Pendulum Leptogenesis

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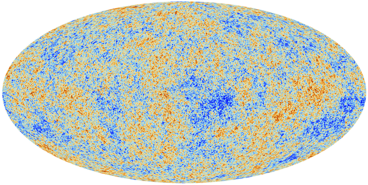
K. Bamba, NDB, A. Sugamoto, T. Takeuchi and K. Yamashita, arxiv:1610.03268 (MPLA), and arxiv:1805.04826 (PLB).

Matter-Antimatter Asymmetry



The asymmetry is described quantitatively by,

$$\eta = \frac{n_b - n_{\bar{b}}}{s} \simeq 8.5 \times 10^{-11}$$

The Sakharov Conditions

1. Baryon number violation
2. $\mathcal{C}$ and $\mathcal{CP}$ violation
3. Period of non-equilibrium

Ratchet Mechanism

Inspired by molecular motors in biological systems, and their ability to generate directed motion.

- Consider an inflaton and complex scalar carrying L charge during reheating.
- A derivative coupling between a complex scalar and inflaton.
- Directed motion in the complex scalar phase gives a non-zero L number density.

The Model

Interplay between the inflaton and complex scalar during reheating,

$$S = \int dx^4 \sqrt{-g} \left[g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi^* - V_0(\phi, \phi^*) \right.$$
$$\left. + \frac{1}{2} g^{\mu\nu} \partial_\mu \Phi \partial_\nu \Phi - U(\Phi) + \frac{i}{\Lambda} g^{\mu\nu} \left(\phi^* \overleftrightarrow{\partial_\mu} \phi \right) \partial_\nu \Phi \right],$$

where $V_0(\phi, \phi^*) = \lambda \phi^* \phi (\phi - \phi^*)(\phi^* - \phi) + ...$

Satisfying the Sakharov Conditions

1. The complex scalar potential.
2. Derivative coupling interaction.
3. Reheating epoch.

$U(1)_L$ Global Symmetry

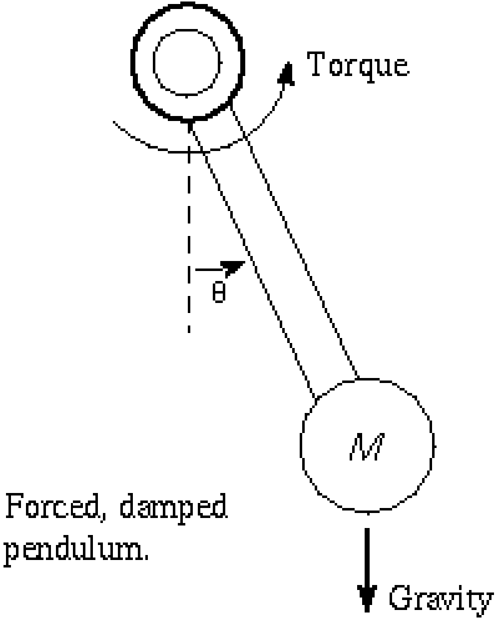
- Identify the $U(1)$ symmetry with L charge, where ϕ has charge 2.
- L preserving interactions, $\mathcal{L}_{\rm int} = g_L \phi^* \bar{\nu}_R^c \nu_R + y_H H \bar{L} \nu_R + h.c.$
- Taking ϕ in polar form, $\phi = \frac{1}{\sqrt{2}} \phi_r e^{i\theta}$,

$$V(\phi_r, \theta) = \lambda \phi_r^4 \sin^2 \theta + \cdots$$

and

$$n_L = j^0 = -2\phi_r^2 \left(\dot{\theta} - \frac{\dot{\Phi}}{\Lambda} \right) \ .$$

A non-zero n_L requires driven motion in $\dot{\theta}$.


Torque
θ
M
Forced, damped
pendulum.
Gravity

Behaviour of the Inflaton

Assuming Starobinsky inflation ($U(\Phi) \approx \frac{1}{2}\mu^2\Phi^2$) during reheating,

$$\ddot{\Phi} + \left(\frac{2}{t}\dot{\Phi} + \Gamma\dot{\Phi}\right) + \mu^2\Phi + \frac{\lambda\phi_r^4}{\Lambda}\sin(2\theta) = 0 \ .$$

Need the inflaton (Torque) to be unaffected by θ, ($\mu^2\Phi \gg \lambda\phi_r^4/\Lambda$),

$$\ddot{\Phi} + \left(\frac{2}{t} + \Gamma\right)\dot{\Phi} + \mu^2\Phi \ = \ 0 \ .$$

When $\Gamma \ll \mu$, the approximate solution to this equation is

$$\frac{\Phi(t)}{\Phi_i} \approx \left(\frac{t_i}{t}\right)\cos\left[\mu(t - t_i)\right] \ .$$

Behaviour of θ

Utilising the inflaton EoM, and neglecting Φ decay term,

$$\ddot{\theta} + (\Gamma_\theta + 3H)\,\dot{\theta} + p\sin(2\theta) \;=\; -q(t)\cos\big[\mu(t - t_i)\big]\;,$$

where

$$p \;=\; \lambda\phi_r^2\;, \qquad q(t) \;=\; \frac{\mu^2\Phi_i}{\Lambda}\left(\frac{t_i}{t}\right)\;.$$

This is analogous to a forced pendulum where

- LHS represents acceleration, damping, and gravitation,
- RHS is a sinusoidal driving torque with amplitude q and frequency μ.

Possible Cases: $p \ll q(t)$

Consider a large torque ($p \ll q(t)$),

$$\ddot{\theta} + 3H\dot{\theta} = \frac{1}{t^2}\frac{d}{dt}\left(t^2\dot{\theta}\right) = -q(t)\cos\left[\mu(t-t_i)\right] \ ,$$

which gives,

$$\dot{\theta}(t) = \frac{\dot{\Phi}}{\Lambda} \quad \Rightarrow \quad j_0 = 0$$

- L violation has vanished, $\dot{\theta}$ oscillates around zero with $\dot{\Phi}$.
- Analogous to an effectively massless pendulum.

Possible Cases: $p \gg q(t)$

Consider a small torque ($p \gg q(t)$),

$$\ddot{\theta} + 3H\dot{\theta} + p\sin(2\theta) = 0$$

Friction term damps $\dot{\theta}$ until θ settles into a minima $\Rightarrow$ no persistent non-zero $\dot{\theta}$.

- $\mathcal{C}$ and $\mathcal{CP}$ breaking term ignored.
- Analogous to a very massive pendulum, unaffected by input torque.

Sweet Spot Condition and Driven Motion

We need $p \simeq q(t) \Rightarrow L$, $\mathcal{C}$ and $\mathcal{CP}$ violating terms all contribute.

$$\text{SSC:}\ \ \lambda\phi_r^2 \ \simeq \ \frac{\mu^2\Phi_i}{\Lambda}\frac{H_d}{H_i}$$

- Equivalent to $F_d \simeq mgl$,
- Torque is time dependent so can only satisfy for a finite time.

Thus we want to consider,

$$\ddot{\theta} + (\Gamma_\theta + 3H_d)\dot{\theta} + p\sin(2\theta) \ = \ -q(t_d)\cos\left[\mu(t-t_i)\right] \ .$$

Phase Locked States

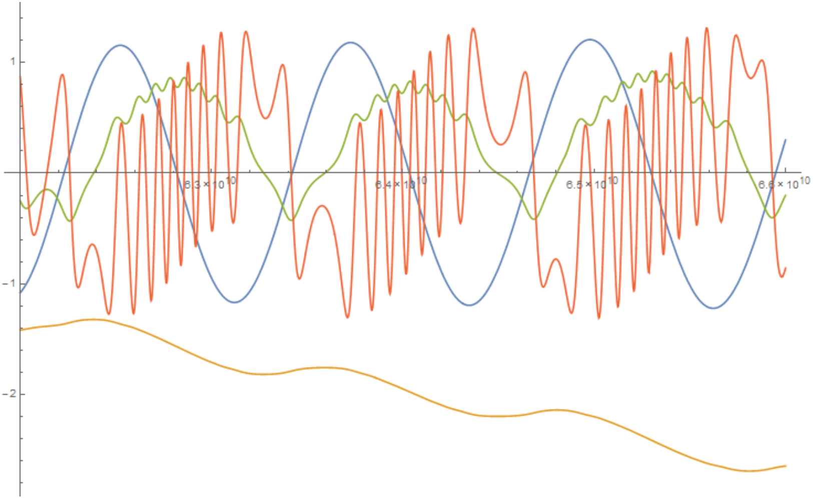
Reparameterise the EoM,

$$\ddot{\Theta} + \frac{1}{Q}\dot{\Theta} + \sin\Theta \ = \ \frac{q(t_d)}{p}\cos(\omega\tau) \ ,$$

- Solutions increasing monotonously in time with small modulations.
- Known as "phase-locked states" in the study of forced pendulum.

General phase-locked state solution,

$$\Theta(\tau) \ = \ \Theta_0 + n\omega\tau - \sum_{m=1}^{\infty} \alpha_m \sin(m\omega\tau - \phi_m) \ \ .$$


1
−1
−2
6.3 × 10^10
6.4 × 10^10
6.5 × 10^10
6.6 × 10^10

Estimation of n and Parameter Constraints

Consistent with our initial assumptions,

$$n \simeq \frac{2\lambda\phi_r^2}{\mu^2} \ .$$

where $n/2 =$ number of rotations of θ per oscillation of Φ.

- After the SSC is violated no simultaneous violation of $\mathcal{C}$, $\mathcal{CP}$ and L ,
- Can approximate the amplitude of oscillations as, $\sin 2\theta \approx \frac{H}{H_d}$.

Parameter constraints (e.g. Φ is unaffected by θ),

$$10^{15}\ \mathrm{GeV} > \phi_r > \mu \ ,$$

$$1 > \lambda > 5 \times 10^{-4} \ ,$$

$$310 > n > 1 \ .$$

The Generated Asymmetry and Neutrino Masses

Assuming no additional entropy production,

$$\eta^{\rm reh} \;=\; \frac{n_L}{s} \;\approx\; 0.04 n \times \left(\frac{\mu\phi_r^2}{T_{\rm reh}^3}\right)\left(\frac{a_d}{a_{rh}}\right)^3 \;.$$

Generated baryon asymmetry,

$$\frac{\eta}{\eta_{obs}} = \frac{T_{rh}}{2\lambda \cdot 10^8 \;{\rm GeV}} \;.$$

L preserving interaction, generates a Majorana mass,

$$10^{14} \;\;{\rm GeV} > m_{\nu_R} > 10^{11} \;\;{\rm GeV} \;.$$

Via the seesaw mechanism,

$$m_\nu = y_H^2 v_h^2 / 2 m_{\nu_R} \;.$$

Conclusion and Future Work

- Interplay between inflaton and scalar lepton during reheating,
- Driven motion can be modelled as a forced pendulum,
- Presence of phase-locked states,
- Asymmetry linearly dependent on the reheating temperature.
- Seesaw mechanism generates active neutrino masses.

Future work

- Investigation of efficiency required.
- Other cosmological implications