Feynman Rules of Massive Gauge Theory in Physical GAGUES

Junmou Chen (KIAS)

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Gauge Choice:

Covariant gauge v.s. Physical gauges

$$\mathcal{L}_\xi = -\frac{1}{2\xi}(\partial^\mu A_\mu^a)^2$$

$$\mathcal{L}_\xi = -\frac{1}{2\xi}(n \cdot A^a)^2$$

Covariant gauge:

1. Manifest Lorentz symmetry
2. Gauge redundancies
3. Ghosts — unitarity is not straightforward

Physical gauges:

1. No manifest Lorentz symmetry
2. Only physical degrees of freedom remain
3. Ghosts decouple

Physical gauges: gauge fixing along a fixed direction

Propagator:

$$< A_a^\mu A_b^\nu > = \frac{-i\delta_{ab}(g^{\mu\nu} - \frac{n^\mu k^\nu + k^\mu n^\nu}{n\cdot k} + \frac{n^2}{(n\cdot k)^2} k^\mu k^\nu)}{k^2 + i\epsilon}$$

$n^2 < 0,$ axial gauge

$n^2 = 0$ light-cone gauge ← our topic today

$n^2 > 0,$ temporal gauge

Physical gauges (axial gauge)

Propagator:

$$< A_a^\mu A_b^\nu > = \frac{-i\delta_{ab}(g^{\mu\nu} - \frac{n^\mu k^\nu + k^\mu n^\nu}{n\cdot k} + \frac{n^2}{(n\cdot k)^2} k^\mu k^\nu)}{k^2 + i\epsilon}$$

$$k^2 k_\mu < A^\mu A^\nu >= ik^2 (\frac{n^\nu}{n \cdot k})$$

goes to 0 when $k^2 \to 0$,
only transverse (physical)
polarizations remain

APPLICATIONS (LIGHT-CONE GAUGE ESPECIALLY)

1. Calculations of collinear splitting functions. e.g. S. Catani and M. Grazzini. Nucl. Phys. B570, 287 (2000) . hep-ph/9908523

2. Proof of factorization: one of the fundamental presumption of perturbative QCD.

   e.g. J. C. Collins, D. E. Soper, and G. F. Sterman. Adv. Ser. Direct. High Energy Phys. 5, 1 (1989) . hep-ph/0409313

3. Impose on massive gauge theory: Effective W approximation.

   e.g. Z. Kunszt and D. E. Soper, . Nucl. Phys. B296, 253 (1988).

4. Applications on Super Yang-Mills Theory and on String Theory

   e.g. S. Mandelstam. Nucl. Phys. B213, 149 (1983)

   Comm. Math. Phys. 92, 455 (1984)

Impose on massive gauge theory

$$\mathcal{L}_{W_a^2} = -\frac{1}{2}\partial^\mu W_a^\nu \partial_\mu W_{a\nu} + \frac{1}{2}\partial^\mu W_{a\mu}\partial^\nu W_{a\nu} + \frac{1}{2}m_W^2 W_{a\mu}W^{a\mu}$$
$$+ \frac{1}{2\xi}(n\cdot\partial\, n\cdot W_a)(n\cdot\partial\, n\cdot W_a)^*$$
$$\mathcal{L}_{\phi_a W^a} = -m_W W^{a\mu}\partial_\mu\phi_a$$
$$\mathcal{L}_{\phi_a^2} = \frac{1}{2}(\partial^\mu\phi_a)^2$$

Not easy to work out the propagator.

Solution: treat gauge fields and goldstone fields as a whole.

Appeared in

Z. Kunszt and D. E. Soper, Nucl. Phys. B296, 253 (1988).

M. S. Chanowitz and M. K. Gaillard, Nucl. Phys. B261, 379 (1985).

Impose on massive gauge theory

$$n^M = (n^\mu, 0),\ W^a_M = (W^a_\mu, \phi^a),\ \partial^M = (\partial^\mu, -m_W),\ \eta^{MN} = \mathrm{diag}(1, -1, -1, -1, -1).$$

Kinetic Lagrangian:

$$\mathcal{L}_{W^2_M} = -\frac{1}{2}\partial_M W^a_N \partial^M W^N_a + \frac{1}{2}(\partial_M W^M_a)^2 + \frac{1}{2\xi}(n\cdot\partial\, n_M W^M_a)(n\cdot\partial\, n_M W^M_a)^*$$

$\partial^M = (\partial^\mu, -m_W)$ gives $k^M = (k^\mu, -im_W)$ for incoming momentum

$k^{M*} = (k^\mu, im_W)$ for outgoing momentum

$k\cdot k^* = \eta^{MN} k_M k^*_N = k^2 - m^2_W$ equals to 0 when on shell

$$\mathcal{L}_{W_M^2} = -\frac{1}{2}\partial_M W_N^a \partial^M W_a^N + \frac{1}{2}(\partial_M W_a^M)^2 + \frac{1}{2\xi}(n\cdot\partial\, n_M W_a^M)(n\cdot\partial\, n_M W_a^M)^*$$

$$n^M = (n^\mu, 0),\ W_M^a = (W_\mu^a, \phi^a),\ \partial^M = (\partial^\mu, -m_W),\ \eta^{MN} = \mathrm{diag}(1,-1,-1,-1,-1).$$

Kinetic Lagrangian and algebra is completely analogue to the massless case.

massless:

$$< A_a^\mu A_b^\nu > = \frac{-i\delta_{ab}(g^{\mu\nu} - \frac{n^\mu k^\nu + k^\mu n^\nu}{n\cdot k} + \xi \frac{k^2}{(n\cdot k)^4} k^\mu k^\nu)}{k^2 + i\epsilon}$$

massive:

$$< W_a^M W_b^N >= \frac{-i\delta_{ab}(g^{MN} - \frac{n^M k^{*N} + k^M n^{*N}}{n\cdot k} + \xi \frac{k\cdot k^*}{(n\cdot k)^4} k^M k^{*N})}{k\cdot k^* + i\epsilon}$$

Propagator:

write gauge components and goldstone components separately

explicite gauge-goldston mixing

$$< (W_a^\mu, \phi_a), (W_b^\nu, \phi_b) >= \frac{i\delta_{ab}}{k^2 - m_W^2 + i\epsilon} \begin{pmatrix} -(g^{\mu\nu} - \frac{n^\mu k^\nu + k^\mu n^\nu}{n\cdot k}) & i\frac{m_W}{n\cdot k} n^\mu \\ -i\frac{m_W}{n\cdot k} n^\nu & 1 \end{pmatrix}$$

same mass for gauge and goldstone modes

when on-shell

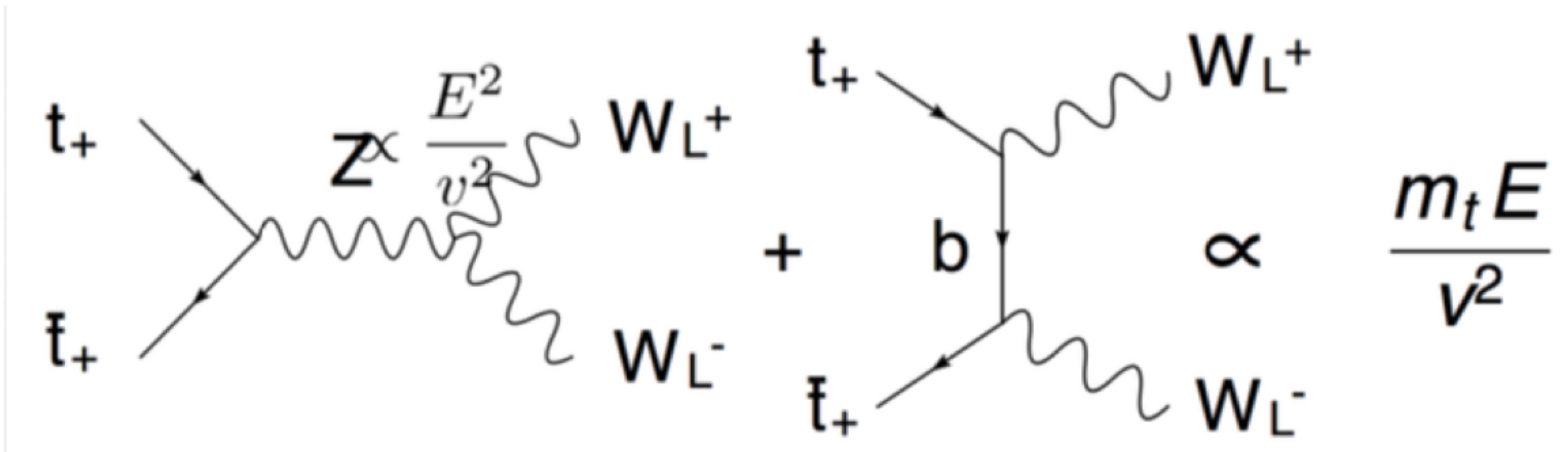
$$< W_a^M W_b^{*N} >= \frac{i\delta_{ab} \sum_{s=\pm,L} \epsilon_s^M \epsilon_s^{N*}}{k \cdot k^* + i\epsilon}$$

with

longitudinal pol. is interpolated by gauge fields and goldstone fiel

$$\epsilon_\pm^{M(*)} = (\epsilon_\pm^{\mu(*)}, 0)$$
$$\epsilon_L^M = (-\frac{m_W}{n\cdot k} n^\mu, i)$$
$$\epsilon_L^{*M} = (-\frac{m_W}{n\cdot k} n^\mu, -i)$$

Longitudinal vector boson $\epsilon_L \sim \frac{k^\mu}{m_W}$, bad energy behavior and huge interference!



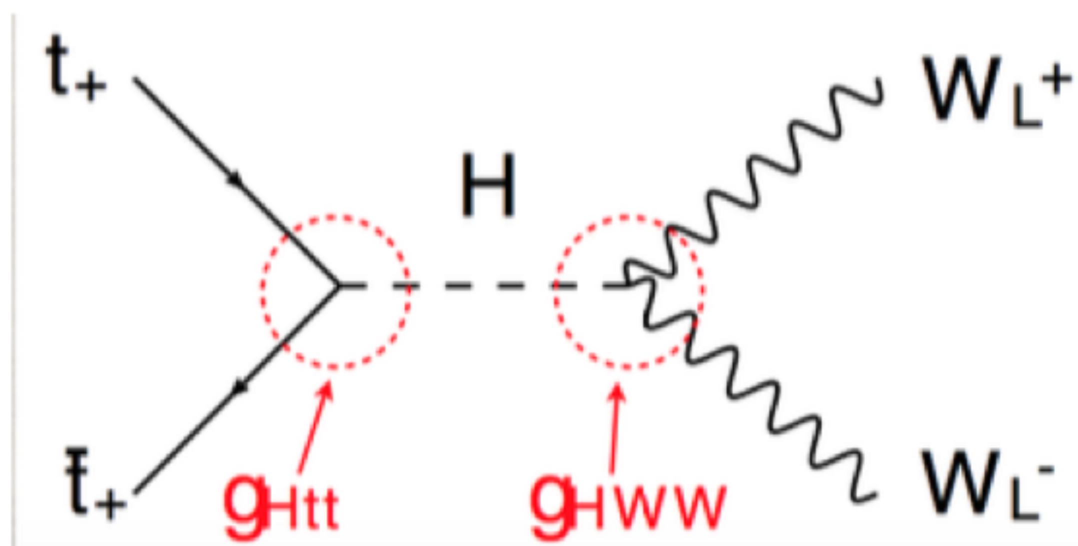


Figure: higgs contribution $\propto \frac{m_t m_h}{v^2}$ fixes the problem.

A Special physical gauge: Goldstone equivalence Gauge

- Hint to the solution: $\epsilon_L \sim \frac{k^\mu}{m_W} \to \phi_W$ in high energy — goldstone equivalence theorem.
- Write $\epsilon_L^\mu(k) = \frac{k^\mu}{m_W} - \frac{m_W}{n \cdot k} n^\mu$, with $n^\mu = (-1, \hat{k})$. So impose gauge-fixing $\frac{1}{2\xi}(n^\mu W_\mu)^2$.
- Consequences:
  1. $n \cdot k \neq 0$: k^μ terms are eliminated – power counting is good
  2. $n \cdot n = 0$, $\epsilon_n \simeq \frac{m_W}{2E}$, VEV corrections to unbroken limit.
  3. $|W_L >= |W_n > +i|\phi >$ —- manifest goldstone equivalence theorem
  4. Broken theory continuously goes to unbroken theory when $v \to 0$

see arXiv: 1611.00788 (B. Tweedie, T. Han, and J.Chen)

Five-component treatment for vertices

Problem with manifest goldstone equivalence:

$$i\mathcal{M}(L) = i\mathcal{M}^{\mu}\epsilon_{n\mu} - i\mathcal{M}^{4}\epsilon_{4}$$

Many diagrams for multiple longitudinal external states.
number of diagrams ~ 2^n

Solution?: Higgs multiplet is a representation of the gauge group

Question: Arrange interactions with goldstone modes in a way similar to gauge ?

Higgs-Kibble model with SU(2) gauge group

$$\mathcal{H} = \frac{1}{\sqrt{2}}(i\sigma_2\Phi^*, \Phi) = \frac{1}{2}(h - i\sigma^a\phi_a)$$

$$D_\mu\mathcal{H} = (\partial_\mu + igW_{a\mu}\frac{\sigma^a}{2})(h\frac{\mathbf{1}}{2} - i\frac{\sigma^b}{2}\phi_b)$$
$$= (\partial_\mu + igW_{a\mu}\frac{\sigma^a}{2}) \cdot h\frac{\mathbf{1}}{2} - \frac{\sigma^a}{2}(\partial_\mu\delta^{ac} - \frac{g}{2}\epsilon^{abc}W^b_\mu)\phi^c + \frac{1}{4}gW^a_\mu\phi_a\mathbf{1}$$

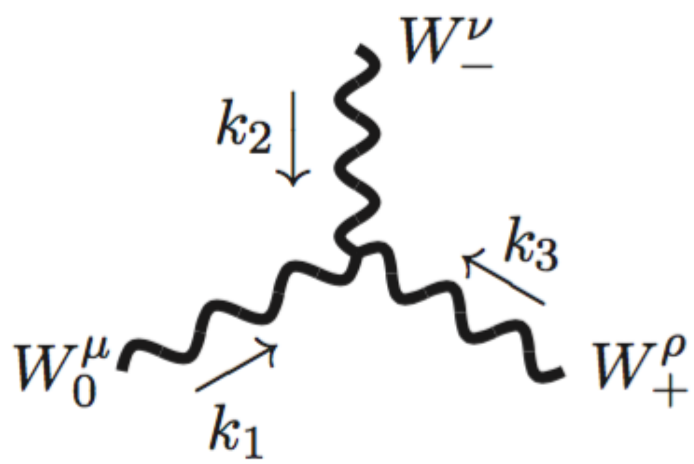
Resulting in, for example the Lagrangian term

$$\mathcal{L}_{W^3_M} = g\epsilon^{abc}\partial_\mu W_N W^{\mu b} W^c_K \eta'^{NK}$$

$$\eta'^{MN} = \mathrm{diag}(\eta^{\mu\nu}, -1/2),\ \eta^{MN} = \mathrm{diag}(\eta^{\mu\nu}, -1).$$

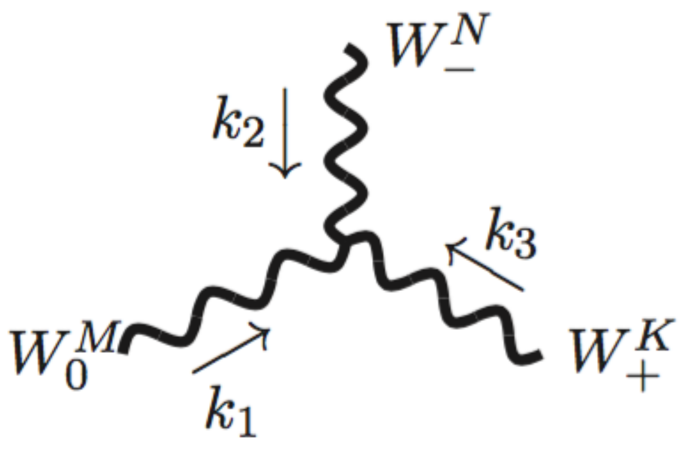
Higgs-Kibble model with SU(2) gauge group

Triple gauge vertices for massless theory



$$= \quad -ig(\eta^{\mu\nu}(k_1-k_2)^\rho + \eta^{\nu\rho}(k_2-k_3)^\mu + \eta^{\rho\mu}(k_3-k_1)^\nu)$$

Triple gauge-goldstone vertices for massive theory



$$= \quad -ig(\eta'^{MN}(k_1-k_2)^\rho + \eta'^{NK}(k_2-k_3)^\mu + \eta'^{KM}(k_3-k_1)^\nu)$$

$$\eta'^{MN} = \mathrm{diag}(\eta^{\mu\nu}, -1/2),\ \eta^{MN} = \mathrm{diag}(\eta^{\mu\nu}, -1).$$

Example: collinear splitting function for $W_L^+ \to W_L^+ W_L^0$

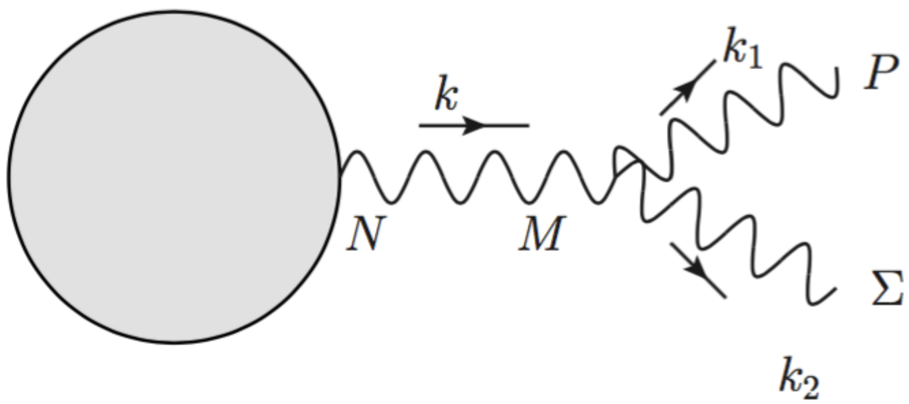


FIG. 5: Factorization for $1 \to 2$ splittings.

$$i\mathcal{M} = i\mathcal{M}^{M}_{split} \cdot \frac{i \sum_{s=\pm,L} \epsilon^0_{sM} \epsilon^{0*}_{sN}}{k^2 - m_W^2} \cdot i\mathcal{M}^N_0 + O(\frac{k_T^2 \text{ or } m_W^2}{E^4})$$

$$\simeq \sum_{s=\pm,L} i\mathcal{M}^s_{split} \cdot \frac{i}{k^2 - m_W^2} \cdot i\mathcal{M}^s_0$$

$$i\mathcal{M}_{W_L^+ \to W_L^+ W_L^0} = \frac{ig^2 v}{2} \frac{z - \bar{z}}{z\bar{z}} (1 + \frac{z\bar{z}}{2})$$

→

$$\frac{dP}{dk_T^2 dz}(W_L^+ \to W_L^+ W_L^0) = \frac{g^4}{4} \frac{(z - \bar{z})^2}{z\bar{z}} (1 - \frac{z\bar{z}}{2})^2 \frac{v^2}{\tilde{k}_T^4}$$

Conclusions

1. Physical Gauges in massive gauge theory gives manifest goldstone equivalence.
2. A re-identification of longitudinal polarization vector: mixture of gauge components and goldstone components
3. A reorganization and rederivation of Feynman rules(in SU(2)).
4. A special physical gauge with is $n^\mu = (1, -\hat{k})$ worth noting.