569 \le \sqrt{s} \le 1.937$	0.00 ± 0.00	0.00 ± 0.00	$\cdots$
$\eta\pi^+\pi^-2\pi^0$	$1.338 \le \sqrt{s} \le 1.937$	0.08 ± 0.04	0.03 ± 0.02	$\cdots$
Other contributions ($\sqrt{s} > 1.937$ GeV)				
Inclusive channel	$1.937 \le \sqrt{s} \le 11.199$	43.67 ± 0.67	82.82 ± 1.05	[56,62,63]
J/ψ	$\cdots$	6.26 ± 0.19	7.07 ± 0.22	$\cdots$
ψ'	$\cdots$	1.58 ± 0.04	2.51 ± 0.06	$\cdots$
$\Upsilon(1S-4S)$	$\cdots$	0.09 ± 0.00	1.06 ± 0.02	$\cdots$
pQCD	$11.199 \le \sqrt{s} \le \infty$	2.07 ± 0.00	124.79 ± 0.10	$\cdots$
Total	$m_\pi \le \sqrt{s} \le \infty$	693.26 ± 2.46	276.11 ± 1.11	$\cdots$

Table from KNT18, Phys. Rev. D97 (2018) 114025

Breakdown of contributions to a_μ(had, LO VP) from various hadronic final states

We have included new data sets from $\sim$ 30 papers, in addition to those included in the HLMNT11 analysis

We have included $\sim$ 30 hadronic final states

At $2 \lesssim \sqrt{s} \lesssim 11$ GeV, we use inclusively measured data

At higher energies $\gtrsim 11$ GeV, we use pQCD

Data Combination

真空偏極 (vacuum polarization) からの寄与を分散関係を使って評価するには、多数の実験データの「平均値」を求める必要がある

普通は χ^2 関数を作って、それを最小化するような$R(s)$ の値を求める

単純には共分散行列 (covariance matrix) V を使って

$$\chi^2(\{\overline{R}_i\}) \equiv \sum_{n=1}^{N_{\text{exp}}} \sum_{i=1}^{N_{\text{bin}}} \sum_{j=1}^{N_{\text{bin}}} (R_i^{(n)} - \overline{R}_i)(V_n^{-1})_{ij}(R_j^{(n)} - \overline{R}_j)\ ,$$

where V_n is the cov. matrix of the n-th exp.,

$$V_{n,ij} = \begin{cases} (\delta R_{i,\text{stat}}^{(n)})^2 + (\delta R_{i,\text{sys}}^{(n)})^2 & (\text{for } i = j) \\ (\delta R_{i,\text{sys}}^{(n)})(\delta R_{j,\text{sys}}^{(n)}) & (\text{for } i \neq j) \end{cases}$$

と定義した $\chi^2(\{\overline{R}_i\})$ を最小にするような $\overline{R}_1, \overline{R}_2, \cdots, \overline{R}_{N_{\text{bin}}}$ を求めればよいように思えるが、実験データに normalization error がある場合には注意が必要 (次ページにつづく)

χ^2 vs normalization error: d'Agostini bias

G. D'Agostini, Nucl. Instrum. Meth. A346 (1994) 306

We first consider an observable x whose true value is 1. Suppose that there is an experiment which measures x and whose normalization uncertainty is 10%.
Now, assume that this experiment measured x twice:

$$\text{1st result:} \quad 0.9 \pm 0.1_{\text{stat}} \pm 10\%_{\text{syst}} \ ,$$
$$\text{2nd result:} \quad 1.1 \pm 0.1_{\text{stat}} \pm 10\%_{\text{syst}} \ .$$

Taking the systematic errors 0.09 and 0.11, respectively, the covariance matrix and the χ^2 function are

$$(\text{cov.}) = \begin{pmatrix} 0.1^2 + 0.09^2 & 0.09 \cdot 0.11 \\ 0.09 \cdot 0.11 & 0.1^2 + 0.11^2 \end{pmatrix} \ ,$$
$$\chi^2 = \begin{pmatrix} x - 0.9 & x - 1.1 \end{pmatrix} (\text{cov.})^{-1} \begin{pmatrix} x - 0.9 \\ x - 1.1 \end{pmatrix} \ .$$

χ^2 takes its minimum at $x = 0.98$: Biased downwards!

d'Agostini bias (2): improvement by iterations

What was wrong? In the previous page,

$$\text{1st result:} \quad 0.9 \pm 0.1_{\text{stat}} \pm 10\%_{\text{syst}} \,,$$
$$\text{2nd result:} \quad 1.1 \pm 0.1_{\text{stat}} \pm 10\%_{\text{syst}} \,.$$

we took the syst. errors 0.09 and 0.11, respectively, which made the downward bias. Instead, we should take 10% of some estimator $\bar{x}$ as the syst. errors. Then,

$$\text{(cov.)} = \begin{pmatrix} 0.1^2 + (0.1\bar{x})^2 & (0.1\bar{x})^2 \\ (0.1\bar{x})^2 & 0.1^2 + (0.1\bar{x})^2 \end{pmatrix} \,,$$

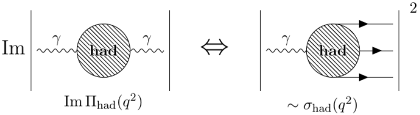
$$\chi^2 = \begin{pmatrix} x - 0.9 & x - 1.1 \end{pmatrix} \text{(cov.)}^{-1} \begin{pmatrix} x - 0.9 \\ x - 1.1 \end{pmatrix} \,.$$

χ^2 takes its minimum at $x = 1.00$: Unbiased!

In more general cases, we use iterations: we find an estimator for the next round of iteration by χ^2-minimization. R.D.Ball et al, JHEP 1005 (2010) 075.

$\sigma^0_{\mathrm{had},\gamma}$: vacuum polarisation corrections

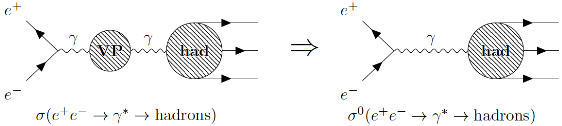
⇒ Reconsider the optical theorem:



$\mathrm{Im}\,\Pi_{\mathrm{had}}(q^2)$ $\sim\sigma_{\mathrm{had}}(q^2)$

⇒ Photon VP corresponds to higher order contributions to $a_\mu^{\mathrm{had,\,VP}}$

→ Must subtract VP:



$\sigma(e^+e^-\to\gamma^*\to\text{hadrons})$ $\sigma^0(e^+e^-\to\gamma^*\to\text{hadrons})$

⇒ Fully updated, self-consistent VP routine: `[vp_knt_v3_0]`, available for distribution

→ Cross sections undressed with full photon propagator (must include imaginary part), $\sigma^0_{\mathrm{had}}(s)=\sigma_{\mathrm{had}}(s)|1-\Pi(s)|^2$

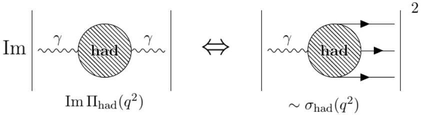
⇒ If correcting data, apply corresponding radiative correction uncertainty

→ Take $\frac{1}{3}$ of total correction per channel as conservative extra uncertainty

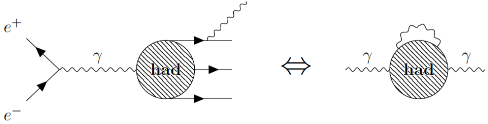
Slide by A. Keshavarzi (Liverpool) at 'Muon $g-2$ Workshop' at Mainz, June 18-22, 2018

$\sigma^0_{\mathrm{had},\gamma}$: final state radiation corrections

⇒ Reconsider the optical theorem:



⇒ Photon FSR formally higher order corrections to $a_\mu^{\mathrm{had,\,VP}}$



⇒ Cannot be unambiguously separated, not accounted for in HO contributions

→ Must be included as part of 1PI hadronic blobs

⇒ Experiment may cut/miss photon FSR → Must be added back

⇒ For $\pi^+\pi^-$, sQED approximation [Eur. Phys. J. C 24 (2002) 51, Eur. Phys. J. C 28 (2003) 261]

⇒ For higher multiplicity states, difficult to estimate correction

∴ Apply conservative uncertainty

Need new, more developed tools to increase precision here

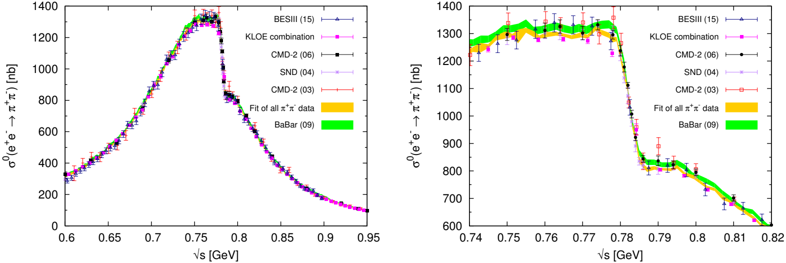
(e.g. - CARLOMAT 3.1 [Eur.Phys.J. C77 (2017) no.4, 254]?)

Slide by A. Keshavarzi (Liverpool) at 'Muon $g-2$ Workshop' at Mainz, June 18-22, 2018

$\pi^+\pi^-$ channel

⇒ Large improvement for 2π estimate

→ BESIII [Phys.Lett. B753 (2016) 629-638] and KLOE combination [arXiv:1711.03085] provide downward influence to mean value



⇒ Correlated & experimentally corrected $\sigma^0_{\pi\pi(\gamma)}$ data now entirely dominant

$$a_\mu^{\pi^+\pi^-}[0.305 \leq \sqrt{s} \leq 1.937\ \text{GeV}] = 502.97 \pm 1.14_{\rm stat} \pm 1.59_{\rm sys} \pm 0.06_{\rm vp} \pm 0.14_{\rm fsr}$$
$$= 502.97 \pm 1.97_{\rm tot}$$

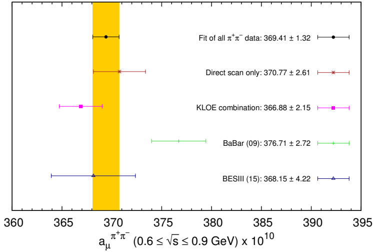
⇒ 15% local $\chi^2_{\rm min}/{\rm d.o.f.}$ error inflation due to tensions in clustered data

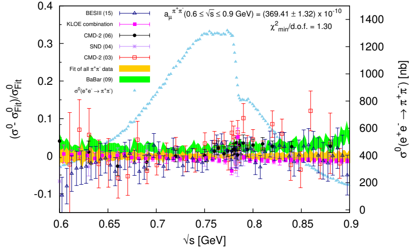
Slide by A. Keshavarzi (Liverpool) at ‘Muon $g-2$ HVP Workshop’ at KEK, Feb. 12-14, 2018

$\pi^+\pi^-$ channel

⇒ Tension exists between BaBar data and all other data in the dominant ρ region.

→ Agreement between other radiative return measurements and direct scan data largely compensates this.





BaBar data alone ⇒ $a_\mu^{\pi^+\pi^-}$(BaBar data only) $= 513.2 \pm 3.8$.

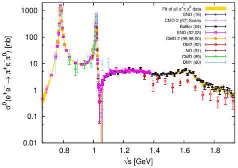
Simple weighted average of all data ⇒ $a_\mu^{\pi^+\pi^-}$(Weighted average) $= 509.1 \pm 2.9$.
(i.e. - no correlations in determination of mean value)

BaBar data dominate when no correlations are taken into account for the mean value
Highlights importance of fully incorporating all available correlated uncertainties

Slide by A. Keshavarzi (Liverpool) at 'Muon $g-2$ HVP Workshop' at KEK, Feb. 12-14, 2018

Other notable exclusive channels [KNT18: arXiv:1802.02995, PRD (in press)]

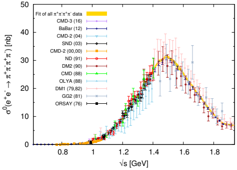
$\pi^+\pi^-\pi^0$



HLMNT11: 47.51 ± 0.99

KNT18: 47.92 ± 0.89

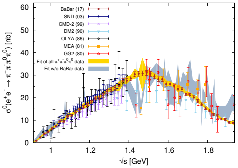
$\pi^+\pi^-\pi^+\pi^-$



HLMNT11: 14.65 ± 0.47

KNT18: 14.87 ± 0.20

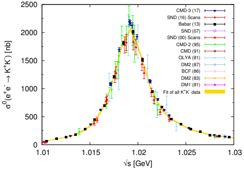
$\pi^+\pi^-\pi^0\pi^0$



HLMNT11: 20.37 ± 1.26

KNT18: 19.39 ± 0.78

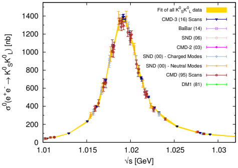
K^+K^-



HLMNT11: 22.15 ± 0.46

KNT18: 23.03 ± 0.22

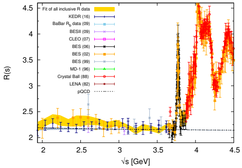
$K^0_S K^0_L$



HLMNT11: 13.33 ± 0.16

KNT18: 13.04 ± 0.19

Inclusive ($\sqrt{s} > 2$ GeV)



HLMNT11: 41.40 ± 0.87

KNT18: 41.27 ± 0.62

Slide by A. Keshavarzi (Liverpool) at 'Muon $g-2$ Workshop' at Mainz, June 18-22, 2018

KNT18 $a_\mu^{\text{had, VP}}$ update

HLMNT(11): 694.91 ± 4.27

$\downarrow$

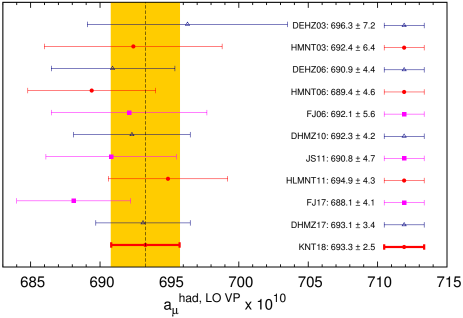
This work: $a_\mu^{\text{had, LO VP}} = 693.27 \pm 1.19_{\text{stat}} \pm 2.01_{\text{sys}} \pm 0.22_{\text{vp}} \pm 0.71_{\text{fsr}}$

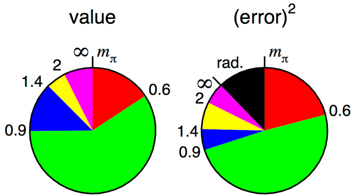
$= 693.27 \pm 2.34_{\text{exp}} \pm 0.74_{\text{rad}}$

$= 693.27 \pm 2.46_{\text{tot}}$

$a_\mu^{\text{had, NLO VP}} = -9.82 \pm 0.04_{\text{tot}}$

⇒ Accuracy better then 0.4%
(uncertainties include all available correlations)





⇒ 2π dominance

Slide by A. Keshavarzi (Liverpool) at 'Muon $g-2$ HVP Workshop' at KEK, Feb. 12-14, 2018

Comparison with other similar works

Channel	This work (KNT18)	DHMZ17	Difference
$\pi^+\pi^-$	503.74 ± 1.96	507.14 ± 2.58	-3.40 ± 3.24
$\pi^+\pi^-\pi^0$	47.70 ± 0.89	46.20 ± 1.45	1.50 ± 1.70
$\pi^+\pi^-\pi^+\pi^-$	13.99 ± 0.19	13.68 ± 0.31	0.31 ± 0.36
$\pi^+\pi^-\pi^0\pi^0$	18.15 ± 0.74	18.03 ± 0.54	0.12 ± 0.92
K^+K^-	23.00 ± 0.22	22.81 ± 0.41	0.19 ± 0.47
$K_S^0 K_L^0$	13.04 ± 0.19	12.82 ± 0.24	0.22 ± 0.31
$1.8 \le \sqrt{s} \le 3.7$ GeV	34.54 ± 0.56 (data)	33.45 ± 0.65 (pQCD)	1.09 ± 0.86
Total	693.3 ± 2.5	693.1 ± 3.4	0.2 ± 4.2

⇒ Total estimates from two analyses in very good agreement

⇒ Masks much larger differences in the estimates from individual channels

⇒ Unexpected tension for 2π considering the data input likely to be similar

→ Points to marked differences in way data are combined

→ From 2π discussion: $a_\mu^{\pi^+\pi^-}$ (Weighted average) $= 509.1 \pm 2.9$

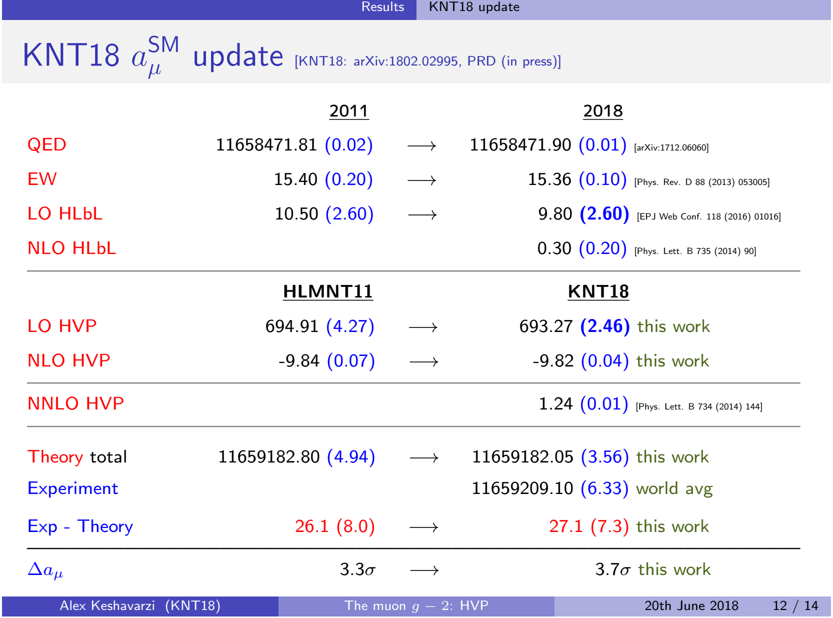
⇒ Compensated by lower estimates in other channels

→ For example, the choice to use pQCD instead of data above 1.8 GeV

⇒ FJ17: $a_{\mu,\,\mathrm{FJ17}}^{\mathrm{had,\,LO\,VP}} = 688.07 \pm 41.4$

→ Much lower mean value, but in agreement within errors

Slide by A. Keshavarzi (Liverpool) at 'Muon $g-2$ HVP Workshop' at KEK, Feb. 12-14, 2018



KNT18 a_μ^{SM} update [KNT18: arXiv:1802.02995, PRD (in press)]

	2011		**2018**	
QED	11658471.81 (0.02)	→	11658471.90 (0.01)	[arXiv:1712.06060]
EW	15.40 (0.20)	→	15.36 (0.10)	[Phys. Rev. D 88 (2013) 053005]
LO HLbL	10.50 (2.60)	→	9.80 **(2.60)**	[EPJ Web Conf. 118 (2016) 01016]
NLO HLbL			0.30 (0.20)	[Phys. Lett. B 735 (2014) 90]
	HLMNT11		**KNT18**	
LO HVP	694.91 (4.27)	→	693.27 **(2.46)**	this work
NLO HVP	-9.84 (0.07)	→	-9.82 (0.04)	this work
NNLO HVP			1.24 (0.01)	[Phys. Lett. B 734 (2014) 144]
Theory total	11659182.80 (4.94)	→	11659182.05 (3.56)	this work
Experiment			11659209.10 (6.33)	world avg
Exp - Theory	26.1 (8.0)	→	27.1 (7.3)	this work
Δa_μ	3.3σ	→	3.7σ	this work

(HVP: Hadronic Vacuum Polarization)
(HLbL: Hadronic Light-by-Light)
Slide by A. Keshavarzi (Liverpool) at ‘Muon $g-2$ Workshop’ at Mainz, June 18-22, 2018

SM prediction vs exp. value of a_μ

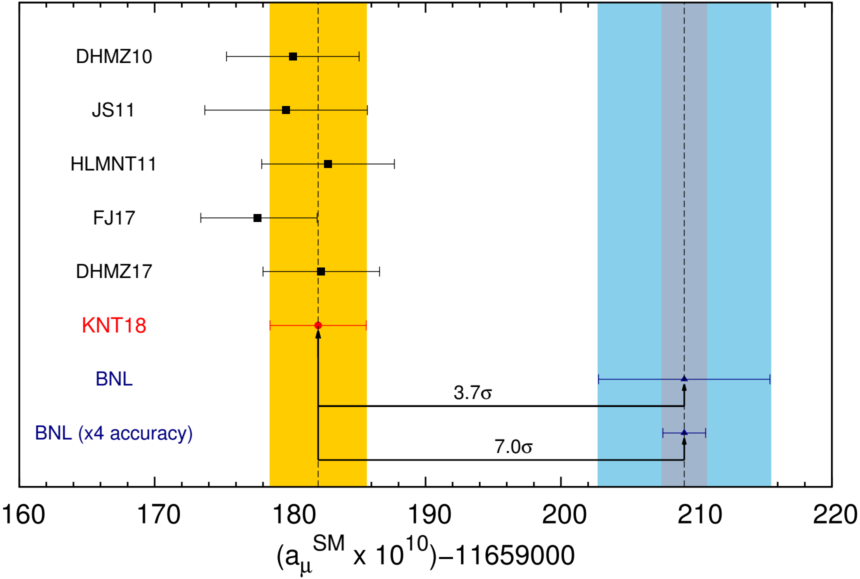
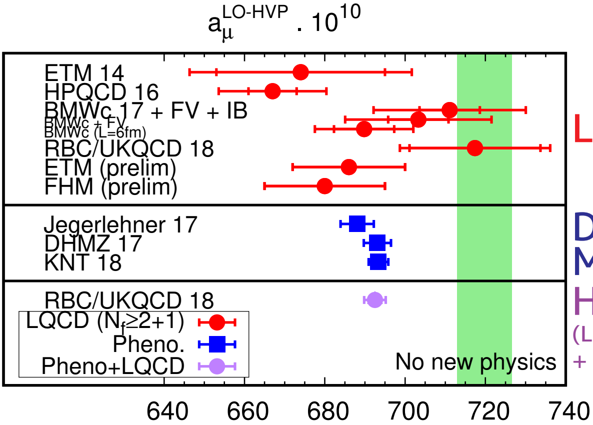


Fig. from KNT18, Phys. Rev. D97 (2018) 114025

Comparison with Lattice Results



Lattice

Dispersive Method

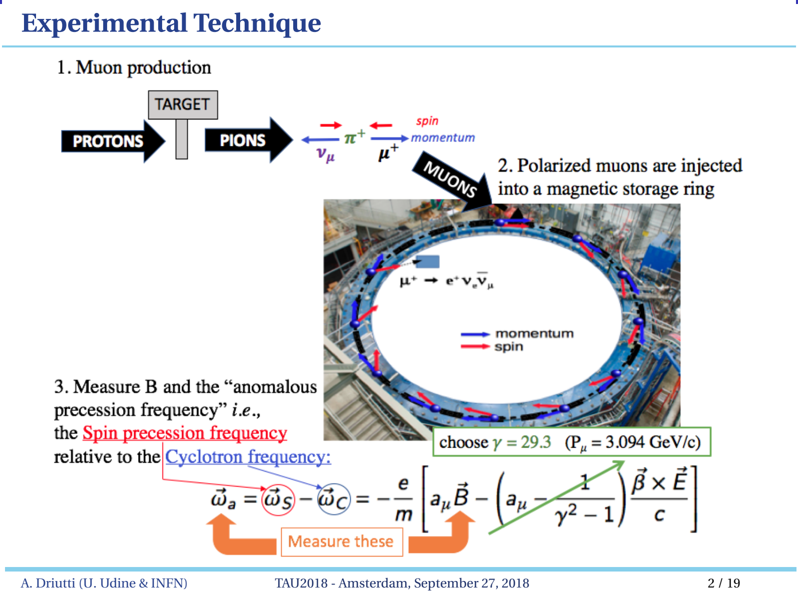
Hybrid

(Lattice + Disp. Method)

- Lattice errors ~ 2% vs phenomenology errors ~ 0.4%
- Some lattice results suggest new physics others not but all compatible with phenomenology

Slide by L. Lellouch (Marseille) at ‘Muon g-2 Workshop’ at Mainz, June 18-22, 2018

New muon g-2 exp. at Fermilab



Experimental Technique

1. Muon production

2. Polarized muons are injected into a magnetic storage ring

3. Measure B and the “anomalous precession frequency” *i.e.*, the Spin precession frequency relative to the Cyclotron frequency:

choose $\gamma = 29.3$ $(P_\mu = 3.094\ \mathrm{GeV/c})$

$$\vec{\omega}_a = \vec{\omega}_S - \vec{\omega}_C = -\frac{e}{m}\left[a_\mu \vec{B} - \cancel{\left(a_\mu - \frac{1}{\gamma^2-1}\right)}\frac{\vec{\beta}\times\vec{E}}{c}\right]$$

Measure these

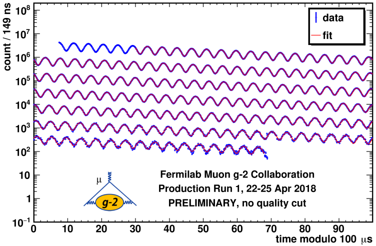
A. Driutti (U. Udine & INFN) TAU2018 - Amsterdam, September 27, 2018 2 / 19

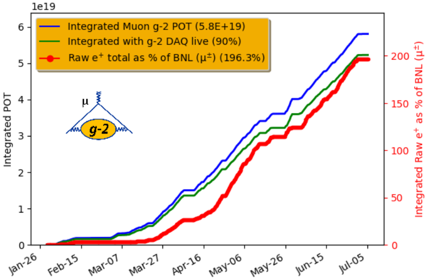
Slide by A. Driutti (Udine U. & INFN) at ‘tau 2018’ at Amsterdam, Sept. 24-28, 2018

Status of muon g-2 exp. at Fermilab

First physics run is successfully over.

- engineering run in Summer 2017
- commissioning run in Fall and Winter 2017
- first physics run from March to July 2018
- next physics run from November 2018 to July 2019





Slide by J. Kaspar (U. of Washington) at 'NuFACT 2018' at Virginia Tech, Aug. 13-18, 2018

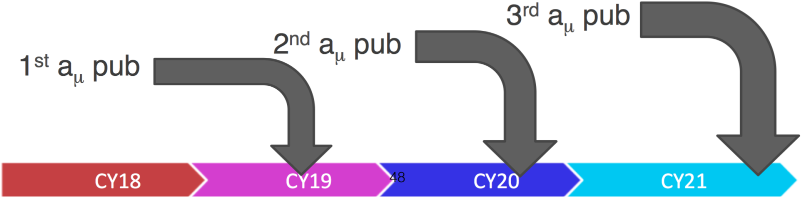
Publication Plan of Fermilab g-2 exp.

publication plan

Planning on three generations of g-2 publications:

- 1-2 x BNL (~400 ppb) collected in FY18 and aiming for publication in 2019.
- 5-10 x BNL (~200 ppb) collected over FY18+FY19 with publication by end of 2020.
- 20+ x BNL (~140 ppb) collected by end of FY20 with final publication at end of 2021 or early 2022

Muon EDM and CPT/LV physics results in at least two generations.



1st a_μ pub

2nd a_μ pub

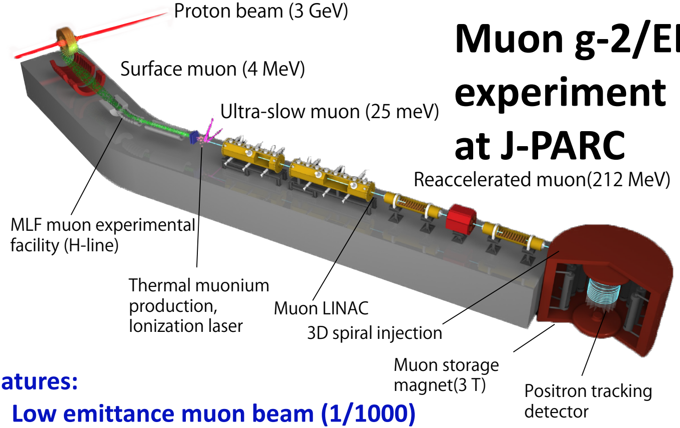
3rd a_μ pub

CY18 CY19 CY20 CY21

Slide by J. Kaspar (U. of Washington) at ‘NuFACT 2018’ at Virginia Tech, Aug. 13-18, 2018

New muon g-2 exp. at J-PARC

Muon g-2/EDM experiment at J-PARC

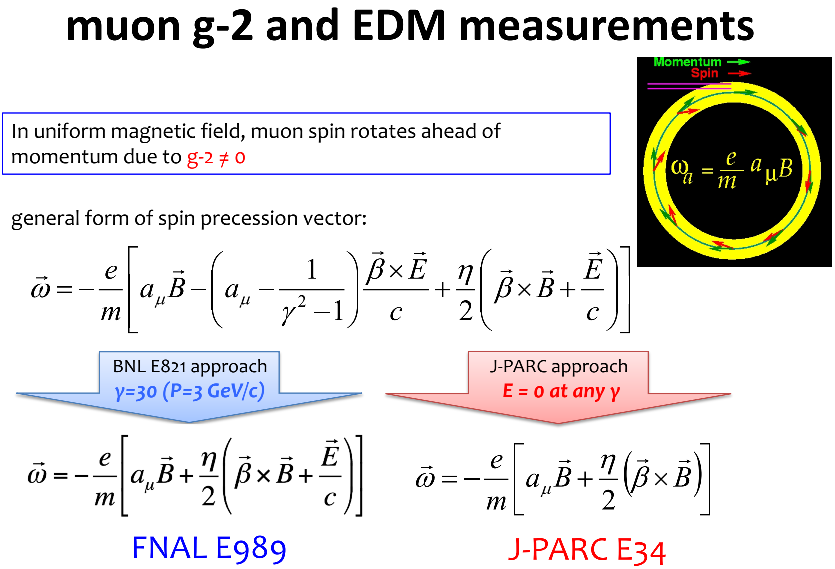


Features:

- **Low emittance muon beam (1/1000)**
- **No strong focusing (1/1000) & good injection eff. (x10)**
- **Compact storage ring (1/20)**
- **Tracking detector with large acceptance**
- **Completely different from BNL/FNAL method**

Slide by T. Mibe (KEK) at ‘tau 2018’ at Amsterdam, Sept. 24-28, 2018

New muon g-2 exp. at J-PARC



muon g-2 and EDM measurements

In uniform magnetic field, muon spin rotates ahead of momentum due to g-2 ≠ 0

general form of spin precession vector:

$$\vec{\omega} = -\frac{e}{m}\left[a_\mu \vec{B} - \left(a_\mu - \frac{1}{\gamma^2-1}\right)\frac{\vec{\beta}\times\vec{E}}{c} + \frac{\eta}{2}\left(\vec{\beta}\times\vec{B} + \frac{\vec{E}}{c}\right)\right]$$

BNL E821 approach
γ=30 (P=3 GeV/c)

J-PARC approach
E = 0 at any γ

$$\vec{\omega} = -\frac{e}{m}\left[a_\mu \vec{B} + \frac{\eta}{2}\left(\vec{\beta}\times\vec{B} + \frac{\vec{E}}{c}\right)\right]$$

$$\vec{\omega} = -\frac{e}{m}\left[a_\mu \vec{B} + \frac{\eta}{2}\left(\vec{\beta}\times\vec{B}\right)\right]$$

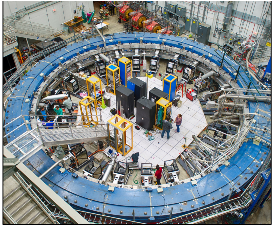
FNAL E989

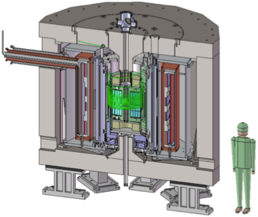
J-PARC E34

5

Slide by T. Mibe (KEK) at 'tau 2018' at Amsterdam, Sept. 24-28, 2018

Summary of new muon g-2 exp.





FNAL

- 7 m radius storage ring
- B = 1.45 T
- weak electric focusing
- high-rate 3 GeV/c beam
- spin polarization 97 %
- data taking 2018 - 2020
- 100 ppb by end of 2021

J-PARC

- 0.33 radius storage bottle
- B = 3 T
- no E -field, week mag. focusing
- 0.3 GeV/c beam
- spin polarization 50 %
- data taking 2020 - 2023
- 400 ppb by end of 2023

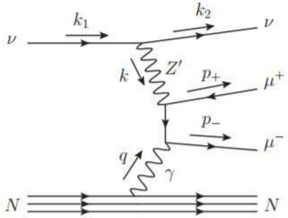
Slide by J. Kaspar (U. of Washington) at 'NuFACT 2018' at Virginia Tech, Aug. 13-18, 2018

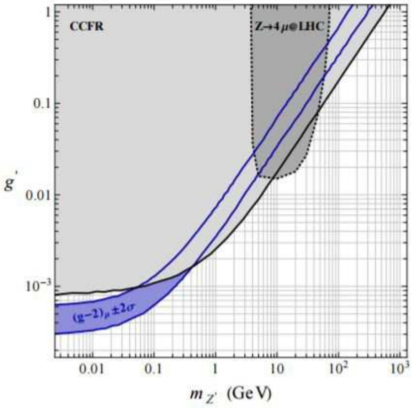
Some Alternative Possibilities for $g_\mu - 2$

- Gauged $L_\mu - L_\tau$: anomaly free

Altmannshofer, Gori, Pospelov, Yavin, 1403.1269, 1406.2332

- Constrained by "trident processes" for low vector masses





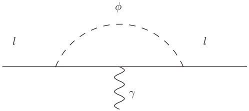
Figures from 1406.2332

Slide by H. Davoudiasl (BNL) at muon g-2 workshop at KEK, Feb. 12-14, 2018

Moments from a Dark Higgs

Chen, HD, Marciano, Zhang, 2015

- Consider a GeV scale "dark" Higgs ϕ as main source of $g_\mu - 2$



- Potentially associated with $U(1)_d$ breaking
- Effective coupling of ϕ with leptons:

$$\mathcal{L}_{\phi\ell\ell} = -\phi\bar{\ell}\left(\lambda_S^\ell + i\lambda_P^\ell\gamma_5\right)\ell$$

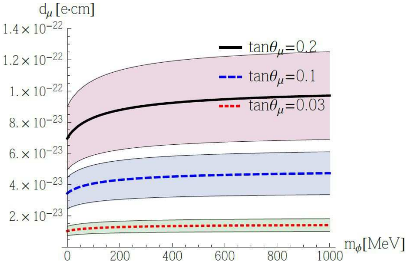
- $\ell = e, \mu, \tau$, and λ_S^ℓ (λ_P^ℓ) CP-even (odd) coupling See also Geng and Ng, 1989

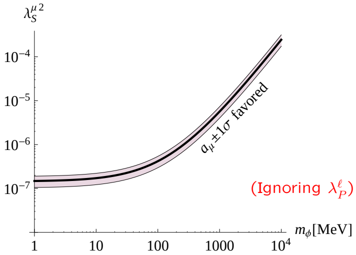
- Induced by heavy vector-like fermions (integrated out at $E \sim m_\mu$)

[Alternate model: ϕ-Higgs mixing (leptonic 2HDM)
Batell, Lange, McKeen, Pospelov, Ritz, 1606.04943]

Slide by H. Davoudiasl (BNL) at muon g-2 workshop at KEK, Feb. 12-14, 2018

μ-EDM, assuming $a_\mu \pm 1\sigma$; tan $\theta_\ell = \lambda_P^\ell/\lambda_S^\ell$





(Ignoring λ_P^ℓ)

Various experimental implications:

- Direct probes: bump hunting in μ decay and capture, K decay including a μ
- Potentially observable muon EDM $\lesssim 10^{-22}$ e cm Semertzidis et al., 2000, 2003
- Current bound $|d_\mu| < 1.8 \times 10^{-19}$ $e \cdot$ cm Muon (g-2) Collaboration, 2008
- Possible lepton flavor violating decays, deviation in BR$(H \to \mu^+\mu^-)_{\text{SM}}$

Slide by H. Davoudiasl (BNL) at muon g-2 workshop at KEK, Feb. 12-14, 2018

- Axion-Like Particles   Marciano, Masiero, Paradisi, Passera, 1607.01022
- Pseudo-scalar a: $\mathcal{L}_a = \frac{1}{4} g_{a\gamma\gamma} a\, F_{\mu\nu} \tilde{F}^{\mu\nu} + i y_{al} a\, \bar{l} \gamma_5 l$

$$[\mathcal{L}_s\colon\ (\tilde{F}, g_{a\gamma\gamma}, i y_{al}) \to (F, g_{s\gamma\gamma}, y_{sl})]$$

- "Barr-Zee" diagram (B) or LBL (C):

$$a_l^{\mathrm{BZ},a,s} \propto g_{a\gamma\gamma} y_{al} \quad ; \quad a_l^{\mathrm{LBL},a} \propto -\, a_l^{\mathrm{LBL},s} \propto g_{a\gamma\gamma}^2$$

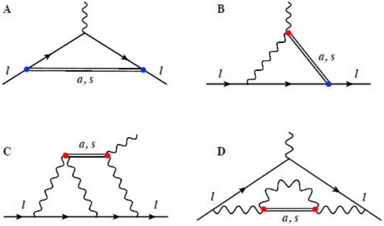
- $g_\mu - 2$: $y_{al} g_{a\gamma\gamma} > 0$; $g_{a\gamma\gamma} \sim 10^{-(2-4)}$ GeV^{-1}
- $g_{a\gamma\gamma}$ large; can be experimentally allowed

See Bauer, Neubert, Thammm, 1708.00443, for a detailed discussion of constraints

- May require non-trivial model building   Marciano, Masiero, Paradisi, Passera, 2016

- VP contribution (D) small

Figures from 1607.01022



20

Slide by H. Davoudiasl (BNL) at muon g-2 workshop at KEK, Feb. 12-14, 2018

Summary

- Standard Model prediction for $(g-2)_\mu$: $\gtrsim 3.5\sigma$ deviation from measured value $\Longrightarrow$ New Physics?
- Recent data-driven evaluations of hadronic vacuum polarization contributions seem convergent (Similar mean values from KNT18 and Davier et al with slightly smaller uncertainty from KNT18.)
- To better establish the $g-2$ anomaly, better data for $e^+e^- \to \pi^+\pi^-$ welcome (from Belle II !?)
- Lattice calculations still suffer from large uncertainties
- New exp. at Fermilab and J-PARC expected to reduce the uncertainty of $(g-2)_\mu$ by a factor of 4