1 November 2018

Flavor Physics Workshop 2018@IPMU

Phys.Lett.B785(2018) 536-542 [arXiv:1805.10793]

Tomography by neutrino pair beam

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Thanks to the remarkable efforts of various experiments neutrino oscillations have been measured accurately.

So we consider seriously

the application of neutrino physics

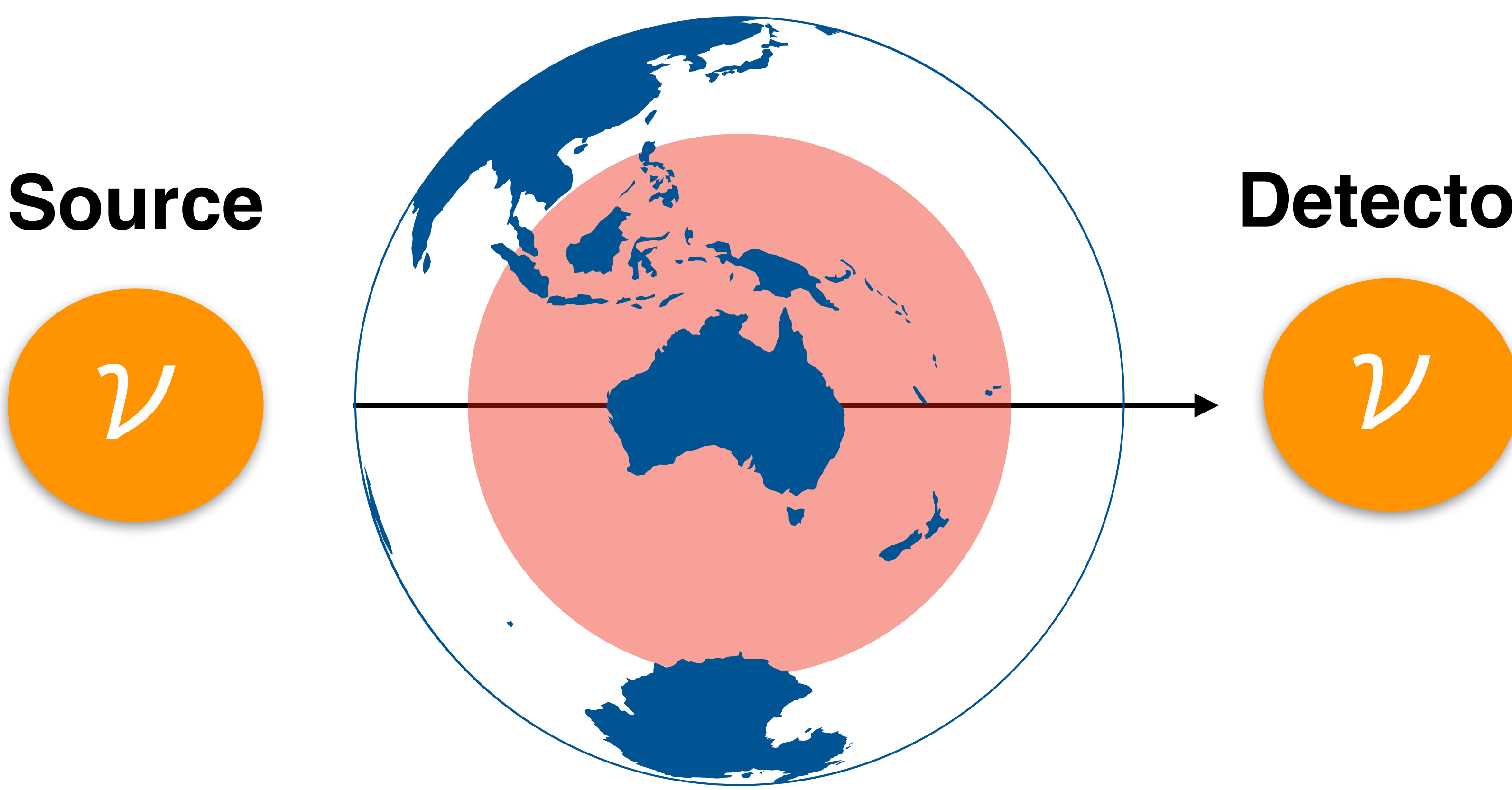
to various fields of basic science.

One of the applications of neutrino physics is

Neutrino Tomography

The idea of neutrino tomography

Imaging of the Earth's interior structure using neutrino.



Neutrino can easily transmit the Earth due to the weakness of its interaction.

Neutrino Tomography

There are three different methods.

Neutrino Absorption Tomography

- L. V. Volkova and G. T. Zatsepin, Bull. Acad. Sci. USSR, Phys. Ser. 38 (1974) 151.
  And more …

Neutrino Diffraction Tomography

- A.D. Fortes, I. G.Wood, and L. Oberauer, Astron. Geophys. 47(2006) 5.31–5.33.
- R. Lauter, Astron. Nachr. 338 (2017) no.1, 111.

Neutrino Oscillation Tomography

- T. Ohlsson andW.Winter, Europhys. Lett. 60 (2002) 34
- E. K. Akhmedov, M. A. Tortola and J.W. F. Valle, JHEP 0506, 053 (2005)
- W.Winter, Nucl. Phys. B 908 (2016) 250
- A.N. Ioannisian and A. Y. Smirnov, Phys. Rev. D 96 (2017) no.8, 083009
  And more …

There is no realistic tomography method.

∵

- There is no powerful source.
- There is no established reconstruction method.

We discuss the neutrino oscillation tomography precisely from now on.

Neutrino Oscillation in Matter

Neutrino oscillation in matter

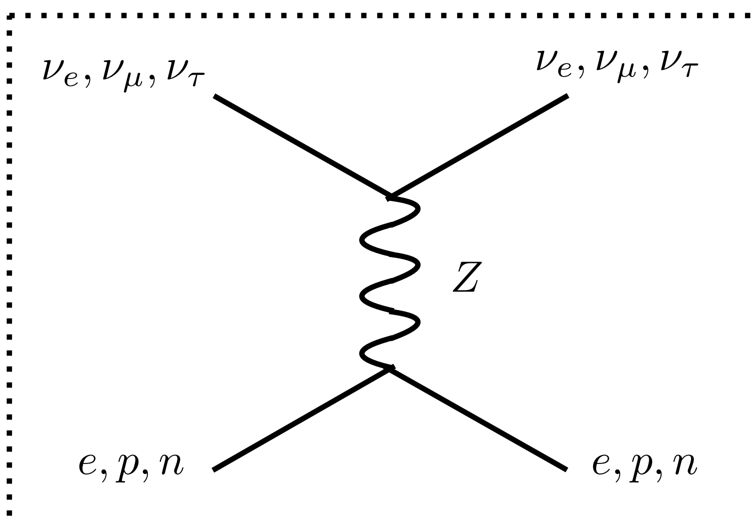
In matter, effective potential is added to the vacuum Hamiltonian.

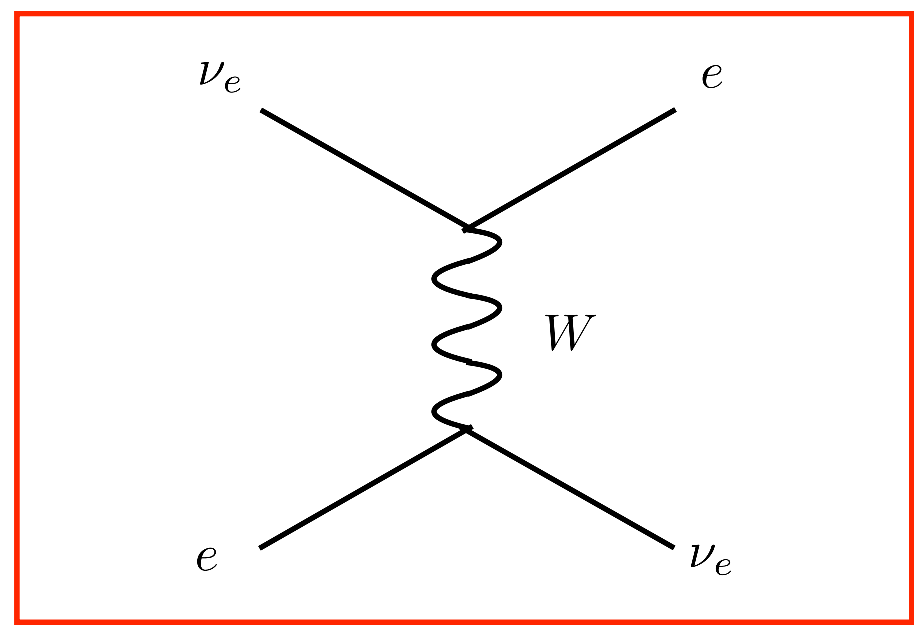
For simplicity, we consider the 2 flavor neutrino oscillation.

$$A_{\nu_\alpha \to \nu_\beta} = \langle \nu_\beta | \nu_\alpha(x) \rangle$$

$$i\frac{d}{dx}\begin{pmatrix} A_{\nu_e \to \nu_e} \\ A_{\nu_e \to \nu_\mu} \end{pmatrix} = \left[U \begin{pmatrix} 0 & 0 \\ 0 & \frac{\Delta m^2}{2E} \end{pmatrix} U^\dagger + \begin{pmatrix} V_{CC}(x) & 0 \\ 0 & 0 \end{pmatrix} \right] \begin{pmatrix} A_{\nu_e \to \nu_e} \\ A_{\nu_e \to \nu_\mu} \end{pmatrix}$$

Vacuum contribution **Additional effective potential**





The main contribution to the potential is the CC interaction and effective potential depends on the electron number density.

Neutrino Oscillation in Matter

For simplicity, we consider the 2 flavor neutrino oscillation.

$$A_{\nu_\alpha \to \nu_\beta} = \langle \nu_\beta | \nu_\alpha(x) \rangle$$

$$i\frac{d}{dx}\begin{pmatrix} A_{\nu_e \to \nu_e} \\ A_{\nu_e \to \nu_\mu} \end{pmatrix} = \left[U \begin{pmatrix} 0 & 0 \\ 0 & \frac{\Delta m^2}{2E} \end{pmatrix} U^\dagger + \begin{pmatrix} V_{CC}(x) & 0 \\ 0 & 0 \end{pmatrix} \right] \begin{pmatrix} A_{\nu_e \to \nu_e} \\ A_{\nu_e \to \nu_\mu} \end{pmatrix}$$

Effective potential is written as

$$V_{CC}(x) = \sqrt{2} G_F n_e(x)$$

The electron number density is translated into the matter density.

$$n_e(x) \simeq \frac{\rho(x)}{2m_p}$$

$$\begin{aligned} \rho &= m_p n_p + m_n n_n + m_e n_e \\ &\simeq m_N (n_p + n_n) \\ &\simeq m_N 2 n_e \\ \therefore n_e &\simeq \frac{\rho}{2m_N} \end{aligned}$$

$$m_p \simeq m_n \gg m_e$$

$$n_e = n_p = n_n$$

Neutrino Oscillation in Matter

For simplicity, we consider the 2 flavor neutrino oscillation.

$$A_{\nu_\alpha \to \nu_\beta} = \langle \nu_\beta | \nu_\alpha(x) \rangle$$

$$i\frac{d}{dx}\begin{pmatrix} A_{\nu_e \to \nu_e} \\ A_{\nu_e \to \nu_\mu} \end{pmatrix} = \left[U \begin{pmatrix} 0 & 0 \\ 0 & \frac{\Delta m^2}{2E} \end{pmatrix} U^\dagger + \begin{pmatrix} V_{CC}(x) & 0 \\ 0 & 0 \end{pmatrix} \right] \begin{pmatrix} A_{\nu_e \to \nu_e} \\ A_{\nu_e \to \nu_\mu} \end{pmatrix}$$

Effective potential is written as

$$V_{CC}(x) = \sqrt{2} G_F n_e(x)$$

The electron number density is translated into the matter density.

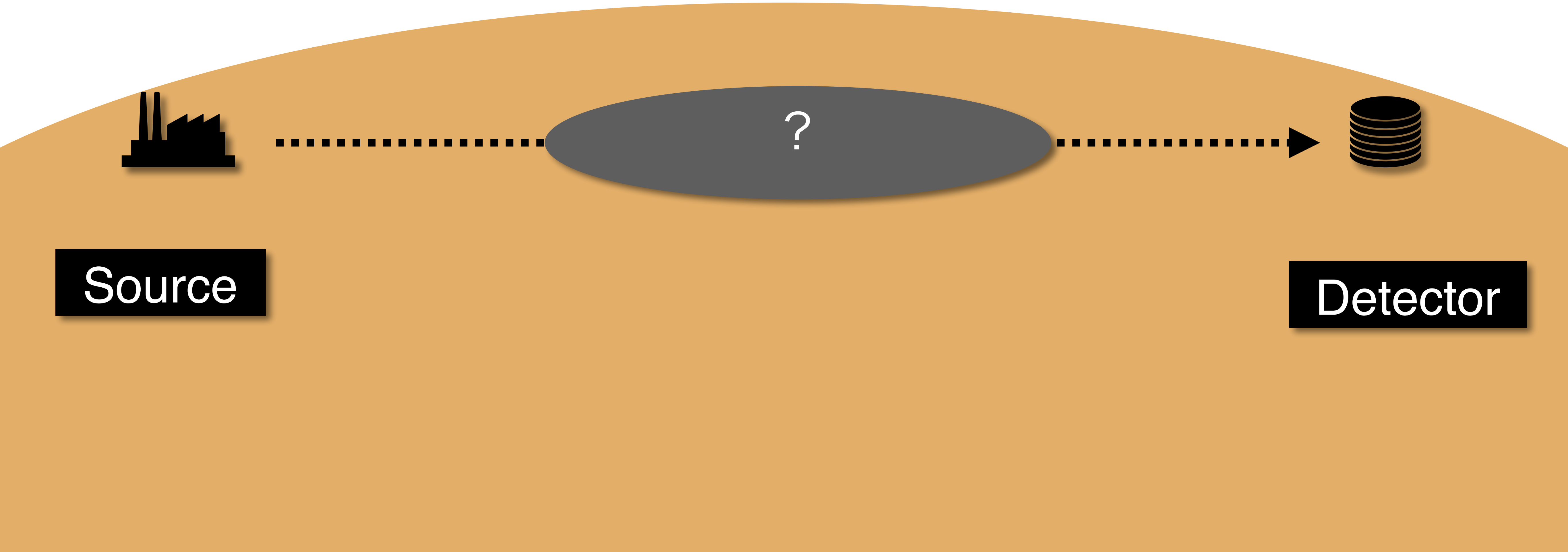
$$n_e(x) \simeq \frac{\rho(x)}{2m_p}$$

Probability is calculated as follow

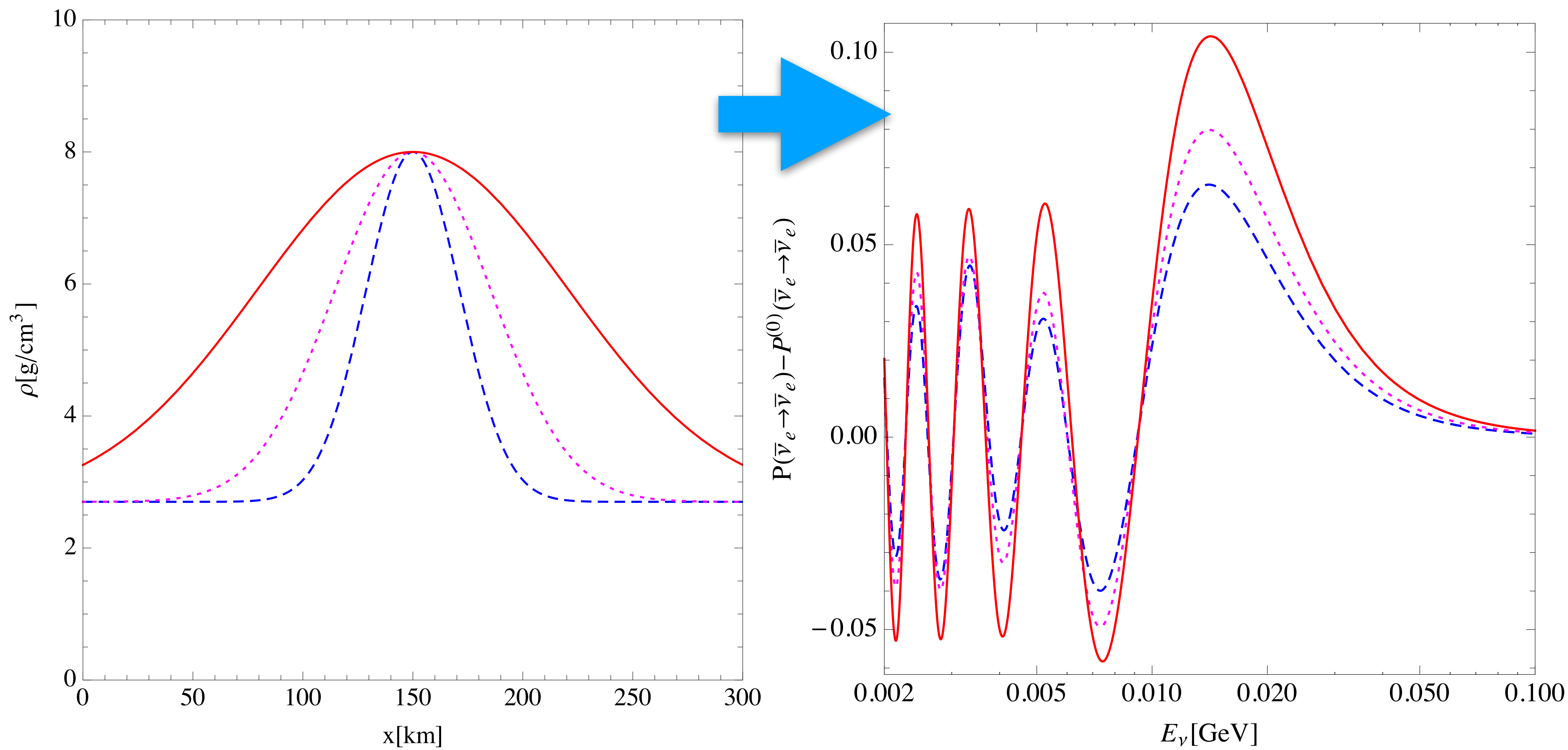
$$P_{\nu_\alpha \to \nu_\beta}(E_\nu, x) = |A_{\nu_\alpha \to \nu_\beta}(E_\nu, x)|^2$$

Neutrino path length

Measurement of energy spectrum



Influence of the density profile

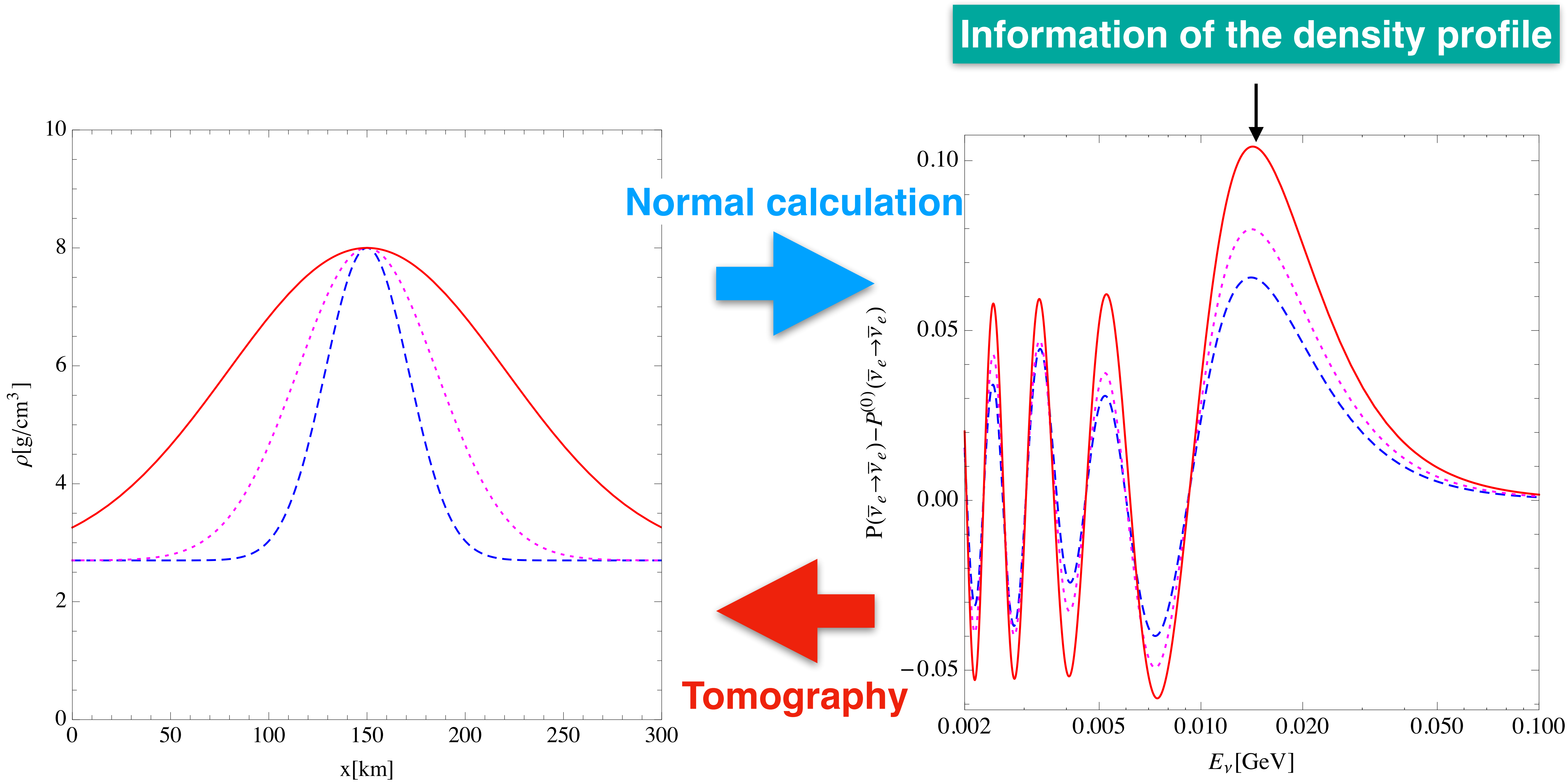


Density Profile

Oscillation Probability (subtracted the vacuum contribution)

Energy spectrum of the oscillation probability changes according to the density profile.

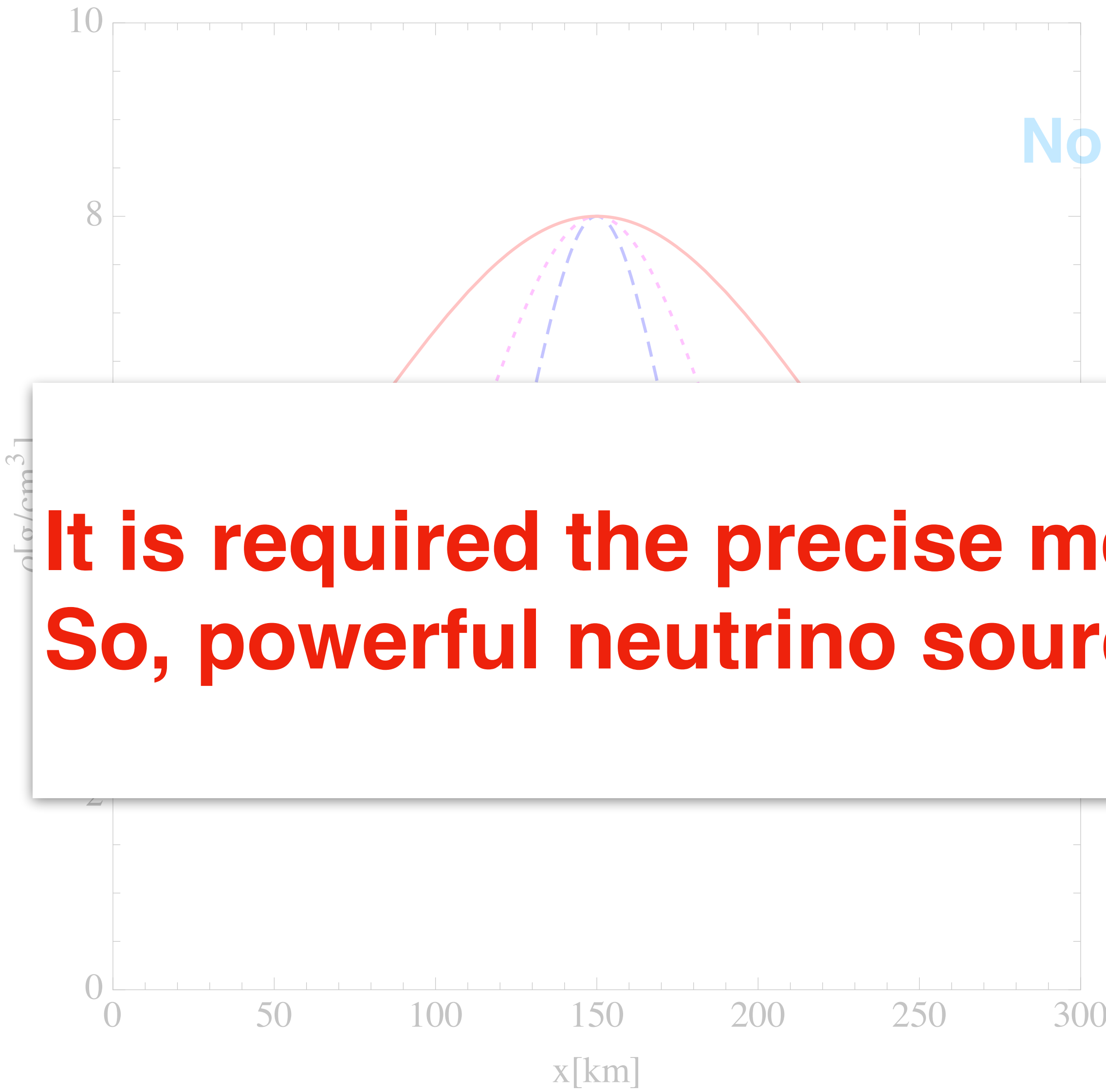
Neutrino Oscillation Tomography



But this effect is a few percent !

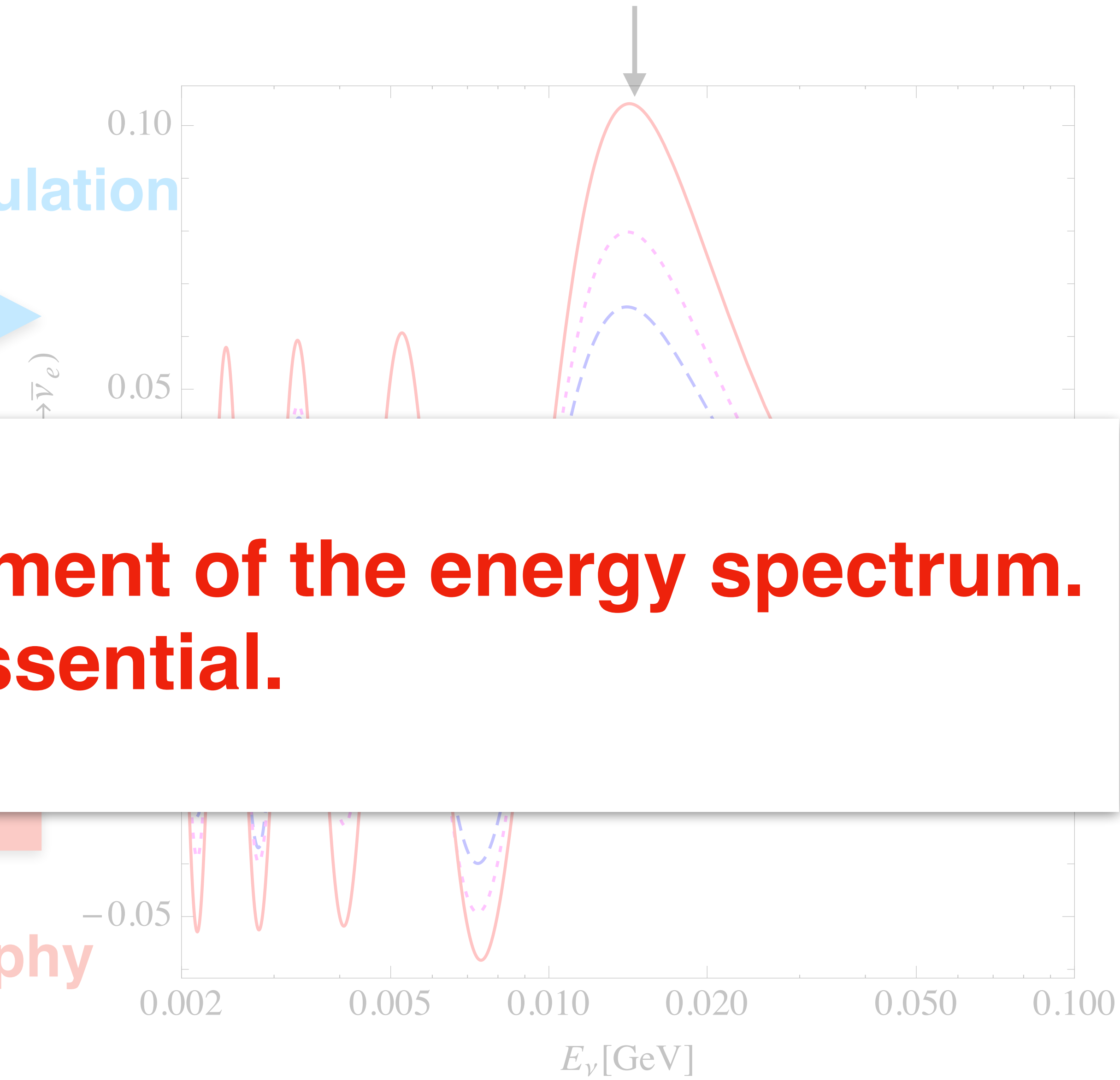
Neutrino Oscillation Tomography

Information of the density profile



Normal calculation

Tomography



It is required the precise measurement of the energy spectrum.
So, powerful neutrino source is essential.

But this effect is few percent !

Open Questions

- **How do we realize accurate energy spectrum measurement ?**
- **How do we reconstruct the Earth's density distribution ?**

Tomography by neutrino pair beam

Phys.Lett.B785 (2018) 536-542 [arXiv:1805.10793]

T Asaka, HO, M Tanaka, M Yoshimura

Open Questions

- **How do we realize accurate energy spectrum measurement ?**

  → Powerful source (Neutrino pair beam)

- **How do we reconstruct the Earth's density distribution ?**

  → Reconstruction method with 2nd order perturbation

Neutrino Source

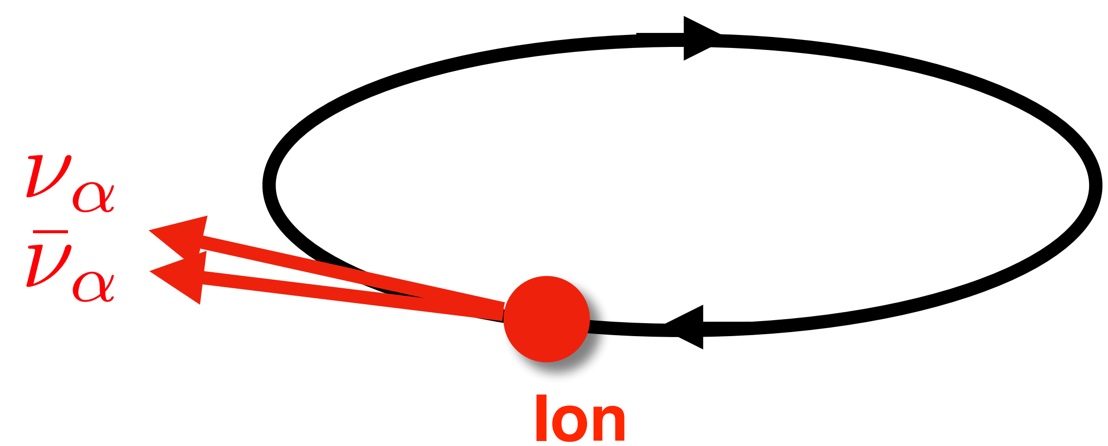
Neutrino Pair Beam

The pair beam, which has been proposed recently, can produced a large amount of neutrino pairs from the circulating partially stripped ions.

[Yoshimura, Sasao, Phys. Rev. D 92, 073015 (2015)]

Characteristics of the Neutrino Pair Beam

- **It generates the all flavor neutrino pairs** $(\nu_e, \overline{\nu_e}), (\nu_\mu, \overline{\nu_\mu}), (\nu_\tau, \overline{\nu_\tau})$
- **Very high intensity flux of neutrino beam**
- **High beam directivity**
- **Mainly electronic type of the pair being generated**



Neutrino Tomography requires the precise measurement of the energy spectrum for the precise reconstruction of the density profile.

This high event rate (high flux) is essential.

Comparison of neutrino flux

Source	Energy	Flux
Atmospheric : ν_μ $(\cos\theta_Z = 0)$	3.2 GeV	$3.6 \times 10^2 [\mathrm{m}^{-2}\mathrm{s}^{-1}]$
Solar	10MeV	$10^4 [\mathrm{m}^{-2}\mathrm{s}^{-1}]$
T2K at SK : ν_μ	1GeV	$2 \times 10^4 [\mathrm{m}^{-2}\mathrm{s}^{-1}]$
Beta beam at 100 km : $\bar{\nu}_e$	581 MeV (average)	$2.1 \times 10^5 [\mathrm{m}^{-2}\mathrm{s}^{-1}]$
Neutrino Pair Beam at 100 km	100 MeV	$\sim 10^{10}\ [\mathrm{m}^{-2}\mathrm{s}^{-1}]$
Neutrino Pair Beam at 300 km	100 MeV	$\sim 10^{9}\ [\mathrm{m}^{-2}\mathrm{s}^{-1}]$

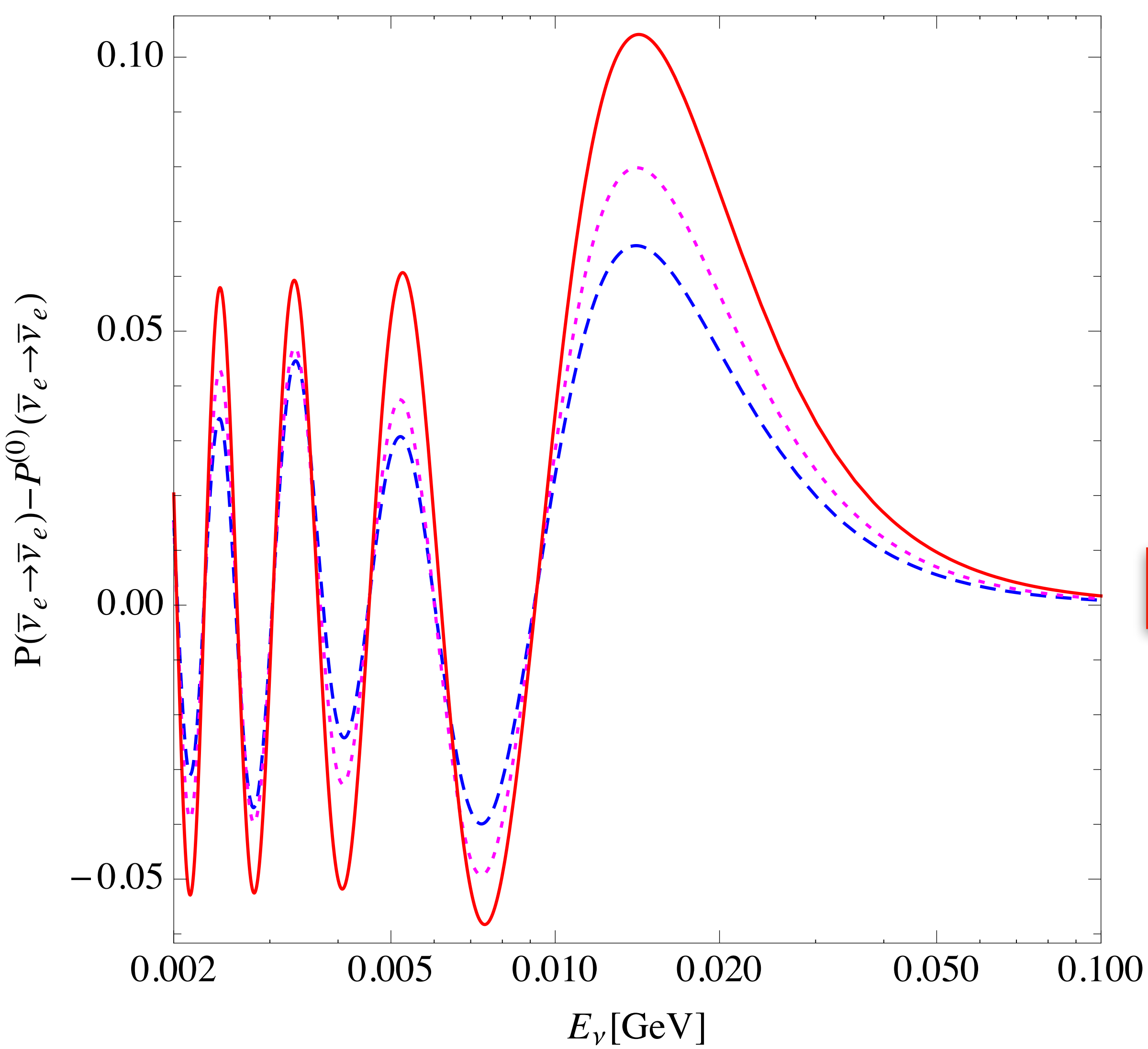
Atmospheric : M. Honda et.al., PhysRevD.92.023004
Solar : J. N. Bahcall et.al., New J. Phys. 6 (2004) 63
T2K : K. Abe et al. Phys. Rev. D 87 (2013) no.1, 012001
Beta beam : P. Zucchelli, Phys. Lett. B 532 (2002) 166
Neutrino Pair Beam : M.Yoshimura, N.Sasao, Phys.Rev.D92(2015) no.7, 073015

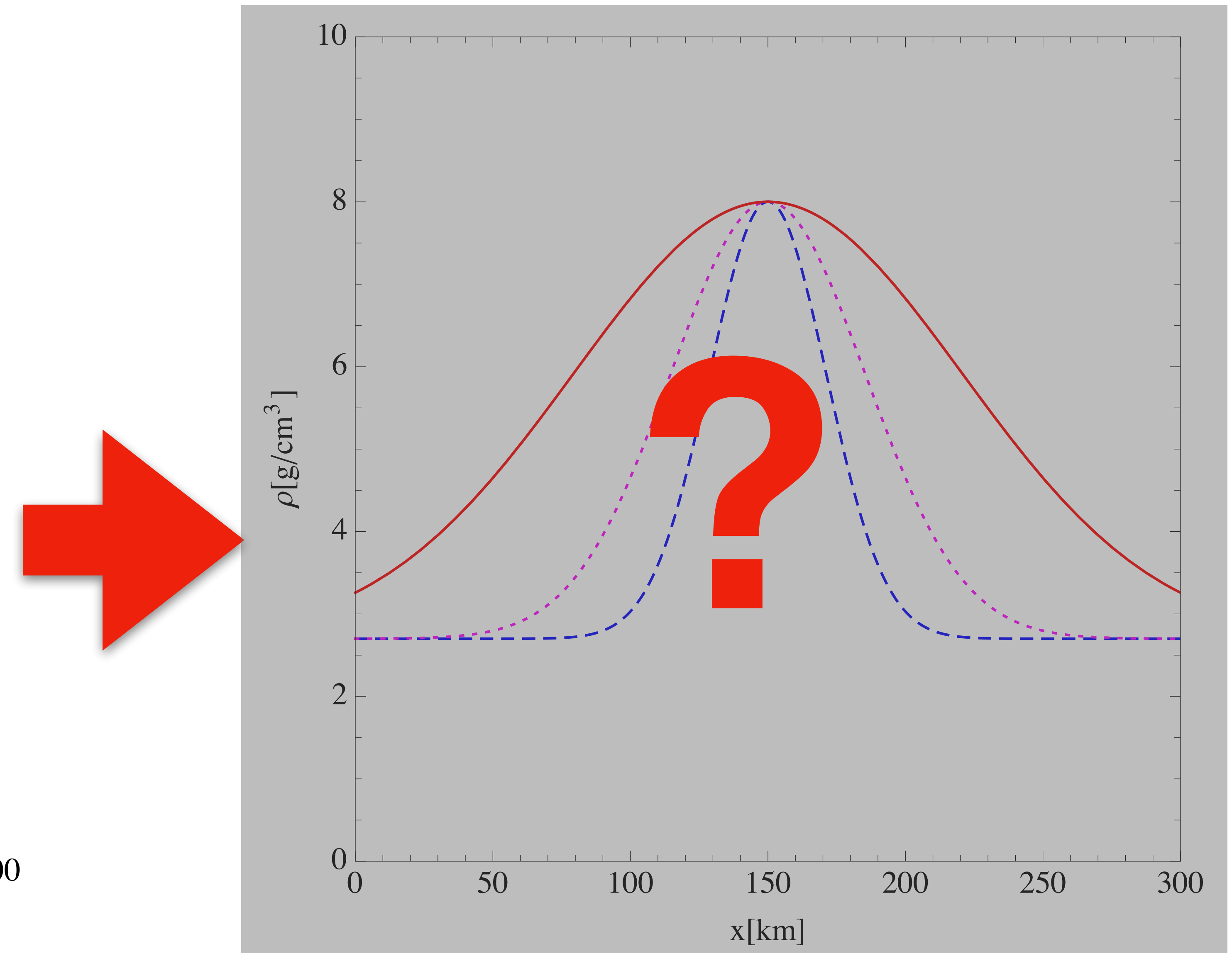
Production amount of neutrino (estimation)

nuMAX (Neutrino Factory) : $\sim 10^{20}$ / yr
NPB : $\sim 10^{22}$ / yr

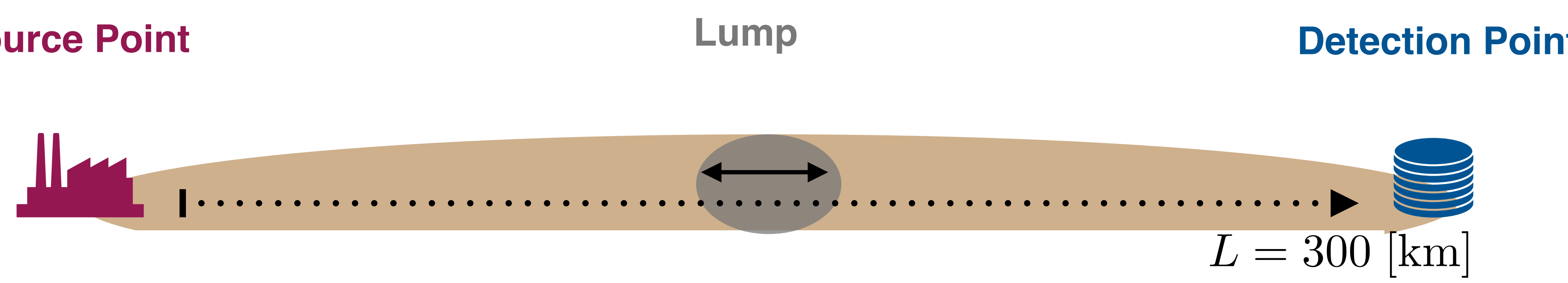
Reconstruction Method

How reconstruct the density profile from the energy spectrum of the neutrino oscillation?





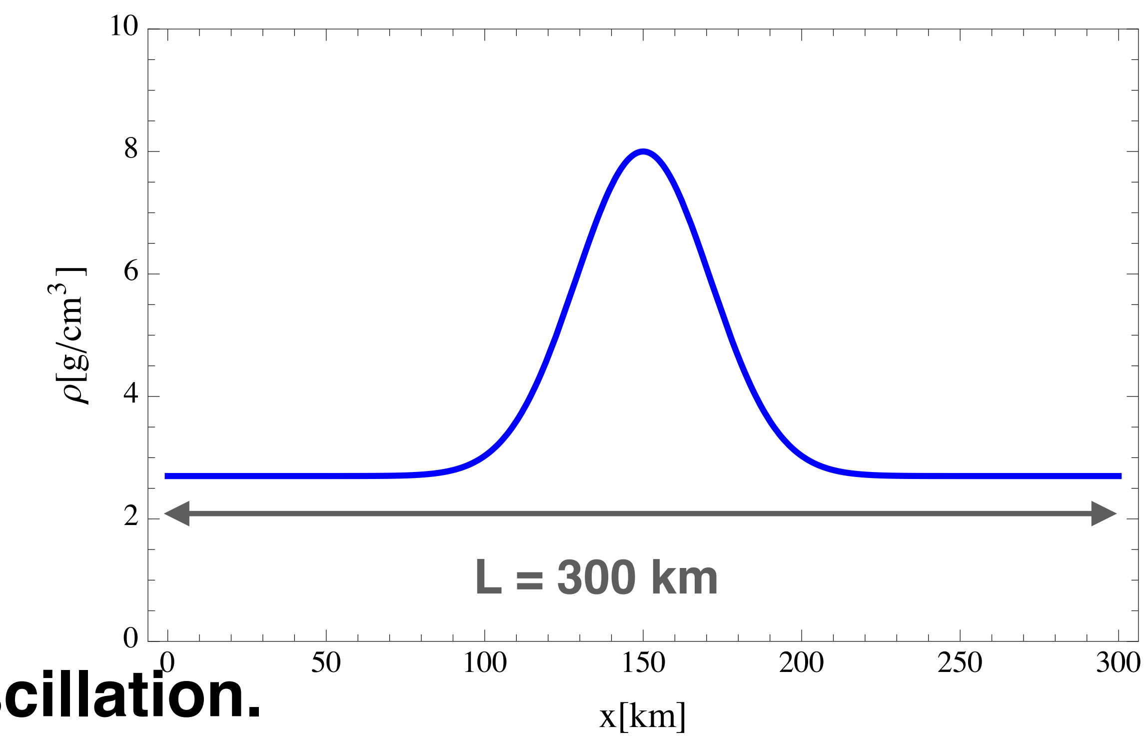
Toy model



- **We consider the symmetric exponential type of the density profile.**

$$\rho(x) = \bar{\rho} + (\rho_l - \bar{\rho}) \exp\left[-\frac{\left(x - \frac{L}{2}\right)^2}{D_l^2}\right]$$

L : length of the baseline
D_l : width of the lump



- **We consider the low energy $\bar{\nu}_e \to \bar{\nu}_e$ oscillation.**

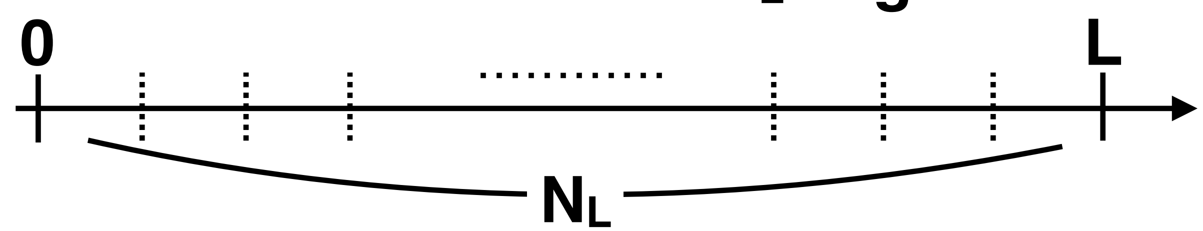
$$E_\nu : 2 \sim 100 \text{ [MeV]}$$

- **We assume the huge liquid Argon as the neutrino detector.**

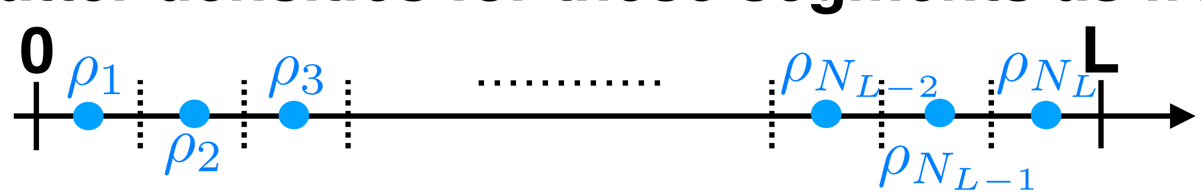
Fiducial volume 10^5 m^3

Density Profile Reconstruction Method

1. We discretize the neutrino baseline into the N_L segments.

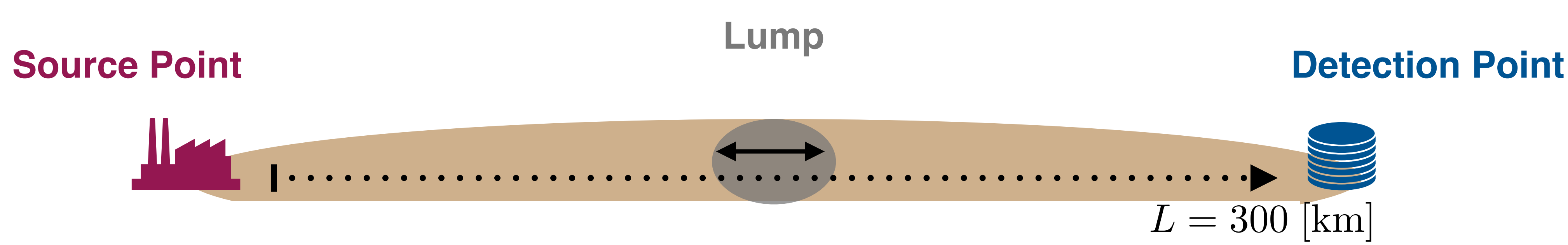


2. We consider the matter densities for these segments as free parameters ρ_j .



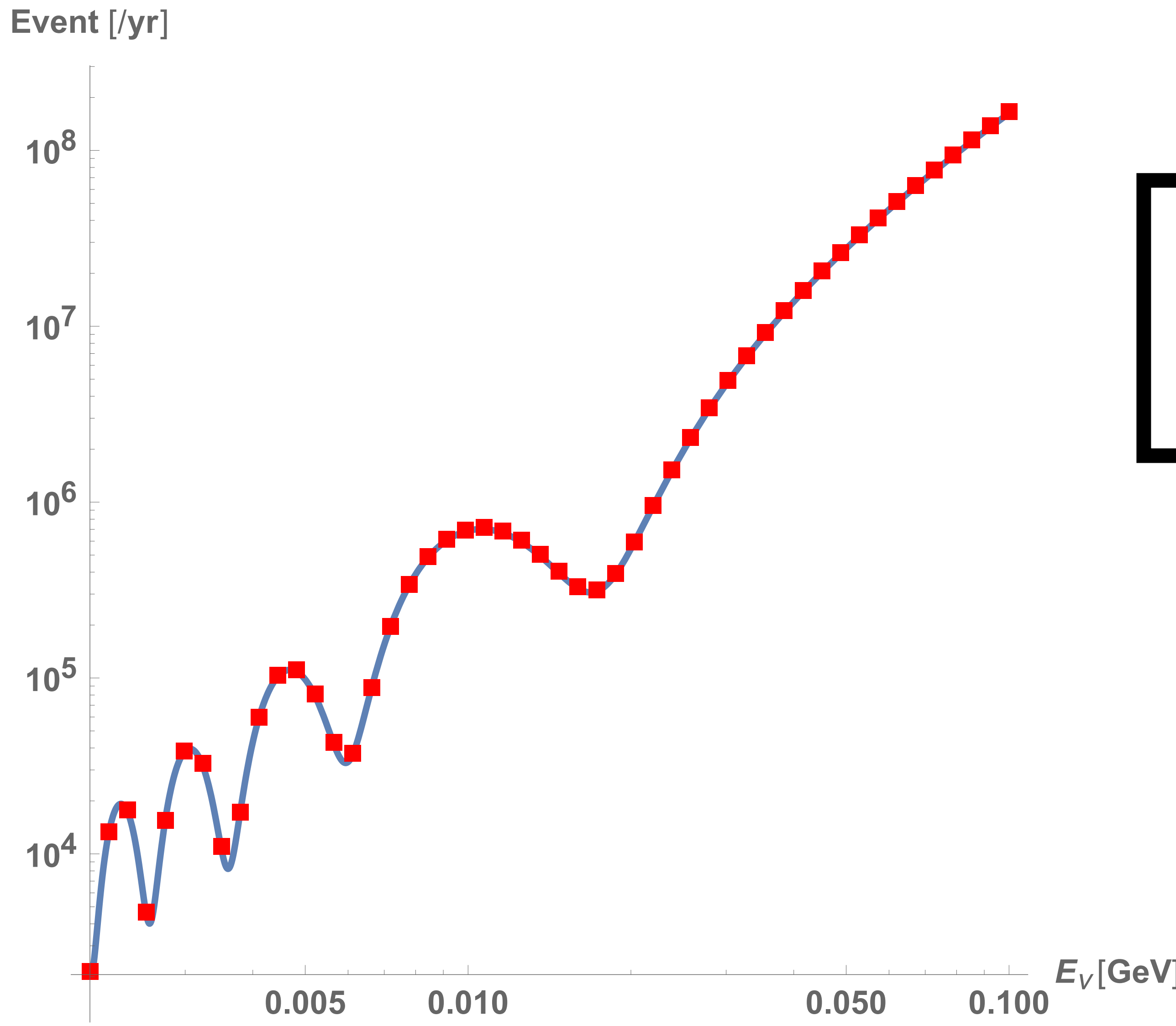
$j = 1, \cdots, N_L$

We assume that the each density is constant within each segment.



Density Profile Reconstruction Method

3. We also divide the energy range into the N_E parts, and define the χ^2 function



$$\chi^2 = \sum_{i=1,N_E} \frac{\left[N^{\rm obs}(E_i) - N^{\rm th}(E_i)\right]^2}{\sigma^2(E_i)}$$

$$\sigma(E_i) = \sqrt{N^{\rm obs}(E_i)}.$$

$$N(E_\nu) \simeq 4.73 \times 10^7 \times P(\bar{\nu}_e \to \bar{\nu}_e; E_\nu) \times \left(\frac{E_\nu}{100\ {\rm MeV}}\right)^{\frac{5}{2}} \left(\frac{\Delta E_\nu}{1\ {\rm MeV}}\right) \left(\frac{300\ {\rm km}}{L}\right)^2 \times \left(\frac{V_d}{10^5\ {\rm m}^3}\right) \left(\frac{T}{1\ {\rm year}}\right).$$

4. We determine those density by minimizing the χ^2 function by comparing the experimental data $N^{\rm obs}(E_i)$ for a given original profile ρ(x) with the theoretical prediction $N^{\rm th}(E_i)$ from unknown parameters ρ_j .

Perturbation Formula

$$N^{\rm th}(E_i) = \text{flux} \times P_{\bar{\nu}_e \to \bar{\nu}_e}(E, L) \times \text{detection rate}$$

Neutrino oscillation probability is calculated from the evolution equation.

$$i\frac{d}{dx}\vec{A}(x) = [H_0^F + V^F]\vec{A}(x)$$

Then we assume the relation $H_0^F > V^F$

And calculate the oscillation probability by perturbation.

$$P_{\alpha\beta} = |A^{(0)}_{\beta\alpha} + A^{(1)}_{\beta\alpha} + A^{(2)}_{\beta\alpha} + \cdots|^2$$

$$= |A^{(0)}_{\beta\alpha}|^2 + A^{(0)*}_{\beta\alpha}A^{(1)}_{\beta\alpha} + A^{(0)}_{\beta\alpha}A^{(1)*}_{\beta\alpha} + |A^{(1)}_{\beta\alpha}|^2 + A^{(0)*}_{\beta\alpha}A^{(2)}_{\beta\alpha} + A^{(0)}_{\beta\alpha}A^{(2)*}_{\beta\alpha} + \cdots$$

0th 1st 2nd

Ex) perturbation formula at 1st order is written as

$$P^{(1)}(E_i) \propto \sum \rho(x_j)\left[\sin\left\{\frac{\Delta m^2}{2E_i}L\right\} - \sin\left\{\frac{\Delta m^2}{2E_i}L\right\}x_j - \sin\left\{\frac{\Delta m^2}{2E_i}(L-x_j)\right\}\right]$$

Reconstruction of density profile

4. We determine these densities by minimizing the χ² function by comparing the experimental data $N^{\rm obs}(E_i)$ for a given original profile ρ(x) with the theoretical prediction $N^{\rm th}(E_i)$ from unknown parameters ρ_j.

$$\chi^2 = \sum_{i=1,N_E} \frac{\left[N^{\rm obs}(E_i) - N^{\rm th}(E_i)\right]^2}{\sigma^2(E_i)}$$

$$N^{\rm th}(E_i) = {\rm flux} \times P_{\bar{\nu}_e \to \bar{\nu}_e}(E, L) \times {\rm detection\ rate}$$

Neutrino Oscillation Probability of the perturbation formulae

We find the 2nd order perturbation is important for the successful reconstruction.

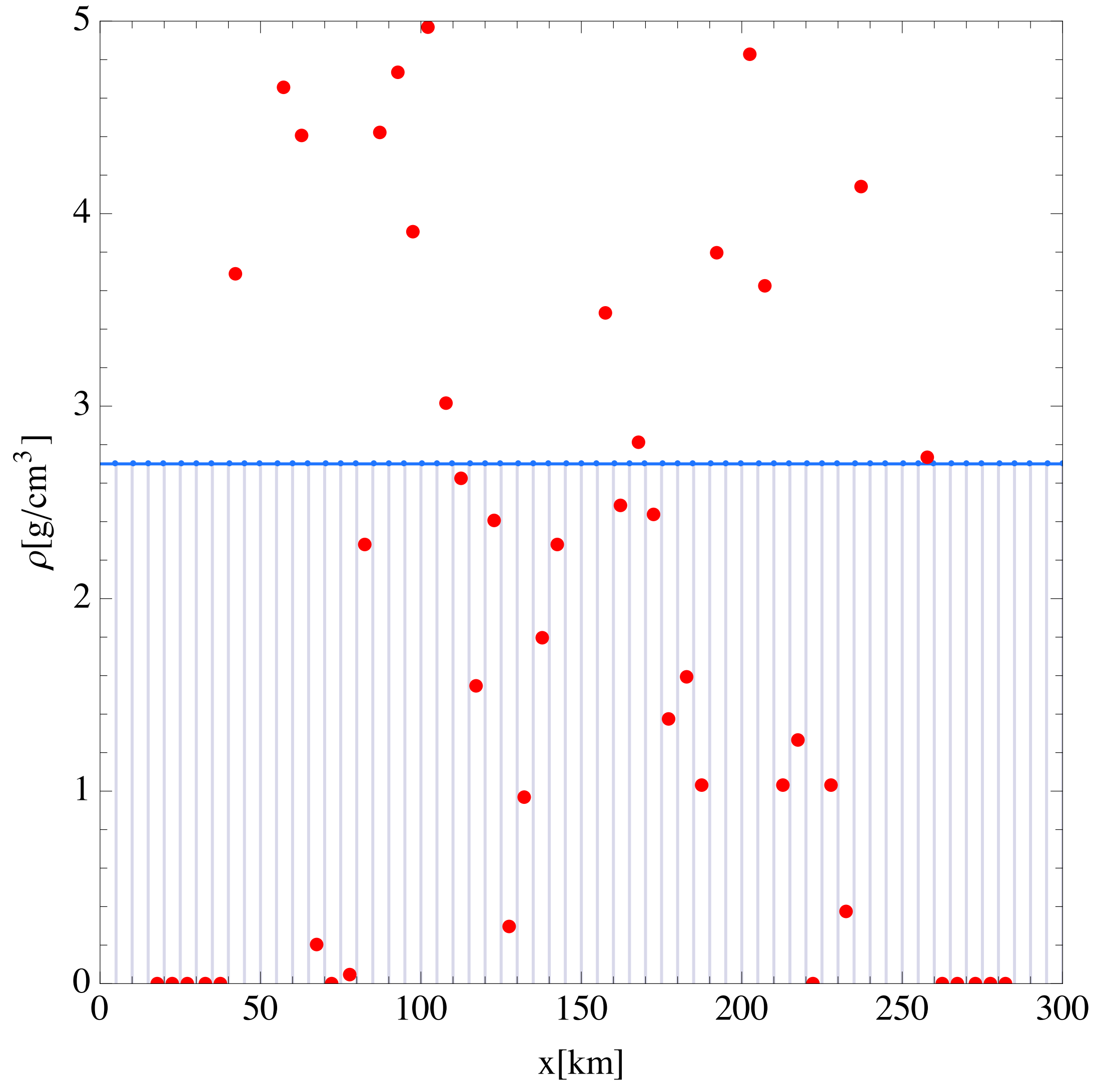
Results of Reconstruction

Result of the flat density

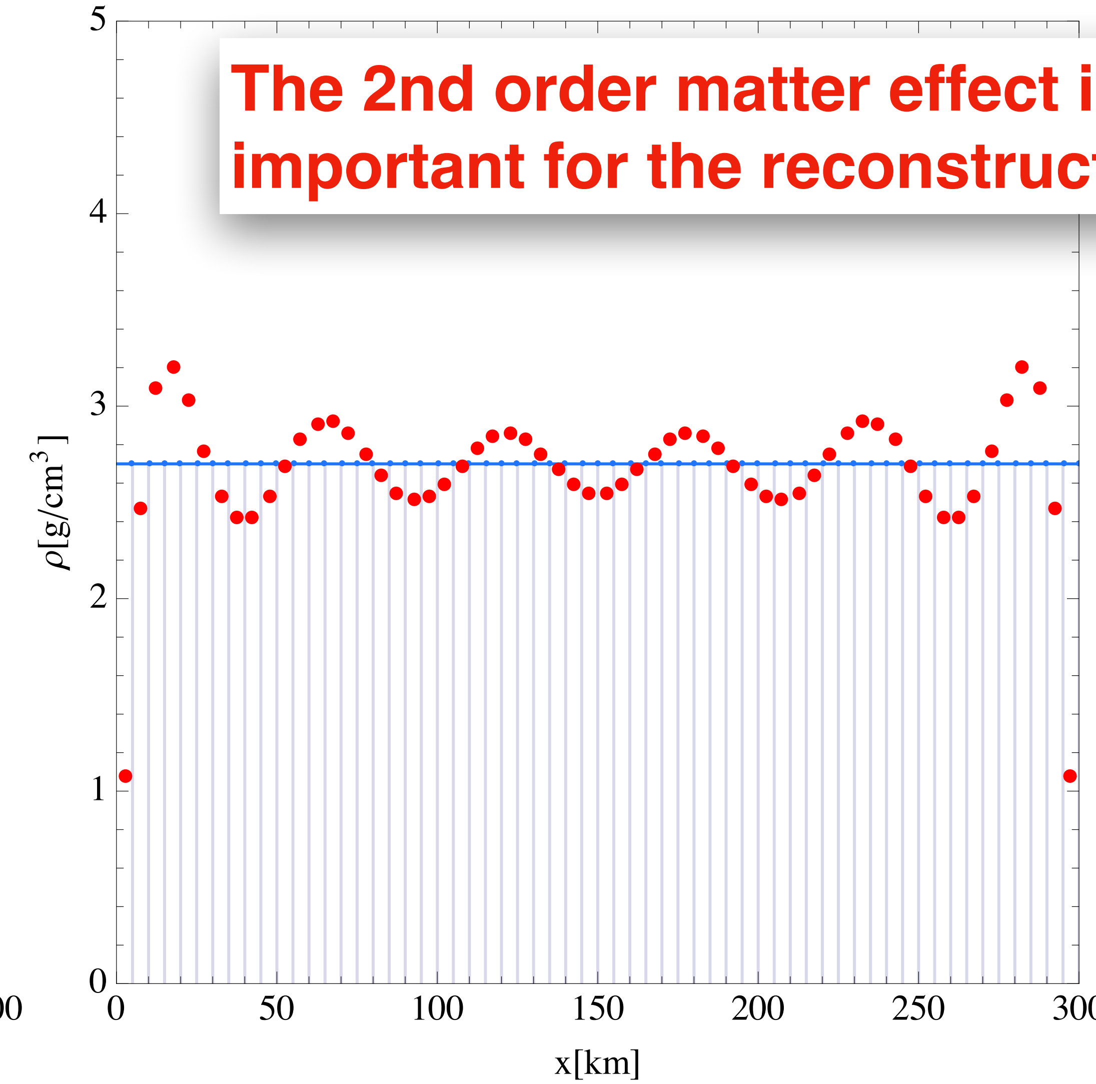
Reconstruction of 60 segment's densities with 100 energy bins

$\bar{\rho} = 2.7 \ [\mathrm{g/cm^3}]$

Result with using the 1st order formula



Result with using the 2nd order formula



- : Original density profile

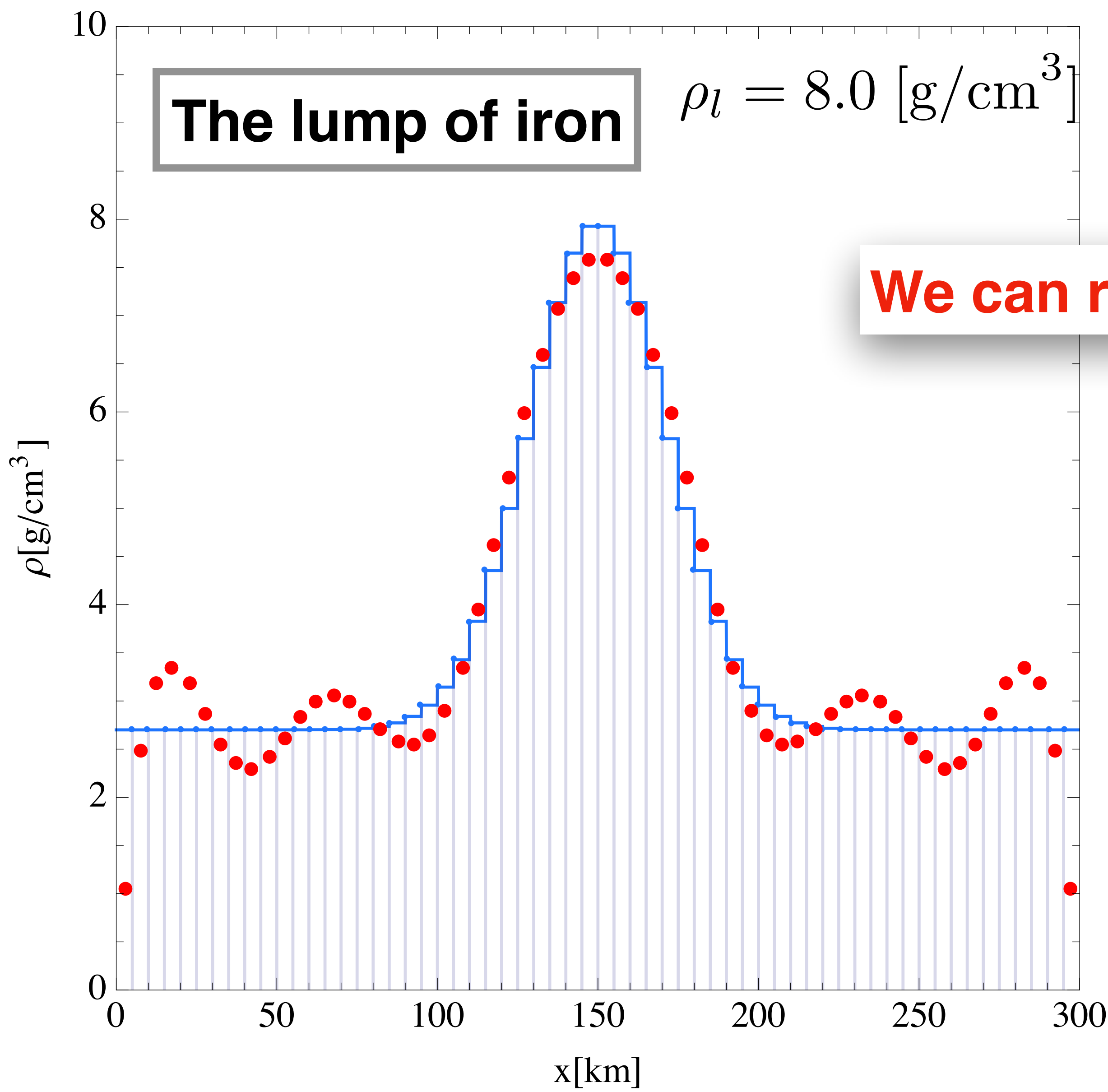
• : Reconstructed density profile

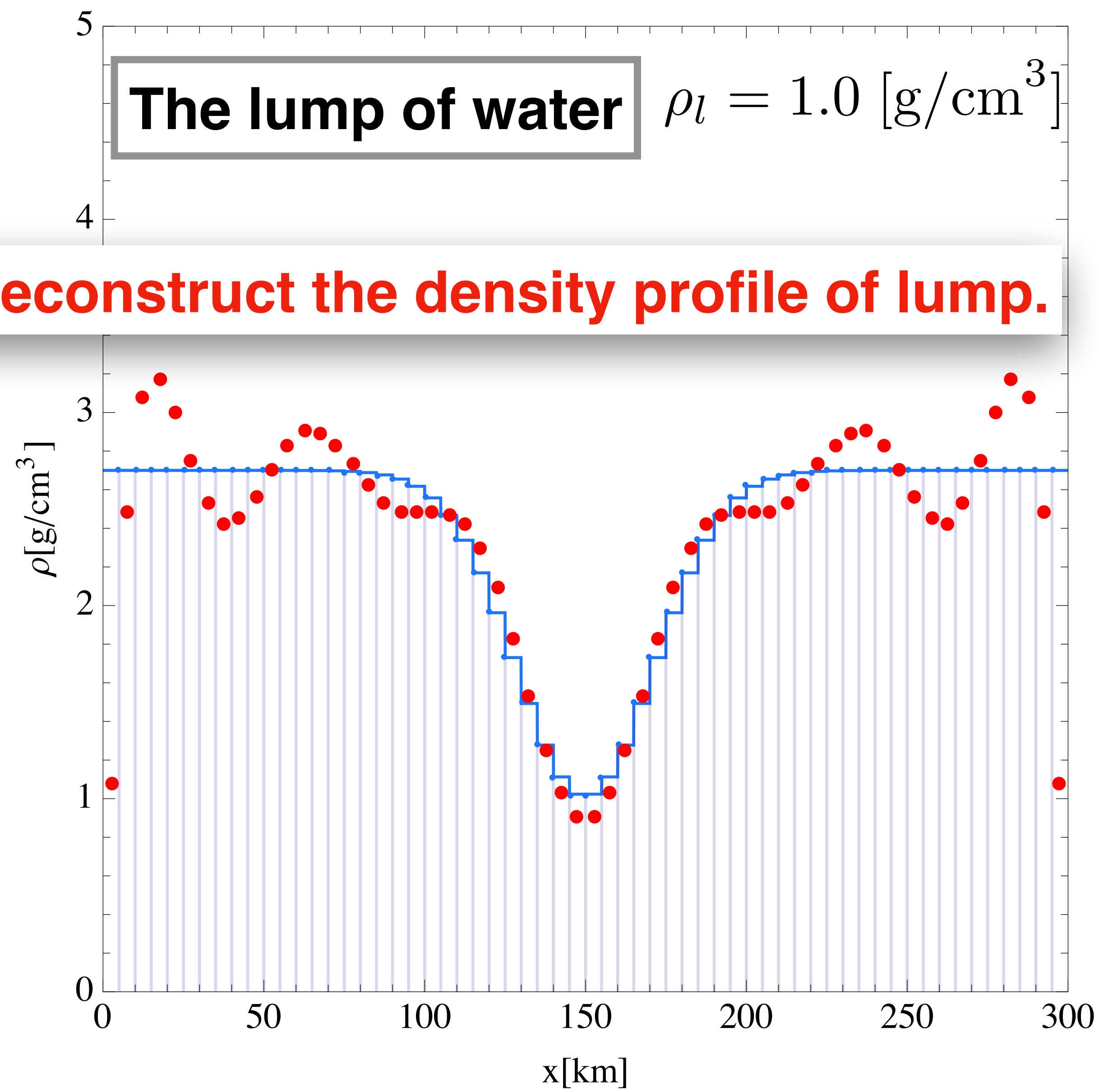
Result with using the 2nd order formula

Reconstruction of 60 segment's densities with 100 energy bins

Original density profile $\bar{\rho} = 2.7\ [\mathrm{g/cm^3}]$

$$\rho(x) = \bar{\rho} + (\rho_l - \bar{\rho}) \exp\left[-\frac{\left(x - \frac{L}{2}\right)^2}{D_l^2}\right]$$





We can reconstruct the density profile of lump.

- : Original density profile

• : Reconstructed density profile

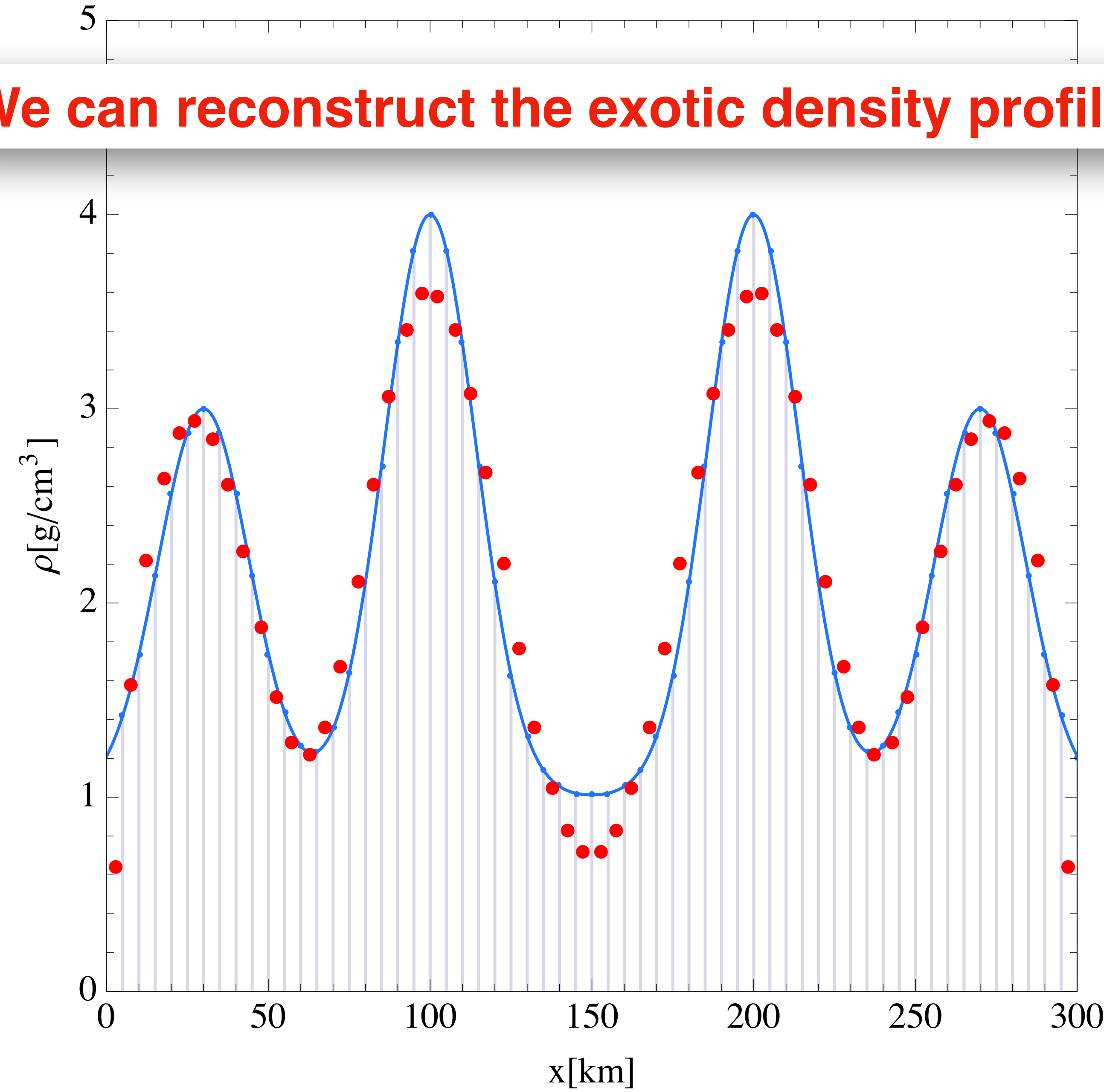
Result of the exotic density profile

Reconstruction of 60 segment's densities

by 100 energy bin data

using the 2nd order formula

4 lump



We can reconstruct the exotic density profile.

- : Original density profile

• : Reconstructed density profile

Conclusions

Conclusions

We have investigated the oscillation tomography by the neutrino pair beam.

In this talk

- The neutrino pair beam is a powerful source to the probe of the Earth's interior, especially the structures inside the crust.
- The reconstruction method with the 2nd order perturbation formula is successful tool.
- We believe that these two ingredients give considerable progress toward the realization of the neutrino tomography.

Conclusions

Toward to the realization of neutrino tomography

- **Realistic 3 flavor oscillation.**
- **Method with the reconstruction of the asymmetric density profile.**
- **More realistic set up.**
  - **We have to consider the realistic systematic error.**
  - **Realistic target of the neutrino tomography.**
  - **Ex) Earth's core and mantle, mineral, oil, etc…**

Back Up

Perturbation formulae

1st order

$$P^{(1)}(\nu_e \to \nu_e) = \frac{G_F}{2\sqrt{2}m_p}\sin^2 2\theta \cos 2\theta \int_0^L dx\ \rho(x)[\sin\{\frac{\Delta m^2}{2E}L\} - \sin\{\frac{\Delta m^2}{2E}x\} - \sin\{\frac{\Delta m^2}{2E}(L-x)\}]$$

2nd order

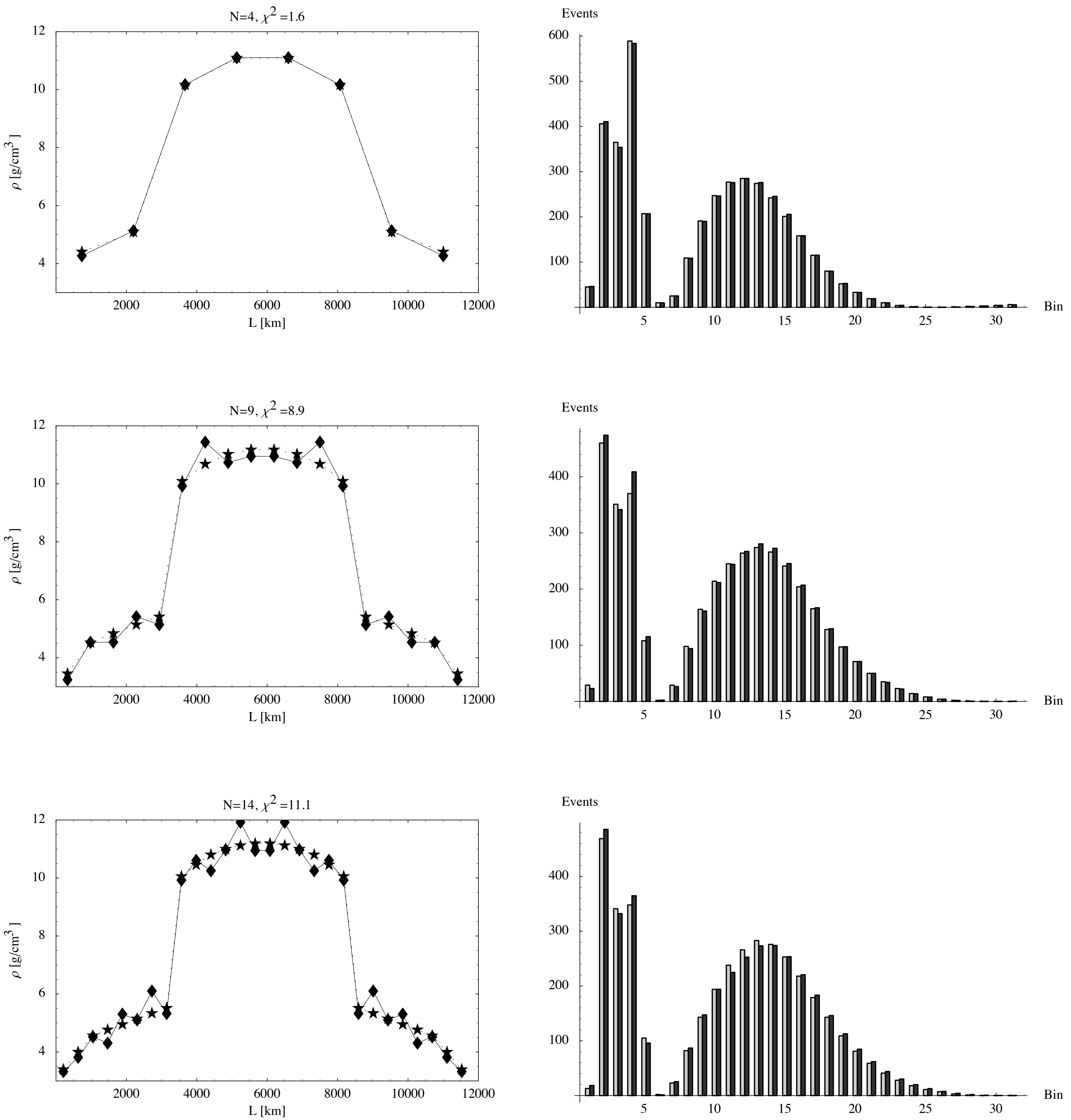
$$P^{(2)}(\nu_e \to \nu_e; t) = P^{(2a)}(\nu_e \to \nu_e; t) + P^{(2b)}(\nu_e \to \nu_e; t)$$

$$\begin{aligned} P^{(2a)}(\nu_e \to \nu_e; t) &= [\cos^8\theta + \sin^8\theta + 2\cos^4\theta\sin^4\theta\cos(\Phi t)]G_1(t)^2 \\ &+ \cos^4\theta\sin^4\theta[G_2(t)^2 + G_3(t)^2] \\ &+ 2(\cos^4\theta + \sin^4\theta)\cos^2\theta\sin^2\theta G_1(t)G_2(t) \end{aligned}$$

$$\begin{aligned} P^{(2b)}(\nu_e \to \nu_e; t) &= -2\int_0^t dt_1 \int_0^{t_1} dt_2 V_{CC}(t_1)V_{CC}(t_2) \\ &\times \{+\cos^8\theta + \sin^8\theta \\ &+ \cos^2\theta\sin^2\theta(\cos^4\theta + \sin^4\theta)[\cos(\Phi t) + \cos(\Phi t_2) + \cos(\Phi(t_2 - t_1)) + \cos(\Phi(t_1 - t))] \\ &+ 2\cos^4\theta\sin^4\theta[\cos(\Phi(t_2 - t)) + \cos(\Phi t_1) + \cos(\Phi(t_2 - t_1 + t))]\} \end{aligned}$$

$$\begin{aligned} G_1(t) &= \int_0^t dt_1 V_{CC}(t_1) \\ G_2(t) &= \int_0^t dt_1 V_{CC}(t_1)[\cos(\Phi t_1) + \cos(\Phi(t - t_1))] \\ G_3(t) &= \int_0^t dt_1 V_{CC}(t_1)[\sin(\Phi t_1) + \sin(\Phi(t - t_1))] \end{aligned}$$

W.Winter et. al.

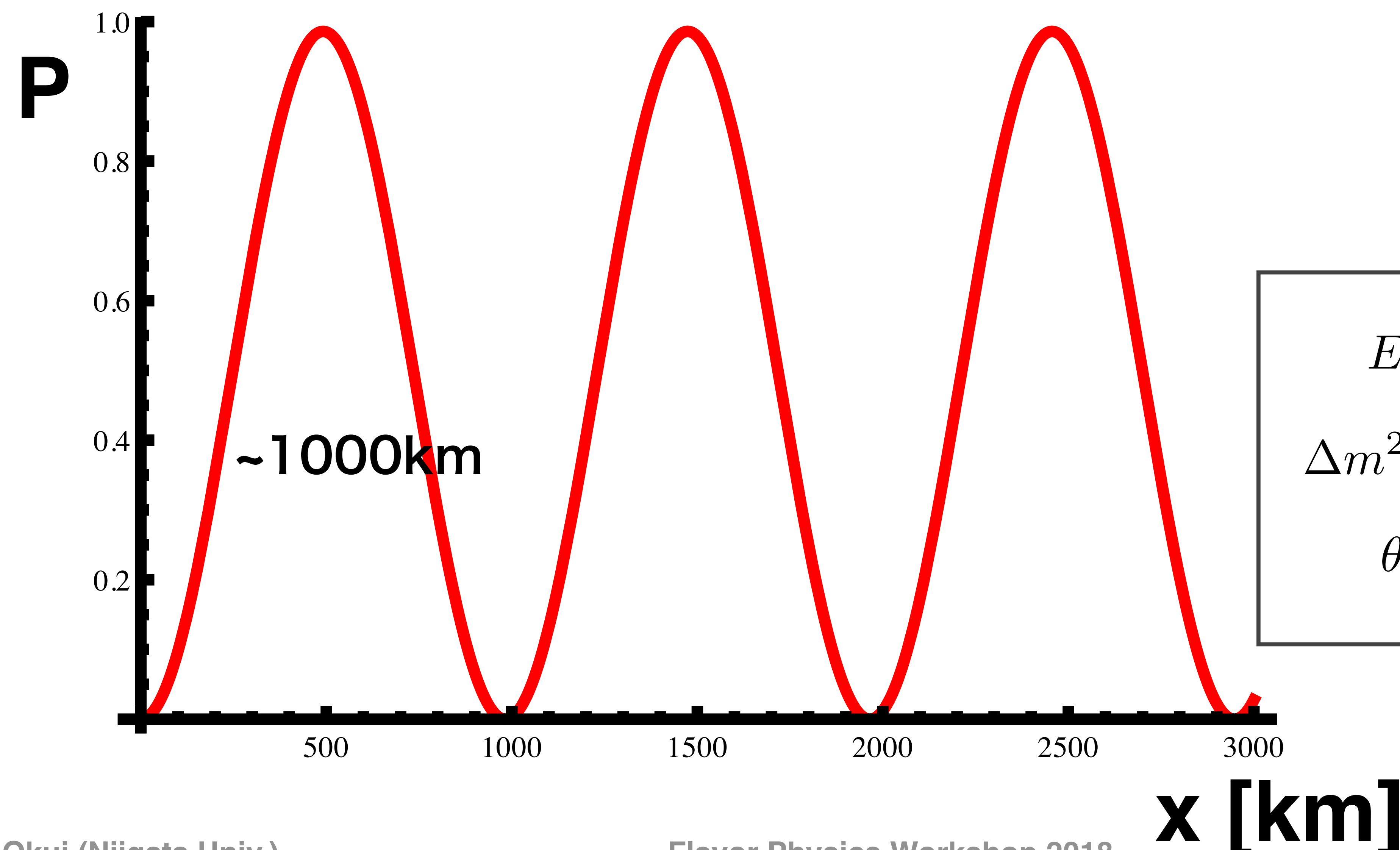


Neutrino Oscillation

Probability is written by

$$P(\nu_e \to \nu_\mu; E, x) = |\langle \nu_\mu | \nu_e(x) \rangle|^2 = \sin^2 2\theta \sin^2\left(\frac{\Delta m^2}{4E} x\right)$$

The flavor transition probability behaves oscillatory.



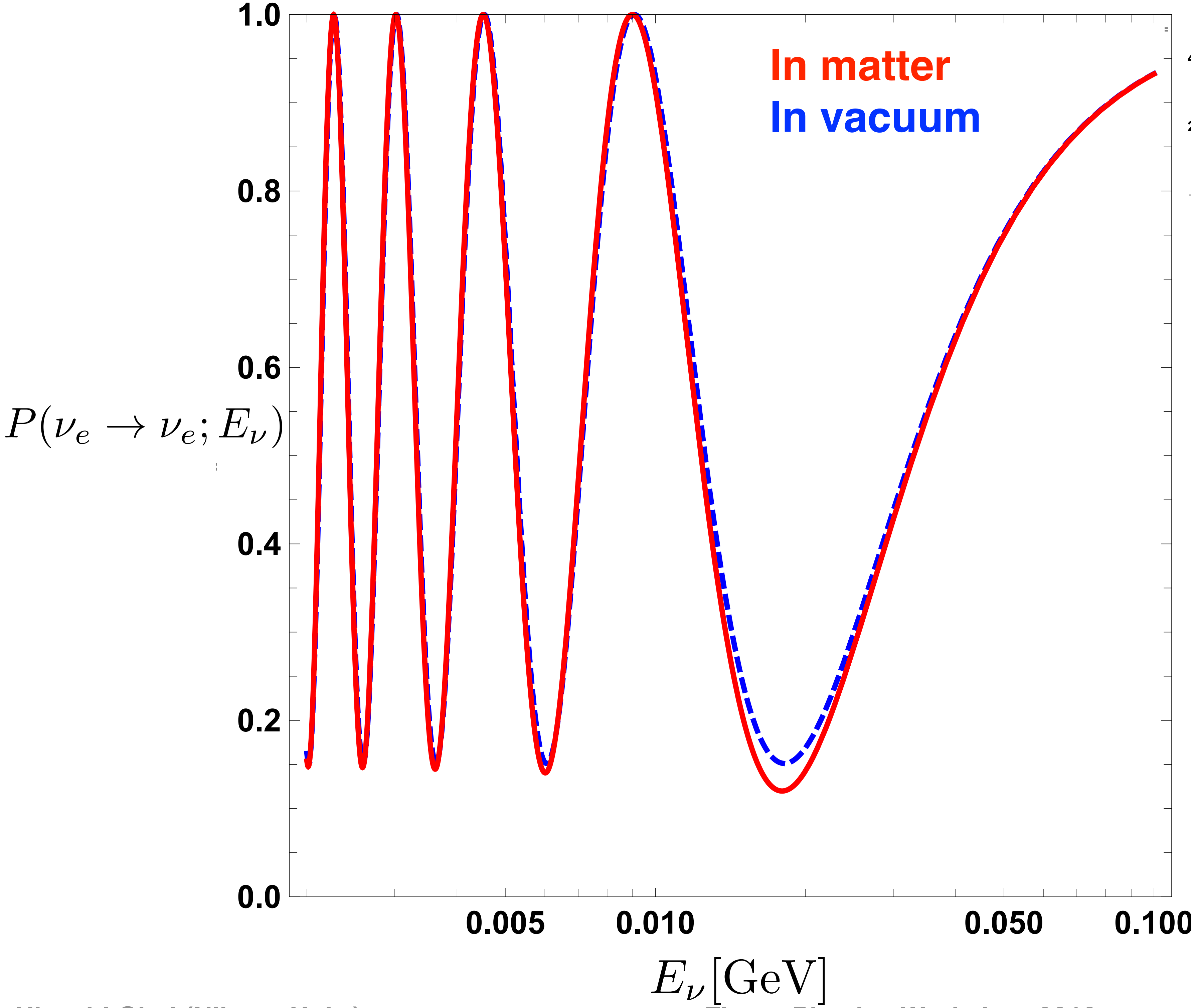
$$E = 1[\mathrm{GeV}]$$

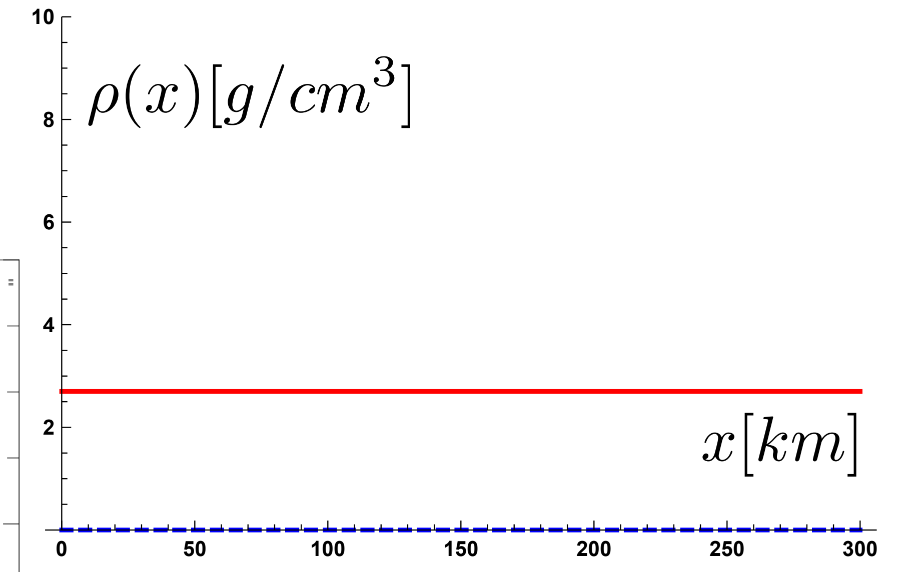
$$\Delta m^2 = 2.524 \times 10^{-3}[\mathrm{eV}^2]$$

$$\theta = \frac{41.6}{180}\pi$$

Neutrino Oscillation in Matter

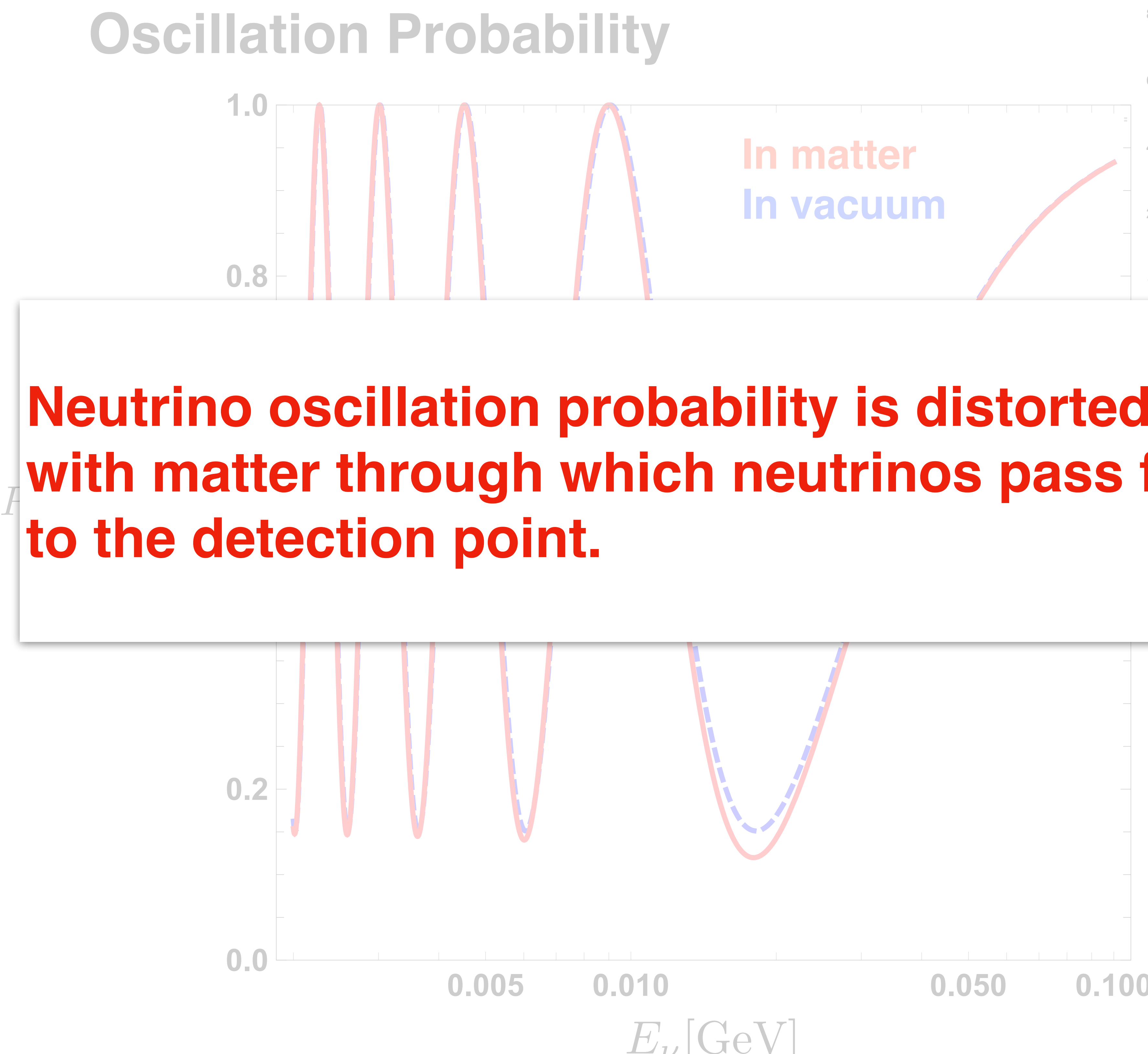
Oscillation Probability





$$x = 300\ [km]$$
$$\Delta m^2 = 7.5 \times 10^{-5}\ [eV^2]$$
$$\theta = \frac{33.56}{180}\pi$$

Neutrino Oscillation in Matter



Neutrino oscillation probability is distorted by the interaction with matter through which neutrinos pass from the production to the detection point.

Degeneracy of the 2 flavor neutrino oscillation

If we consider the 2 flavor oscillation, probability degenerate by the Unitarity.

$$P(\nu_e \to \nu_e) + P(\nu_e \to \nu_\mu) = 1$$

$$P(\nu_e \to \nu_e) + P(\nu_\mu \to \nu_e) = 1$$

$$\therefore P(\nu_\mu \to \nu_e) = P(\nu_e \to \nu_\mu)$$

If there is no CP phase, time reversal transformation is equal to these transformation.

$$V(x) \leftrightarrow V(L - x)$$

So, we can find the degeneracy of the 2 flavor neutrino oscillation.

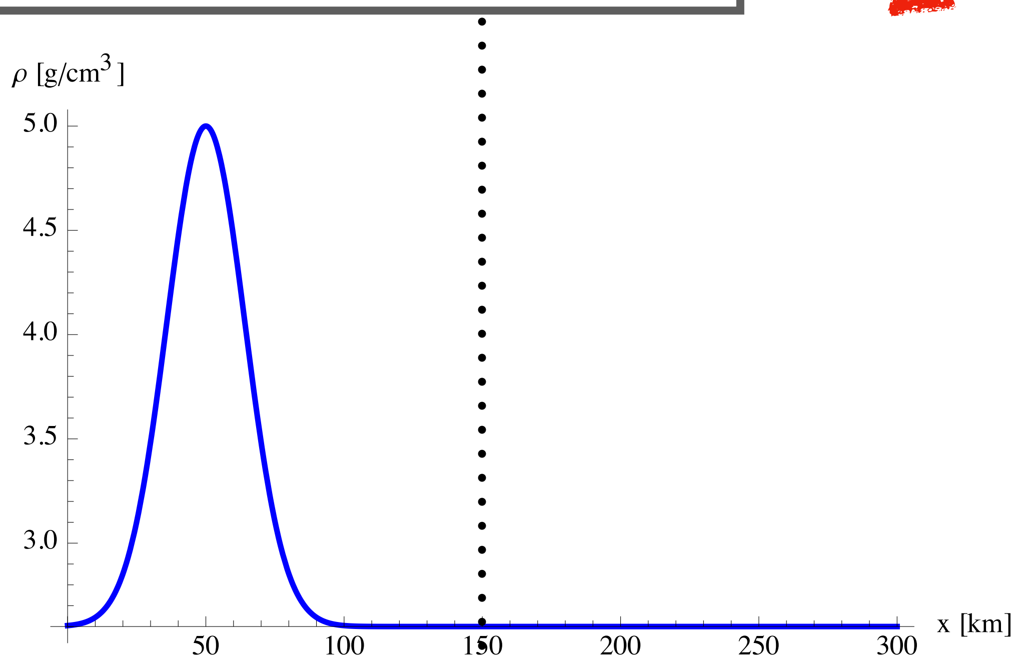
$$\begin{aligned} P(\nu_\alpha \to \nu_\beta)|_{V(x)} &= P(\nu_\beta \to \nu_\alpha)|_{V(x)} \\ &= P(\nu_\alpha \to \nu_\beta)|_{V(L-x)} \end{aligned}$$

Degeneracy of the 2 flavor neutrino oscillation

The oscillation probability with asymmetric density profile coincide with the another one.

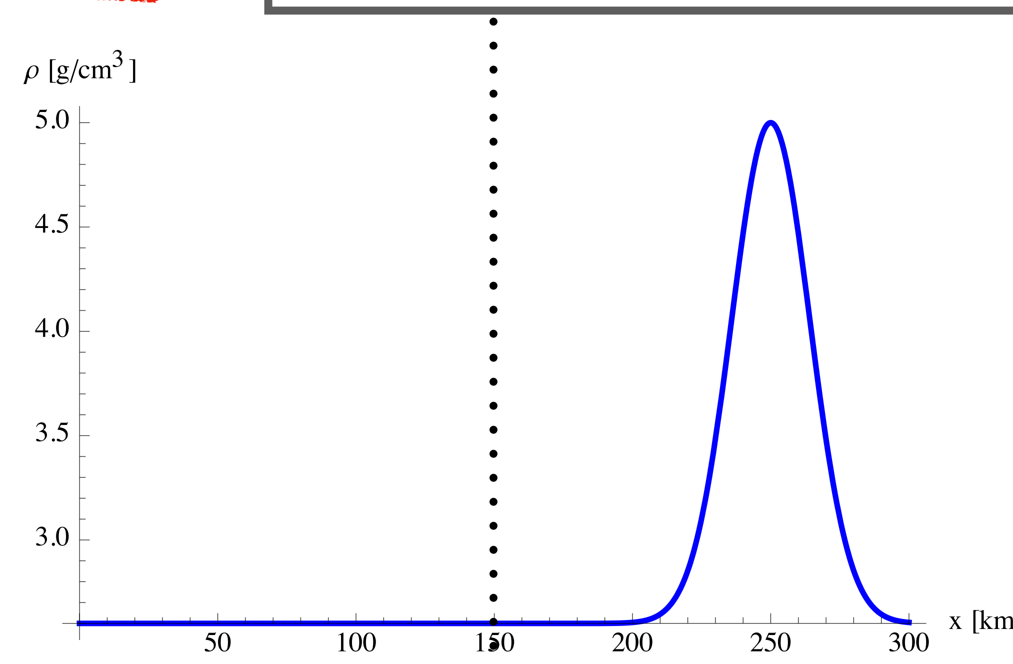
Same Oscillation Probability

Density profile with left side lump



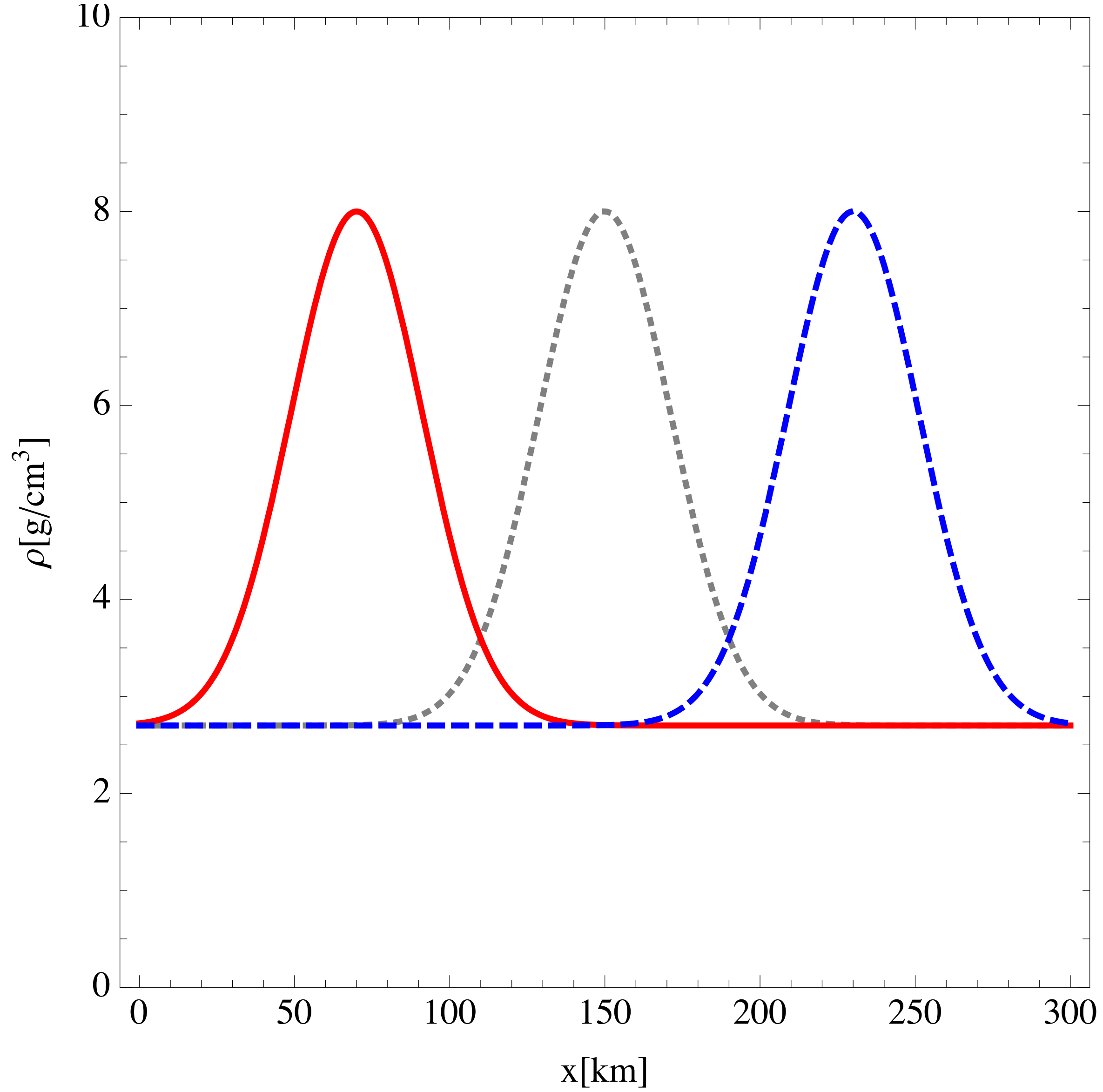
$$V(x)$$

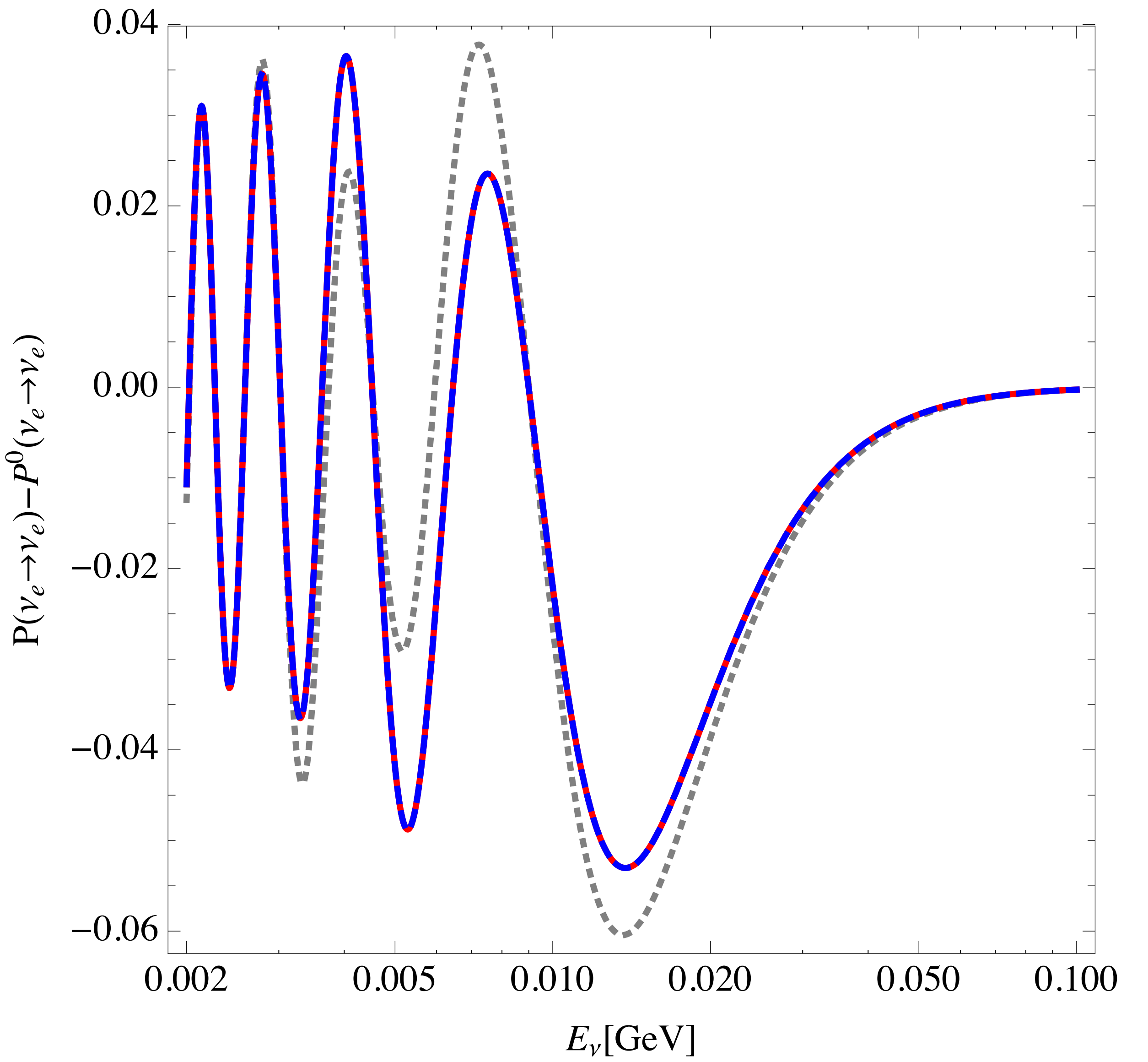
Density profile with right side lump



$$V(L-x)$$

Degeneracy of the 2 flavor Neutrino Oscillation





Assumptions

- **There is degeneracy of the probability in 2 flavor case**

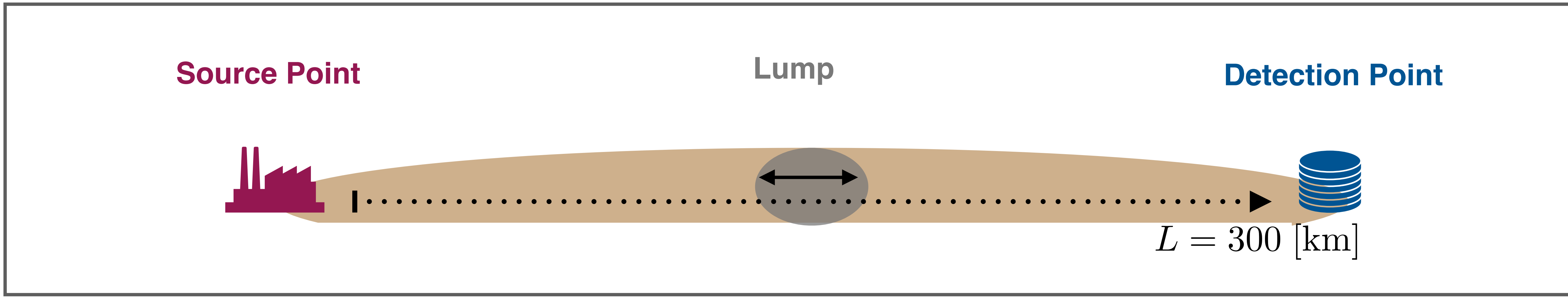
$$P(\nu_\alpha \to \nu_\beta)|_{\rho(x)} = P(\nu_\alpha \to \nu_\beta)|_{\rho(L-x)}$$

- **Here we focus on the reconstruction of the symmetric density profile with** $\rho(x) = \rho(L - x)$ **.**

- **And we provide a useful procedure of its reconstruction.**

  - **The advantage of ours is that the reconstruction with a sufficient spatial resolution is possible even with a low numerical cost.**

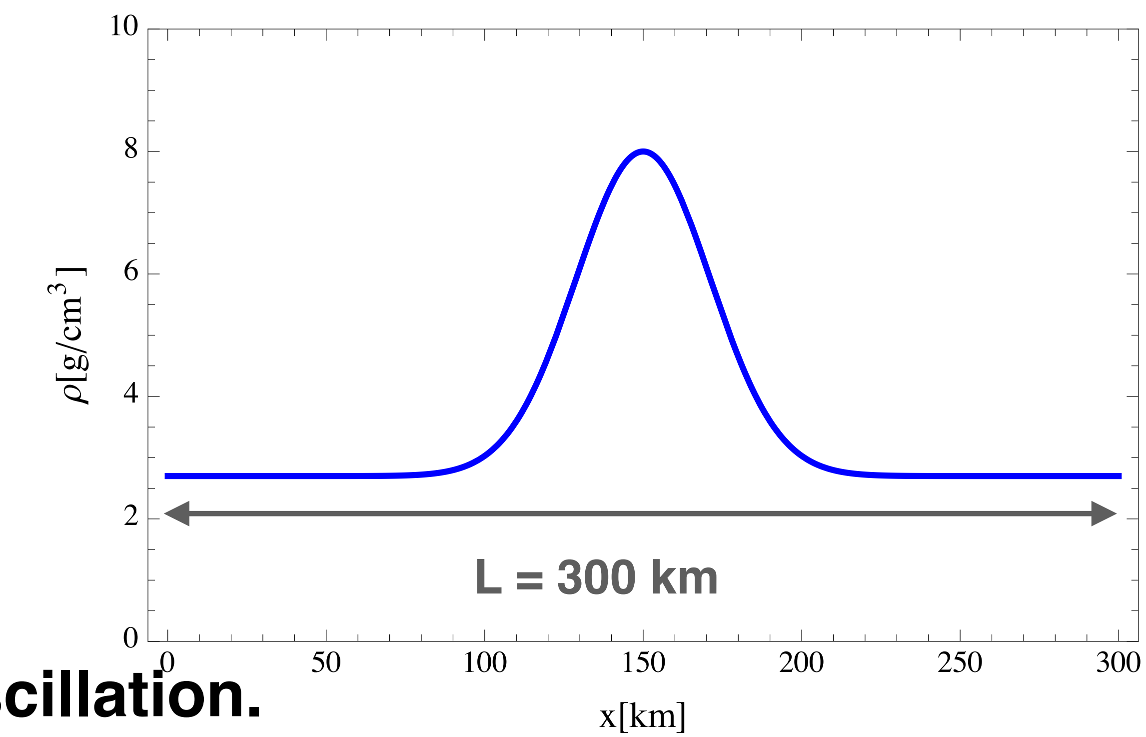
Toy model



- **We consider the symmetric exponential type of the density profile.**

$$\rho(x) = \bar{\rho} + (\rho_l - \bar{\rho}) \exp\left[-\frac{\left(x-\frac{L}{2}\right)^2}{D_l^2}\right]$$

L : length of the baseline
D_l : width of the lump



- **We consider the low energy $\bar{\nu}_e \to \bar{\nu}_e$ oscillation.**

$$E_\nu : 2 \sim 100 \text{ [MeV]}$$

- **We assume the huge liquid Argon as the neutrino detector.**

Fiducial volume 10^5 m^3

Statistical Test

We estimate how precisely the width(D_*) and density(ρ_*) of the lump can be reconstructed under this set-up.

$$\rho(x) = \bar{\rho} + (\rho_l - \bar{\rho}) \exp\left[-\frac{\left(x - \frac{L}{2}\right)^2}{D_l^2} \right] \qquad \bar{\rho} = 2.7[\mathrm{g/cm^3}]$$

We perform the χ^2 analysis.

$$\Delta\chi^2 = \sum_{i=1}^{N_b} \frac{\left[N(E_i)|_{D_*,\rho_*} - N(E_i)|_{D_l,\rho_l} \right]^2}{\sigma^2(E_i)}$$

N_b = 100 : the number of energy bin

N (Event number) = flux × oscillation probability × detection rate

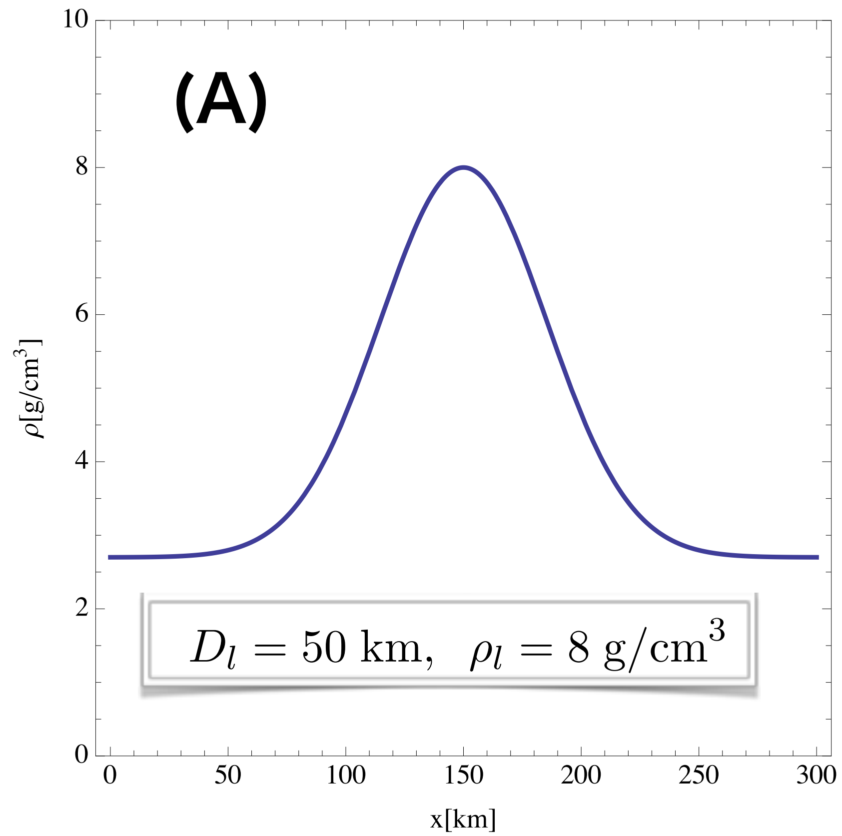
We assume 1 year as experimental running time.

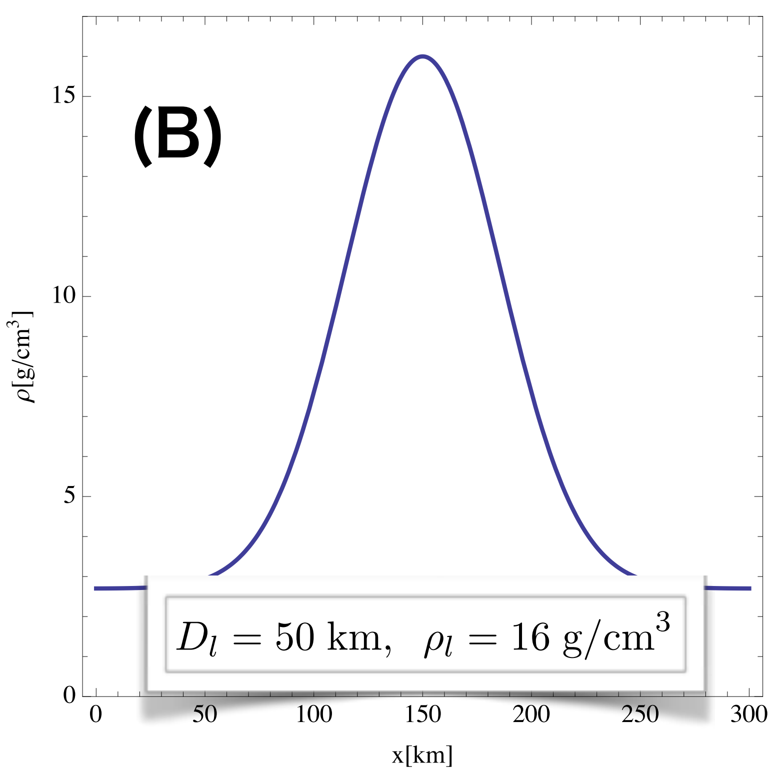
- **We only consider the statistical error in this calculation.**
- **(It is not included the systematic error)**

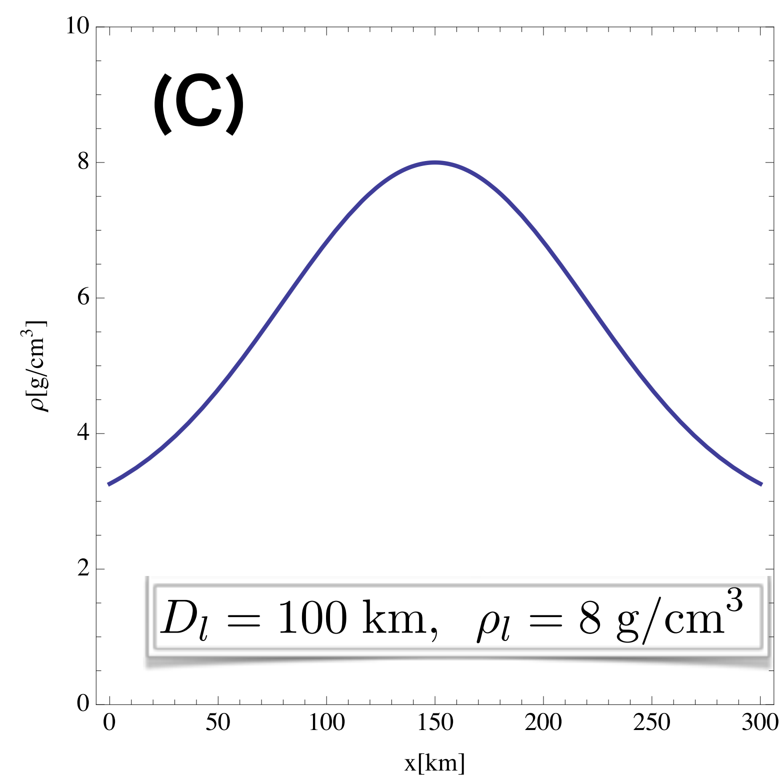
Statistical Test

We assume the 3 density profiles.

$$\rho(x) = \bar{\rho} + (\rho_l - \bar{\rho}) \exp\left[-\frac{\left(x-\frac{L}{2}\right)^2}{D_l^2}\right]$$

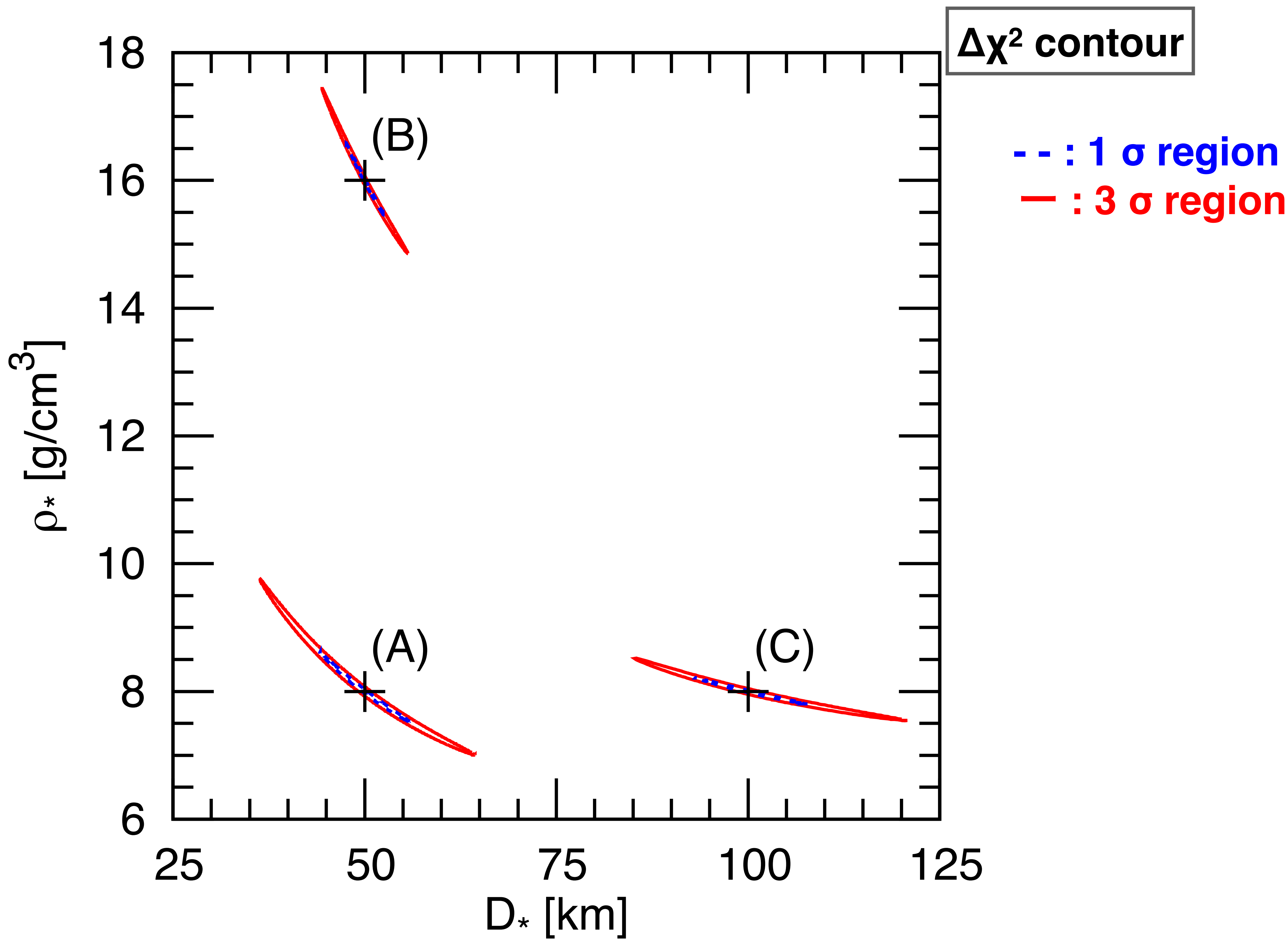






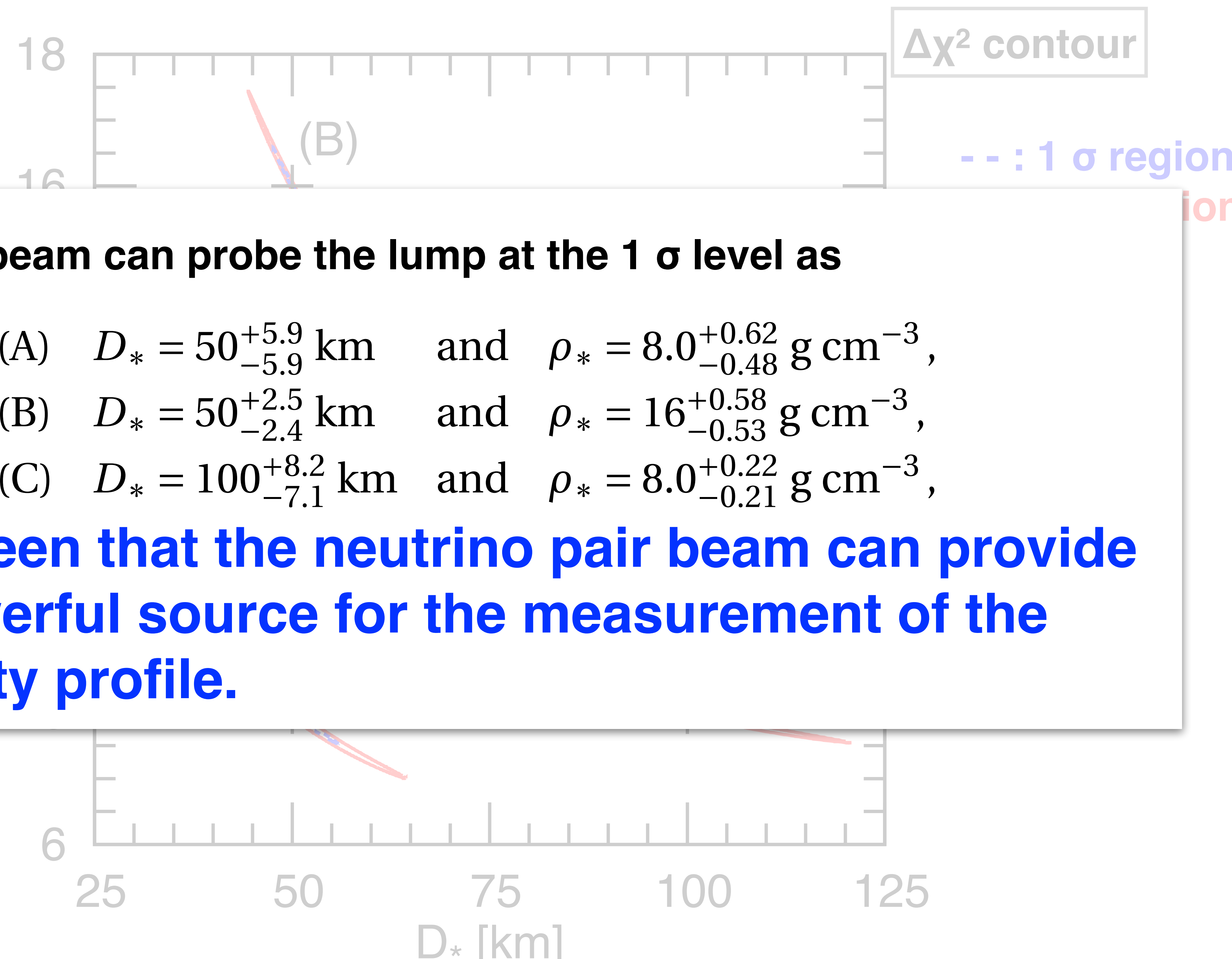
Statistical Test

We assume the 3 density profiles.



Statistical Test

We assume the 3 density profiles.



The pair beam can probe the lump at the 1 σ level as

$$
\begin{aligned}
&\text{(A)} \quad D_* = 50^{+5.9}_{-5.9}\,\text{km} \quad \text{and} \quad \rho_* = 8.0^{+0.62}_{-0.48}\,\text{g cm}^{-3}\,, \\
&\text{(B)} \quad D_* = 50^{+2.5}_{-2.4}\,\text{km} \quad \text{and} \quad \rho_* = 16^{+0.58}_{-0.53}\,\text{g cm}^{-3}\,, \\
&\text{(C)} \quad D_* = 100^{+8.2}_{-7.1}\,\text{km} \quad \text{and} \quad \rho_* = 8.0^{+0.22}_{-0.21}\,\text{g cm}^{-3}\,,
\end{aligned}
$$

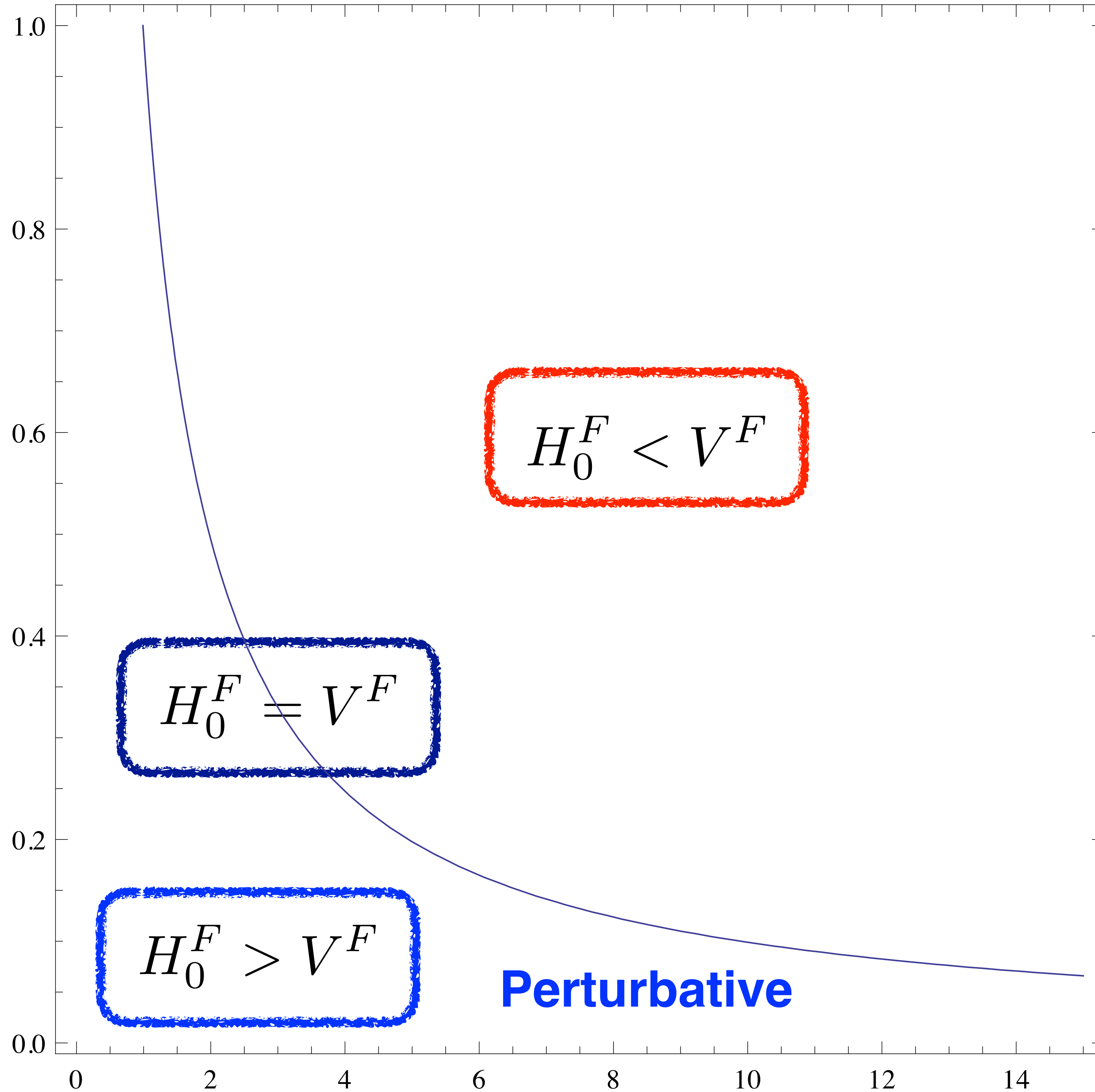
It is seen that the neutrino pair beam can provide a powerful source for the measurement of the density profile.

Condition of Perturbation

$$\Delta m^2_{SOL} = 7.5 \times 10^{-5} [\mathrm{eV}^2]$$

Neutrino Energy

$E_\nu \ [GeV]$



$\rho \ [g/cm^3]$

Density

Evolution equation

$$i \frac{d}{dx} \vec{A}(x) = [H_0^F + V^F] \vec{A}(x)$$

Condition of perturbation

$$H_0^F > V^F$$